Massachusetts Institute of Technology

Mathematical Methods for Materials Scientists and Engineers

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Problem Set 2: Due Friday Sept. 23, Before 5PM: email to the TA.

Individual Exercise I2-1

Kreyszig MATHEMATICA® *Computer Guide:* problem 6.2, page 77

Individual Exercise I2-2

Kreyszig MATHEMATICA® *Computer Guide:* problem 6.10, page 77

Individual Exercise I2-3

Kreyszig MATHEMATICA® *Computer Guide:* problem 6.12, page 78

Group Exercise G2-1

A crack in a thin elastic material gives a stress concentration when the material is loaded in "mode I" as illustrated:

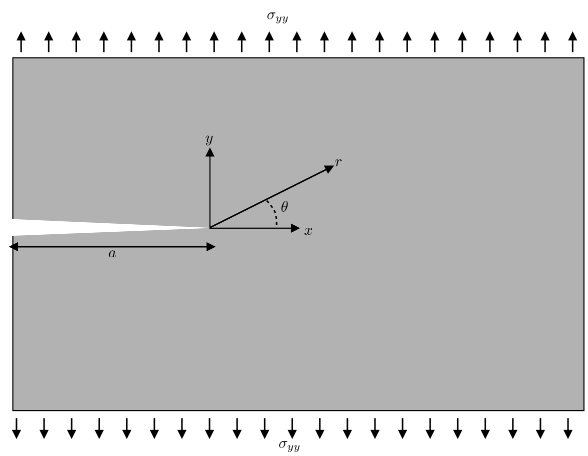

Illustration of crack in thin sheet being loaded in mode I.

The displacements in the x- and the y- direction of each point in the material located at a distance r from the crack tip and at an angle θ as illustrated are given by:

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix} = \frac{K_I(1+\nu)}{2E}\sqrt{\frac{r}{2\pi}} \begin{pmatrix} \frac{5-3\nu}{1+\nu}\left(\cos\frac{\theta}{2} - \cos\frac{3\theta}{2}\right) \\ \frac{7-\nu}{1+\nu}\left(\sin\frac{\theta}{2} - \sin\frac{3\theta}{2}\right) \end{pmatrix}$$

where $K_I = \sigma_{yy}^{\infty}\sqrt{\pi a}$ is called the "stress intensity factor" and E is the "Young's elastic modulus." Assume that the Poisson's ratio, ν, is 1/4.

1. Assuming that the crack is very sharp (i.e., very thin), plot the shapes of the crack if the material is loaded to $\sigma_{yy}^{\infty} = 0.1$, 0.25, and 0.5 E.

The strains in a material indicate how far two points have separated depending on the original separation between the points. The units of strain are (Δlength)/(length); in other words, dimensionless. Because there are two coordinates, x and y, there are different kinds of strain:

ϵ_{xx} The xx strain indicates the rate of separation in the x-direction with original separation in the x-direction: this is the "x-stretch."

ϵ_{yy} The yy strain indicates the rate of separation in the y-direction with original separation in the y-direction: this is the "y-stretch."

ϵ_{xy} The xy strain indicates the rate of separation in the x-direction with original separation in the y-direction: this is the "shear."

ϵ_{yx} The yx strain indicates the rate of separation in the y-direction with original separation in the x-direction: this is the same as ϵ_{xy}.

The strains are calculated from the displacements as follows:

$$\epsilon_{\eta\zeta} = \frac{1}{2}\left(\frac{\partial u_\eta}{\partial \zeta} + \frac{\partial u_\zeta}{\partial \eta}\right)$$

e.g.,

$$\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

2. Find an expression for the strains for the mode-I problem.

The stresses in a material indicate how much force is applied across a plane, per unit area of plane. Stresses have the same units as pressure and as Young's modulus E. Because forces can point in two independent directions and the planes can be oriented in two independent directions, there are different kinds of stress:

σ_{xx} The xx stress is the force in the x-direction per unit area of a plane with normal in the x-direction: this is the "x tensile stress"

σ_{yy} The yy stress is the force in the y-direction per unit area of a plane with normal in the y-direction: this is the "y tensile stress"

σ_{xy} The xy stress is the force in the x-direction per unit area of a plane with normal in the y-direction: this is the "shearing stress"

σ_{yx} The yx stress is the force in the y-direction per unit area of a plane with normal in the x-direction: this is the same as σ_{xy} if the material is in elastic equilibrium.

In an isotropic linear elastic material in a state of plane stress, the strains are linearly related to the stresses through the compliance matrix:

$$\begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{pmatrix} = \frac{1}{E} \begin{pmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{pmatrix} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix}$$

3. Find the corresponding compliance matrix that linearly relates the stresses to the strains.

For plane stress, the hydrostatic pressure is given by $2(\sigma_{xx} + \sigma_{yy})/3$.

4. Plot contours of constant hydrostatic pressure.

5. Plot contours of constant magnitude of shear stress.

Group Exercise G2-2

In two dimensions there are a set of symmetry operations on points $\vec{v}$:

$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

that can be represented by matrix operations on vectors:

$$\underline{M}\vec{v} = \begin{pmatrix} m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

Among the possible symmetry operation are:

Mirror Reflection across the x-axis

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mirror Reflection across the y-axis

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Rotation by θ about the origin

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Use these operations and modify the MATHEMATICA® example in ps2_setup.nb available in the Assignments section and illustrate an object that has:

1. Mirror symmetry across the x-axis.
2. A 2-fold rotation symmetry and mirror symmetry across the y-axis.
3. A mirror plane rotated by 45° from the x-axis.