

Lecture 6

Energy and WorkLast Time**Topic**

Topic

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Energy

In physics, you are used to dividing energy into two (or more) parts:

$$E^{\text{total}} = \text{Kinetic Energy} + \text{Potential Energy} + \dots \quad (6-1)$$

for example: $E^{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \dots + mgh$

Typically, in thermodynamics, the body remain in the same inertial frame as the observer, so the velocity and rotation of the body can be taken to be zero. The kinetic energy of the system is typically ignored.⁸

⁸Questions about whether to include kinetic energy in the internal energy of a body can lead to considerable confusion in classical thermodynamics. One can choose to add the total kinetic energy of body to the total internal energy and keep track of the relative reference frames of the body and the observer. However, it is quite reasonable to ask “Should one add the kinetic energies of all the individual particles that make up a

It is useful, but not necessary, to divide the potential energy into parts that do not depend on its position in space and those that do, for instance:

$$U = U_{int} + Mgh \quad (6-3)$$

In the majority of cases in materials thermodynamics, changes in the potential energy due to gravity are not considered. If they are required to account for the balance of internal energy, then they must be included and terms like h become a degree of freedom for the system.⁹ We will not consider gravitational energy any further and what we mean by the internal energy U will be intrinsic to the material system under consideration.

In other words, the internal energy of a body is associated with all the internal degrees of freedom of a body.

Internal energy

How can the internal energy of a body be increased?

How can the internal energy of a body be decreased?

body?" To be specific, consider a balloon filled with Helium that is floating upwards at constant velocity \vec{v}_0 . Suppose that, at some particular moment, each of the Helium atoms has a velocity \vec{v}_i . A reasonable way to divide up the kinetic energy of the N He atoms would be:

$$E_{\text{kin}} = \frac{1}{2} N m_{\text{He}} |\vec{v}_0|^2 + \sum_{i=1}^N \frac{1}{2} m_{\text{He}} |\vec{v}_i - \vec{v}_0|^2 \quad (6-2)$$

where the last term includes all the velocities which taken together have zero average velocity. The first term has an old scholarly latin phrase associated with it—the *vis viva* kinetic energy. The kinetic energy associated with the summation is not “readily observable” by experiment and is associated with the internal energy U . For an ideal gas, *all* of the internal energy is associated with the sum.

⁹An excellent example of an application in which the gravitational term *must* be included is the calculation of partial pressure of Oxygen as a function of altitude.

We consider all possible internal changes to the body as making up the total internal energy U . There are two ways to change the internal energy: those that involve work, and everything else.

Everything else is defined as heat. Heat is defined as the transfer of energy to a body that does not involve work—or those transfers of energy that occur only because of a difference in temperature. As the Bellman would say, “Repetition being the apotheoses of pedagogy”—therefore, heat is the workless transfer of energy.

So, by definition and as indefatigably observed by Joule:

$$dU = dq + dw \quad (6-4)$$

Note: You will see this written up to four different ways: $dU = dq + dw$, $dU = dq - dw$, $dU = -dq + dw$, and $dU = -dq - dw$. They are *all* correct! They only differ in the definitions of the heat and work. If there is a minus sign in front of the dq , then dq is the heat out of the body; if a minus sign is in front of dw , it is the work one by the body. One consequence is that the student has to either pay close attention to the definition that are being used or use common sense—adding heat to a body should increase its internal energy!

The flow of heat is fairly obvious; we can always refer to the changes due to a difference in temperatures of an object that has no work performed on it and the agent that is performing the work.

However, it is instructive to list the various ways that one can do work on a body—in other words, the ways that a body can store energy internally when you do work on the body.

Types of Work

Thermodynamics is almost always taught for fluids only—i.e. systems that do only PdV work. I think this is not terribly useful because it gives the impression that thermo can *only* be applied to fluids and this is a mistake! Even more, because, as students, professional scientists

were taught thermodynamics as fluids, they tend to reduce every system they study to a fluid—and this can be a mistake as well!

However, there is a simple reason for this historical method of teaching thermodynamics: the language and symbols become very complicated if we develop thermodynamics in the most general way. And this makes it difficult to learn anything at all—so, my object is to teach you how to apply thermodynamics and extend the lessons to more general cases. To extend it to more general cases, we need to consider other ways that work can be done on a body, then you can work out useful relations in new cases by following the methods that we will develop in this course.

The rate at which work is done is always of the form Fdx where dx represents the change in the *extent* of some quantity and F is a force that resists the change. This should be familiar for the case of a springs and mechanical objects. Perhaps, it is not so familiar to think of dx as a generalization. For instance, we might want to think of dx as the ‘change in number of water molecules in a polymer’ or we might want to think of dx as the ‘change in the the charge held in a capacitor’, or the ‘change in the number of Lithium ions in an anode.’ Each one of these examples represents a different way for a material to store internal energy—and each one has an intensive variable (a generalized) force associated with it.

What follows is a discussion of some other types of work that can be done on a system. The details about polarization, magnetization, and elasticity are abstracted from the very useful book by Nye.

Work of Polarization

A material is said to be polarized if the positive and negative charges within the material become slightly displaced from one another due to an electric field.

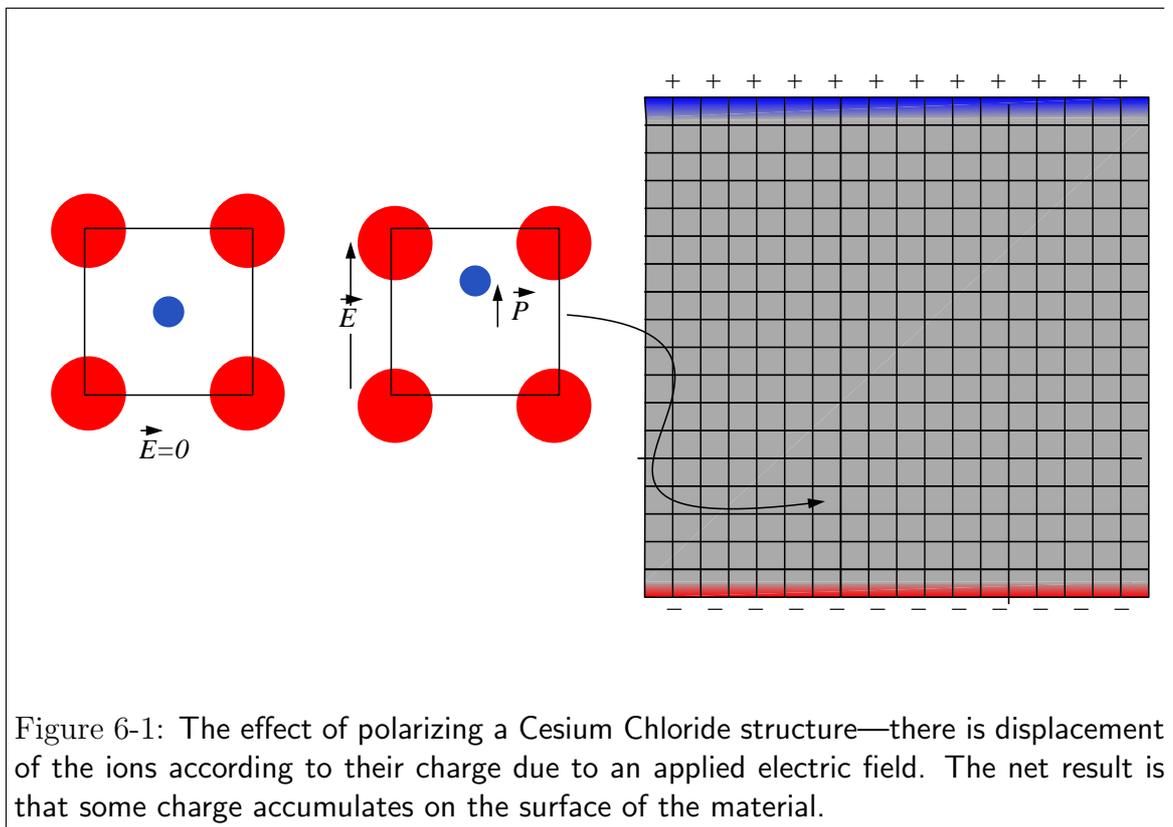


Figure 6-1: The effect of polarizing a Cesium Chloride structure—there is displacement of the ions according to their charge due to an applied electric field. The net result is that some charge accumulates on the surface of the material.

For electric work due to polarizing a material:

\vec{E}	Electric Field
\vec{P}	Polarization in material due to local charge displacement
\vec{D}	Electric Displacements

$$\vec{D} = \kappa_0 \vec{E} + \vec{P} \quad (6-5)$$

The electric displacement, \vec{D} , is the total polarization per unit volume. The electric displacement is the sum of two terms: the electric field in free space and the contribution due to the polarization of the material. The electric displacement \vec{D} can be thought of as the net polarization. κ_0 is the permittivity of free space; it is a conversion between the units of electric field and polarization and its value reflects how an electromagnetic wave travels through the vacuum.¹⁰

The polarization work done is:

$$dw = V \vec{E} \cdot d\vec{D} \quad (6-6)$$

¹⁰In SI units, the permittivity of free space is approximately 8.85410×10^{-12} F/m (F/m = m kg/(s²coulomb²))

Linear Isotropic Material Properties

The simplest model for polarizable material is that the induced polarization density \vec{P} is linearly related to the applied field \vec{E} :

$$\vec{P} = \kappa_o \chi \vec{E} \quad (6-7)$$

χ is the dielectric susceptibility. χ is a measure of the extent to which a material forms internal dipoles.

An isotropic material is one that the response of a material does not depend on the *direction* of the applied field with respect to the *orientation* of the crystal—that is the polarization direction is parallel to the applied field \vec{E} .¹¹

In an isotropic material, then, the polarization of the material is in the same direction as the direction of electric field that is trying to polarize it:

$$\vec{D} = \kappa \vec{E} \quad (6-8)$$

$$\kappa = \kappa_o(1 + \chi) \quad (6-9)$$

κ is called the permittivity of a material.

If κ does not depend on \vec{E} and the material generates no heat as it is polarized, then

$$dw = V \kappa \vec{E} \cdot d\vec{E} \quad \text{and} \quad w = \frac{1}{2} V \kappa E^2 \quad (6-10)$$

Magnetic Work

The magnetic work has a form that is very similar to that of the polarization work. However, there are important differences between magnetic properties. One of the most important is that the linear material that relates the induced magnetization to the applied field (by analogy to Eq. 6-7) can be either positive or negative.

Physically, it is imagined that the material is composed of magnetic moments that align (or anti-align) in response to an applied field.

There are two categories of magnetism:

Ferromagnets Materials that can maintain a magnetization even when there is no applied magnetic field. These are the magnets you find in toy stores, electric motors, and on refrigerators. Very few materials exhibit ferromagnetism; if they do it is likely that they contain iron, cobalt, or nickel.

¹¹We will discuss *anisotropic* properties later. An anisotropic material is one that does not satisfy the definition for isotropy.

Ferromagnets tend to align their own magnetic field with an applied field—this is the magic trick that allowed them to be discovered by the Chinese about 1000 years ago and Europeans about 500 years ago.

Non-Ferromagnets Non-ferromagnets do not sustain their own magnetic field when an applied field is removed. There are two important sub-categories of such *non-permanent* magnetic behavior:

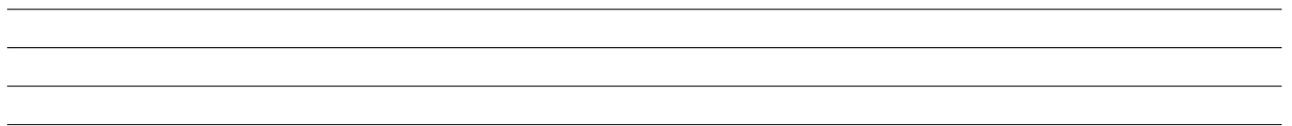
Paramagnetism The induced magnetic field in the material tends to align parallel with an applied field.

Diamagnetism The induced magnetic field in the material tends to align at right angles with an applied field.

The relation between the applied field \vec{H} , the induced magnetic field density \vec{I} and the total magnetic field density is analogous to the case of the electric field (c.f. Eq. 6-5):

$$\vec{B} = \mu_0 \vec{H} + \vec{I} \quad (6-11)$$

\vec{H} is the applied magnetic field, μ_0 is called the permeability of vacuum and relates the magnetization of empty space to the applied field.¹² \vec{I} is the magnetization of the material, it is the magnetic moment per unit volume (a derived intensive quantity). \vec{B} is the magnetic induction of the local net field density.



Exactly similar to the polarization work, the magnetic thermodynamic work is:

$$dw = V \vec{H} \cdot d\vec{B} \quad (6-12)$$

_____ Models for magnetic materials _____

Linear Isotropic Non-Ferromagnetic Materials

$$\vec{I} = \mu_0 \psi \vec{H} \quad (6-13)$$

ψ is the magnetic susceptibility, is relates the local microscopic alignment of the individual magnetic moments to the applied field.

If $\psi > 0$, the magnetic moments align with the field—this is called a *paramagnetic material*

If $\psi < 0$, the magnetic moments align anti-parallel with the field—this is called a *diamagnetic material*

Combining Eq. 6-11 with Eq. 6-13, we get the definition of magnetic permeability μ for an isotropic magnetic material:

¹²In SI (MKS) units, $\mu_0 = 4\pi \times 10^{-7}$ newtons/(ampere)².

$$\vec{B} = (1 + \psi)\mu_0\vec{H} = \mu\vec{H} \quad (6-14)$$

And from this follows the associated magnetic work for this model:
