HST.582J / 6.555J / 16.456J Biomedical Signal and Image Processing
Spring 2007

Harvard-MIT Division of Health Sciences and Technology
HST.582J: Biomedical Signal and Image Processing, Spring 2007
Course Director: Dr. Julie Greenberg

HST 582 / 6.555

Image Processing II

2D and 3D Image Processing

- Useful linear signal processing for medical images
  - Interpolation
  - Down sampling
  - Hierarchical filtering
  - Computed Tomography (CT),
    - Projection Slice Theorem

2D and 3D Image Processing

- Useful non-linear processing techniques
  - Histogram equalization
  - Homomorphic filtering
  - Edge detection

Applications of Signal Processing in Medical Images

- Linear signal processing
  - Image reconstruction (tomography, MRI)
  - Image enhancement
  - Noise reduction
  - Artifact reduction
- Non-linear signal processing
  - Non-linear, adaptive filters
    - Tube enhancing filters
  - Segmentation and beyond

Interpolating Images

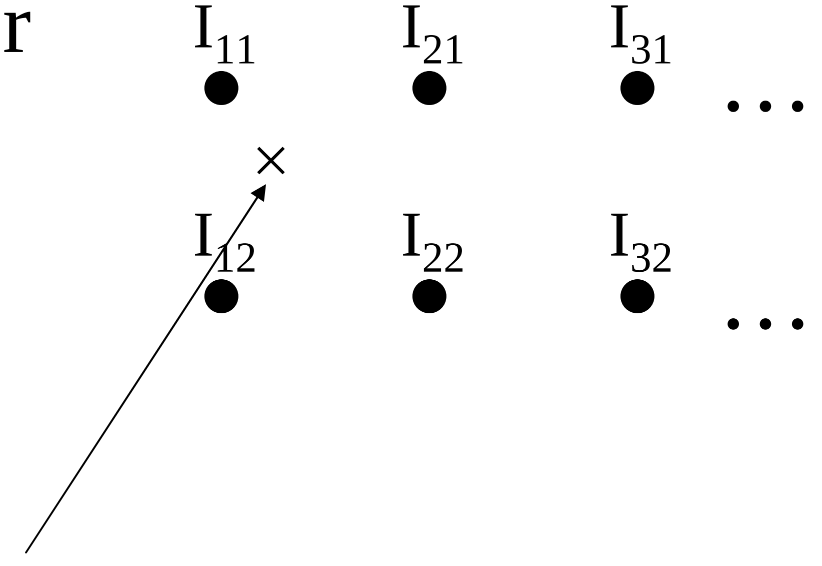
- Why
  - Prepare synthetic multichannel data
  - Prepare data sets for parallel trips down the same processing pipeline
  - Comparing images during Registration

Interpolation

- Optimal
  - Reconstruct the *bandlimited* original signal using sinc functions
  - Re-sample the reconstruction
- Nearest Neighbor
- Linear

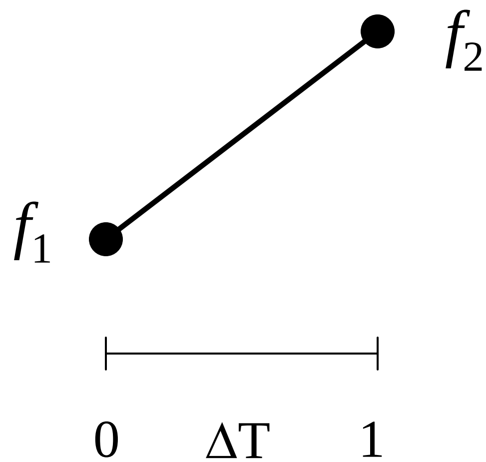
Nearest Neighbor

- Easy interpolator



  - To interpolate here, assign the intensity of the closest original pixel (in this case, I_{11})
  - leads to blocky results

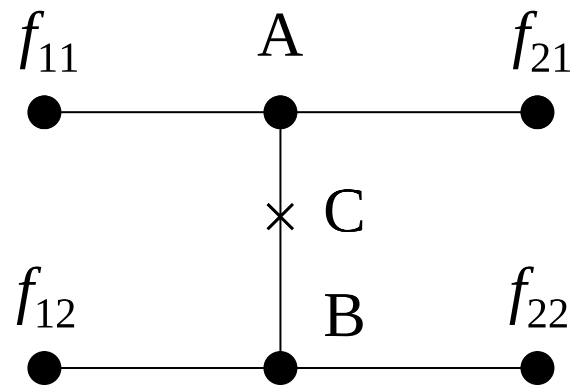
Linear Interpolator (1D)



$$f(\Delta T) = f_1 + \Delta T\,(f_2 - f_1)$$

Bilinear Interpolator (2D)

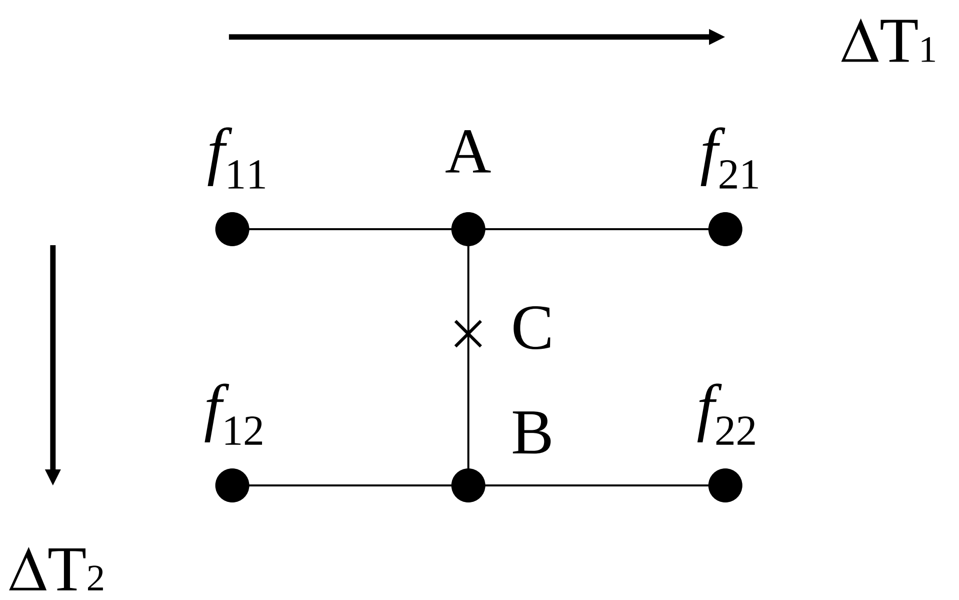
- 2D linear interpolator



use 1D linear interpolation for A and B, use 1D linear interpolation among A and B to get C

- Similar for 3D

Bilinear Interpolator...

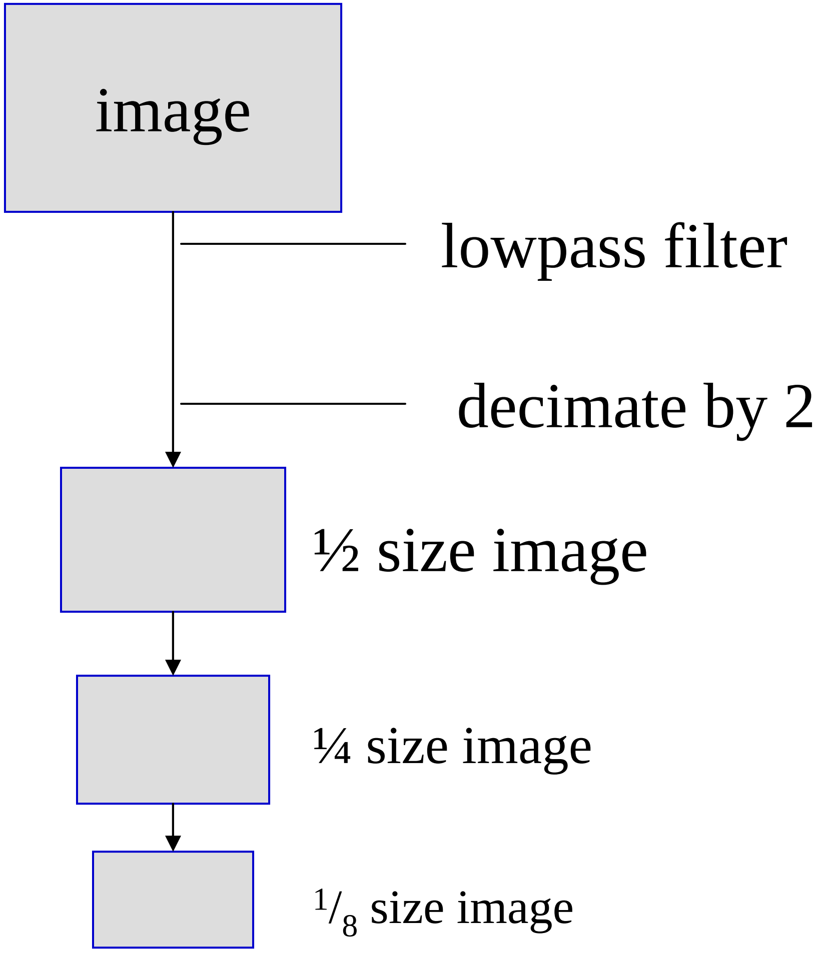


$$f(\Delta T_1, \Delta T_2) = f_{11} + \Delta T_1 (f_{21} - f_{11}) + \Delta T_2 (f_{12} - f_{11})$$
$$+ \Delta T_1 \Delta T_2 (f_{22} - f_{21} - f_{12} + f_{11})$$

Down Sampling

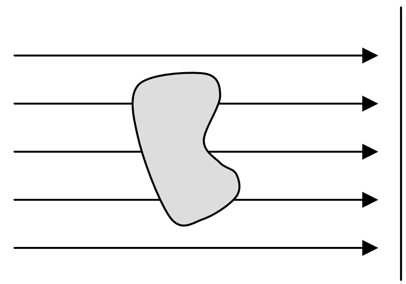
- Whenever a lowpass filter is applied, it may be possible to discard alternating pixels without much loss of information (down-sampling, or decimation)
- If down-sampling is desired, it may be best to do some lowpass filter to avoid aliasing
  - Reasonable LPF to use: $\frac{1}{16}[1,4,6,4,1]$

Burt (Peter) Filter



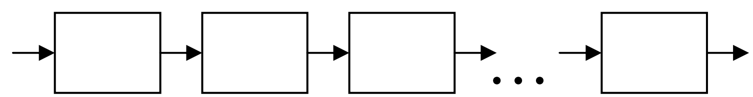
Projection Slice Theorem and CT

x-rays



Focus on one ray: (discrete)

source



sensor

photons leaving = A_n • photons entering

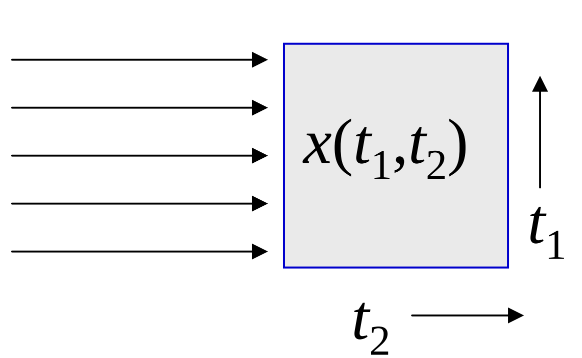
$$\boxed{\frac{\text{photons at sensor}}{\text{photons at source}} = \prod A_N}$$

$$\boxed{\log \frac{\text{photons at sensor}}{\text{photons at source}} = \Sigma \log A_N}$$

Continuous Analog...

$$y = \log \frac{photons\_at\_sensor}{photons\_at\_source} = \int x(t)dt$$

x is a log-attenuation-density called:
Linear Attenuation Coefficient

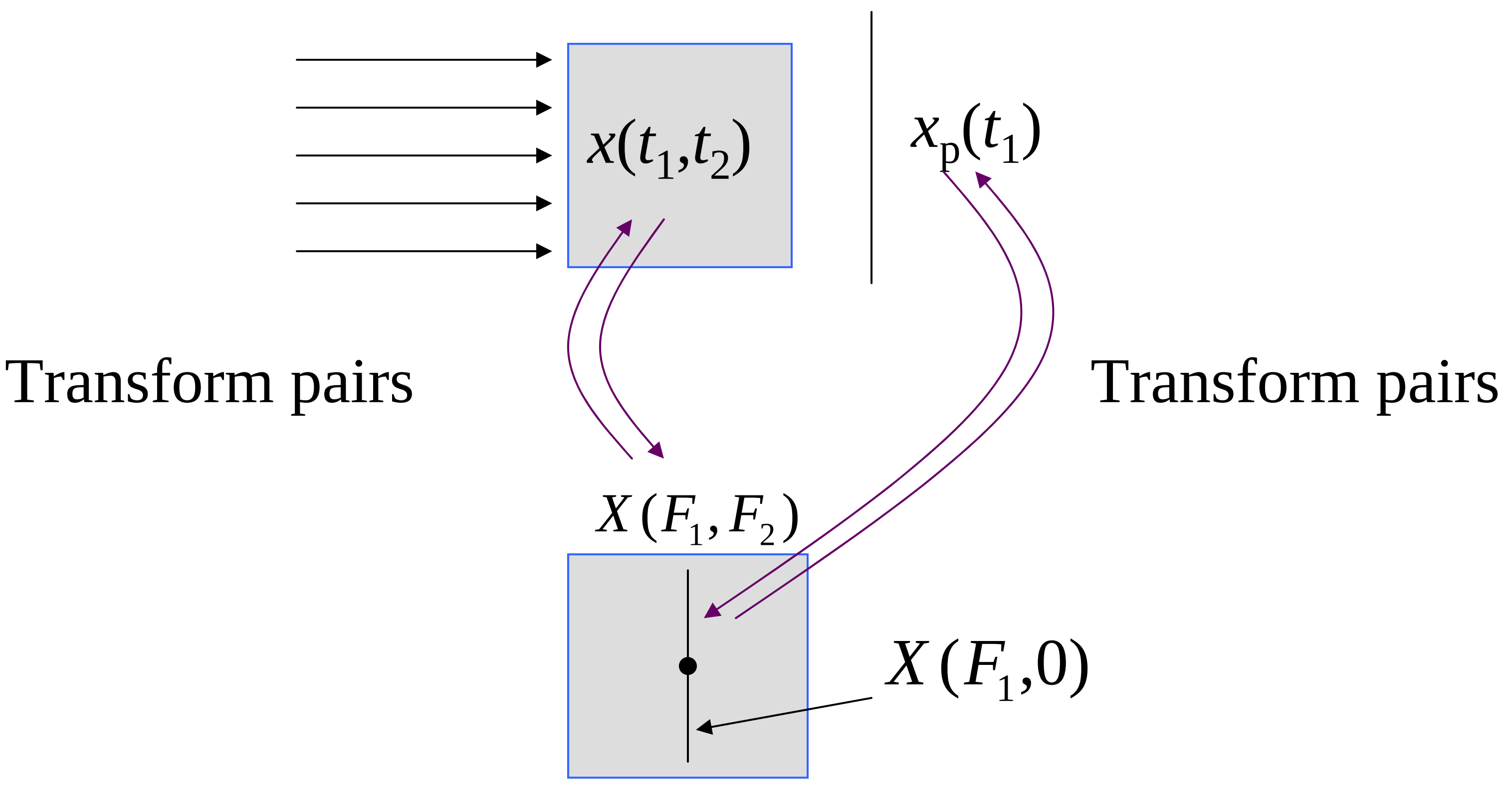


$$x_p(t_1) = \int x(t_1, t_2)dt_2$$

we can model x-ray imaging by line integrals…

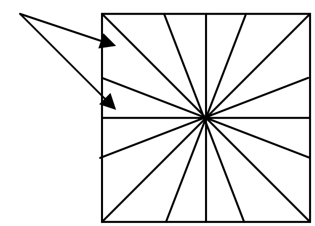
Fourier Transform...

$$X_p(F_1) = X(F_1, 0)$$



CT Reconstruction using FT Methods

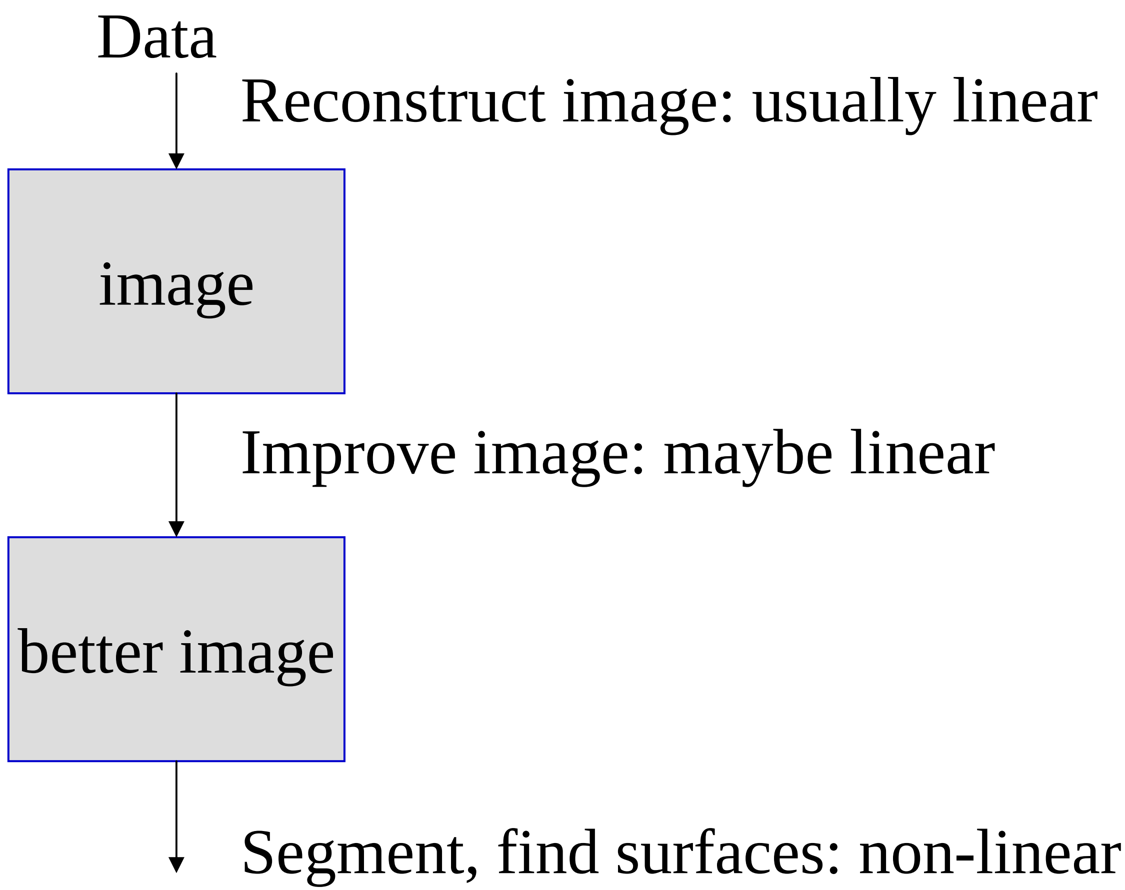
- Rotate the x-ray apparatus to many angles
  - Take projections
- FT the projections
- Assemble the transformed projections in frequency space:
  - Get frequency data on these lines at the angles the projections were taken



- Interpolate the full frequency space
- Inverse FT to get back reconstruction

There is a related method: Filtered Back-projection which is usually used

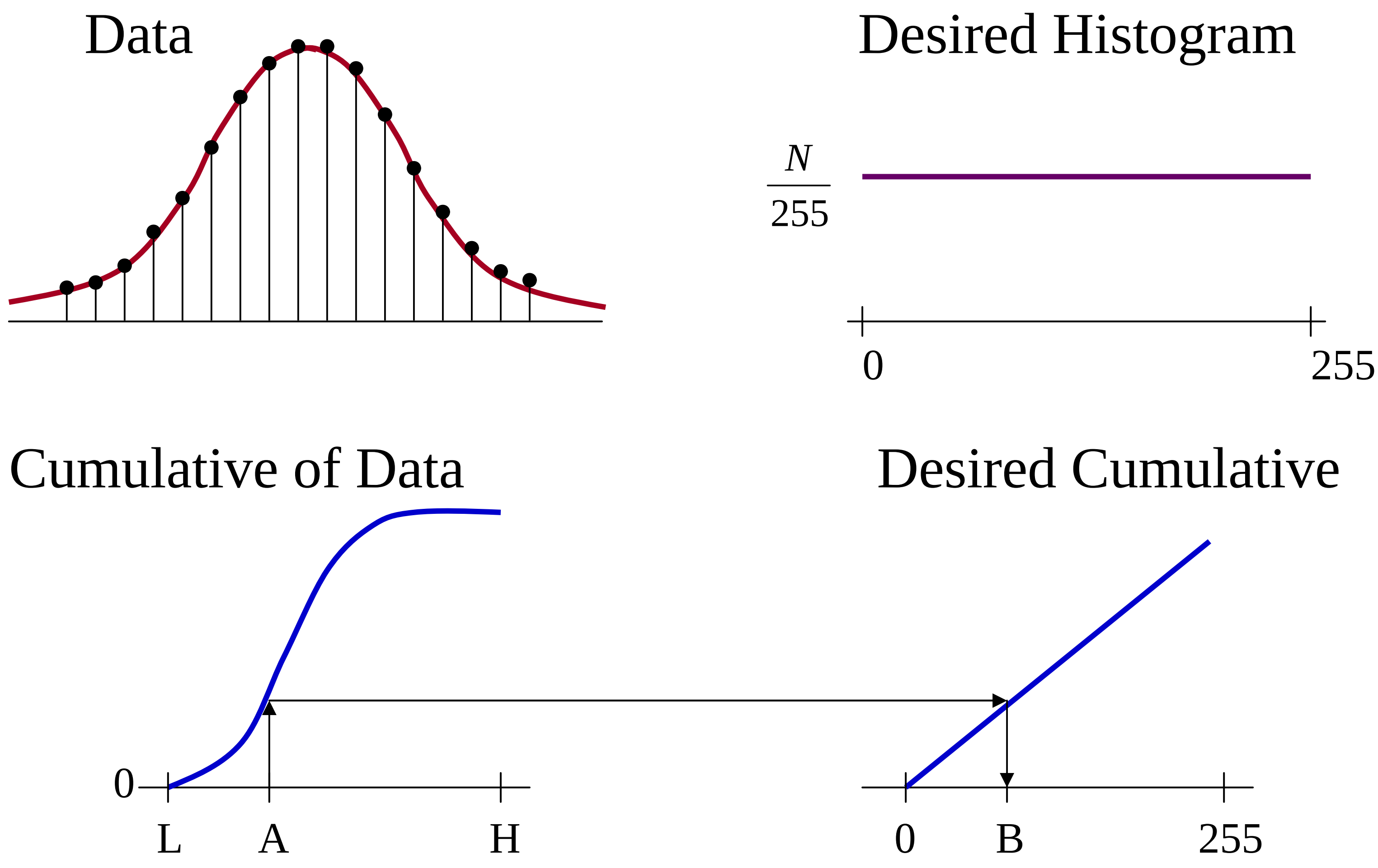
Linear and Non-Linear Processing for Medical Images



Histogram Equalization

- A method for automatically setting intensity lookup table
- Apply a monotonic transformation to the data so that after the transformation, the result has a (nearly) uniform intensity histogram
- Tends to increase contrast in areas where the data is concentrated

Histogram the Image Intensities



Histogram: Examples

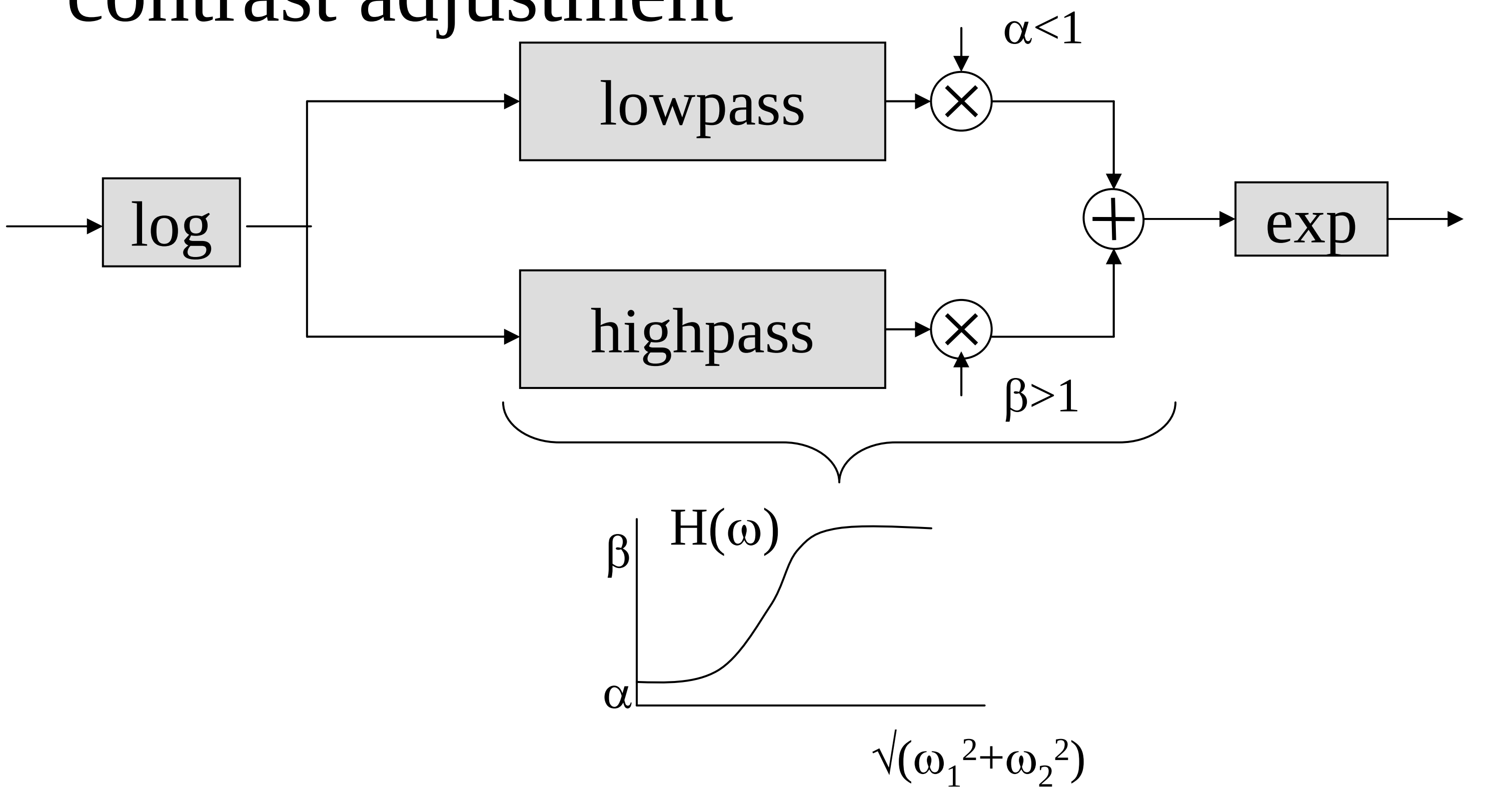
Figures removed due to copyright restrictions.

Image and its intensity histogram

Histogram equalization

Homomorphic Filtering

- An automatic method for spatially varying contrast adjustment



Homomorphic Processing: Example

Figure 8.11 in: Lim, J. S. *Two-Dimensional Signal and Image Processing*. Upper Saddle River, NJ: Prentice Hall, 1989. ISBN: 9780139353222

Original image

Image after homomorphic processing

related: *unsharp masking*

From **Lim**

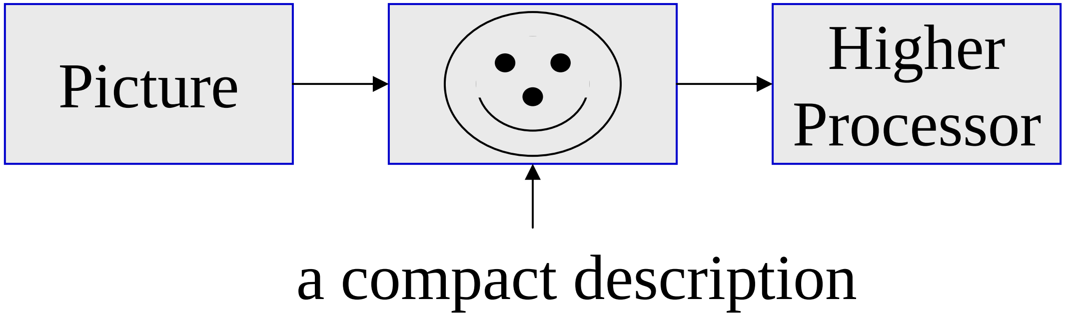
Duality

- Focus on regions → Segmentation
- Focus on boundaries → Edges

- Sometimes best to do both.

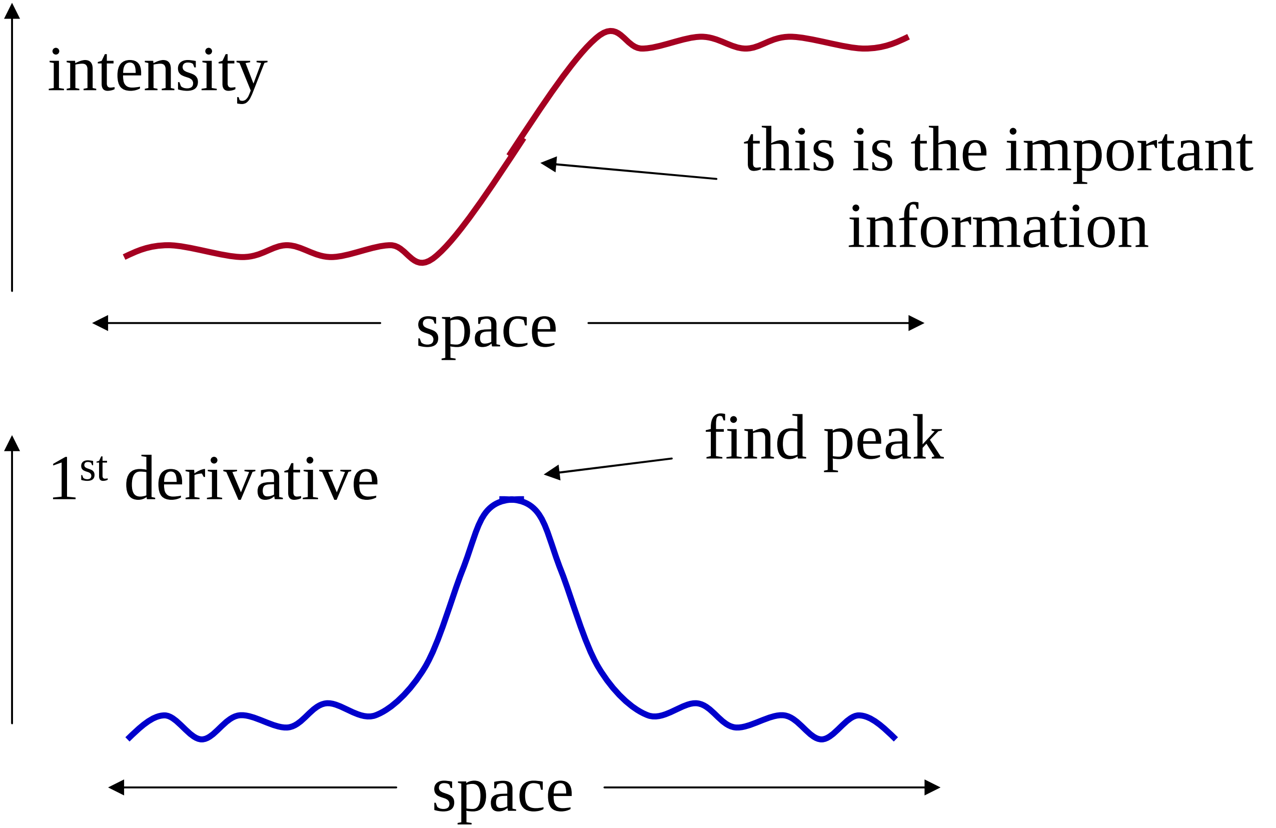
Edge Detection

- From computer vision
  - Roberts 1965 Lincoln Lab
  - Horn 1972 MIT AI
  - Marr and Hildreth 1980 MIT AI
  - Canny 1983 MIT AI

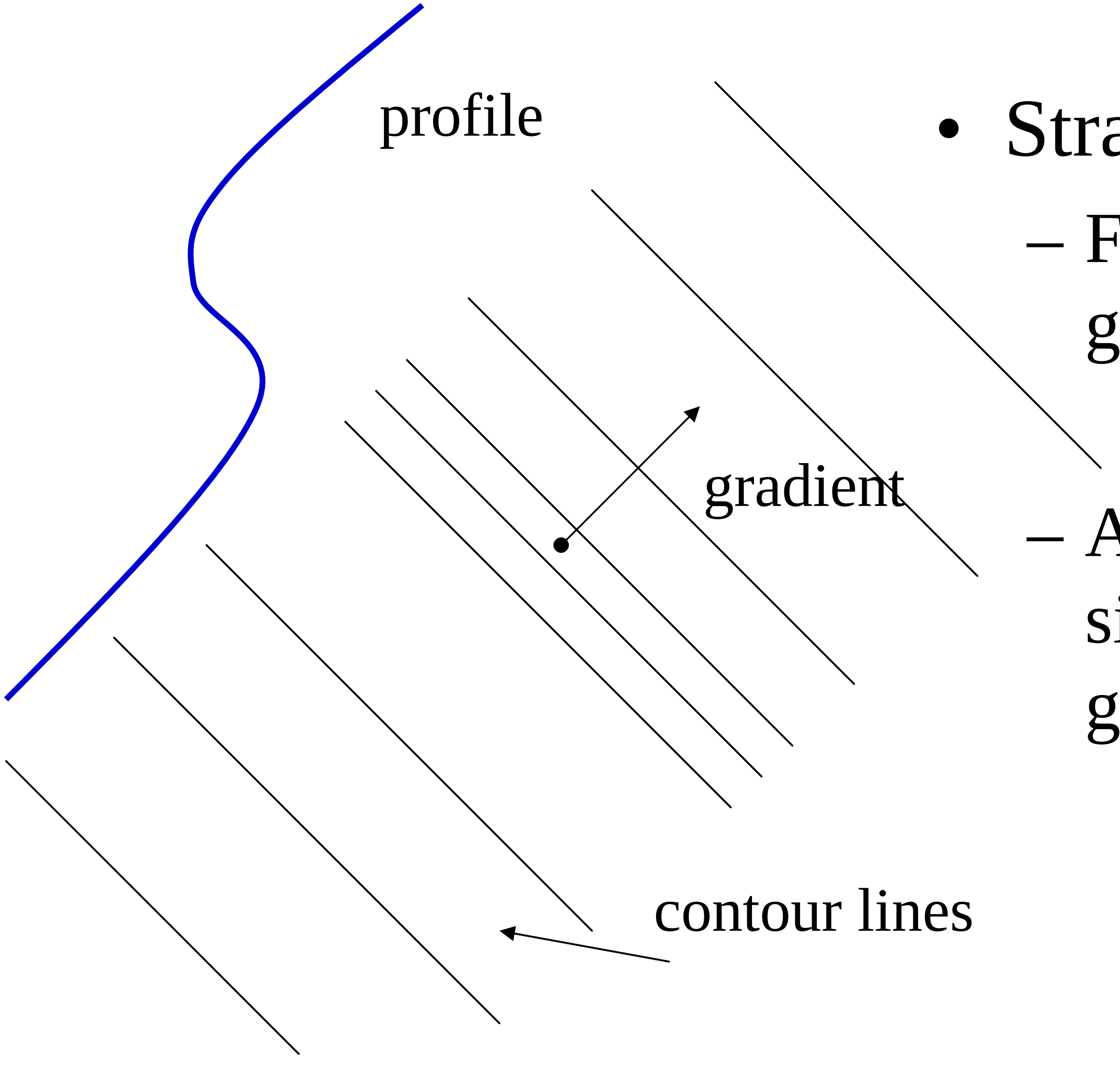


Basic Idea: 1D Edge

- Motivation: ideal step edge + noise

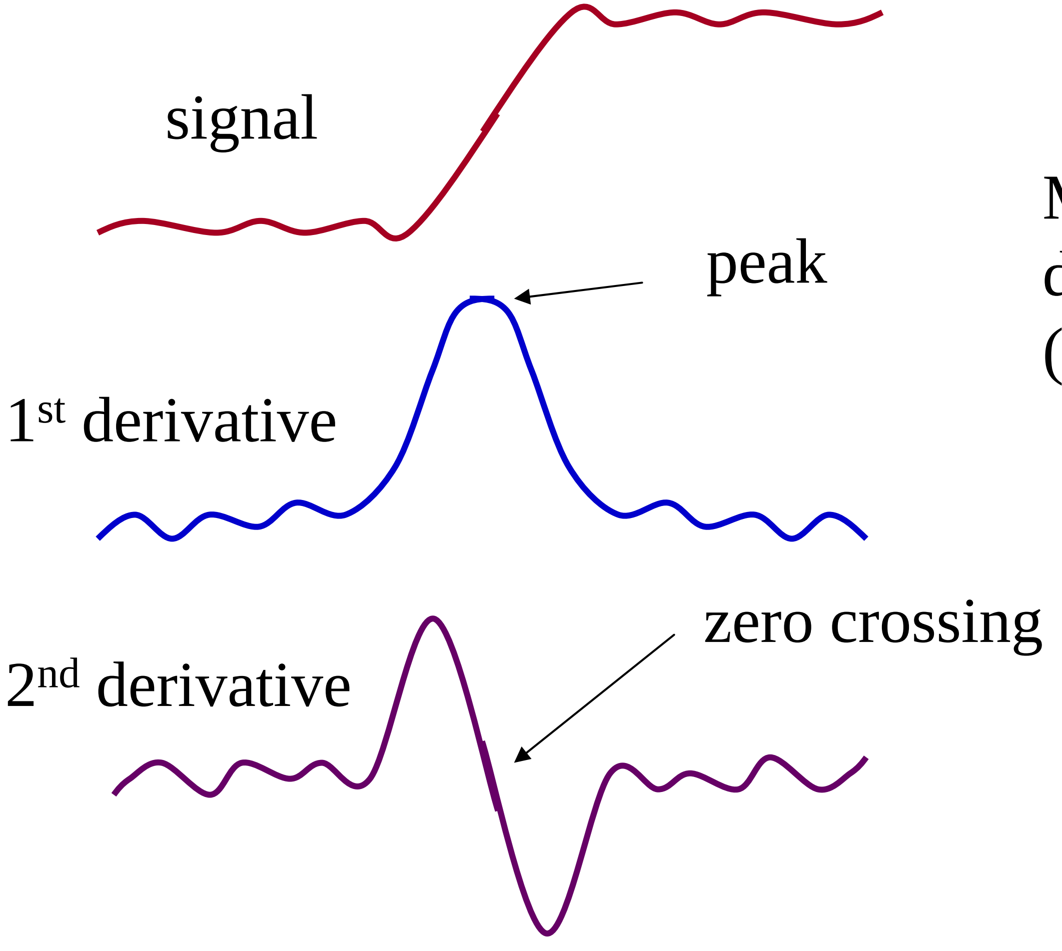


Edges in 2D



- Strategy:
  - Find direction of gradient
  - Analyze derivative of signal in direction of gradients

Zero Crossing Style Edges: 1D



Mark edges where 2nd derivative = 0 (vigorously)

Noise

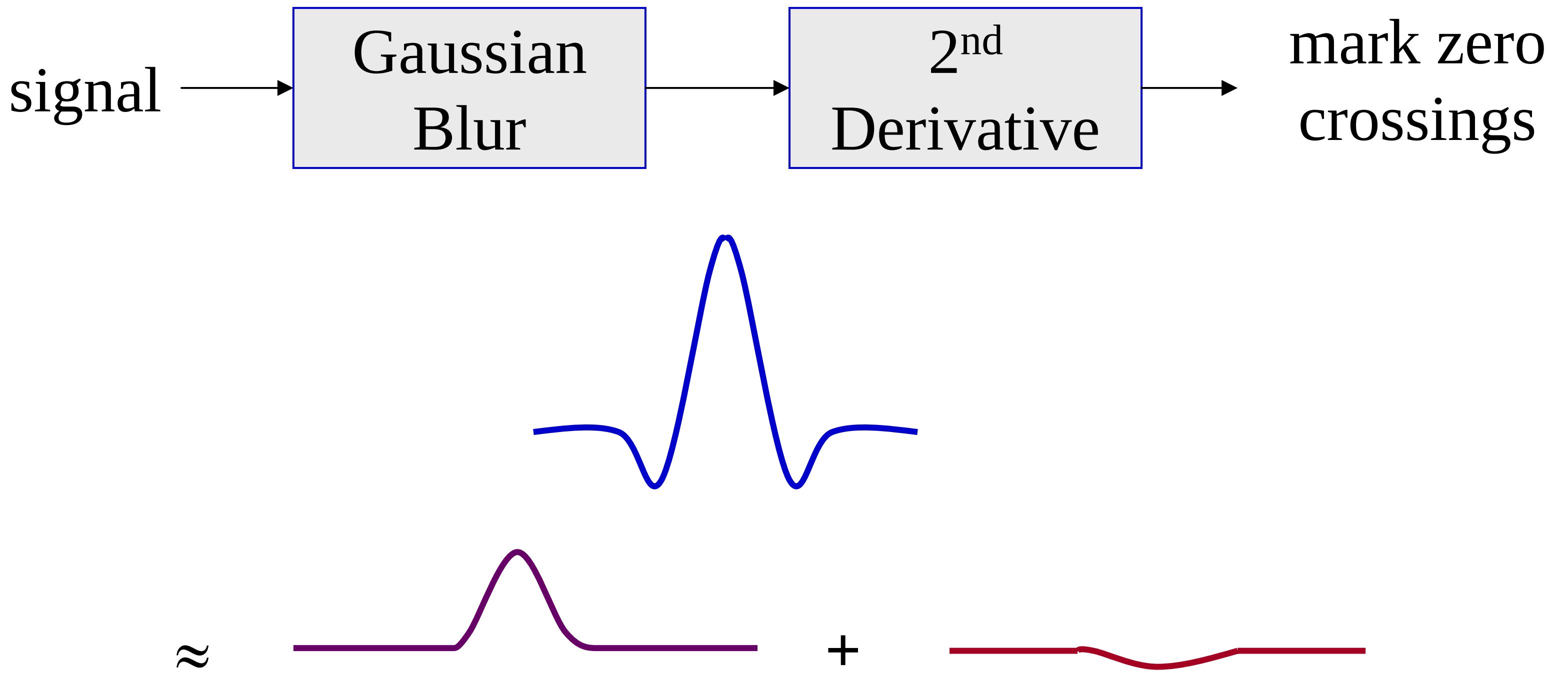
- Noise is an issue with derivative operators
  - How do we fight the noise
    - Assume white noise
    - Assume good stuff has important low frequency content
    - *recall Wiener filtering...*

Noise

- Noise is an issue with derivative operators
  - How do we fight the noise
    - Assume white noise
    - Assume good stuff has important low frequency content

    - *USE LOWPASS FILTER* (Gaussian ?)

Difference of Gaussians



Works in 2D, 3D, …

- Gaussian separates
- There exists a rotationally-symmetric 2nd derivative operator:

Laplacian

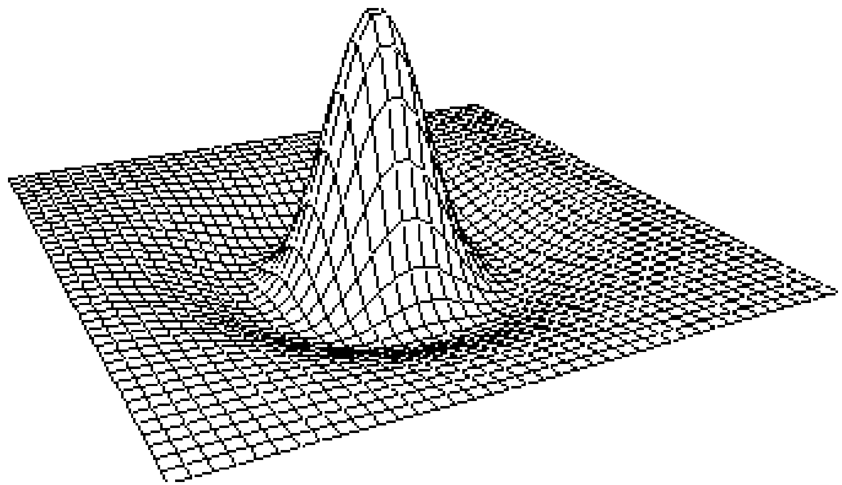
$$\boxed{\nabla^2 f(x,y) \equiv \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}}$$

similar for 3D

Discrete analog:

-1	-1	-1
-1	8	-1
-1	-1	-1

2D $\nabla^2 G$ Operator



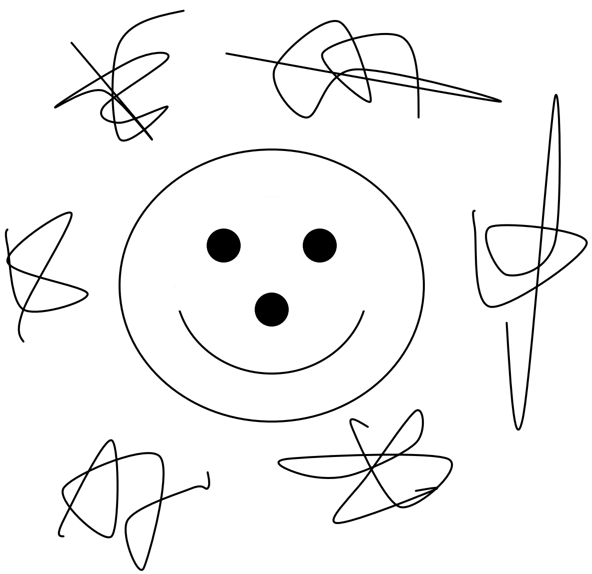
"Mexican hat"

- The zero crossing contours of the result of an operator like this form closed contours.

Seminal Edge References...

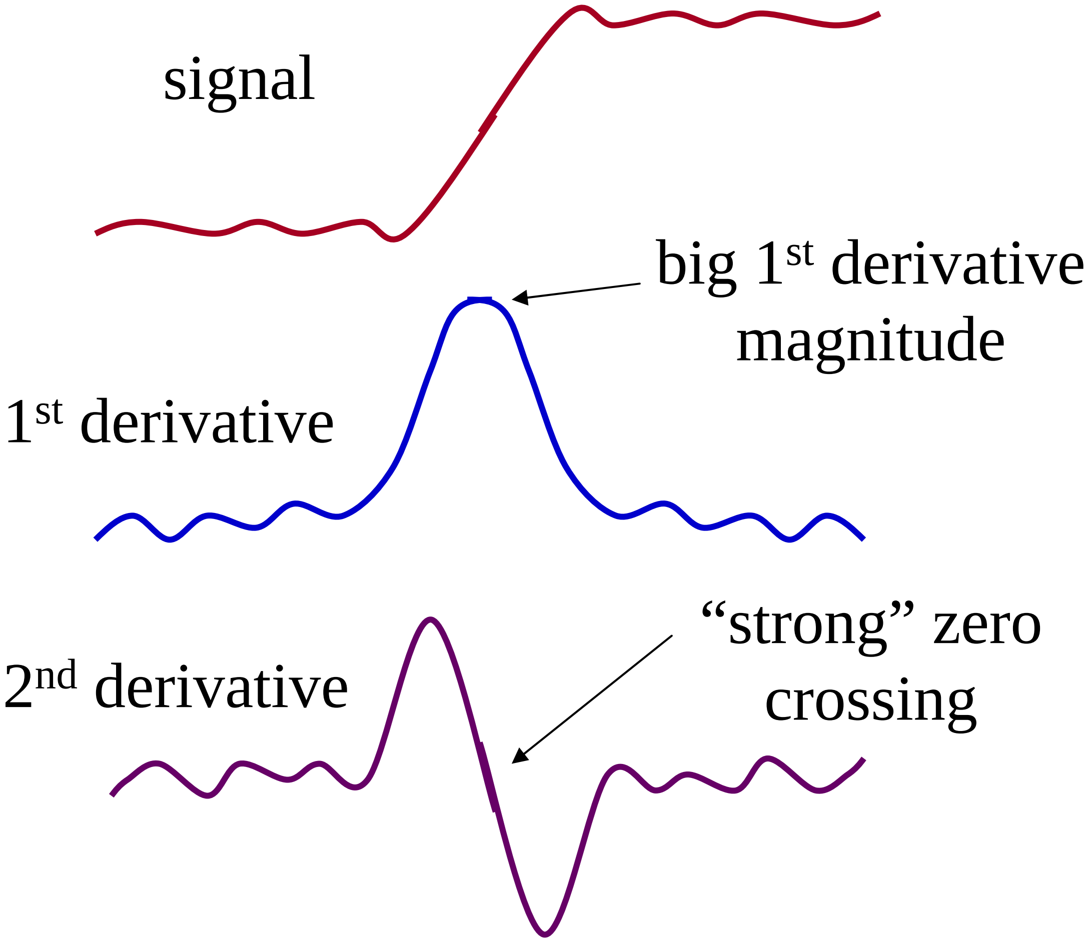
- "Theory of Edge Detection", David Marr, Ellen Hildreth, Proc. Royal Statistical Society of London, B, vol 207, pp 187 -- 217, 1980
  – good paper, often cited
- **VISION**, David Marr
  – good book!

- Excess zero crossings from noise, etc.
  - Usual fix: threshold edges based on “strength.”



- Threshold Bug…

Edge Detection



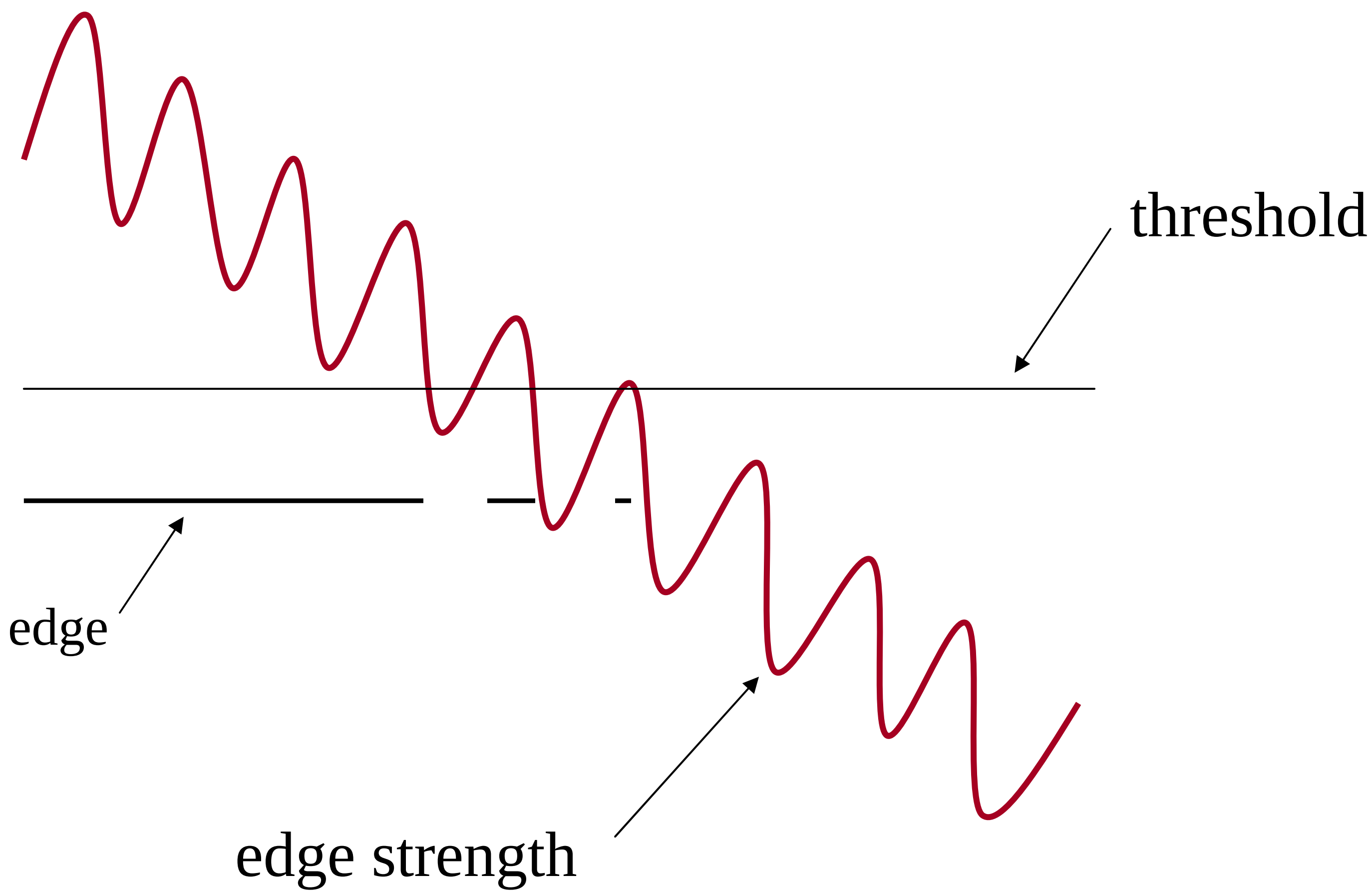
In 2D: threshold based on gradient strength

$$\left[\frac{\partial}{\partial x} f\right]^2 + \left[\frac{\partial}{\partial y} f\right]^2$$

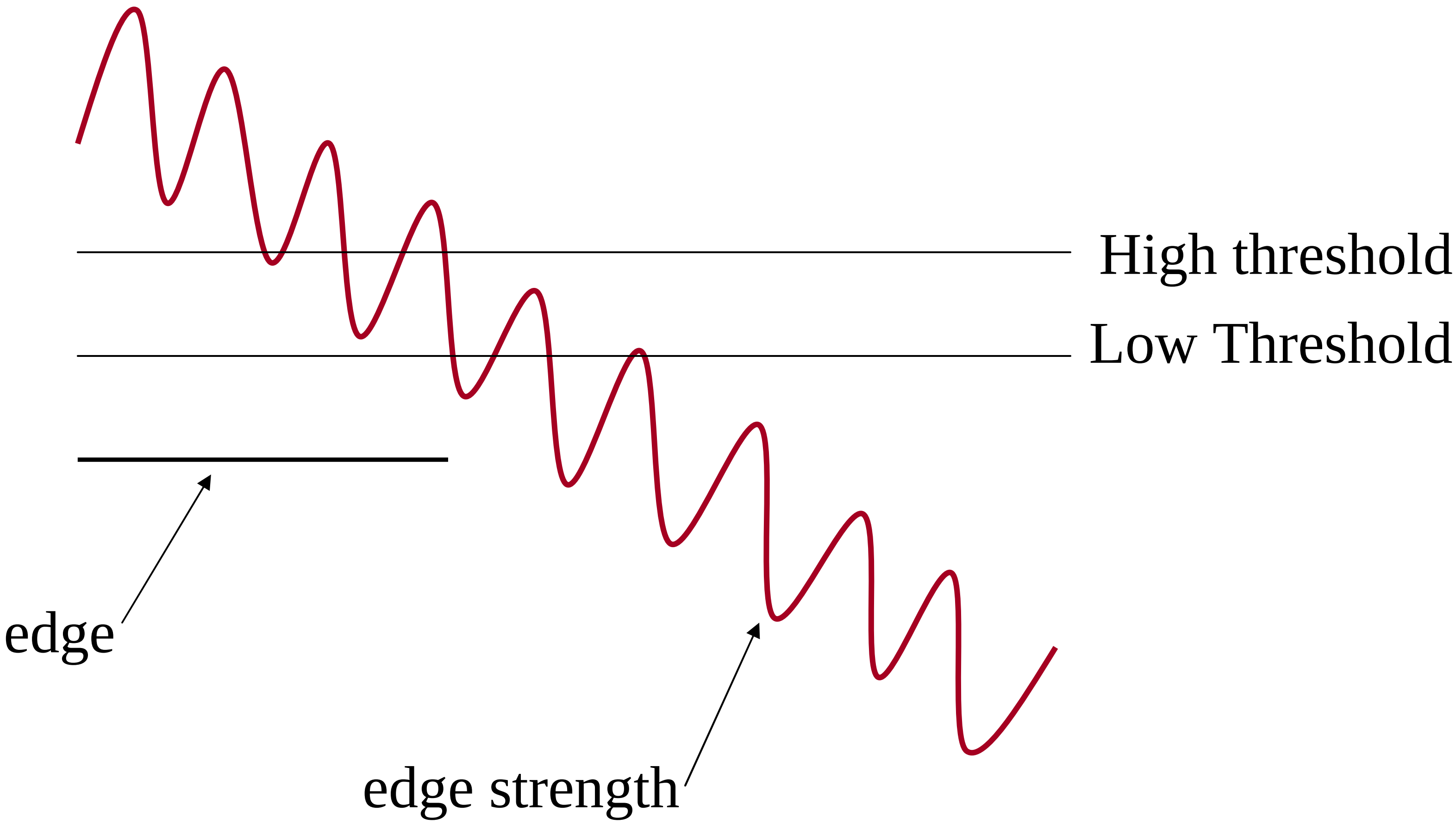
Ultimate Edge Operator: Canny

- For improved noise behavior go back to 1D directional derivatives
- For less fragmentation
  - Use Hysteresis thresholding
- The problem:

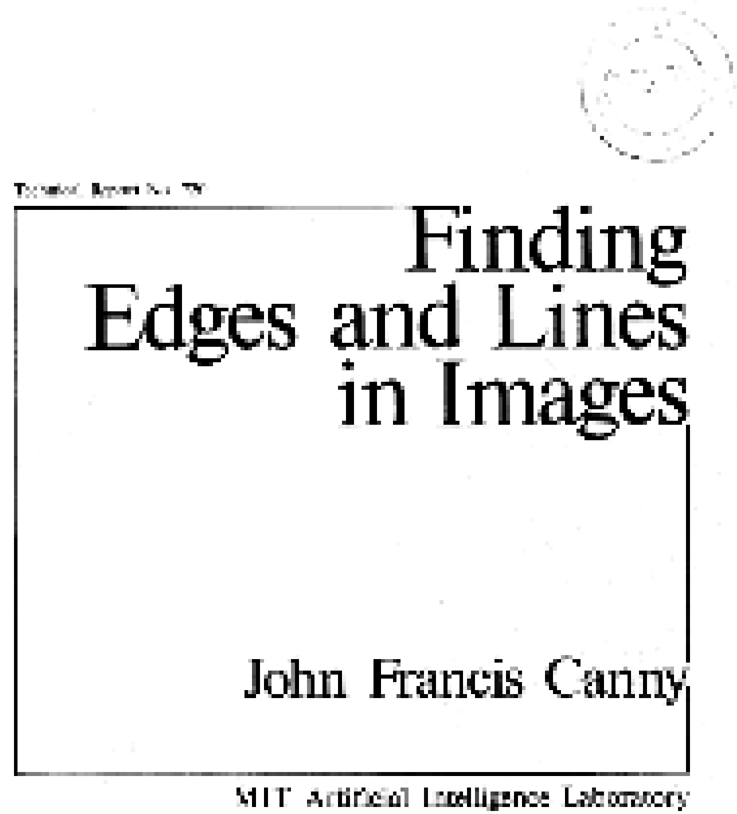
In 1D Along 2D Contour



Schmidt Trigger uses Hysteresis



John Canny MIT MS Thesis


Technical Report No. [illegible]

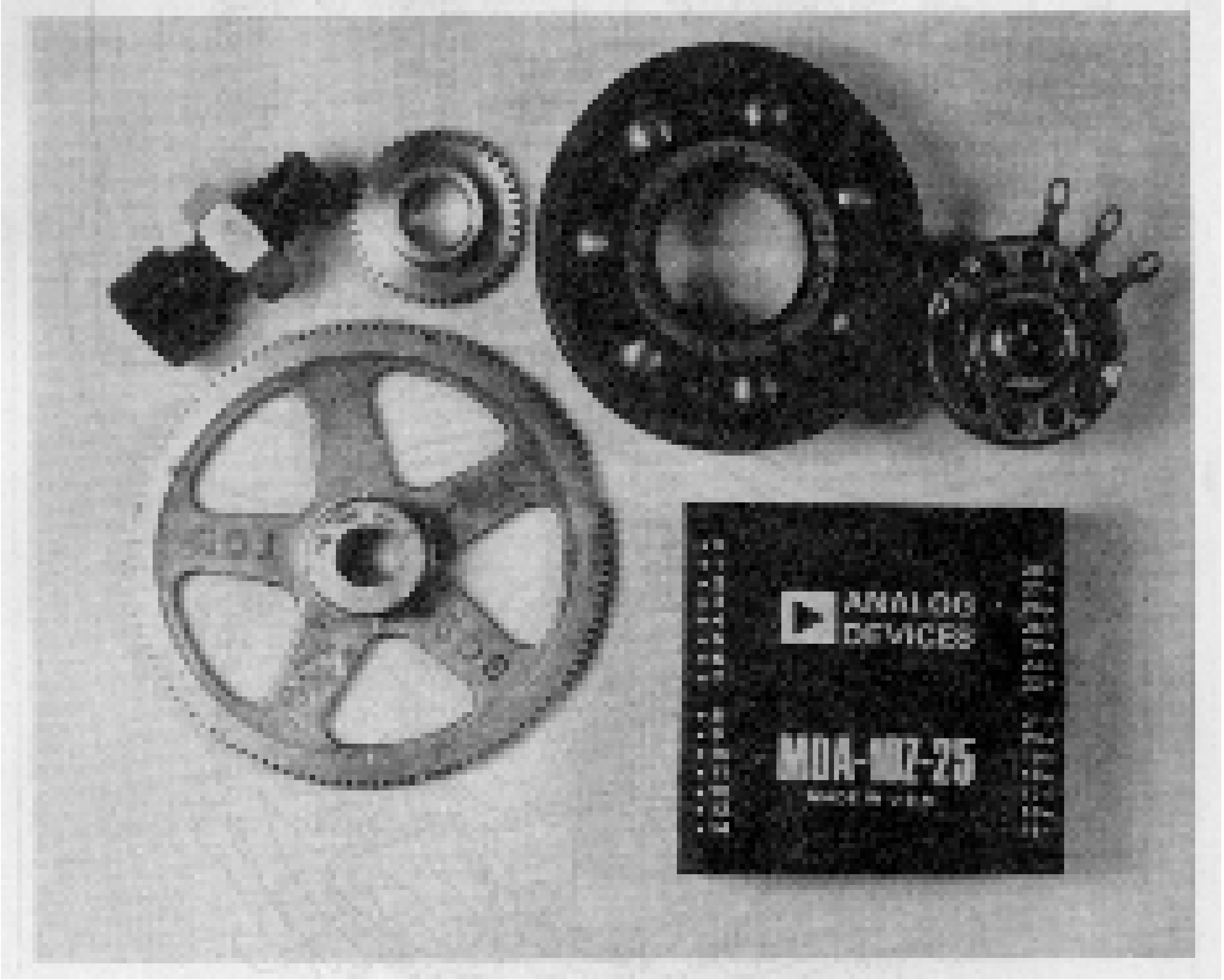
Finding Edges and Lines in Images

John Francis Canny

MIT Artificial Intelligence Laboratory

Edge Detection: Example

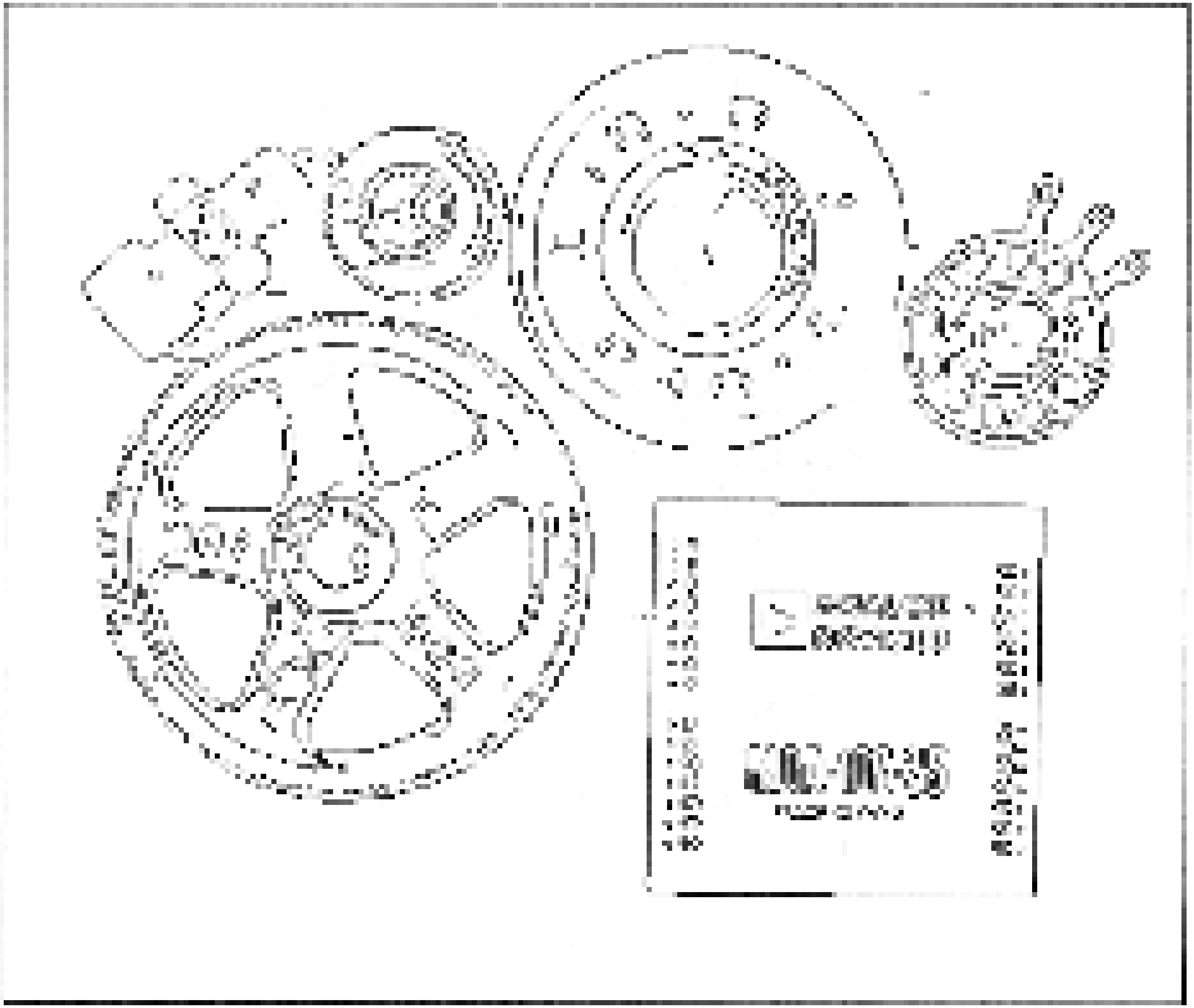
Original image



Source: Canny, J. F. “The complexity of robot motion planning.” MIT Ph.D. thesis, 1987.

Edge Detection: Example

Detected edges



Source: Canny, J. F. “The complexity of robot motion planning.” MIT Ph.D. thesis, 1987.

Edges in 3D are Surfaces

- Somewhat useful for finding organ boundaries.
  – Simple.
  – May leave the problem of figuring out which boundary is what.

3D Medical Edge Finding...

- Recursive Filtering and Edge Tracking: Two Primary Tools for 3D Edge Detection
  - Olivier Monga, Rachid Deriche, Gergoire Malandain, Jean Pierre Cocquerez
  - Image and Vision Computing Vol 9, Nr. 4, 1991

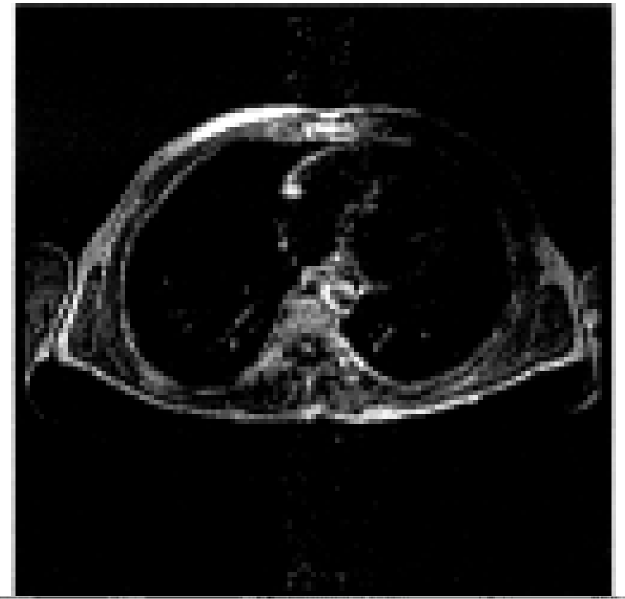


Figure 17. Original cross-section of NMR image corresponding to the diastolic cardiac phase

Figure 18. 3D edges after hysteresis thresholding, Deriche filter, $\alpha = 0.6$

Figure 19. 3D edges after hysteresis thresholding, Shen filter, $\alpha = 0.6$

the end.