

# **Engineering, Economics & Regulation of the Electric Power Sector**

ESD.934, 6.974

**Session 7. Spring 2010**

## **Generation & wholesale markets Basic economic functions**

**Prof. Ignacio J. Pérez-Arriaga**

### **Study material**

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- PB Power, “The cost of generating electricity”, 2004 (*just to learn the cost components*)
- Javier García, “Decision support models in electric power systems”, IIT working paper, Comillas University, 2007 (*no need to go into details on the specific formulations*)

## Readings

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- M. Ventosa, “Electricity market modeling trends”, 2005 (*a good taxonomy of power system models, including oligopolistic ones*)
- F. Galiana & A. Conejo, “Economics of electricity generation”, Chapter 5 of the book “Electric energy systems”, edited by A. Gomez, 2009 (*an alternative reading on power system optimization models, including network effects*)
  - See also Annex B of the same book, by A. Conejo, on a compact review of mathematical optimization techniques

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## Generation costs

## Generation costs

- The different concepts of cost & the concept of levelized cost
  - Source: Royal Academy of Engineering (UK), “The costs of generating electricity”, 2004
  - Capital expenditure (depreciation & interest on equity & debt)
  - Fuel
  - Carbon emissions
  - General overhead
  - Standby generation (to back-up lack of output from plant) ~ very questionable concept
- To be assumed: economic life, interest rates, depreciation method, fuel costs, price of CO<sub>2</sub>, fraction of overhead costs, unavailability or intermittency

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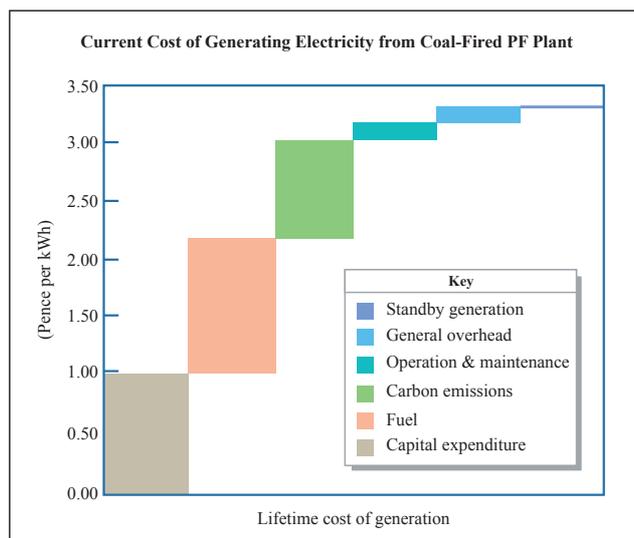


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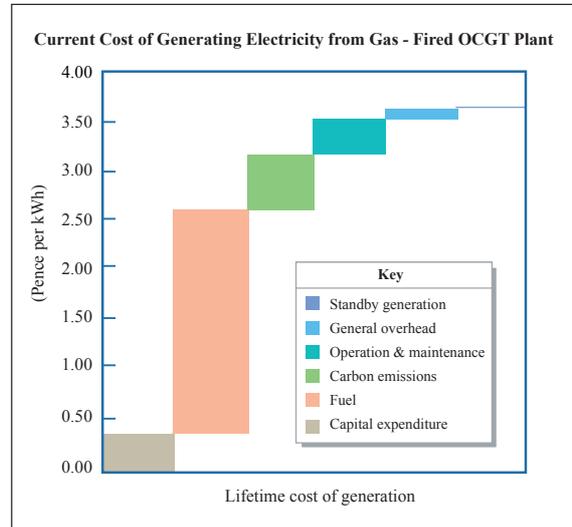


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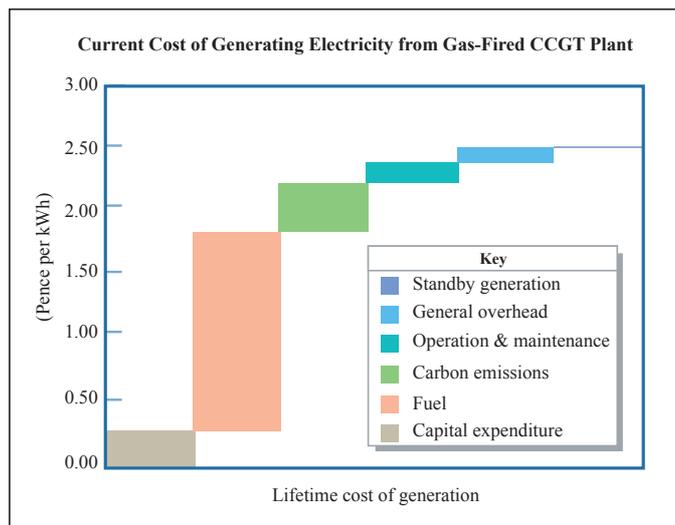


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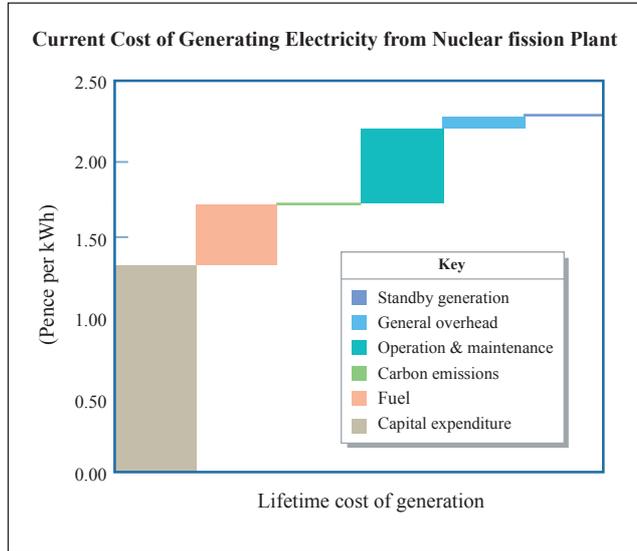


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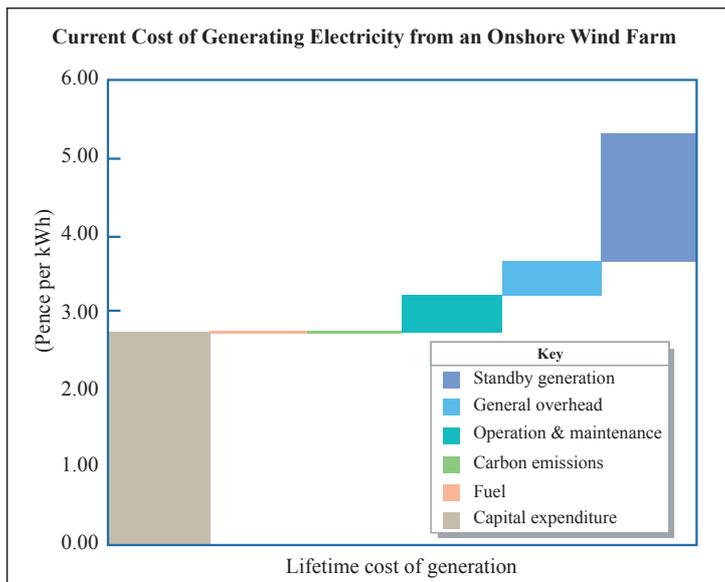


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## Generation costs

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- Large uncertainty in the estimation of future costs
  - Source: IEA (OCDE) *Projected costs of generating electricity, 2005*
  - Source: EU Commission, “An energy policy for Europe”, January 2007
  - Source: IEH CERA Power Capital Costs Index, <http://energy.ihs.com>

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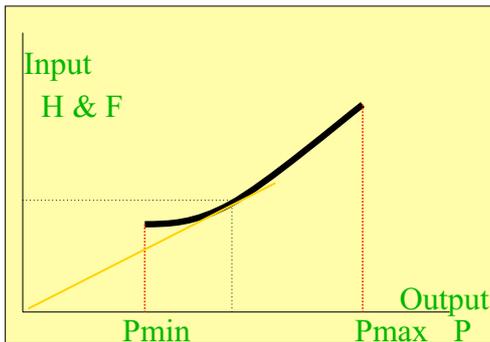
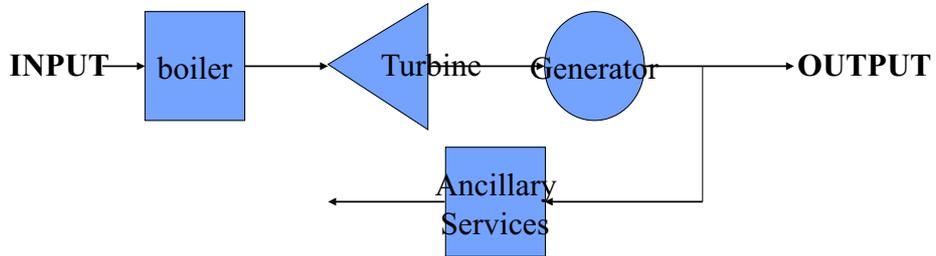
## But reality is even more complex...

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- Minimum load requirements
  - besides, obviously, maximum load requirements
- Start-up costs (*should we shut-down plants when demand is low, therefore incurring later in start-up costs, or should we rather keep them at minimum technical level, even if they are not the lowest cost available plant?*)
  - plus start-up & shut-down minimum times
  - & ramps
- & non-linearity of production cost curves

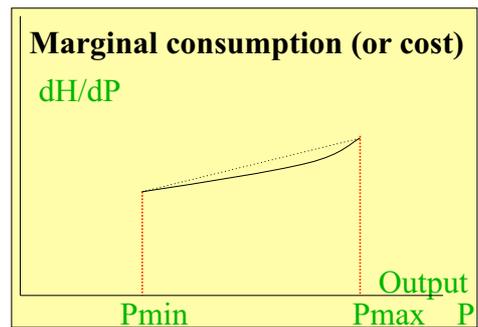
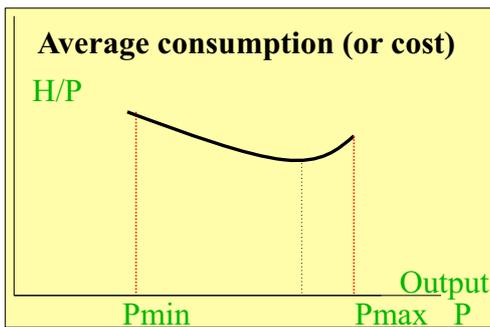
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## INPUT/OUTPUT CURVES OF A THERMAL UNIT



$H = \frac{\text{kcal}}{\text{hour}}$	CONSUMPTION
$F = \frac{\$}{\text{hour}}$	COST
$\frac{H}{P} = \frac{\text{kcal}}{\text{MWh}}$	AVERAGE CONSUMPTION
$\frac{dH}{dP} = \frac{\text{kcal}}{\text{MWh}}$	MARGINAL CONSUMPTION

## INPUT/OUTPUT CURVES OF A THERMAL UNIT



### TYPICAL CURVES FOR A THERMAL UNIT

- CONSUMPTION:  $A \cdot P^2 + B \cdot P + C$
- AVERAGE CONSUMPTION:  $A \cdot P + B + C / P$
- MARGINAL CONSUMPTION:  $2 \cdot A \cdot P + B$

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# Models & some basic power system functions

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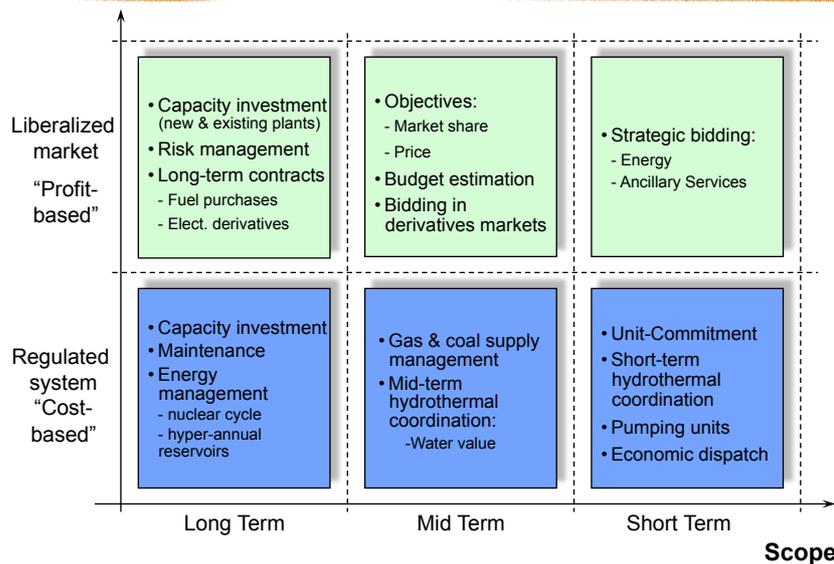
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## Representative functions & models

- Analysis of electromagnetic transients
- Protection coordination
- Short circuit analysis
- Stability analysis
- **Load flow \*\*\***
- State estimation
- Security / contingency analysis
- Load forecasting
- **Economic dispatch \*\*\***
- **Optimal load flow \*\*\***
- **Unit commitment \*\*\***
- **Hydrothermal coordination \*\*\***
- Reliability / adequacy analysis
- Risk assessment
- Investment (generation / transmission) planning

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## Broad review of production models



Borrowed from Javier García González presentation (Master Power Sector, IIT, Comillas University)

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## What is to be optimized?

- The **traditional** objective of electric system planning & operation functions is to supply electricity demand at **minimum cost** with **acceptable levels of reliability** and **environmental impact**
  - The concept of cost does not need much explanation
  - Reliability means different things at different time scales & can be represented in diverse formats
  - Environmental impacts are multiple & difficult to translate in terms of costs

The following slides on reliability have been adapted from a presentation by Prof. Andres Ramos (Master Power Sector, IIT, Comillas University)

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## Reliability aspects & criteria

- Reliability encompasses
  - **Adequacy** of installed capacity
  - **Firmness** of available capacity for operation
  - **Security** of available capacity to respond in real time
- No reliability index covers all aspects; typically indices represent
  - **Number** or frequency of failures
  - **Duration** of failures
  - **Incidence** of failures

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## Formats of representing reliability

Minimize:

$\Sigma$  Operation costs + (if this is the case) Investment costs

Subject to:

- Load supply
- **Reliability criterion**: within prescribed value of an index

Minimize:

$\Sigma$  Operation costs + (if this is the case) Investment costs +  
+ Costs associated to **Non served energy**

Subject to:

- Load supply

Minimize:

Objective function #1:  $\Sigma$  Operation costs +  
+ (if this is the case) Investment costs  
Objective function #2: **Reliability index**

Subject to:

- Load supply

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## Examples of reliability indices (i)

### A deterministic index

- **Reserve Margin (RM):**
  - **Excess** of **generation** capacity available to satisfy yearly load demand

$$RM (MW) = \text{Avaliable generation} - \text{Maximum demand}$$

$$RM (pu) = \frac{\text{Avaliable generation} - \text{Maximum demand}}{\text{Maximum demand}}$$

- Main characteristic is simplicity:
  - Intuitive, easy to compute
  - Limited because **does not consider water reserves**, sizes, technologies, or outage rates

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## Examples of reliability indices (ii)

### Probabilistic indices

- **Loss Of Load Probability (LOLP):**
  - This is the **probability** of being unable to satisfy all the **demanded power** with the available generation
- **Loss Of Load Expectation (LOLE):**
  - This is the expected **number of hours** or **days** in a year with insufficient generation (i.e., a measure of expected frequency, rather than a probability of failure) to meet the total demand: e.g. **1 day in 10 years**

$$LOLP = \frac{LOLE}{365 \text{ days} \quad \text{o} \quad 8760 \text{ hours}}$$

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## Examples of reliability indices (iii)

### Probabilistic indices

- Loss Of Energy Expectation (*LOEE*) widely known as **Expected Energy Non Served *EENS*** (*Expected Unserved Energy EUE*):
  - It is defined as **energy** expected not to be supplied in a year because of generation **unavailability** or by **lack of primary energy**
- **Loss Of Energy Probability (*LOEP*)**:
  - It is defined as **probability** of not supplying **one kWh** with the available generation
  - As it is expressed in per unit, it allows to compare systems of different sizes

$$LOEP = \frac{EENS}{Total\ load}$$

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## Reliability-related references

- Billinton, R. and Allan, R.N. *Reliability Evaluation of Power Systems*, Springer, 1996.
- Billinton, R. and Li, W. *Reliability Assessment of Electrical Power Systems Using Monte Carlo Methods*, Springer, 1994
- Billinton, R. and Allan, R.N. *Reliability Evaluation of Engineering Systems Concepts and Techniques*, Springer, 1992
- Billinton, R. and Allan, R.N. *Reliability Assessment of Large Electric Power Systems*, Springer, 1988.
  - Good reference about electric system reliability
  - Scope and insight that exceeds this presentation
- [IAEA, 84] "Chapter 7. Generating System Reliability", *Expansion planning for electrical generating systems, A guidebook*. International Atomic Energy Agency Technical report No 241. Vienna, Austria, 1984
- A.J. Wood, B. F. Wollenberg *Power generation, operation, and control 2nd edition*. John Wiley & Sons, 1996.
- Vardi, J. and Avi-Itzhak, B. *Electric energy generation. Economics, reliability and rates*. The MIT Press, 1981.

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# Economic dispatch

This section is based on Chapter 5 of “Electric energy systems; Analysis and operation”, edited by A. Gómez-Expósito et al., CRC Press, 2009

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## Economic dispatch

- The economic dispatch (ED) problem consists in allocating the total demand among generating units so that the production cost is minimized
- The allocation is made basically on real-time (*1 to 5 minutes time horizon; ED may also set guidelines on which units have to address primary frequency control*)

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## Economic dispatch

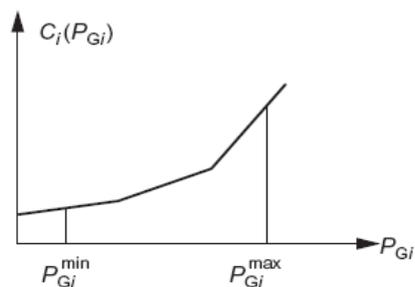
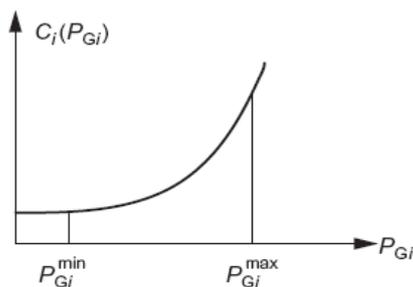
- Each generating unit is assigned a function,  $C_i(P_{Gi})$ , characterizing its generating cost in \$/h in terms of the power produced in MW,  $P_{Gi}$ , during 1 h
- This function is obtained by multiplying the heat rate curve, expressing the fuel consumed to produce 1MW during 1 h, by the cost of the fuel consumed during that hour.

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## Economic dispatch

- The cost function is generally approximated by a convex quadratic or piecewise linear function

$$C_i(P_{Gi}) = C_{0i} + a_i P_{Gi} + \frac{1}{2} b_i P_{Gi}^2$$



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## Economic dispatch: Problem formulation

The Economic Dispatch problem consists of minimizing the total production cost of  $n$  generating units

$$C(P_G) = \sum_{i=1}^n C_i(P_{Gi})$$

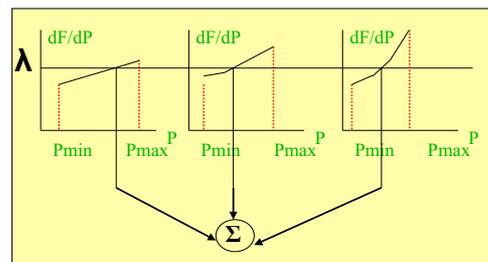
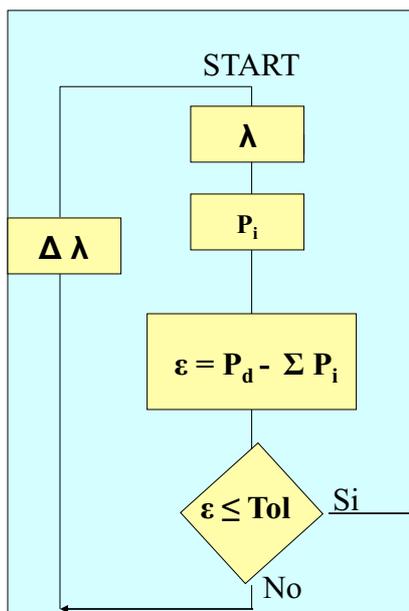
with respect to the unit generation outputs,  $P_{Gi}$ , subject to the power balance

$$\sum_{i=1}^n P_{Gi} = P_D^{\text{total}} + P_{\text{loss}}$$

where  $P_{\text{loss}}$  are the transmission losses, and subject to the generating unit operational limits,

$$P_{Gi}^{\text{min}} \leq P_{Gi} \leq P_{Gi}^{\text{max}}$$

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- The ED algorithm should provide “the system  $\lambda$ ”
- Typically most units will be either operating at full capacity or shut down. Only one (or a few, at most) will be marginal

## Accounting for network effects

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- We should rather learn about the network first, and then we can introduce the most y convenient approximations

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# Load flow

This section is based on Chapter 3 of “Electric energy systems; Analysis and operation”, edited by A. Gómez-Expósito et al., CRC Press, 2009

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## Load flow

- The power system is assumed to be stable & at rated frequency. The “load flow” is like a snapshot of the power inputs, outputs & flows in the network
- The load flow is the workhorse of power system networks operators & planners
- The load flow problem consists of determining the value of all significant variables in a power system network:
  - node voltages in magnitude  $U$  & angle  $\theta$
  - active power  $P$  & reactive power  $Q$  flows in lines & transformers
  - active power  $P$  & reactive power  $Q$  supplied by generators & consumed by loads

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## Load flow: Problem statement

- Power system with
  - $n_L$  **load nodes** (PQ buses) where both the active  $P^{sp}$  & reactive  $Q^{sp}$  power are specified; unknowns are the voltage magnitude  $V$  & angle  $\theta$  at the node
  - $n_G$  **generation nodes** (PV buses) where the active power  $P^{sp}$  power is specified by some higher economic function & the generator can maintain the voltage at some specified value  $V^{sp}$ ; unknowns are the injected reactive power  $Q$  & angle  $\theta$  at the node
- As the network losses resulting from the line flows are not known a priori, at least the active power  $P$  at one node (some arbitrarily chosen slack bus) cannot be specified; if  $N$  is the number of nodes in the system then

$$n_G + n_L = N - 1$$

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## Load flow: Problem formulation

$$P_i^{\text{sp}} = V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad i = 1, 2, \dots, n_L + n_G$$

$$Q_i^{\text{sp}} = V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad i = 1, 2, \dots, n_L$$

where

$G_{ij}$  &  $B_{ij}$ : elements of the admittance between the buses  $i$  &  $j$

$\theta_{ij}$ : difference between the voltage angles  $\theta_i$  &  $\theta_j$  at the buses  $i$  &  $j$

Solving the load flow consists of finding the set of phase angles  $\theta_i, i=1, 2, \dots, n_L + n_G$ , and the set of voltage magnitudes  $V_i, i=1, 2, \dots, n_L$ , satisfying these  $2n_L + n_G$  equations.

As the resulting equation system is nonlinear, its solution necessarily involves an iterative process, for which adequate initial values should be given to the state variables.

## DC load flow

A reasonable linear approximation to the nonlinear load flow equations is the so-called *DC load flow*. It is assumed that  $V_i = 1$  at all buses (reactive power flows & voltage differences are ignored) the load flow equations become

$$P_{ij} = G_{ij}(\cos \theta_{ij} - 1) + B_{ij} \sin \theta_{ij}$$

Further simplifications

$$\cos \theta_{ij} \approx 1 \text{ \& \; } \sin \theta_{ij} \approx \theta_i - \theta_j$$

$$B_{ij} = x_{ij}/(r_{ij}^2 + x_{ij}^2) \approx 1/x_{ij}$$

lead to the simple & linear expression

$$P_{ij} = (\theta_i - \theta_j)/x_{ij}$$

only in terms of the reactances of the branches, the voltage angles & the branch active power flows.

Although the DC model is lossless, actual power losses can be estimated in terms of active power flows by conveniently adding terms of the form  $R_{ij} \cdot P_{ij}^2$

## Even simpler approximations

In some cases the size of the model or the lack of sufficient information on the network justify even further simplifications of the network representation.

A model that has been frequently used makes only use of the first Kirkhoff's law, i.e. the equality of the sum of the power flows entering each network node to the sum of the power flows exiting the same node. Constraints on the maximum flows for each individual line could be added to the model.

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# Optimal load flow

This section is based on Chapters 5 & 6 of “Electric energy systems; Analysis and operation”, edited by A. Gómez-Expósito et al., CRC Press, 2009

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## Optimal load flow (OPF)

- The objective of the OPF is the same as for the ED, but now the network is explicitly included in the model
- Industrial OPFs typically include some kind of contingency analysis (preventive security control)

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## Optimal load flow: Problem formulation

$$\text{Minimize } P_G, \theta \quad \sum_{i=1}^n C_i(P_{Gi})$$

subject to

$$P_G^{\min} \leq P_G \leq P_G^{\max}$$

$$P_i = V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad i = 1, \dots, n$$

$$Q_i = V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad i = 1, \dots, n$$

*Inequality constraints:*

- Limits on control variables: generator active and reactive power, transformer turn ratios, capacitor and/or reactor banks, FACTS, etc.
- Operational constraints: limits on bus voltages and power flows.

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## Optimal load flow: Problem formulation with DC load flow approximation

$$\text{minimize } P_G, \delta \quad \sum_{i=1}^n C_i(P_{Gi})$$

subject to

$$P_G^{\min} \leq P_G \leq P_G^{\max}$$

$$P_i = \sum_{j=1}^n \frac{\theta_{ij}}{x_{ij}} \quad i = 1, \dots, n$$

where  $x_{ij}$  is the branch reactance between nodes  $i$  and  $j$

*Inequality constraints:*

- Limits on control variables: generator active and reactive power, transformer turn ratios, capacitor and/or reactor banks, FACTS, etc.
- Operational constraints: limits on bus voltages and power flows.

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## Additional features of actual OPF algorithms

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- In order for the outcome of an OPF to be qualified as “secure”, it has to be able to maintain this status against predictable changes (*demand & generation evolution*) & unpredictable events called contingencies
- OPF must include
- Operating reserves
  - Operating constraints that are derived from contingency analysis of potential transmission failures

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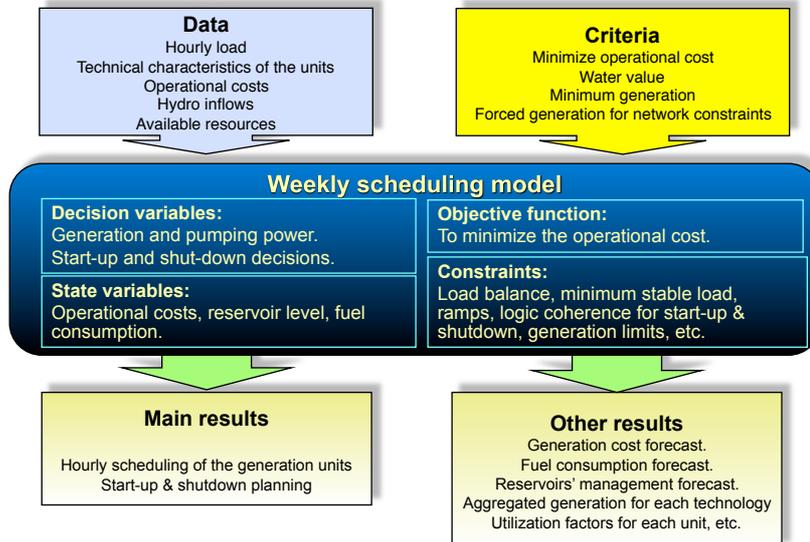
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## Unit commitment

This section is based on Chapter 5 of “Electric energy systems; Analysis and operation”, edited by A. Gómez-Expósito et al., CRC Press, 2009

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## Weekly unit-commitment model



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## Unit commitment: problem formulation

The minimization extends now over  $p$  time periods (24 hours or more, up to a week, typically). The operating status of a generating unit is then expressed by the couple  $(P_{Gi}, u_i)$ , where  $u$  can be 0 (unit off) or 1 (unit on). The generating limits of unit  $i$ ,  $P_{\min Gi}$  and  $P_{\max Gi}$ , are multiplied by the binary variable  $u_i$ . And every time  $u_i$  changes from 0 (unit off) to 1 (unit on) the start-up cost  $C_{0i}$  is incurred. In this formulation network effects are ignored.

$$\text{subject to} \quad \text{Minimize } (u, P_G) \quad \sum_p \sum_{i=1}^n C_i(u_i, P_{Gi})$$

$$C_i(u_i, P_{Gi}) = u_i C_{0i} + a_i P_{Gi} + \frac{1}{2} b_i P_{Gi}^2$$

$$u_i P_{Gi}^{\min} \leq P_{Gi} \leq u_i P_{Gi}^{\max}; \quad i = 1, \dots, n$$

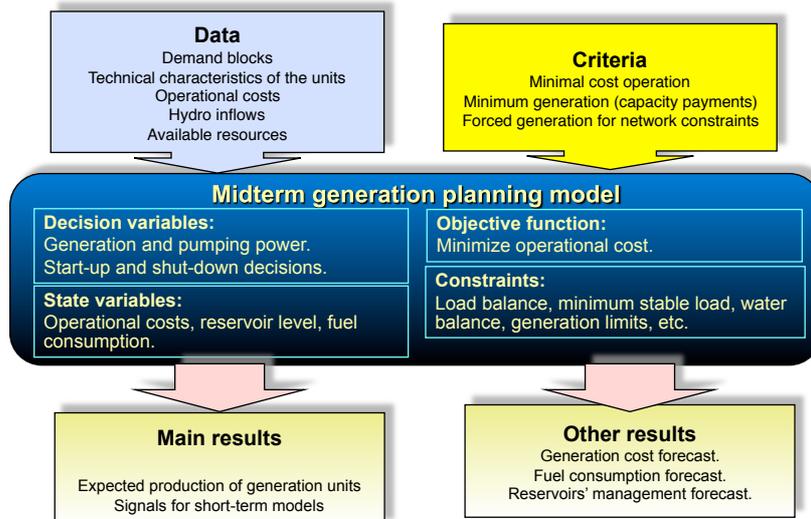
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# Annual production cost models

This section is based on presentations by A. Ramos & J. García González in the Master for the Electric Power Sector, IIT, Comillas University, Madrid, Spain

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## Midterm generation planning model



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## Midterm generation planning

- **Objective**

- To obtain the minimum-cost optimal schedule of the generating units in order to satisfy the supply-demand balance equation and subject to all the other constraints of the system (technical, environmental, regulatory, etc.)
- This model could be adapted for a market participant: maximum profit.

- **Planning...**

- **the operation**

- To find the signals for the short-term models.
- To estimate the share of each generation technology in the final dispatch

- **economical**

- To forecast the operational and marginal costs. This is very relevant when preparing the annual budgets.

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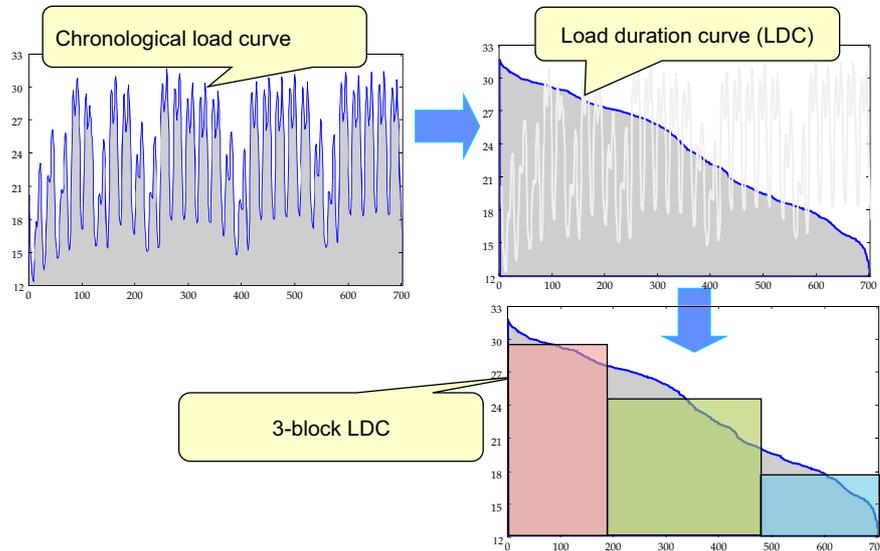
## Midterm generation planning

- **Typical modeling hypothesis:**

- It is not necessary to model the **transmission network**
- It is not necessary to model the hourly **chronological** evolution of the system:
  - Use a representation based in load blocks &/or load duration curves
- Thermal plants **start-up or shutdown** decisions are only made in the transitions from working days to weekends, and vice versa.
- It is not necessary to consider a detailed representation of **hydroelectric** systems

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## Load Duration Curve: illustrative example



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## Major modeling options

- Uncertainty is of essence in the annual horizon: Two approaches that can be combined
  - Decomposition in time periods & deterministic scenarios to represent hydro uncertainty, with possibility to include sequential decision making
  - Less detail in time period decomposition & “probabilistic simulation” to represent demand, hydro input & unit failure uncertainties

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## Temporal representation

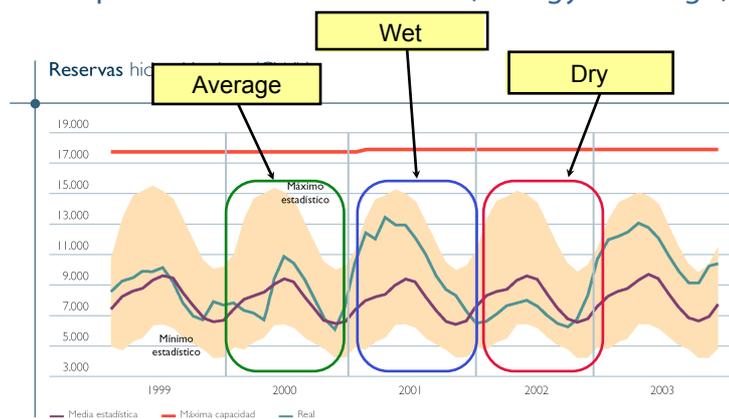
- Time horizon
  - Midterm (**one year**).
- Structure:
  - **Periods**  $p$ : 1 week or 1 month
  - **Subperiods**  $s$ : working and non-working days
  - **Load levels**  $n$ : peak, off-peak hours

The hourly chronological modeling is not suitable for the midterm horizon

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## Scenarios Hydro inflows

- ! Example of historical data (energy storage)



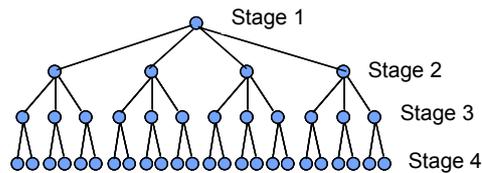
Fuente: REE, Informe 2003, El Sistema Eléctrico Español, <http://www.ree.es>

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## Scenarios Uncertainty modeling

- The uncertainty (demand, inflows, fuel costs, etc.) can be taken into account in the midterm model by means of scenarios & sequential modeling

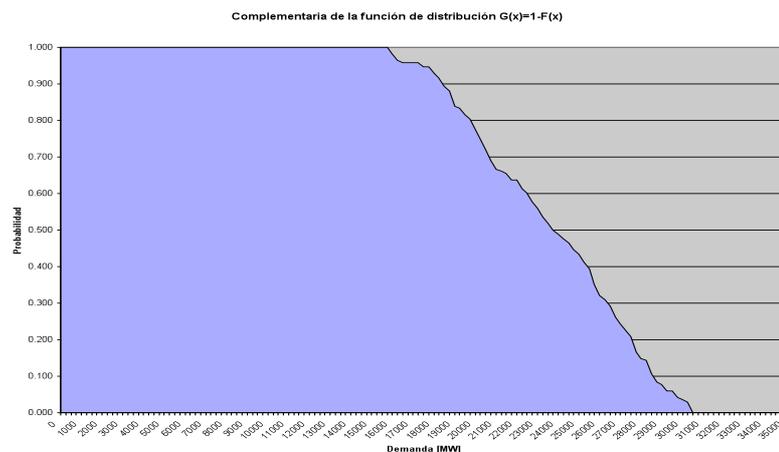


- It is necessary to distinguish between:
  - “here and now” decisions taken in the first stage
  - recourse variables for subsequent stages that represent the strategies that must be followed when uncertainty is being unveiled

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## Probabilistic simulation: Complementary cumulative distribution function (CCDF)

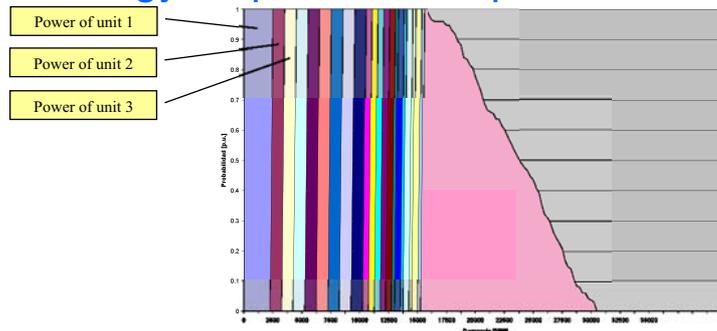
- Probability of the demand is **greater or equal** to a certain value during a given time period  $G_X(x) = 1 - F_X(x)$



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## Dispatch WHITOUT thermal units' failure

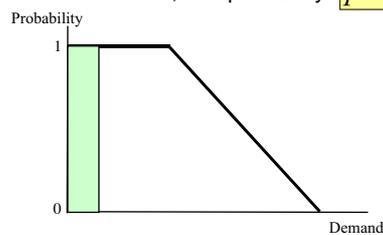
- Thermal units are dispatched from **left to right** under the complementary cumulative distribution function curve
- Energy output = area x period duration**



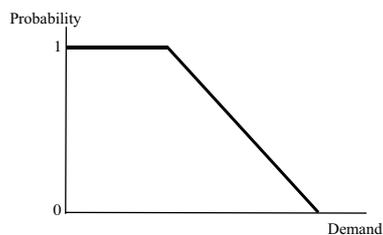
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## Dispatch WITH thermal unit failure (i)

Unit **does NOT** fail, with probability  $p = 1 - q$



Unit **fails**, with probability  $q$

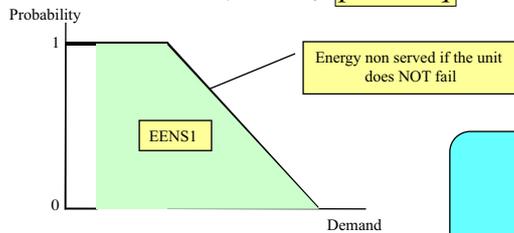


Unit energy output =  
area x duration x probability

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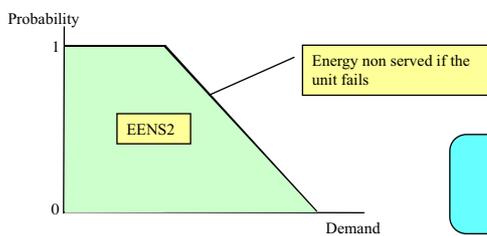
## Dispatch WITH thermal unit failure (ii)

Unit does NOT fail, with probability  $p = 1 - q$



Unit energy output =  
EENS before unit dispatch –  
EENS after unit dispatch

Unit fails, with probability  $q$

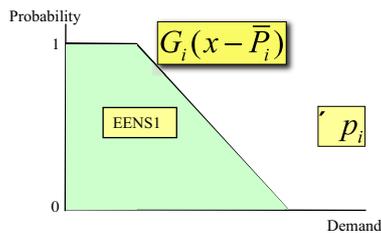


EENS after unit dispatch =  
 $EENS1 \cdot p + EENS2 \cdot q$

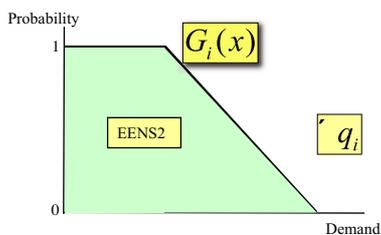
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## Thermal unit convolution (i)

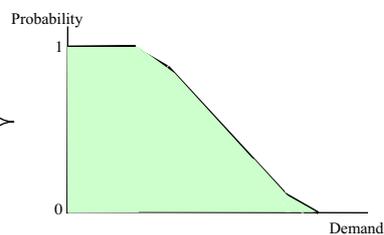
EENS if the unit does NOT fail



EENS if the unit fails



EENS after unit dispatch



$$G_{i+1}(x) = p_i G_i(x - \bar{P}_i) + q_i G_i(x)$$

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## Reliability measures (i)

- *EENS* Expected energy non served

$$EENS = E_{N+1} = T \int_0^{\bar{D}} G_{N+1}(x) dx$$

- *LOLP* Loss of load probability

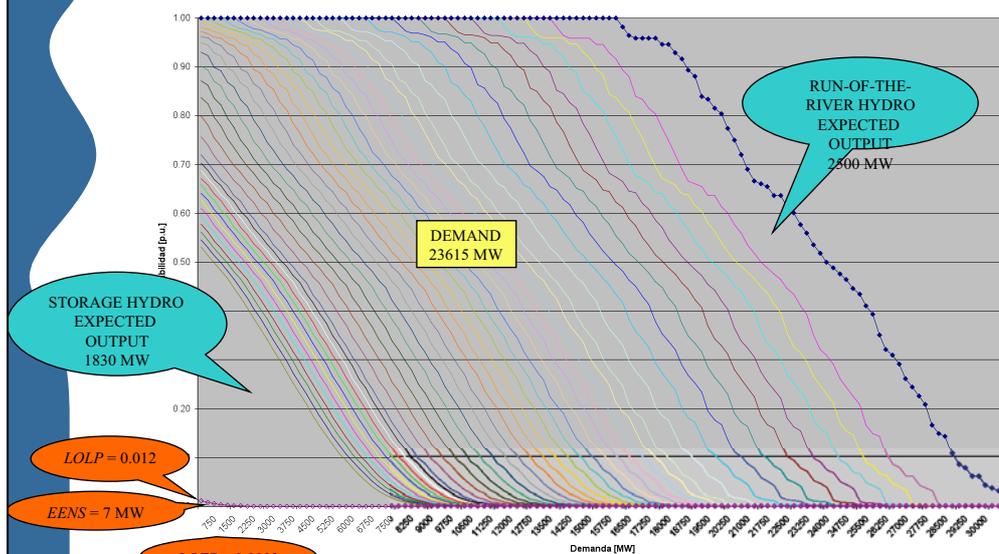
$$LOLP = G_{N+1}(0)$$

- *LOEP* Loss of energy probability

$$LOEP = \frac{EENS}{\int_0^{\bar{D}} G_1(x) dx}$$

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## Actual power system case (ii)

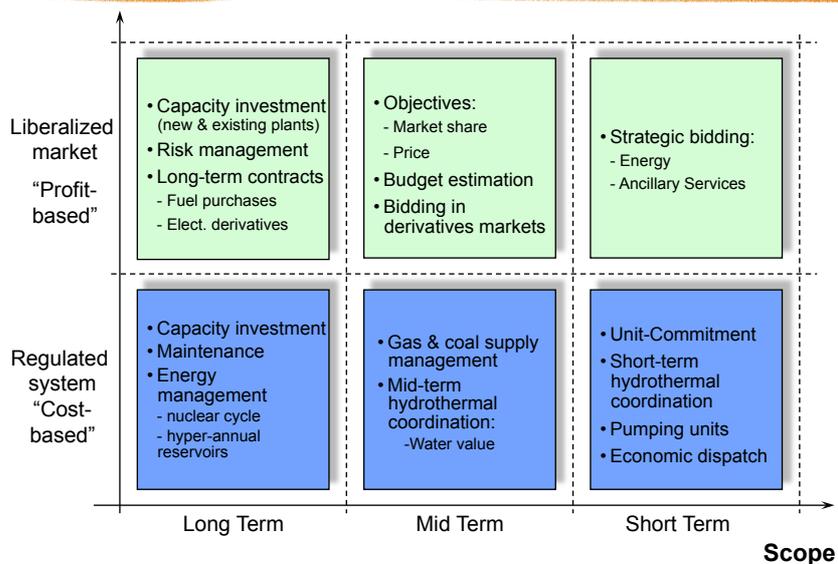


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# A broader vision of the production models being currently used in the power sector

Most of this section is based on M. Ventosa et. al. "Electricity market modeling trends", Energy Policy 33 (2005) 897-913

## Broad review of production models



Borrowed from Javier García González presentation (Master Power Sector, IIT, Comillas University)

## Market models versus centralized optimization models

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- In a market environment the possible applications of models is very diverse
  - Market clearing algorithms for spot markets might be seen as ED or UC by replacing cost by bids (*only in auctions with complex bids; short-term markets may adopt many different formats*)
  - Models may be used by an individual market agent to determine its strategy for bidding, contracting or investing
  - Or by regulators or individual market agents to estimate the future behavior of the market for any given time range

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## Market models in imperfect markets

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- Optimization models focus on the profit maximization problem for one of the firms that compete in the market
- Equilibrium models represent the overall market behavior where all participants compete
- Simulation models are an alternative to equilibrium models when the problem to solve is too complex
- Models can be also classified based on: level of competition, time scope, uncertainty modeling, interperiod links, transmission constraints or market representation

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