ESD: Recitation #6

Revisions

- Four Steps to Happiness
- Z-transforms and s-transforms
- Common PMFs and PDFs
- Poisson processes and random incidence
- Convolution
- Sampling problems
- Spatial models
- Markov processes and queuing systems

Four Steps to Happiness

- Define the random variables
- Identify the joint sample space
- Determine the probability law over the sample space
- Work in the sample space to answer any question of interest
  - Derive the CDF of the RV of interest working in the original sample space whose probability law you know
  - Take the derivative to obtain the desired PDF

Transforms

- Z-transform:

$$p_X^T(z) \equiv \sum_{x=0}^{\infty} p_X(x) z^x, \quad |z| \leq 1, \text{ where}$$

$$p_X(x) \equiv P\{X = x\}, x = 0,1,2,\ldots$$

Nearest neighbor

- Euclidean distance:

$$P\{X(circle)=k\}=\frac{(\gamma.\pi.r^2)^k.e^{-\gamma.\pi.r^2}}{k!},\forall k\in \mathrm{N}$$

- What changes for taxicab distance?

Little's Law

- In steady state:

  $L = \lambda . W$

  $L_q = \lambda . W_q$

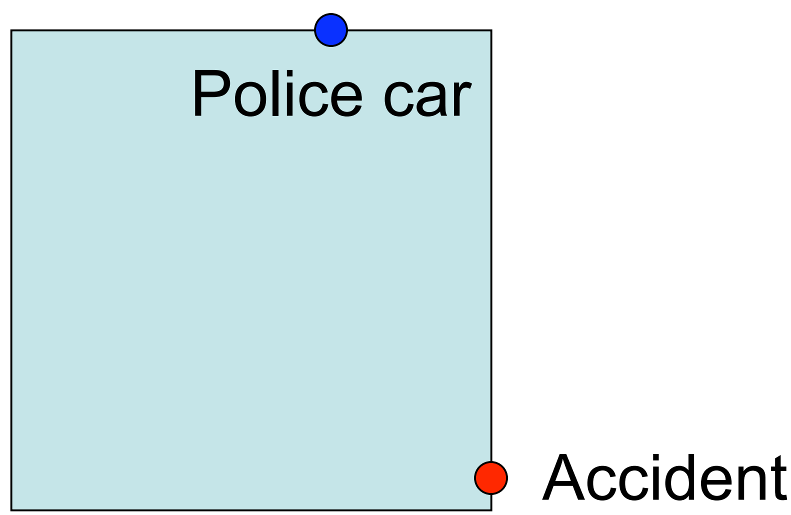
  $W = 1/\mu + W_q$

  $L = L_q + \lambda/\mu$

- Conditions?

Test exercises (1)

- Police car and accident independently and uniformly located on the perimeter of a square (1 x 1 km).



Around a square

- Travel only possible around the square.

1) PDF of travel distance if the police car can make U-turns anywhere?

2) PDF of travel distance if U-turns are impossible?

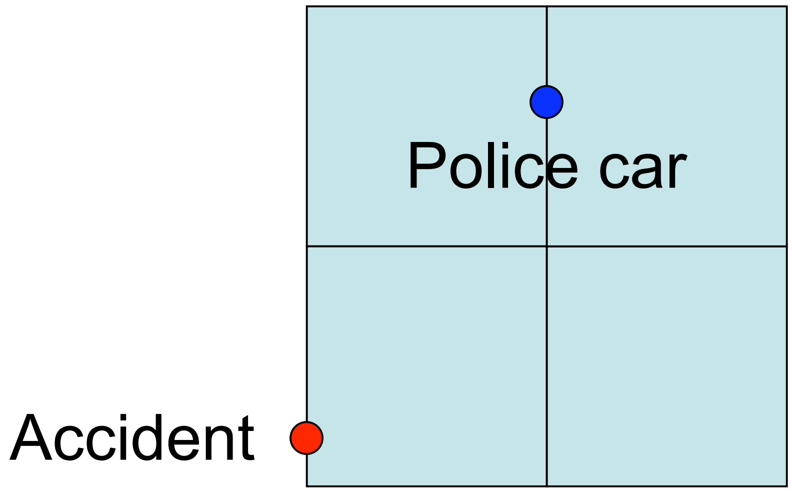
Solving

1) Let us fix X_1. X_2 is uniformly distributed over the sides of the square: Travel distance uniformly distributed between 0 and 2 km.

2) Idem, except that travel distance is now uniformly distributed between 0 and 4 km.

Continued…

- What if we now have four blocks around which the accident and the police car can be?



Test exercises (2)

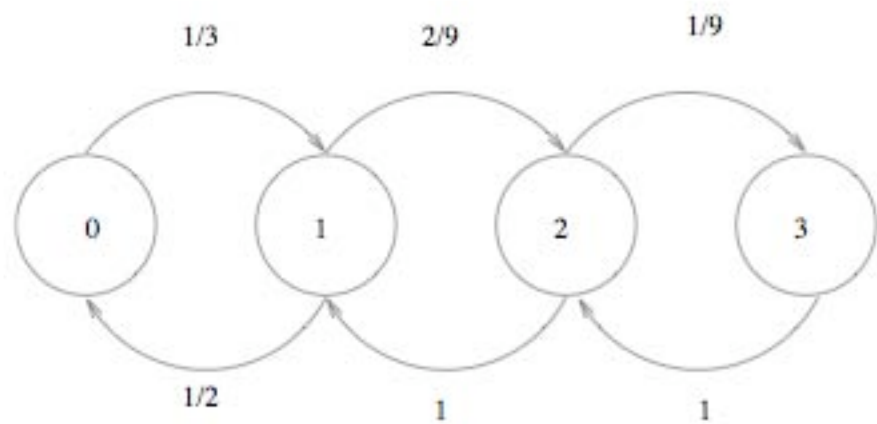
- Consider a small factory that has 3 machines subject to breaking down (independently of each other).
- Whenever a machine breaks down, it is sent to the factory's repair shop, which has two parallel and identical repair stations. Repair is done in a FIFO order. The time needed to repair a machine at a repair station has an exponential pdf with:

  $E[R] = 2$ hours.
- The time until a repaired machine breaks down again has an exponential pdf with: $E[B] = 9$ hours.
- Find the expected number of machines that are operating at this factory in steady state.

Small factory

- The small factory has 3 machines, therefore the total population is three. Our Birth-and-death chain has therefore only a 4 states, that is all machines can be running, one can be broken down, two can be broken down or all can be broken down.

Modeling



Solving (1)

- Steady-state equations:

$$\frac{1}{3}P_0 = \frac{1}{2}P_1$$

$$\frac{2}{9}P_1 = P_2$$

$$\frac{1}{9}P_2 = P_3$$

$$P_0 + P_1 + P_2 + P_3 = 1$$

Solving (2)

- Therefore:

$$P_0 = \frac{243}{445}$$

$$P_1 = \frac{162}{445}$$

$$P_2 = \frac{36}{445}$$

$$P_3 = \frac{4}{445}$$

Solving (3)

- Expected number of machines that are operating:

  $$P\{Operating\} = 3 - L$$
  $$= 3 - (0 \times P_0 + 1 \times P_1 + 2 \times P_2 + 3 \times P_3)$$
  $$\approx 2.45$$