ESD.86 - Recitation 1

The Bernoulli Distribution (1)

- Let’s have an experiment that can only have two outcomes: “success” (labeled n = 1), with probability p; and “failure” (labeled n = 0), with probability q.

- It therefore has the following probability distribution:

$$P(n) = \begin{cases} 1 - p, \_ for \_ n = 0 \\ p, \_ for \_ n = 1 \end{cases}$$

$$P(n) = p^n (1 - p)^{1-n}$$

The Bernoulli Distribution (2)

- The Bernoulli distribution function is:

$$D(n) = \begin{cases} 1 - p,\_ \; for\_n = 0 \\ 1,\_ \; for\_n = 1 \end{cases}$$

- Mean: $\mu = p$

- Variance: $\sigma^2 = p\,(1 - p)$

The Binomial Distribution (1)

- Start with an experiment that can have only two outcomes: "success" and "failure" or {0, 1} with probabilities p and q, respectively.
- Consider N "trials," i.e., repetitions of this experiment with constant q (***Bernoulli*** *trials)*
- Define a new DRV: X = number of 1's in N trials
- Sample space of X: {0,1,2,...,N}
- What is the probability that there will be k 1's (failures) in N trials?

The Binomial Distribution (2)

$$\Pr[X = k] = \binom{N}{k} q^k (1-q)^{N-k}$$

- This is the probability mass function of the Binomial Distribution.
- It is the probability of ***exactly*** k failures in N demands.
- The ***binomial coefficient*** is***:***

$$\binom{N}{k} = \frac{N!}{k!(N-k)!}$$

The Binomial Distribution (3)

- Mean number of failures: q.N
- Variance: q.(1 - q).N
- Normalization: $\sum_{k=0}^{N} \binom{N}{k} q^k (1-q)^{N-k} = 1$
- P[at most m failures]:

$$\sum_{k=0}^{m} \binom{N}{k} q^k (1-q)^{N-k} = F(m)$$

The Geometric Distribution (1)

- The Geometric distribution is a discrete distribution for n = 0, 1, 2, … having the following probability function:

$$P(n) = p(1-p)^n$$

$$P(n) = pq^n$$

  where $0 < p < 1$, and $q = 1 - p$

- Its distribution function is:

$$D(n) = \sum_{k=0}^{n} P(k) = 1 - q^{n+1}$$

The Geometric Distribution (2)

- The geometric distribution is the only discrete memoryless random distribution. It is the discrete distribution of the exponential distribution.
- Normalization: $$\sum_{n=0}^{\infty} P(n) = \sum_{n=0}^{\infty} q^n p = p \sum_{n=0}^{\infty} q^n = \frac{p}{1-q} = \frac{p}{p} = 1$$
- Mean: $\mu = \dfrac{1-p}{p}$
- Variance: $\sigma^2 = \dfrac{1-p}{p^2}$

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- Wolfram's Mathworld:
  http://mathworld.wolfram.com/