ESD.86
Models, Data, Inference for Socio-Technical Systems
Massachusetts Institute of Technology
Engineering Systems Division
Problem Set #4
Issued: Wednesday March 7, 2007. Due: Wednesday March 14, 2007.

1. Pedestrian Light, revisited. Apply Little's Law to the 77 Massachusetts Avenue Pedestrian Crossing light problem, to find L for each of the 3 ways of operating the light. Recall, for each of the 3 modes of operation, you have already derived W or W_q, so this should be easy!

2. Squaring. We'll be doing lots of squaring of random variables in the 2nd half of this subject. So, let's get started now.
   (a) Suppose we know all about random variable X, having pdf
      $f_X(x)$ for all $x, -\infty < x < +\infty$. Define new r.v. $Y=X^2$. Show that the following is
      true: $f_Y(y) = \dfrac{1}{2\sqrt{y}}[f_X(\sqrt{y}) + f_X(-\sqrt{y})]$. Explain in words what this means.
   (b) Apply your result to a situation in which X is uniformly distributed over the interval [-1, +2].

3. M/M/k Queue. Let's examine the queue with homogeneous Poisson arrivals (rate λ), negative exponential service times (mean $1/\mu$), k identical servers, infinite queue capacity and no balking or reneging. We assume the steady state exists, implying that $\lambda < k\mu$. Let the state of the system be the number of customers in the system, both in service and in queue.
   (a) Find the steady state probability P_n=P{n customers in the system}, n=0,1,2,…
   (b) Find W. (Hint: think of Little's Law and that finding any one of 4 quantities gets you the other 3 at 'no extra charge.' So, to find W all you need to do is compute the easiest one of the 4 quantities related to Little's Law.)
   (c) The systems designers of call centers and other systems involving queueing like to enjoy economies of scale. Yet they have service performance standards to meet. To many in the services industries, the quantity (1-ρ) is '*paid lost time*', that is, time that a server is not working serving customers. Suppose a service system that can be modeled as an M/M/k Queue has a performance standard that states that the mean wait in queue shall be no greater than one minute. Assume the mean service time is $1/\mu$ = one minute. Using your results above and an Excel or similar spreadsheet, examine how (1-ρ) can be made smaller as the call volume λ increases, the number of servers k increases, while at all times meeting the service standard of one minute or less mean queue wait time. Show that (1-ρ)

can become much smaller than the value associated with one server as call volume increases, the number of servers increases, but not in direct proportion to call volume.

(d) Suppose you designed your system as in Part (c), and you have 20 servers in your system. How sensitive are you to errors in forecasted call volumes? What happens if tomorrow's call volume is 10% over forecasts? This is a matter of 'ilities,'' reliability, robustness, resilience. Did you follow in the media what happened to stock traders and brokers and even stock indices as trading volume zoomed far above routine averages on Tuesday February 27?

4. M/M/k Queue with Discouraged Voters. We now want to look at a variation of the queue of problem 3. Suppose it is Election Day and we focus on a given voting precinct. Systems managers have a choice before Election Day: how many voting machines (servers) to place in each precinct. Too many in one precinct may mean no queues at that precinct but that another precinct is short-changed. Too few implies long lines of voters, some of whom may not join the line because it is too long.

(a) Suppose arrivals that see a line of queued voters can get discouraged and not join the line. Suppose field research has shown that the probability that a potential voter will join the system and vote is equal to

$$p_V(n_q) = e^{-0.1n_q}, \text{ where } n_q \equiv \text{ number of voters in queue upon arrival}, n_q = 0,1,2,...$$
.

For simplicity we assume no reneging, that is, once someone joins the voters' line, they stay and eventually cast their vote. Model this system as a Markov Birth and Death Queueing system. Find the steady state probabilities, i.e.,
$P_n \equiv \{n \text{ voters in the system, both in queue and in voting booths}\}, n = 0,1,2,...$

You can do this either by explicit formula or by applying the theory to generate numerical results for various plausible values of parameters.

(b) If you interviewed a large sample of voters leaving the polling place, what is the mean time in queue that they would report to you (assuming everyone is accurate and that you have a large sample size).

(c) Is the answer you found in part (b) indicative of the quality of service provided by this voting precinct? Why or why not?

(d) Some say that long lines producing discouraged voters is a form of 'stealth disenfranchisement.' That is, the long lines represent a subtle and difficult-to-track method of removing a certain fraction of voters from the voting process. For your model here, can you estimate the fraction of voters who, by balking at joining the line, may be said to have suffered from such stealth disenfranchisement? (There are lawsuits pending in Ohio that allege just this phenomenon, claiming that the stealth disenfranchisement in a recent federal election was deliberate manipulation of the system.)