Multidisciplinary System Design Optimization (MSDO)

Mixed-Integer Continuous Problems

Recitation 7

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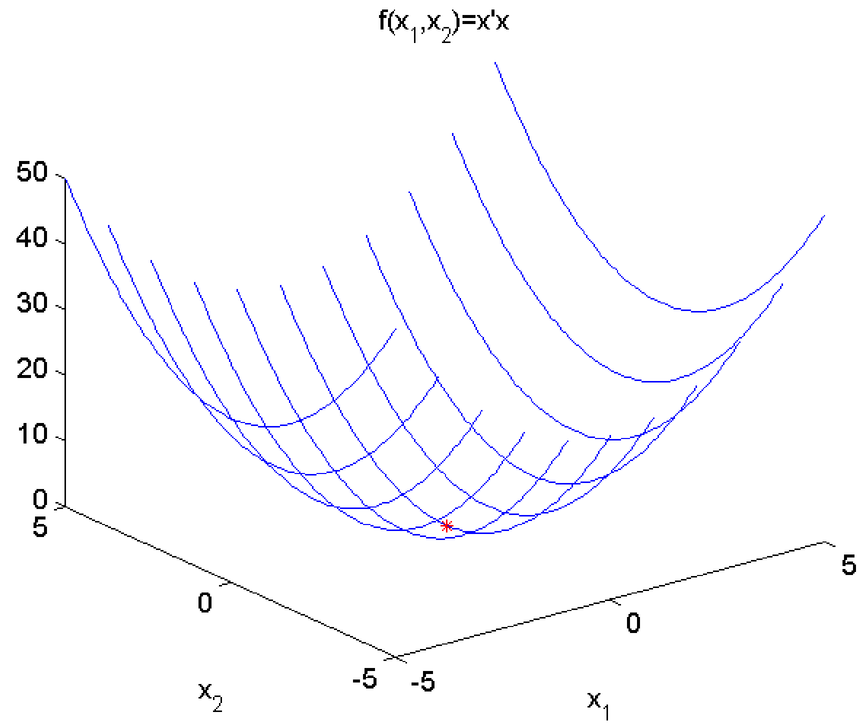
Today's Topics

- What a mixed-integer problem looks like
- I am skipping the theory…
- Options to solve them
  - Direct Search
  - Gradient-Based methods
    - Direct
    - Indirect
  - Heuristic Techniques
    - SA
    - GA
- PSet 4, A2 Discussion

What a MIO Looks Like

- At least one discrete variable
- At least one continuous variable
- Example:

$$\begin{aligned} &\min f(x) = x^T x \\ &s.t. \quad -5 \le x_1 \le 5, x_1 \in \Im \\ &\qquad\;\; -5 \le x_2 \le 5 \end{aligned}$$



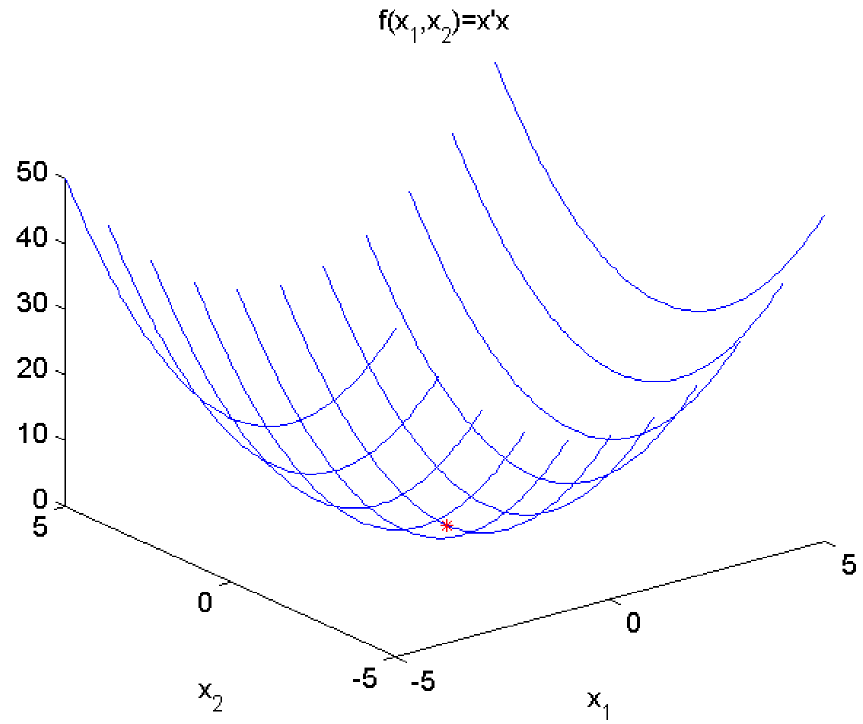
Problem:

- No gradient in discrete direction.
  - What are optimality conditions?

$$\min f(x) = x^T x$$

$$s.t. \quad -5 \le x_1 \le 5, x_1 \in \Im$$

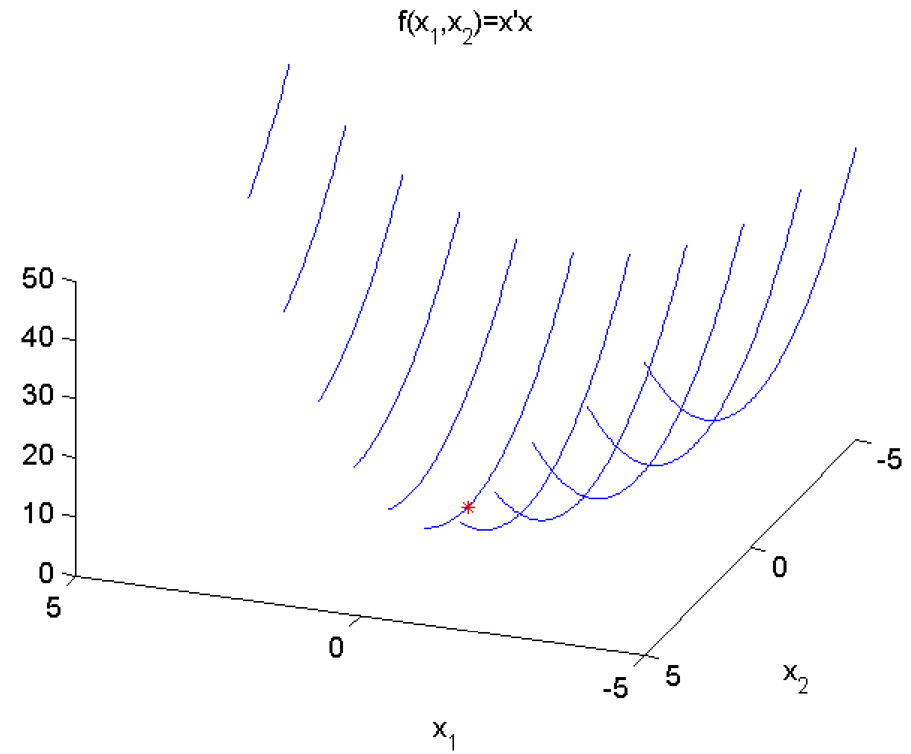
$$-5 \le x_2 \le 5$$



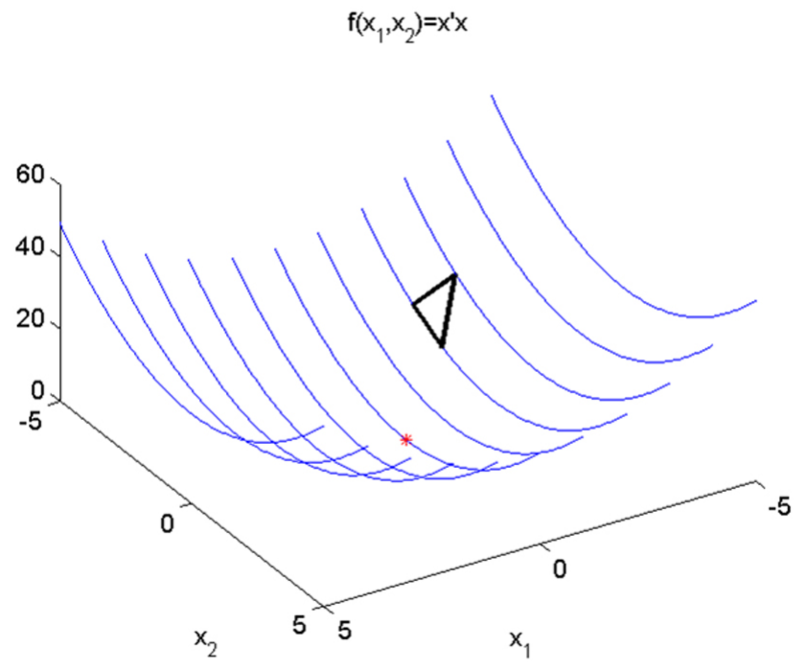
Constrained Version

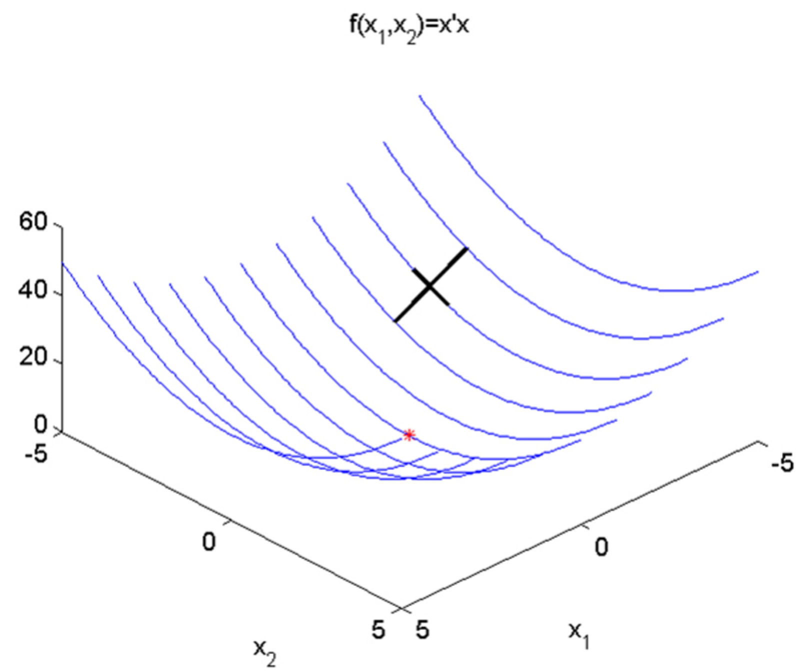
- No gradient in discrete direction.
  - What are optimality conditions?

$$\min f(x) = x^T x$$
$$s.t. \quad -5 \le x_1 \le 5, x_1 \in \Im$$
$$-5 \le x_2 \le 5$$
$$x_1 + x_2 \le 2$$



Solution Techniques: Direct Search

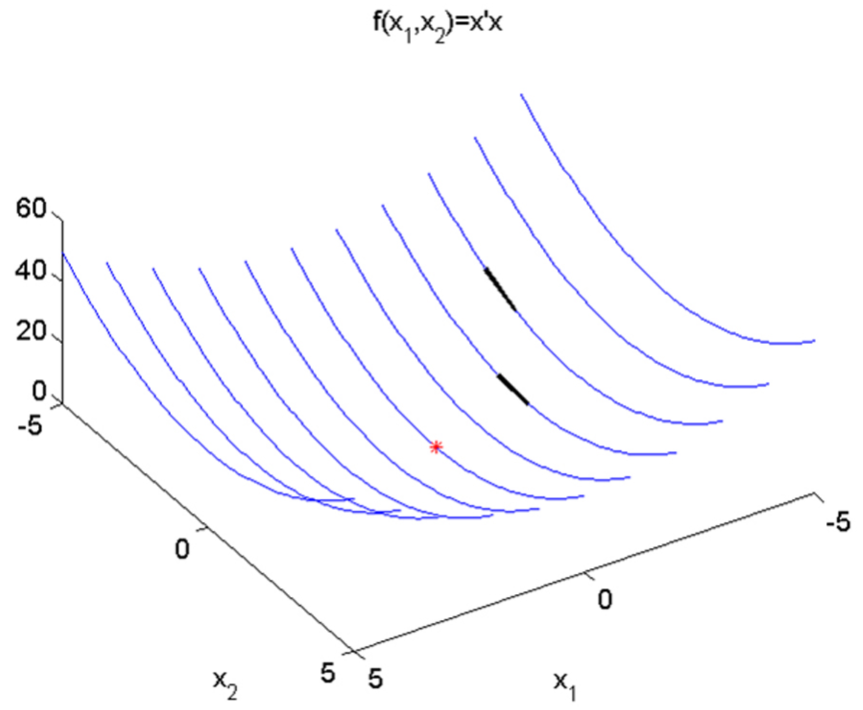




- Use a direct search method
  - Simplex method, where one direction always takes a unit step
  - Compass search (easy to set-up)

Gradient-Based Algorithm

- Fix discrete variables and optimize continuous
- Use a DoE technique for discrete variables
  - Full-factorial expansion
  - Latin-Hypercube
  - Random starting points
- Gradient-based optimization for continuous.



Gradient-Based: Indirect

1. Convert all discrete variables to continuous variables
2. Use typical gradient based algorithms on continuous variables
3. Round final continuous variables to nearest feasible discrete value

- Problems:
  - Not possible for all problems.
  - Finding nearest feasible discrete value may be difficult
  - Answer might be quite poor.

Gradient-Based: Response Surface

1. Generate a response surface:

x_{ij} i=dimension
j=sample point #

$$X = \begin{bmatrix} 1 & x_{11} & x_{21} & x_{11}x_{21} & x_{12}^2 & x_{21}^2 \\ 1 & x_{12} & x_{22} & x_{12}x_{22} & x_{12}^2 & x_{22}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{1n} & x_{2n} & x_{1n}x_{2n} & x_{1n}^2 & x_{2n}^2 \end{bmatrix}$$

$$\beta = [\beta_1 \quad \beta_2 \quad \beta_3 \quad \beta_4 \quad \beta_5 \quad \beta_6]^T$$

$$F = [f(x_{11}, x_{21}) \quad f(x_{12}, x_{22}) \quad \cdots \quad f(x_{1n}, x_{2n})]^T$$

Solve for β:

$$X\beta = F$$

Least-Squares Solution:

$$X^T X\beta = X^T F$$

2. Optimize the response surface
3. Round discrete variables, and check function value/convergence.
   a. Recalibrate response surface locally and repeat?

Heuristic Techniques Overview

- Generally easy to setup MIO problems.
- Brute force?
- No convergence guarantee.

Simulated Annealing

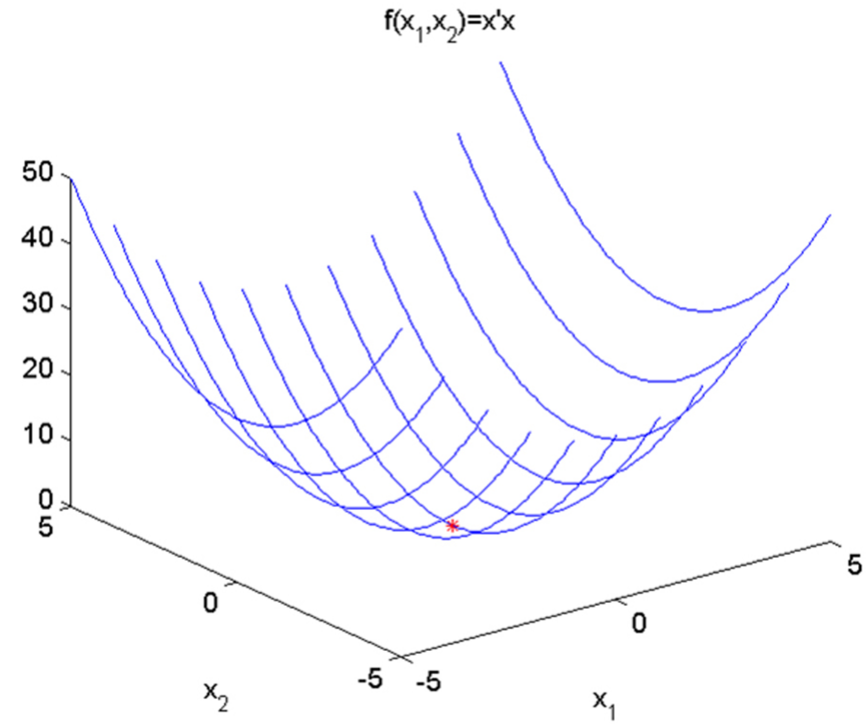
- Easy to accommodate integer variables:
- Example:

$$\min f(x) = x^T x$$

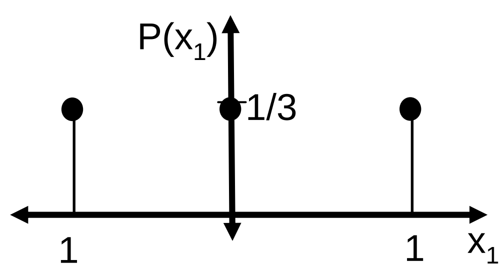
$$s.t. \quad -5 \le x_1 \le 5, x_1 \in \Im$$

$$-5 \le x_2 \le 5$$

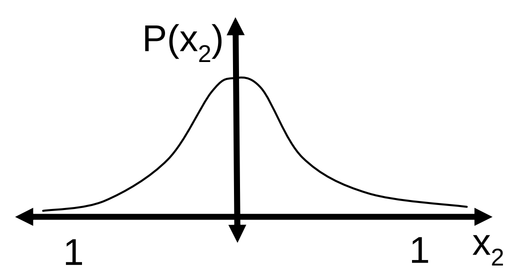
- Possible random perturbations:
  - – $x_1 = x_1 + C(-1,0,1)$
  - – $x_2 = x_2 + N(0,1)$



Categorical Distribution



Normal Distribution



Genetic Algorithm

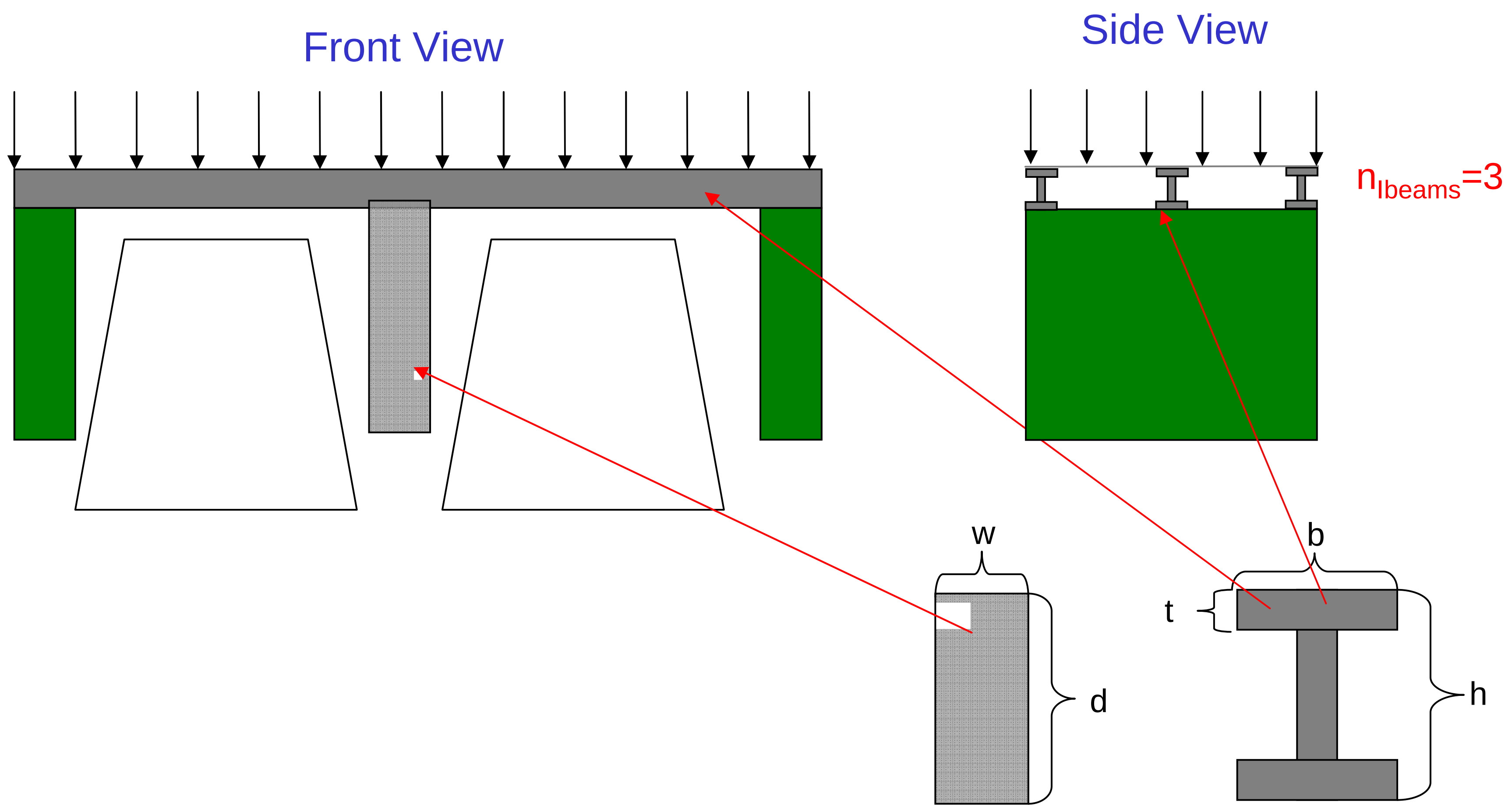
- Can you use a real-valued-GA?

- Can you use a discrete-GA, for instance binary encoded?
  - Could use ternary, etc.

Binary Encoding Example

- Binary encoded GA (Lecture 11)
- $$nbits = \frac{\ln\left(\frac{x_{UB} - x_{LB}}{\Delta x}\right)}{\ln 2}$$
- $$\Delta x = \frac{(x_{UB} - x_{LB})}{2^{nbits}}$$
- How many bits are needed?
  - Continuous variables on a computer?
  - Integer variable, $x \in \{1,2,3,4\}$?
  - Integer variable, $x \in \{1,2,3,4,5\}$?
  - Continuous variable, $x \in [0,1]$, $\Delta x=0.1$?
  - Continuous variable, $x \in [0,1]$, $\Delta x=0.01$?

Problem 2 on A4



3 Discrete Variables

nIbeams

I_{beam} Material

Support Material

5 Continuous Variables

t, b, h, w, d

Summary

- Mixed-discrete continuous problems can be complicated.
- Formal theory is complex, and gradient-based methods have difficulty.
  - Can be smart with DoE techniques
- Heuristic algorithms can be considered brute force, but typically can be made to optimized mixed-discrete continuous problems.
  - Just need to let them run forever.

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