Space Shuttle External Tank Optimization

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Abstract

A simplified model of the Space Shuttle External Tank was used to set up a Multidisciplinary System Design Optimization problem. First, Return of Investment (ROI) was established as single objective. Both gradient based and heuristic methods were used to solve this problem. Sequential Quadratic Programming (SQP) was the gradient based method chosen. In the other hand, a Genetic Algorithm was used as the heuristic optimization tool. In addition, sensitivity analysis was performed to the optimal solution found. Finally, a multi-objective problem was set up adding the total tank weight (TW) as second objective. Adaptive Weighted Sum (AWS) was the method selected to solve the problem.

Introduction

After the Cold war, and especially in times of economic crisis, manned space programs have been questioned per its high costs and the associated safety risks. Reusable launch vehicles (RLV's) have emerged as an alternative to reduce expenses by reusing equipment and commercializing space flights. Recent discussions in Obama's administration about the financials of future human space exploration have motivated us to explore the use of Multidisciplinary System Design Optimization (MSDO) as a tool to improve the business case of current space systems.

The most successful RLV has been, without question, NASA's Space Shuttle. At a high level, the elements of the Space Shuttle (at launch) are: the external tank, two solid rocket boosters and the orbiter vehicle. The external tank has several functions: to provide the fuel and the oxidizer for the main engines (liquid hydrogen and liquid oxygen) and to serve as structure to the system (the solid rocket boosters and the obiter vehicle are attached to the tank at launch). The tank is the only element that is not reused and is also the heaviest.

Framing the optimization problem

In this analysis we will use a simplified model of the external tank as described in Figure 1. This model assumes the tank is divided in three main sections: the hemisphere, the cylinder and the nose cone. Table 1 shows the six design variables considered in this problem and Table 2 shows the parameters assumed.

Figure 1. Graphic representation of the External Tank simplified model

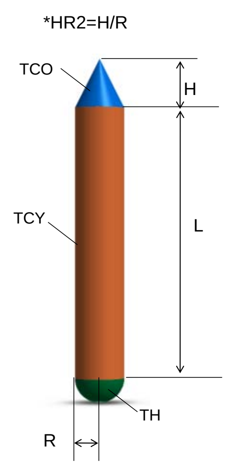


Table 1. Design Variables

Symbol	Variable	Variable name
x_1	HR2	Height /radius ratio
x_2	L	Length of cylindrical body
x_3	R	Radius of the hemisphere

x_4	TCO	Nose cone thickness
x_5	TCY	Cylinder thickness
x_6	TH	Hemisphere thickness

Table 2. Design Parameters considered

#	Parameter name	
1	Cost of material / unit	6.00 dollar/kg
2	Cost of seam / unit	12 dollar/m
3	Material weight / unit	0.003 kg/cm^3
4	Pressure	70 N/cm^2
5	Payload 1	14300 N
6	Payload 2	5000 N
7	Profit ratio	0.05
8	Nominal payload	30000 N
9	Fixed cost per weight unit	2x10^4 dollar/kg

In addition, this model assumes five constraints as described in Table 3.

Table 3. Model constraints

Symbol	Constraint description
g_1	Vibration factor ≥ Nominal vibration factor
g_2	Tank volume ≥ Nominal volume
g_3	Eq. stress in cylinder ≤ Allowed Stress
g_4	Eq. stress in hemisphere ≤ Allowed Stress
g_5	Eq. stress in nose cone ≤ Allowed Stress

The main objective we decided to pursue is to maximize the Return of Investment (ROI) of the system. We selected this objective because it is one of the most common metrics to measure profitability in projects. As a second objective we will minimize the weight of the tank.

This optimization problem requires a multidisciplinary approach as different aspects in design affect the main output function (ROI). Some of these disciplines are: material science, structures engineering, aerodynamics, finance, manufacturing engineering, mechanical vibrations and chemistry.

Based on the interactions between internal design variables a modularization was proposed for this optimization model. Figure 2 shows the N^2 diagram showing the modules identified: Surfaces and volumes, seam lengths, weight and material cost, seam cost, stress, total cost, payload and return of investment .

Figure 2. N^2 diagram with proposed modularization

Modules

Design Vector	Surfaces and Volumes	Seam lengths	Weight and material costo	Seam cost	Stress	Total Cost	Payload	Retur of Investment	Constraints
Design Vector	1, 2, 3	1, 2, 3	4, 5, 6		3, 4, 5, 6		1, 3		1, 2, 3, 4
	Surfaces and Volumes		6, 7, 11				13		
		Seam lengths		15, 16, 17, 18, 19					
			Weight and material costo			22			21
				Seam cost		23			
					Stress				24, 25, 26
						Total Cost		27	
							Payload	28	
								Retur of Investment	
									Constraints

Model implementation

As initial step, the modules in Figure 2 were implemented in an Excel spreadsheet in order to understand how the model behaves. By trying different combinations in the design vector, we were able to get our first feasible solution (a design vector that meets all the constraints). Expression (1) shows the initial design vector and (2) the output of the objective function (ROI) at x_0.

$$x_0 = [2 \quad 4800 \quad 435 \quad 0.75 \quad 0.7 \quad 0.86] \qquad (1)$$
$$ROI = 0.0605 \qquad (2)$$

Although the design above is meeting all the constraints, the ROI output is not acceptable. As next step we used Design of Experiments as a tool to explore design space.

Design of experiments

Our experiment plan started by assuming non-linearity in the factor effects. Therefore, we decided to implement a design of experiments with three levels per factor. By trial and error we defined rough feasibility ranges for each design variable. Using this information, the levels shown on Table 4 were proposed:

Table 4. DOE factors and levels

Factors	Level 1	Level 2	Level 3
HR2	1	2	3
L	4600	4800	5000
R	420	435	450

TCO	0.66	0.75	0.84
TCY	0.66	0.7	0.74
TH	0.76	0.86	0.96

For a full factorial experiment, with 6 factors and 3 levels, we would have $3^6 = 729$ experiments. In order to keep the number of experiments in a manageable quantity, we decided to use orthogonal arrays. Per the number of factors in our model and the required resolution, we decided to implement a L_{18} orthogonal array. Table 5 shows our experiment plan for DoE.

Table 5. Experiment plan L_{18} orthogonal array

Exp	HR2	L	R	TCO	TCY	TH
1	1	4600	420	0.66	0.66	0.76
2	1	4800	435	0.75	0.7	0.86
3	1	5000	450	0.84	0.74	0.96
4	2	4600	420	0.75	0.7	0.86
5	2	4800	435	0.84	0.74	0.76
6	2	5000	450	0.66	0.66	0.86
7	3	4600	435	0.66	0.74	0.86
8	3	4800	450	0.75	0.66	0.96
9	3	5000	420	0.84	0.7	0.76
10	1	4600	450	0.84	0.7	0.86
11	1	4800	420	0.84	0.74	0.96
12	1	5000	435	0.75	0.66	0.76
13	2	4600	435	0.84	0.66	0.96
14	2	4800	450	0.66	0.7	0.76
15	2	5000	420	0.75	0.74	0.86
16	3	4600	450	0.75	0.74	0.76
17	3	4800	420	0.84	0.66	0.86
18	3	5000	435	0.66	0.7	0.96

After calculating the ROI for each experiment, we evaluated the main effects for each factor level. Table 6 summarizes these results. Levels with the best effect are highlighted in gray. As we want to maximize ROI we selected the levels with the maximum effect.

Table 6. DoE output: Main effects

Variable	Level	Factor	Mean	Main Effect
HR2	1	1	-0.2455	0.2555
	2	2	-0.4377	0.0633
	3	3	-0.8197	-0.3187
L	1	4600	-0.3959	0.1050
	2	4800	-0.6122	-0.1112
	3	5000	-0.4948	0.0062
R	1	420	-0.6945	-0.1936
	2	435	-0.1060	0.3950
	3	450	-0.7024	-0.2014
TCO	1	0.66	-0.7426	-0.2416
	2	0.75	-0.4401	0.0609
	3	0.84	-0.3806	0.1204
TCY	1	0.66	-0.5596	-0.0586
	2	0.7	-0.6043	-0.1034
	3	0.74	-0.3390	0.1620
TH	1	0.76	-0.5986	-0.0976
	2	0.86	-0.3494	0.1515
	3	0.96	-0.5959	-0.0950

If we set all design variables to the levels with the best main effects we obtain a starting point for further optimization analysis:

$$x_0 = [1 \quad 4600 \quad 435 \quad 0.84 \quad 0.74 \quad 0.86] \quad (3)$$

$$ROI = 0.0903 \quad (4)$$

Gradient-based optimization

Algorithm selection

To carry on the shuttle external's tank optimization we decided to implement the Sequential Quadratic Programming (SQP) method. Nowadays, this gradient-based algorithm is considered one of the most efficient approaches to obtain the optimal solution in Non Linear Programming (NLP). Similarly to Newton's unconstrained optimization method, SQP creates in each step towards the objective a local model of the problem and solves it. Then, based in that result, continues its path to an optimal solution.

The main difference of SQP relative to other methods is that the former tries to solve the nonlinear program directly instead of transforming it in a sequence of unconstrained minimization problems. Furthermore, as explained in class, SQP is widely applied in engineering problems and it can be easily handled in MATLAB using the *fmincon* function in the optimization toolbox.

Return of Investment optimization

We selected ROI as the single objective for which to optimize our system. We selected this objective per its relevance in real life design projects. A company will not invest in executing a given design if it will not yield any benefit.

Our approach to implement SQP algorithm was to use MATLAB optimization toolbox (*fmincon* function). This algorithm minimizes a given objective function within the constraints determined by the user. To fit our model to this algorithm we redefined our objective function:

$$J(x) = -ROI \tag{5}$$

In addition, constraints in Table 3 were redefined as inequality constraints ≤ 0. As initial point, we used the vector obtained from DoE exploration.

$$x_0 = \begin{bmatrix} HR2 \\ L \\ R \\ TCO \\ TCY \\ TH \end{bmatrix} = \begin{bmatrix} 1 \\ 4600 \\ 435 \\ 0.84 \\ 0.74 \\ 0.86 \end{bmatrix} \tag{6}$$

The model found a minimum after 10 iterations and all constraints were successfully satisfied. The optimal design vector found is shown below:

$$x^* = \begin{bmatrix} 4.559 \\ 4200.74 \\ 426.032 \\ 0.646 \\ 0.646 \\ 0.7416 \end{bmatrix} \tag{7}$$

ROI improved:

$$ROI = -J(x^*) = 0.22657 = 22.66\% \tag{8}$$

This was a significant improvement compared to our first "guess". All constraints are met and we have a positive ROI that would make our project viable from the investment standpoint.

As the nominal volume required was reduced, the optimizer was able to reduce the magnitude of all design variables and improved the ROI.

Scaling

The gradient-based algorithm used in A3 to optimize ROI was Sequential Quadratic Programming (SQP). The optimal solution found in A3 with this method was:

$$x^* = \begin{bmatrix} HR2 \\ L \\ R \\ TCO \\ TCY \\ TH \end{bmatrix} = \begin{bmatrix} 4.559 \\ 4200.74 \\ 426.032 \\ 0.646 \\ 0.646 \\ 0.7416 \end{bmatrix} \tag{9}$$

Using finite differencing, the Hessian matrix at x^* was calculated:

$$H(x^*) = \begin{bmatrix} 0.0016 & -0.0000 & 0.0002 & 0.0546 & -0.0001 & -0.0000 \\ -0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0003 & -0.0000 \\ 0.0002 & 0.0000 & 0.0000 & 0.0012 & 0.0026 & 0.0005 \\ 0.0546 & -0.0000 & 0.0012 & 0.0001 & -0.0001 & -0.0000 \\ -0.0001 & 0.0003 & 0.0026 & -0.0001 & -0.0003 & -0.0000 \\ -0.0000 & -0.0000 & 0.0005 & -0.0000 & -0.0000 & 0.0000 \end{bmatrix} \tag{10}$$

The diagonal of this hessian matrix is:

0.0016 -0.0000 0.0000 0.0001 -0.0003 0.0000 (11)

Scaling factors

The values of the diagonal of the Hessian calculated in (1) lie outside the limits defined (within 10^2 and 10^{-2}). Therefore, scaling is required for all variables.
The proposed scaling method is:

$$y_i = D_i x_i, \text{ where } D_i = (s)^{\frac{1}{2}} \tag{12}$$

Where s is the power of ten (10^x) closest to the elements of the diagonal of the Hessian computed on (9). The proposed scaling factors (D) are shown in Table 7.

Table 7. Proposed scaling factors.

Design variable	Scaling factor	Scaled design variable
Height-radius ratio HR2 (x_1)	10^{-1}	$y_1=(10^{-1})x_1$; $x_1=10y_1$
Length L (x_2)	10^{-5}	$y_2=(10^{-5})x_2$; $x_2=10^5y_2$
Radius R (x_3)	10^{-3}	$y_3=(10^{-3})x_3$; $x_3=10^3y_3$
Cone thickness TCO (x_4)	10^{-2}	$y_4=(10^{-2})x_4$; $x_4=100y_4$
Cylinder thickness TCY (x_5)	10^{-2}	$y_5=(10^{-2})x_5$; $x_5=100y_5$
Hemisphere thickness TH (x_6)	10^{-2}	$y_6=(10^{-2})x_6$; $x_6=100y_6$

The Hessian matrix was recalculated after scaling the design variables, the new values of the diagonal are shown below:

0.1560 -0.5405 8.2226 0.7626 -3.2605 0.3673 (13)

Using the scaled design variables and a scaled initial vector, SQP algorithm was re-run. The *fmincon* function in MATLAB was used to run SQP. The optimal design vector changed and the objective function output was improved:

$$x^* = \begin{bmatrix} 2.3697 \\ 4933.417 \\ 410.378 \\ 0.6227 \\ 0.6219 \\ 0.7182 \end{bmatrix} \qquad (14)$$

A new value for the objective function was found:

$$ROI = -J(x^*) = 0.2833 = 28.3\% \qquad (15)$$

The ROI value is better than the 0.22657 we obtained after the first intent with the SQP algorithm. The Hessian matrix was re-calculated at the new optimal design vector. The new values for the diagonal were:

4.0033 -0.4911 7.1591 0.5199 -4.2683 0.3691 (16)

The values in (16) lie within the limits we initially assumed. Therefore, there is no need for further scaling.

Heuristic Optimization

As an initial attempt, we tried to implement a Simulated Annealing algorithm to this optimization problem. The approach was to implement all the constraints in the perturbation function of the algorithm. We encountered several complications in this attempt, but the most significant was the complexity of the perturbation function. The intent of this function was to generate a neighboring design vector within the constraints of the problem. Unfortunately, the time for computing this vector was very unpredictable and caused the algorithm to stall.

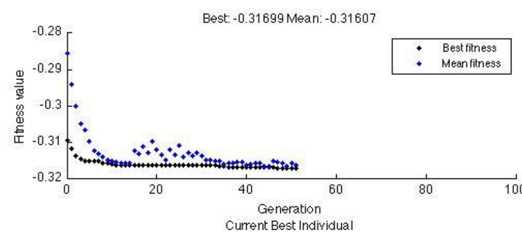
Therefore, we decided to switch to a different heuristic method. We selected the GA algorithm. Our decision was based on the qualities and characteristics of this specific robust technique that are suitable for searches in high-dimensional problems and complex design spaces, as this problem presents.

To implement the Genetic Algorithm, our first approach was to use the MATLAB optimization toolbox (*ga* function). Typically, GA algorithms do not allow implementing constraints directly. So, we decided to use the constraints as penalties to the fit function. The structure of the fit function used in this GA is described below:

$$f = -J_1(x_1, x_2, \dots, x_6) + g_1 + 2.85g_2 + g_3 + g_4 + g_5 \qquad (17)$$

Also, lower and upper boundaries for each design variable were determined based on what we learned from the system in previous analysis. Figure 3 shows the evolution of the fitness value.

Figure 3. Fitness value vs. generation



The algorithm converged after 50 iterations. The optimal design vector found was:

$$x^* = \begin{bmatrix} HR2 \\ L \\ R \\ TCO \\ TCY \\ TH \end{bmatrix} = \begin{bmatrix} 2.4 \\ 4932.893 \\ 409.997 \\ 0.62 \\ 0.619 \\ 0.763 \end{bmatrix} \qquad (18)$$

$$ROI = -J(x^*) = 0.2836 = 28.4\% \qquad (19)$$

This result is very similar to the one obtained with the SQP after scalling.

Sensitivity analysis

To do a sensitivity analysis of our output function, we calculated the normalized gradient for the output function at the optimal solution. As first step we calculated the numerical gradient of $J(x^*)$ using the finite difference method. In this analysis, we considered (14) as the optimal design vector. The gradient at the optimal is:

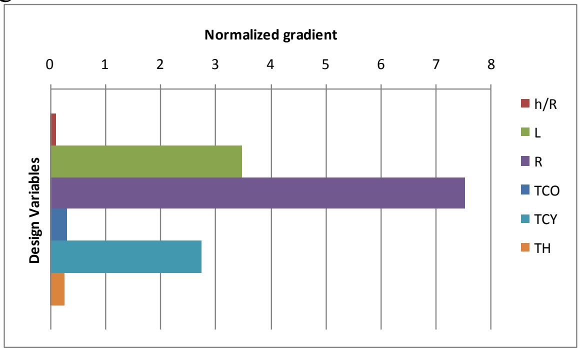
$$\nabla J(x^*) = \begin{bmatrix} 0.0123 \\ 0.0002 \\ 0.0052 \\ 0.1334 \\ 1.2469 \\ 0.1037 \end{bmatrix} \quad (20)$$

Then we calculated a normalized gradient vector:

$$\nabla \bar{J} = \frac{x^*}{J(x^*)} \nabla J = \begin{bmatrix} 0.102885 \\ 3.482822 \\ 7.532529 \\ 0.293216 \\ 2.737194 \\ 0.262892 \end{bmatrix} \quad (21)$$

Figure 4 shows a graphic comparison between the normalized gradients.

Figure 4. Tornado chart with normalized gradient



The variable that seems to have the highest impact on the output is the Radius (R) followed by the Length (L) and the thickness in the cylinder (TCY). These results somehow match intuition as we found the design variable R affect most of the calculations in the model.

Relative to the sensitivity of the optimal vector x^* with respect to the fixed parameters, we explored fixed cost to launch (FL) and cost of seam per length unit (C). In order to calculate the sensitivity, we considered the following equation:

$$\frac{d}{dp}\left(\frac{dJ}{dx}\right) = \frac{\partial\left(\frac{dJ}{dx}\right)}{\partial p} + \frac{\partial\left(\frac{dJ}{dx}\right)}{\partial x}\frac{dx}{dp} = 0 \quad (22)$$

The expression above can be rewritten as:

$$\frac{dx}{dp} = -\frac{\partial\left(\frac{dJ}{dx}\right)}{\partial p}\left[\frac{\partial\left(\frac{dJ}{dx}\right)}{\partial x}\right]^{-1} \quad (23)$$

Sensitivity of x^* to changes in cost seam per length unit (C):

$$\frac{d(x)}{d(C)} = \begin{bmatrix} \frac{dHR2}{dC} \\ \frac{dL}{dC} \\ \frac{dR}{dC} \\ \frac{dTCO}{dC} \\ \frac{dTCY}{dC} \\ \frac{dTH}{dC} \end{bmatrix} = \begin{bmatrix} -0.02576 \\ \sim 0 \\ \sim 0 \\ -0.00413 \\ \sim 0 \\ \sim 0 \end{bmatrix} \quad (24)$$

Sensitivity of x^* to changes in fixed cost to launch (FL):

$$\frac{d(x)}{d(FL)} = \begin{bmatrix} \frac{dHR2}{dFL} \\ \frac{dL}{dFL} \\ \frac{dR}{dFL} \\ \frac{dTCO}{dFL} \\ \frac{dTCY}{dFL} \\ \frac{dTH}{dFL} \end{bmatrix} = \begin{bmatrix} -1.993 * 10^{-5} \\ \sim 0 \\ 3.164 * 10^{-6} \\ -5.155 * 10^{-6} \\ -2.206 * 10^{-8} \\ -9.482 * 10^{-8} \end{bmatrix} \quad (25)$$

From the results shown above, FL does not have a significant impact on the location of the optimal vector x^*. In the other hand, the cost of the seam per unit of length does have small impact on the optimal vector especially on the height to radius ratio (HR2).

To identify the active constraints we evaluated the optimal vector x^* in each of the five inequality constraints in this model. We found that all of them are approximately zero which means all constraints are active. Table 8 shows the summary:

Table 8. Active constraints at x^*.

Constraint	Form	Value at x*	Active?
Vibration constraint	1-VF/VFallowed ≤0	~ 0	Yes
Volume constraint	1-Vtank/Vnominal ≤0	~ 0	Yes
Eq. Cylinder stress constraint	Scyl/Sallowed-1≤0	~ 0	Yes
Eq. Hemisphere stress constraint	Shem/Sallowed-1≤0	~ 0	Yes
Eq. Cone stress constraint	Scon/Sallowed-1≤0	~ 0	Yes

Although the five constraints show values very close to zero, we estimate the volume constraint is the most important one as it shows the smallest value. To evaluate the change in the objective function output and in the optimal design vector, we modified the nominal volume value (relaxed nominal value 5% to reach 2780080380). The new optimal design vector after relaxing the constraint g_2:

$$x^* = \begin{bmatrix} 2.249 \\ 4933.18 \\ 401.073 \\ 0.609 \\ 0.608 \\ 0.702 \end{bmatrix} \quad (26)$$

ROI if relaxing Volume nominal value by 5%:

$$ROI = -J(x^*) = 0.356 \quad (27)$$

Multi-objective Optimization

In the first part of this paper, we optimized the system using the single objective function Return of Investment (ROI). For a multi-objective optimization, we decided to use the tank total weight as the second objective. In this case, our intent is to minimize the weight function. We selected the Adaptive Weighted Sum method to solve this optimization problem. To implement this algorithm the following expression was used:

$$J_{MO} = \sum_{i=1}^{z} \frac{\lambda_i}{sf_i} J_i \quad (28)$$

Specifically:

$$J_1 = -ROI \quad (29)$$
$$J_2 = TW \quad (30)$$

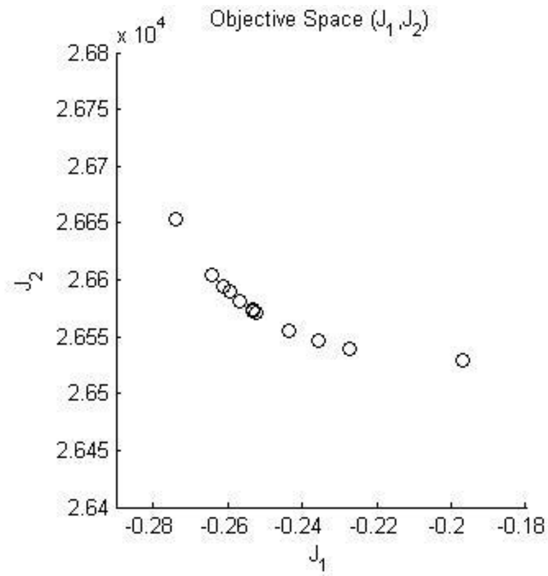
Scale factors to normalize the objective functions:

$$sf_1 = J_1^* = 0.2739 \quad (31)$$
$$sf_2 = J_2^* = 2.6364 * 10^4 \quad (32)$$

To find the pareto front, SQP was used iteratively to find the optimal values for J_1 and J_2 at different values of λ. Figure 5 shows the pareto front found with this algorithm. All constraints are satisfied at all points.

The pareto front plot shows that the objectives are mutually opposing. As we maximize ROI, we increase the Total Weight of the tank. In the other hand, if we minimize the weight we reduce the ROI. We considered this result counter-intuitive as we expected mutually supporting objectives (that minimizing weight would maximize ROI).

Figure 5. Pareto front. ROI (J_1) vs. total tank weight TW (J_2).



To further investigate this result we traced back the ROI and TM at the extremes in the Pareto frontier. It was found that in order to increase ROI we should increase the amount of payload that the customer pays. To do this we should reduce the tank cross section considered in the aerodynamic penalty. Per the stress constraints, the material thickness increases, as well as the cone height. This combination drives the weight up.

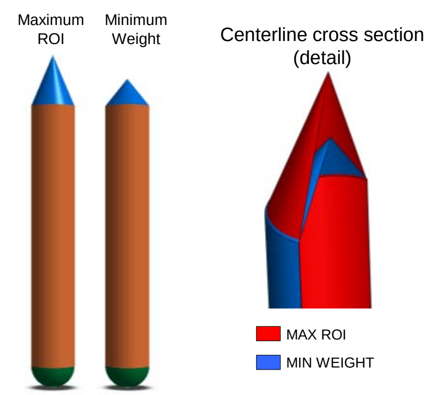
In the opposite scenario, minimizing weight reduces the size of the cone and also the material thickness. Therefore, the tank cross section considered for aerodynamic drag increases. This translates in less amount of payload that is paid by the customer. To confirm our hypothesis we created a CAD model for the two designs at the extremes:

$$x^*_{(\text{MAX ROI})} = \begin{bmatrix} 2.3709 \\ 4933 \\ 410.3862 \\ 0.6227 \\ 06220 \\ 0.7182 \end{bmatrix} ; \quad x^*_{(\text{MIN TW})} = \begin{bmatrix} 1.299 \\ 4933 \\ 415.9526 \\ 0.6386 \\ 0.6304 \\ 0.7279 \end{bmatrix}$$

(33)

Figure 6 shows a CAD model of the extreme designs described in (22). These CAD models are congruent with our hypothesis.

Figure 6. Comparison between designs at the extremes of the Pareto frontier.



Conclusions

After several iterations using different methods we are confident about our exploration of design space. Within the given constraints we believe we found the global optimum design. For our specific problem, the gradient based method used (SQP) demonstrated to be very effective and quick. But, it was trapped in local optimal values when we implemented SQP for the first time. This issue was eliminated after scaling.

The heuristic model was useful to expand the design space exploration, but the result was very similar to the one obtained with the gradient based method.

In real-world problems, where multimodal functions exist, a Hybrid Optimization strategy is highly recommended. For example, heuristic optimization methods such as Genetic Algorithms (GA) or Simulated Annealing might be used to manage the initial steps when seeking for solutions to widely explore the design space. Then, the utilization of a method such as SQP, results extremely useful and efficient to explore thoroughly around the solutions found with the heuristic tool.

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Appendix

A1- Modularized master table

		Description	Symbol	Unit of measurement	Inputs	Outputs
Design Vector	1	Height / Radius ratio	HR2	cm		11, 12, 17, 28
	2	Length of center cylindrical body	L	cm		9, 10, 15
	3	Radius of hemisphere	R	cm		7,8,9,10,11,12,16, 17, 18, 19, 24, 25, 26, 28, 32
	4	Nose cone thickness	T_{cone}	cm		21, 22, 26
	5	Cylinder thickness	$T_{cylinder}$	cm		21, 22, 24
	6	Hemisphere thickness	$T_{hemisphere}$	cm		21, 22, 25
Surfaces and Volumes	7	Hemisphere Surface	HS	cm^2	3	13, 21, 22
	8	Hemisphere Volume	HV	cm^3	3	14
	9	Cylinder Surface	CS	cm^2	2,3	13, 21, 22
	10	Cylinder Volume	CV	cm^3	2,3	14
	11	Cone Surface	CnS	cm^2	1,3	13, 21, 22
	12	Cone Volume	CnV	cm^3	1,3	14
	13	Tank surface	TS	cm^2	7, 9, 11	28
	14	Tank volume	TV	cm^3	8, 10, 12	36
Seams length	15	Seam length in Cylinder	S1	cm	2	20, 23
	16	Seam length in Hemisphere	S2	cm	3	20, 23
	17	Seam length in Cone	S3	cm	1, 3	20, 23
	18	Seam length cylinder & hemisph	S4	cm	3	20, 23
	19	Seam length cylinder & cone	S5	cm	3	20, 23
	20	Total Seam length	St	cm	15-19	
Weight and material cost	21	Tank weight	TW	kg	9,5,7,6,11,4	30, 32
	22	Tank material cost	$C_{material}$	dollar	9,5,7,6,11,4	27
Seam Cost	23	Cost of seams	C_{seam}	dollar	15-19	27
Stress	24	Cylinder Eq. stress	E	N/cm sq	3, 5	34
	25	Sphere Eq. stress	SE	N/cm sq	3, 6	35
	26	Cone Eq. stress	CE	N/cm sq	3, 4	33
Total cost	27	Total Cost	TC	dollar	22, 23	27
Payload	28	Aerodynamic drag penalty	A	kg	1,3,13	30
Return On Investment	29	True Launch cost	TLC	dollar	27	31
	30	Customer pays	CP	dollar	21, 28	31
	31	ROI	ROI	dollar	29, 30	
Constraint	32	Vibration Constraint	g1		1,2,3,4,21	
	33	Stress constraint cone	g5	N/cm^2	26	
	34	Stress constraint cylinder	g3	N/cm^2	24	
	35	Stress constraint hemisphere	g4	N/cm^2	25	
	36	Volume constraint	g2	cm^3	14	