Goal Programming and Isoperformance

Lecture 13

Olivier de Weck

de Weck, O.L. and Jones M. B., "Isoperformance: Analysis and Design of Complex Systems with Desired Outcomes", *Systems Engineering*, 9 (1), 45-61, January 2006

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Why not performance-optimal ?

"The experience of the 1960's has shown that for military aircraft the cost of the final increment of performance usually is excessive in terms of other characteristics and that the overall system must be optimized, not just performance"

Ref: Current State of the Art of Multidisciplinary Design Optimization (MDO TC) - AIAA White Paper, Jan 15, 1991

→ TRW Experience

Industry designs not for optimal performance, but <u>according to targets</u> specified by a requirements document or contract - thus, optimize design for a set of GOALS.

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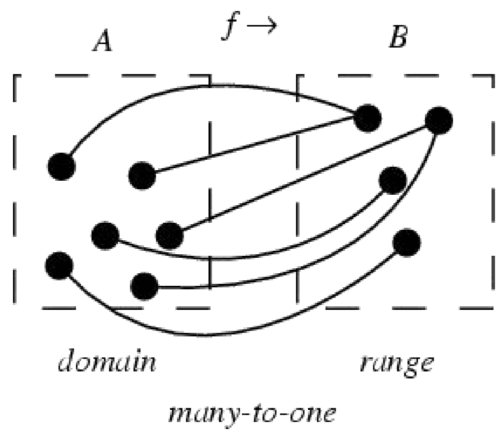
Lecture Outline

- Motivation - why goal programming ?
- Example: Goal Seeking in Excel
- Case 1: Target vector **T** in Range
  = Isoperformance
- Case 2: Target vector **T** out of Range
  = Goal Programming
- Application to Spacecraft Design
- Stochastic Example: Baseball

Target Vector

T

x **J**



Forward Perspective

Choose **x** ⟶ What is **J** ?

Reverse Perspective

Choose **J** ⟶ What **x** satisfy this?

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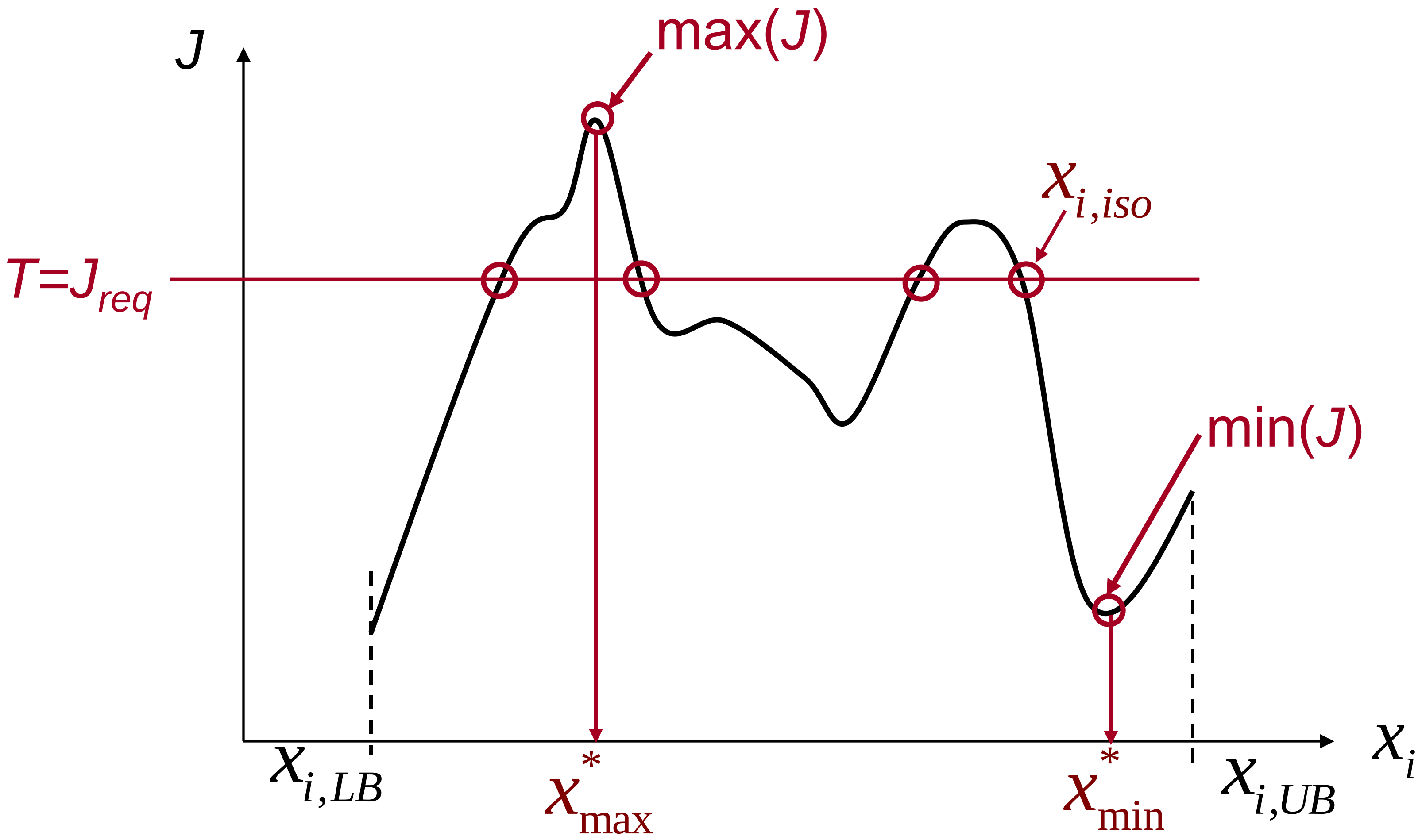
Goal Seeking



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Excel: Tools – Goal Seek

"About Goal Seek" description from Microsoft Excel.
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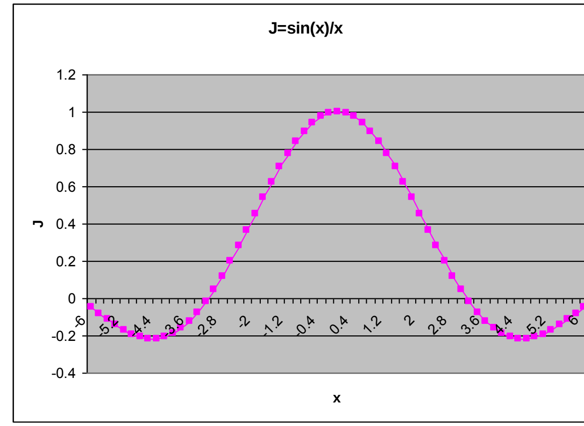
Excel - example



sin(x)/x - example

- single variable x
- no solution if *T* is out of range

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Goal Seeking and Equality Constraints

- Goal Seeking – is essentially the same as finding the set of points **x** that will satisfy the following "soft" equality constraint on the objective:

$$\text{Find all } \mathbf{x} \text{ such that } \left| \frac{J(\mathbf{x}) - J_{req}}{J_{req}} \right| \leq \varepsilon$$

Example Target Vector:

$$J_{req}(x) = \begin{bmatrix} m_{sat} \\ R_{data} \\ C_{sc} \end{bmatrix} \equiv \begin{bmatrix} 1000kg \\ 1.5Mbps \\ 15M\$ \end{bmatrix} \begin{matrix} \leftarrow \text{Target mass} \\ \leftarrow \text{Target data rate} \\ \leftarrow \text{Target Cost} \end{matrix}$$

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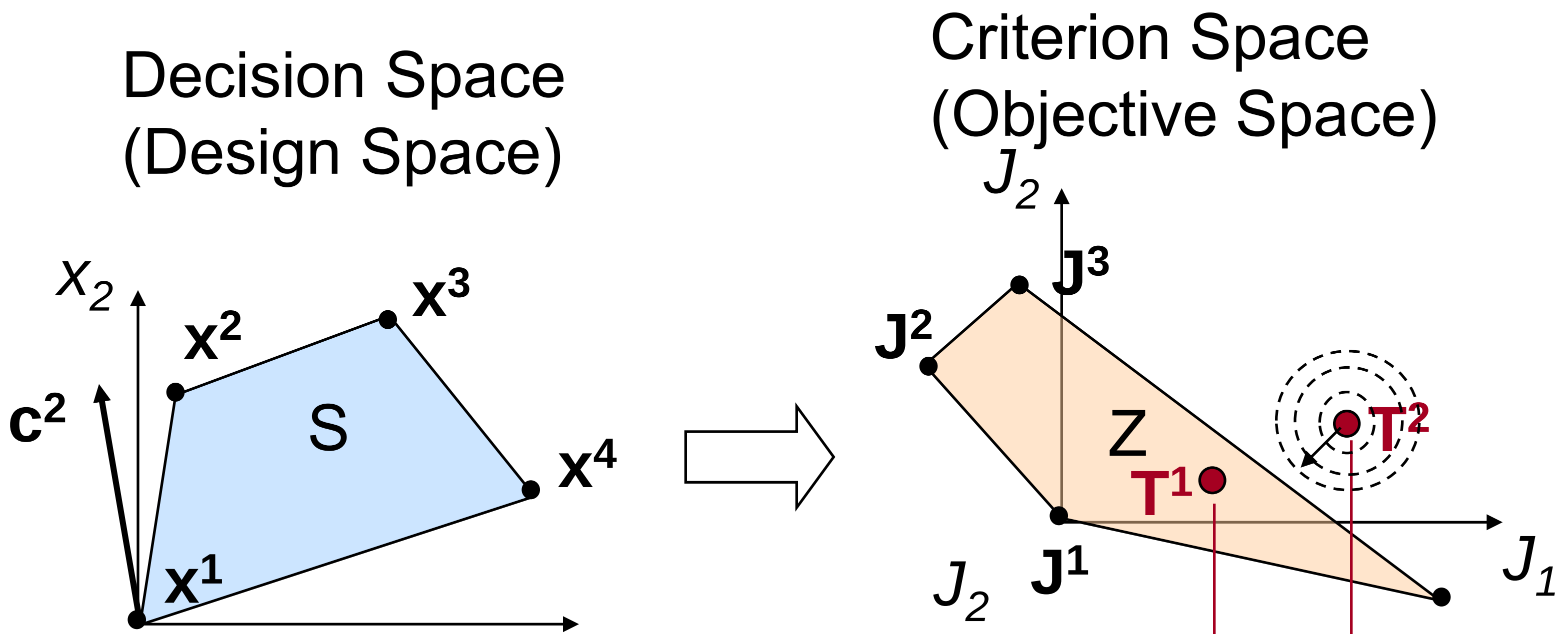
Goal Programming vs. Isoperformance



Case 1: The target (goal) vector is in Z - usually get non-unique solutions = Isoperformance

Case 2: The target (goal) vector is not in Z - don't get a solution - find closest = Goal Programming

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Isoperformance Analogy

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Non-Uniqueness of Design if n > z

Performance: Buckling Load

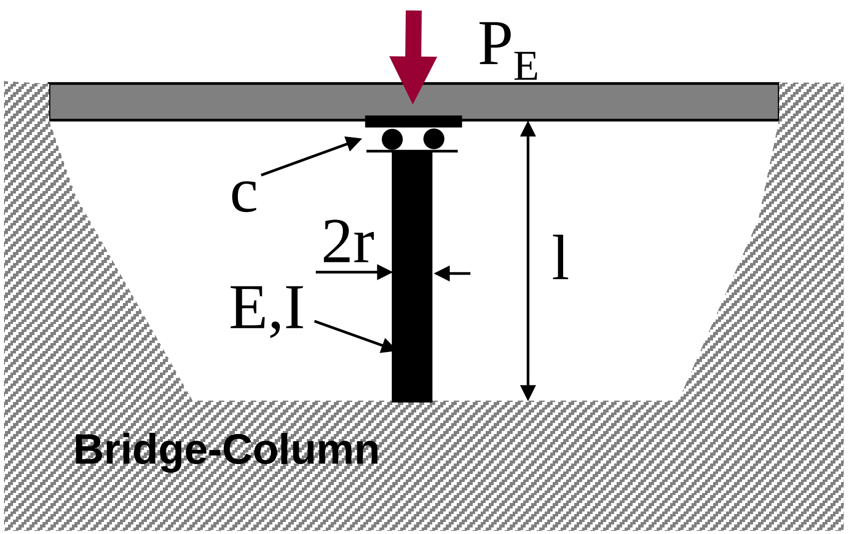
Constants: l=15 [m], c=2.05

$$P_E = \frac{c\pi^2 EI}{l^2}$$

Variable Parameters: *E, I(r)*

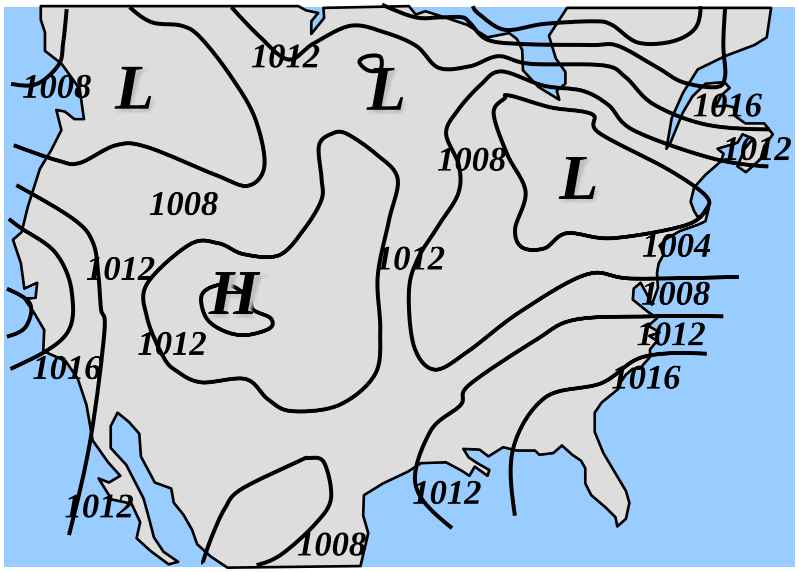
Requirement: $P_{E,REQ} = 1000 \text{ metric tons}$

Solution 1: V2A steel, r=10 cm , E=19.1e+10
Solution 2: Al(99.9%), r=12.8 cm, E=7.1e+10



Analogy: Sea Level Pressure [mbar]
Chart: 1600 Z, Tue 9 May 2000

Isobars = Contours of Equal Pressure
Parameters = Longitude and Latitude



Isoperformance Contours = Locus of constant system performance
Parameters = e.g. Wheel Imbalance Us, Support Beam I_{xx}, Control Bandwidth ω_c

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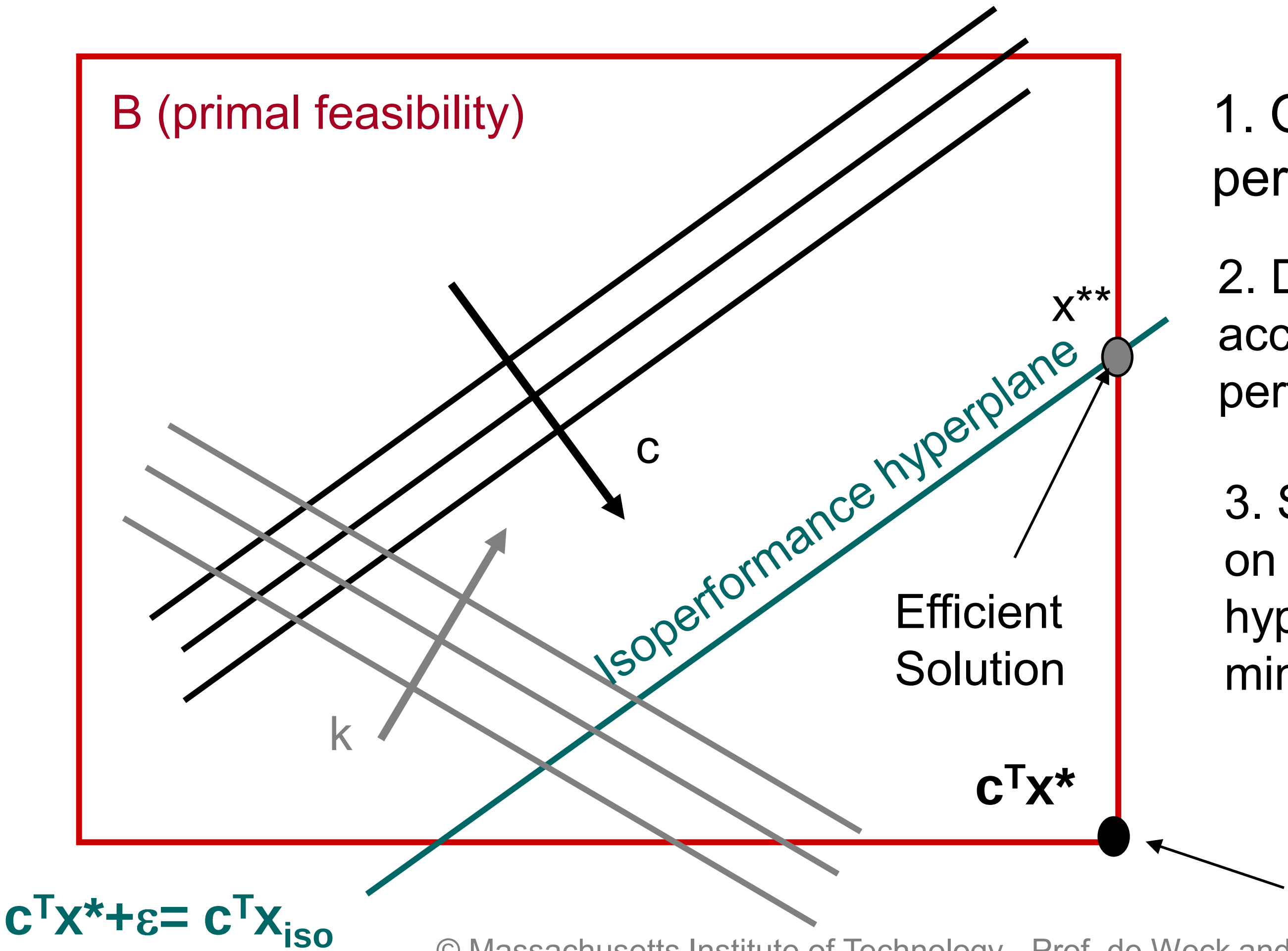
Isoperformance and LP

- In LP the isoperformance surfaces are hyperplanes
- Let $c^T x$ be performance objective and $k^T x$ a cost objective

$$\min \quad c^T x$$
$$s.t. \quad x_{LB} \le x \le x_{UB}$$



1. Optimize for performance $c^T x^*$
2. Decide on acceptable performance penalty ε
3. Search for solution on isoperformance hyperplane that minimizes cost $k^T x^*$

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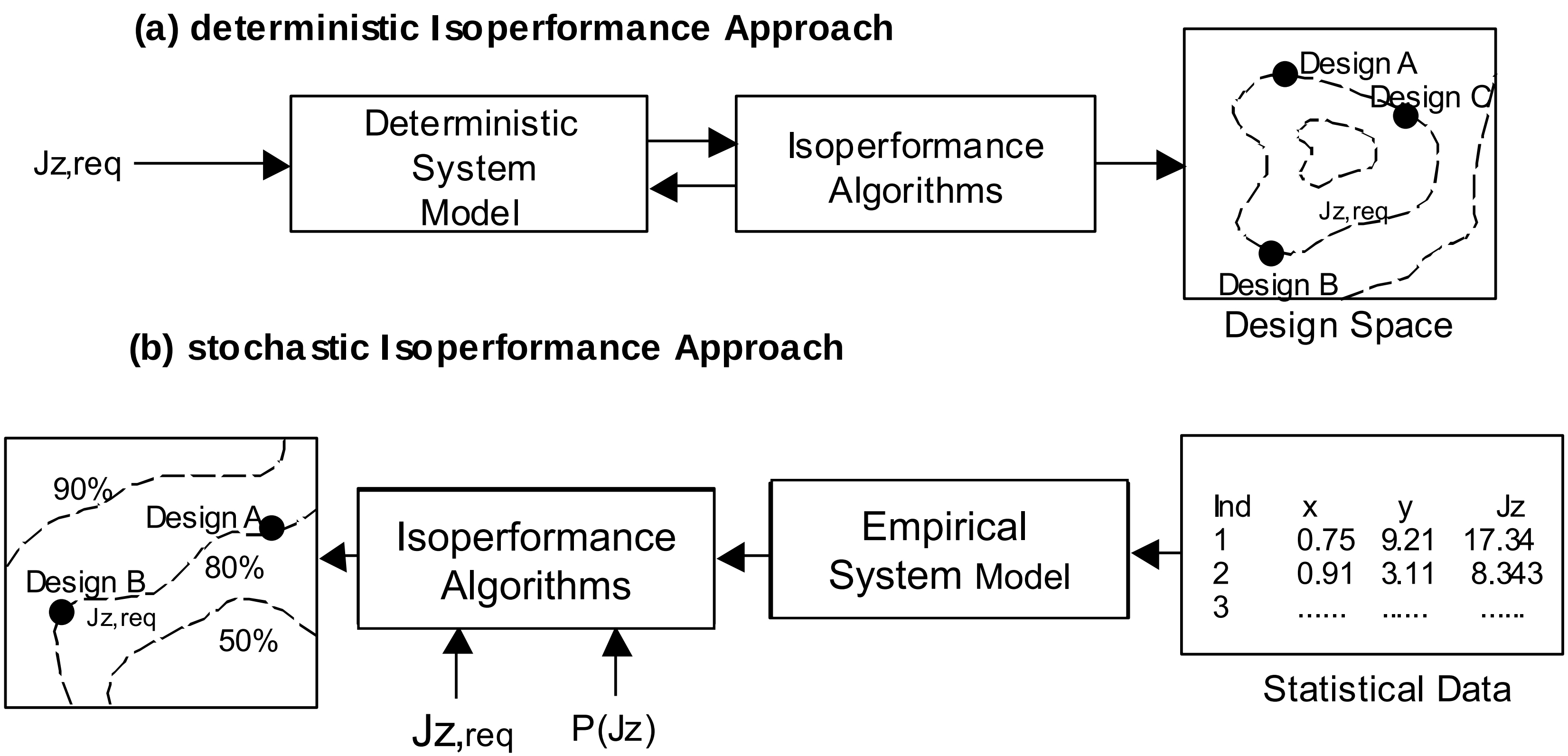
Isoperformance Approaches



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Nonlinear Problem Setting

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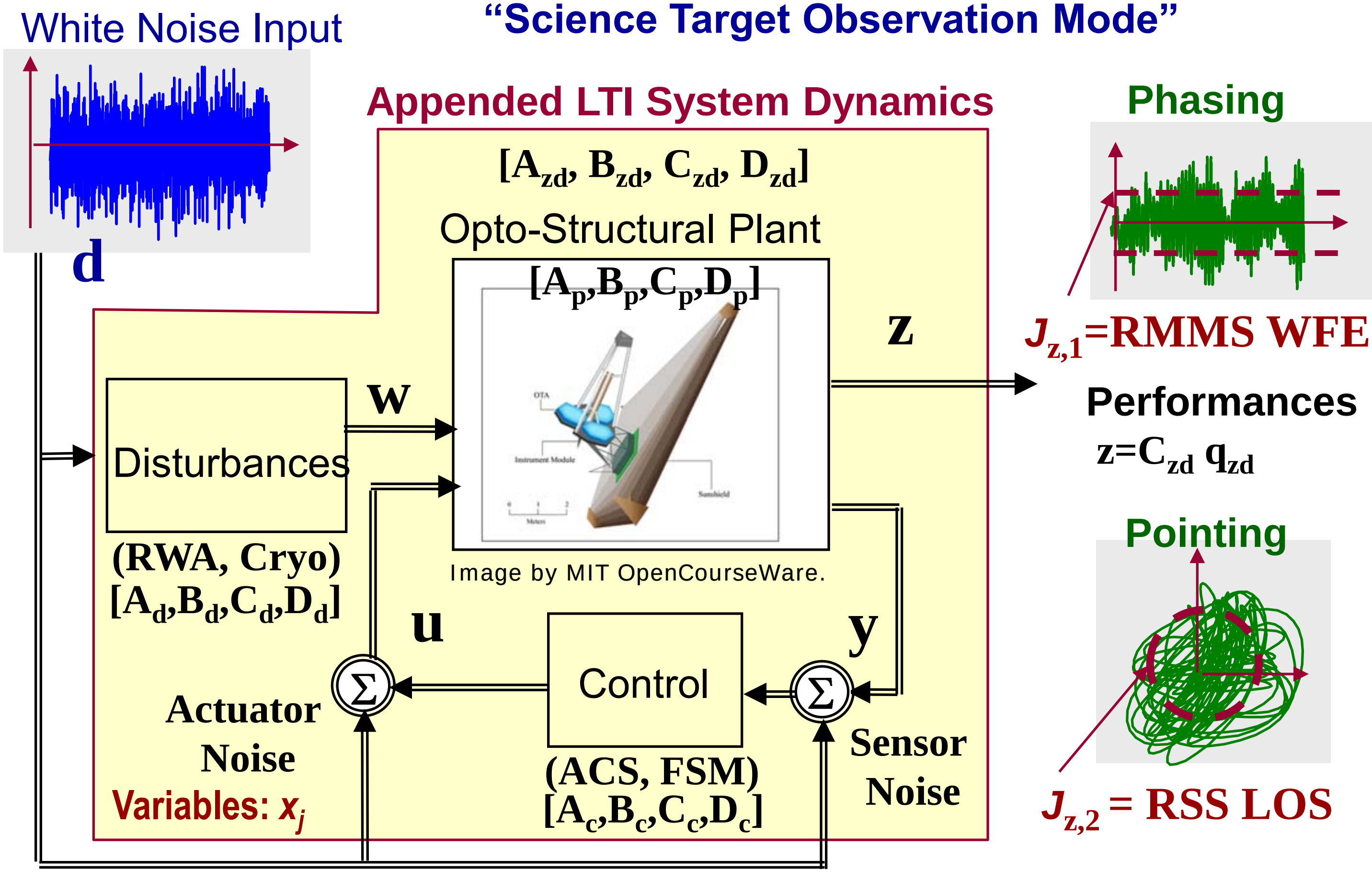
"Science Target Observation Mode"



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Problem Statement

Given

$$\dot{q} = A_{zd}\;\; x_j\;\; q + B_{zd}\;\; x_j\;\; d + B_{zr}\;\; x_j\;\; r$$ **LTI System Dynamics**

$$z = C_{zd}\;\; x_j\;\; q + D_{zd}\;\; x_j\;\; d + D_{zr}\;\; x_j\;\; r\;\;, \text{where } j = 1,2,\ldots,n_p$$

And Performance Objectives

$$J_z = F\;\; z\;\;,\; \text{e.g. } J_{z,i} = \|z\|_2 = \mathrm{E}\left[z^T z\right]^{1/2} = \left(\frac{1}{T}\int_0^T z(t)^2\,dt\right)^{1/2} \quad \text{RMS}$$

Find Solutions x_{iso} such that

$$J_{z,i}\;\; x_{iso}\;\; \equiv J_{z,req,i} \quad \forall\;\; i = 1,2,\ldots,n_z$$

Assuming $n - z \geq 1$ **and** $x_{j,LB} \leq x_j \leq x_{j,UB} \quad \forall\;\; j = 1,2,\ldots,n$

Subject to a numerical tolerance $\tau :\;\; \left|\dfrac{J_z\;\; x\;\; - J_{z,req}}{J_{z,req}}\right| \leq \dfrac{\tau}{100} = \varepsilon$

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Bivariate Exhaustive Search (2D)

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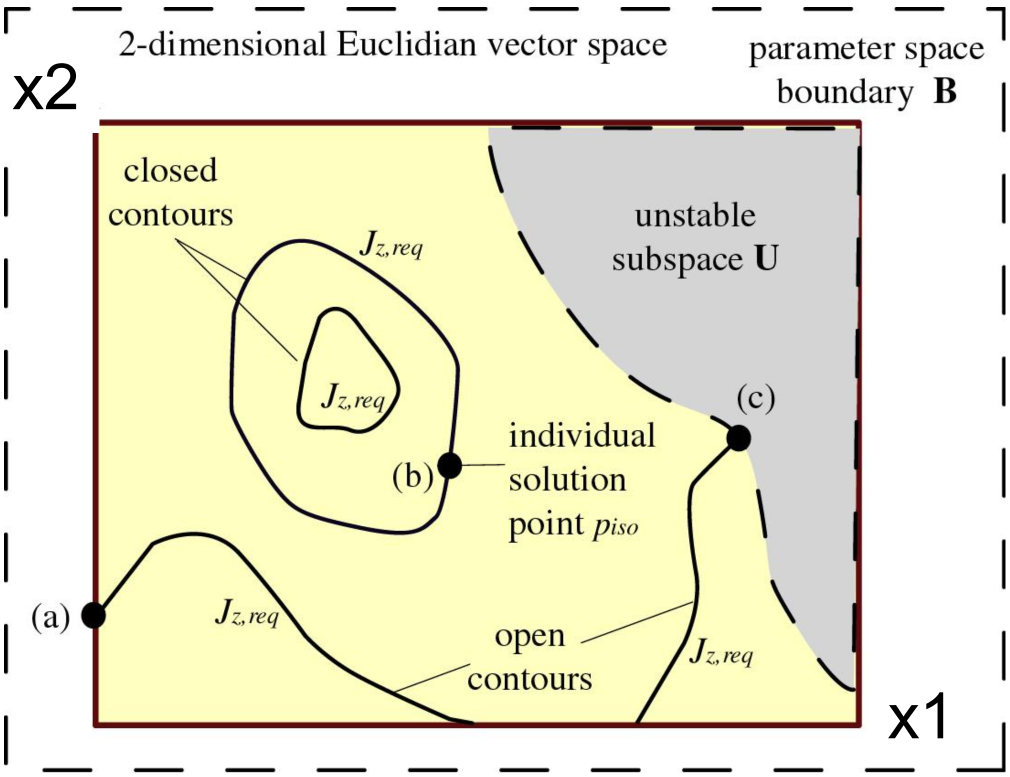
"Simple" Start: Bivariate Isoperformance Problem

$$\text{Performance}\ \ J_z(x_1, x_2):\ z = 1$$

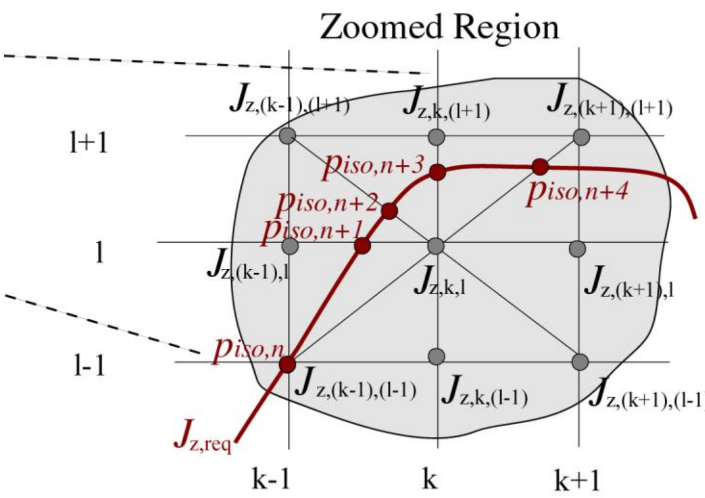
$$\text{Variables}\ \ x_j,\ j = 1,2:\quad n = 2$$



First Algorithm: **Exhaustive Search** coupled with bilinear interpolation

Number of points along j-th axis:

$$n_j = \left\lceil \frac{x_{j,UB} - x_{j,LB}}{\Delta x} \right\rceil$$



Can also use standard contouring code like MATLAB® `contourc.m`

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Contour Following (2D)

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k-th isoperformance point:

Taylor series expansion

$$J_z(x) = J_z(x^k) + \underbrace{\nabla J_z^T\Big|_{x^k} \Delta x}_{\text{first order term}} + \underbrace{\frac{1}{2}\Delta x^T H\Big|_{x^k} \Delta x}_{\text{second order term}} + \text{H.O.T.}$$

$$\nabla J_z = \begin{bmatrix} \dfrac{\partial J_z}{\partial x_1} \\ \dfrac{\partial J_z}{\partial x_2} \end{bmatrix}$$

$$\nabla J_z^T\Big|_{p^k} \Delta x \equiv 0$$

$$\alpha_k = \left[2\frac{\tau J_{z,req}}{100} \left(t_k^T H\Big|_{x^k} t_k \right)^{-1} \right]^{1/2}$$

H: Hessian

$$t^k = \Re \cdot \underbrace{\frac{-\nabla J_z}{\left\|\nabla J_z\right\|}\Bigg|_{p^k}}_{n^k} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot n^k$$

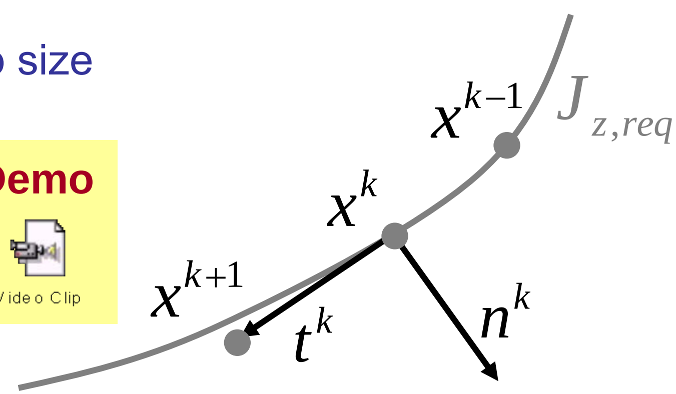
t^k: tangential step direction

α^k : Step size

$$\Delta x = \alpha^k \cdot t^k$$

Demo

Video Clip



k+1-th isoperformance point: $x^{k+1} = x^k + \Delta x$

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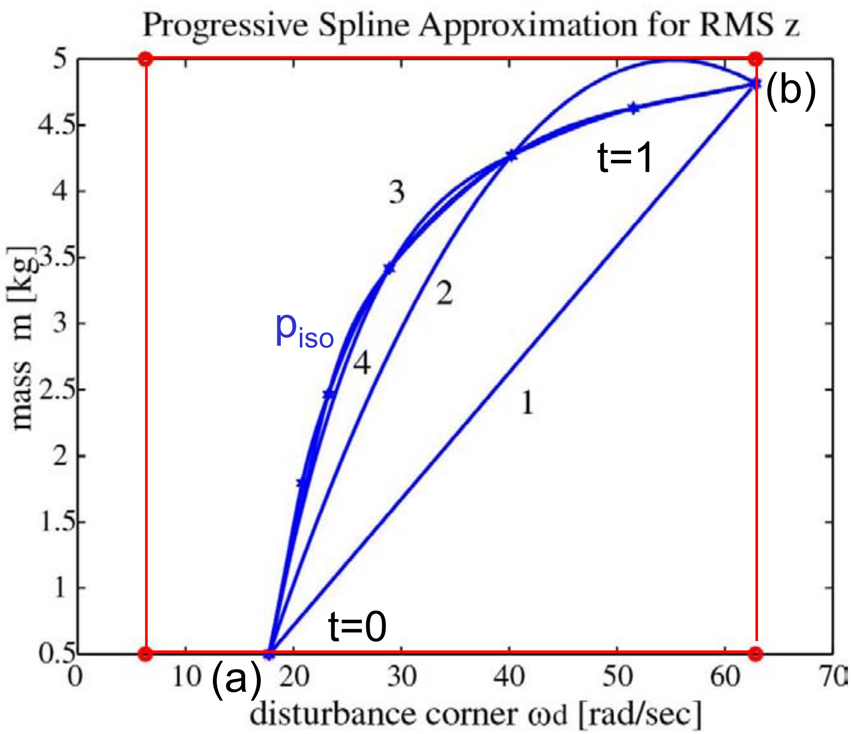
Progressive Spline Approximation (III)



- First find iso-points on boundary
- Then progressive spline approximation via segment-wise bisection
- Makes use of MATLAB spline toolbox , e.g. function **`csape.m`**

$$t \mapsto P_l(t) = \begin{bmatrix} x_{iso,1}(t) \\ x_{iso,2}(t) \end{bmatrix} = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$$

$$t \in [0,1] \mapsto P_l(t) \in [a,b]$$

Use cubic splines: k=4

$$f_{j,l}(t) = \sum_{i=1}^{k} \frac{\left(t-\zeta_l\right)^{k-i}}{(k-i)!} c_{j,l,k}, \quad t \in [\zeta_l \ldots \zeta_{l+1}]$$

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Bivariate Algorithm Comparison

Metric	Exhaustive Search (I)	Contour Follow (II)	Spline Approx (III)
FLOPS	2,140,897	783,761	377,196
CPU time [sec]	1.15	0.55	0.33
Tolerance τ	1.0%	1.0%	1.0%
Actual Error γ_{iso}	0.057%	0.347%	0.087%
# of isopoints	35	45	7

Results for SDOF Problem

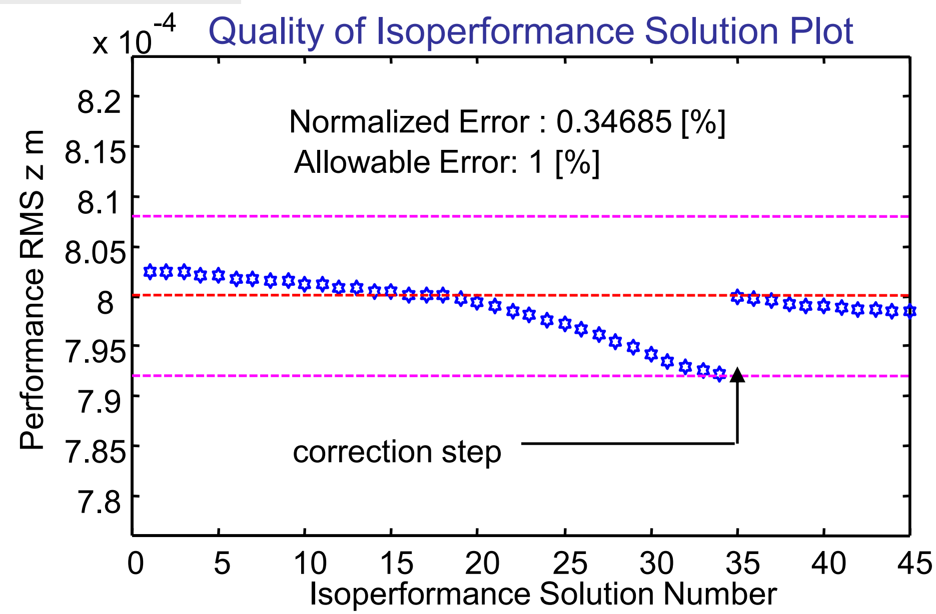
Conclusions:

(I) most general but expensive
(II) robust, but requires guesses
(III) most efficient, but requires monotonic performance J_z

Isoperformance Quality Metric

"Normalized Error"

$$\Upsilon_{iso} = \frac{100}{J_{z,req}} \left[\frac{\sum_{r=1}^{n_{iso}} \left(J_z\left(x_{iso,k}\right) - J_{z,req} \right)^2}{n_{iso}} \right]^{1/2}$$



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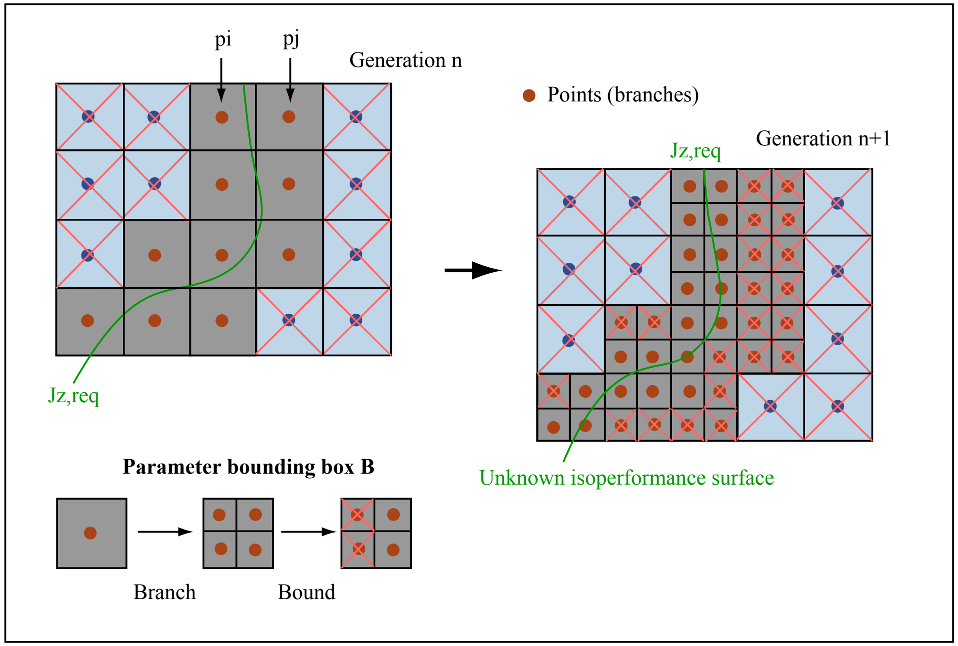
Multivariable Branch-and-Bound



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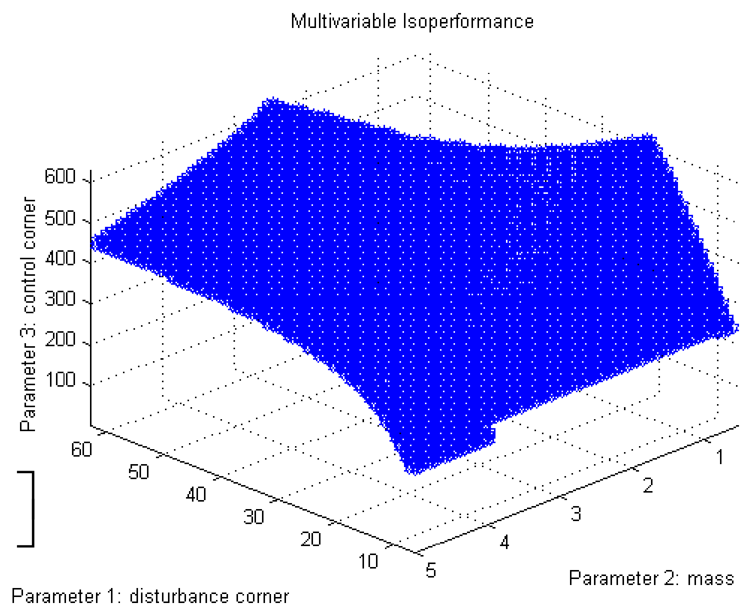
Exhaustive Search requires n_p-nested loops -> NP-cost: e.g.

$$N = \prod_{j=1}^{n_p} \left\lceil \frac{x_{UB,j} - x_{LB,j}}{\Delta x_j} \right\rceil$$



Branch-and-Bound only retains points/branches which meet the condition:

$$\left[J_z \;\; x_i \;\; \geq J_{z,req} \geq J_z \;\; x_j \;\; \right] \cup \left[J_z \;\; x_i \;\; \leq J_{z,req} \leq J_z \;\; x_j \;\; \right]$$

Expensive for small tolerance τ

Need initial branches to be fine enough

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Tangential Front Following

$$U = \underbrace{\begin{bmatrix} u_1 & \cdots & u_{n_z} \end{bmatrix}}_{zxz}$$

$$\Sigma = \underbrace{\begin{bmatrix} \text{diag}\ \sigma_1 & \cdots & \sigma_{n_z} & 0_{n_z x (n_p - n_z)} \end{bmatrix}}_{zxn}$$

$$V = \begin{bmatrix} \underbrace{v_1 \ \cdots \ v_z}_{\text{column space}} & \underbrace{v_{z+1} \ \cdots \ v_n}_{\text{nullspace}} \end{bmatrix}$$

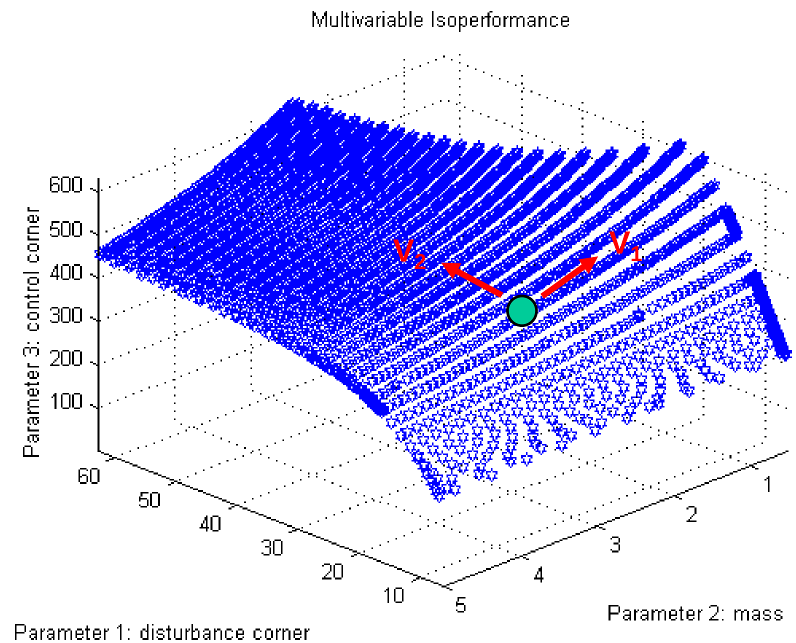
$$U \Sigma V^T = \nabla J_z^T$$

$$\nabla J_z = \begin{bmatrix} \frac{\partial J_1}{\partial x_1} & & \frac{\partial J_z}{\partial x_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial J_1}{\partial x_n} & & \frac{\partial J_z}{\partial x} \end{bmatrix}$$

$$\Delta x = \alpha \cdot \left(\beta_1 v_{z+1} + \ldots + \beta_{n-z} v_n \right) = \alpha V_t \beta$$

SVD of Jacobian provides V-matrix
V-matrix contains the orthonormal vectors of the nullspace.

Isoperformance set I is obtained by following the nullspace of the Jacobian !



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Vector Spline Approximation

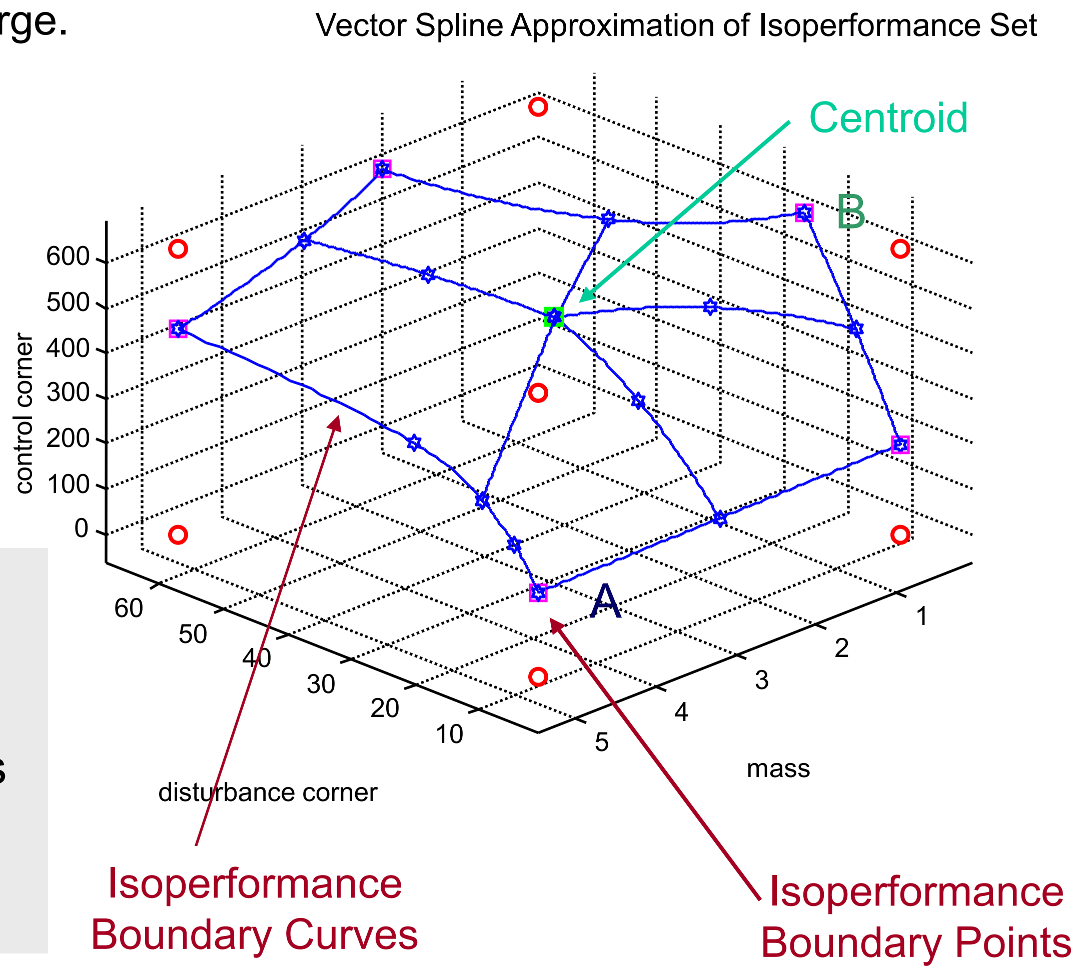
Tangential front following is more efficient than branch-and-bound but can still be expensive for n_p large.

Idea: Find a representative subset off all isoperformance points, which are different from other.

"Frame-but-not-panels" analogy in construction

<u>Algorithm</u>:

1. Find Boundary (Edge) Points
2. Approximate Boundary curves
3. Find Centroid point
4. Approximate Internal curves



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Multivariable Algorithm Comparison

Challenges if $n_p > 2$

- Computational complexity as a function of [n_z n_d n_p n_s]
- Visualization of isoperformance set in n_p-space

Problem Size:

z = # of performances

d = # of disturbances

n = # of variables

n_s = # of states

Table: Multivariable Algorithm Comparison for SDOF (n_p=3)

Metric	Exhaustive Search	Branch-and-Bound	Tang Front Following	V- Spline Approx
MFLOPS	6,163.72	891.35	106.04	1.49
CPU [sec]	5078.19	498.56	69.59	4.45
Error Y_{iso}	0.87 %	2.43%	0.22%	0.42%
# of points	2073	7421	4999	20

From Complexity Theory: Asymptotic Cost [FLOPS]

Exhaustive Search: $\log J_{exs} \rightarrow n_p \log\alpha + 3\log n_s + c$

Branch-and-Bound: $\log J_{bab} \rightarrow n_g \; n_p \log 2 + \log\beta \; + 3\log n_s + c$

Tang Front Follow: $\log J_{tff} \rightarrow n_p - n_z \; \log\gamma + \log \; 1 + n_z \; + 3\log n_s + c$

V-Spline Approx: $\log J_{vsa} \rightarrow n_p \log 2 + 3\log n_s + \log(n_z + 1) + c$

Conclusion: Isoperformance problem is non-polynomial in n_p

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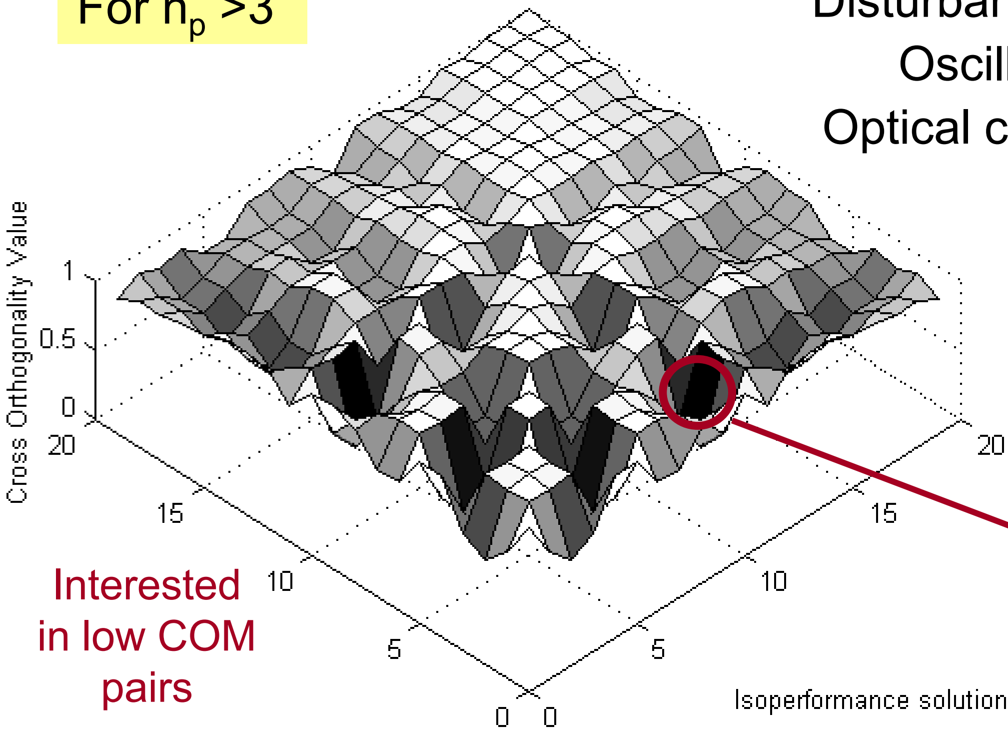
Graphics: Radar Plots

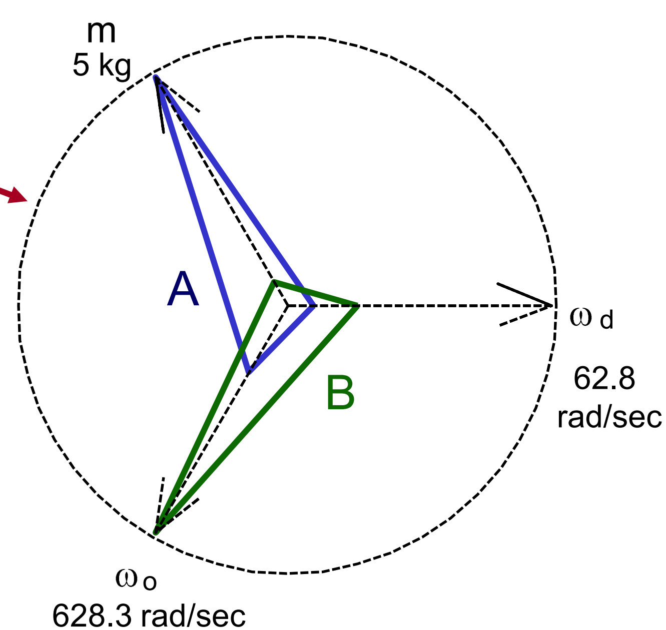
For $n_p > 3$



Interested in low COM pairs

	A	B
Disturbance corner ω_d	6.2832	21.3705
Oscillator mass m	5.0000	0.5000
Optical control bw ω_o	186.5751	628.3185

Multi-Dimensional Comparison of Isoperformance Points



Cross Orthogonality Matrix

$$COM(i, j) = \frac{p_{iso,i} \cdot p_{iso,j}}{\left|p_{iso,i}\right| \cdot \left|p_{iso,j}\right|}$$

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Nexus Case Study

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Purpose of this case study:

Demonstrate the usefulness of Isoperformance on a realistic conceptual design model of a high-performance spacecraft

The following results are shown:

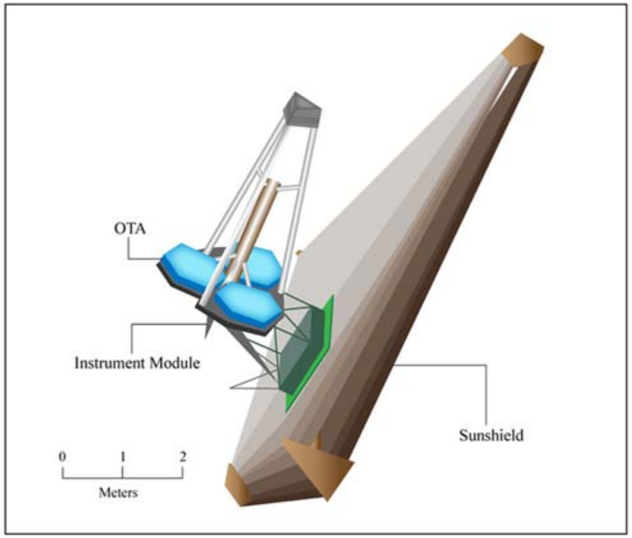
- Integrated Modeling
- Nexus Block Diagram
- Baseline Performance Assessment
- Sensitivity Analysis
- Isoperformance Analysis (2)
- Multiobjective Optimization
- Error Budgeting

Details are contained in CH7

Nexus Spacecraft Concept

on-orbit configuration

Pro/E models © NASA GSFC



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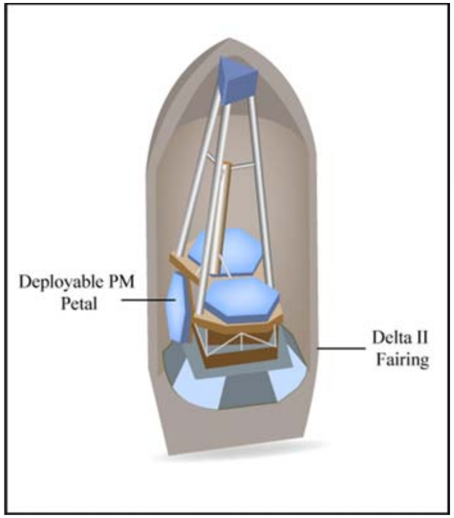
launch configuration



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NGST Precursor Mission
2.8 m diameter aperture
Mass: 752.5 kg
Cost: 105.88 M$ (FY00)
Target Orbit: L2 Sun/Earth
Projected Launch: 2004

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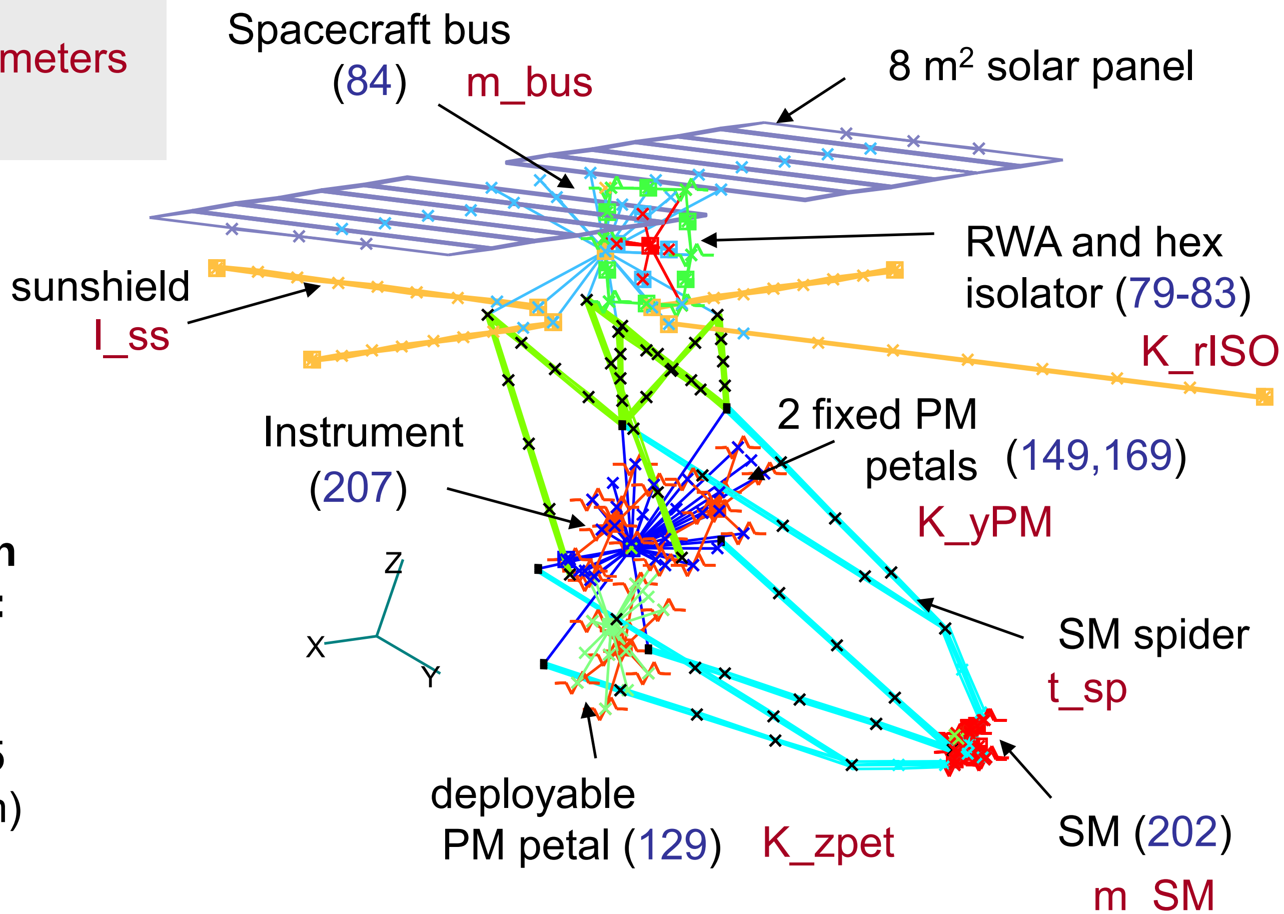
Nexus Integrated Model

Legend:
Design Parameters
(I/O Nodes)



Cassegrain Telescope:

PM (2.8 m)
PM f/# 1.25
SM (0.27 m)
f/24 OTA

Structural Model (FEM)
(Nastran, IMOS) → Ω, Φ

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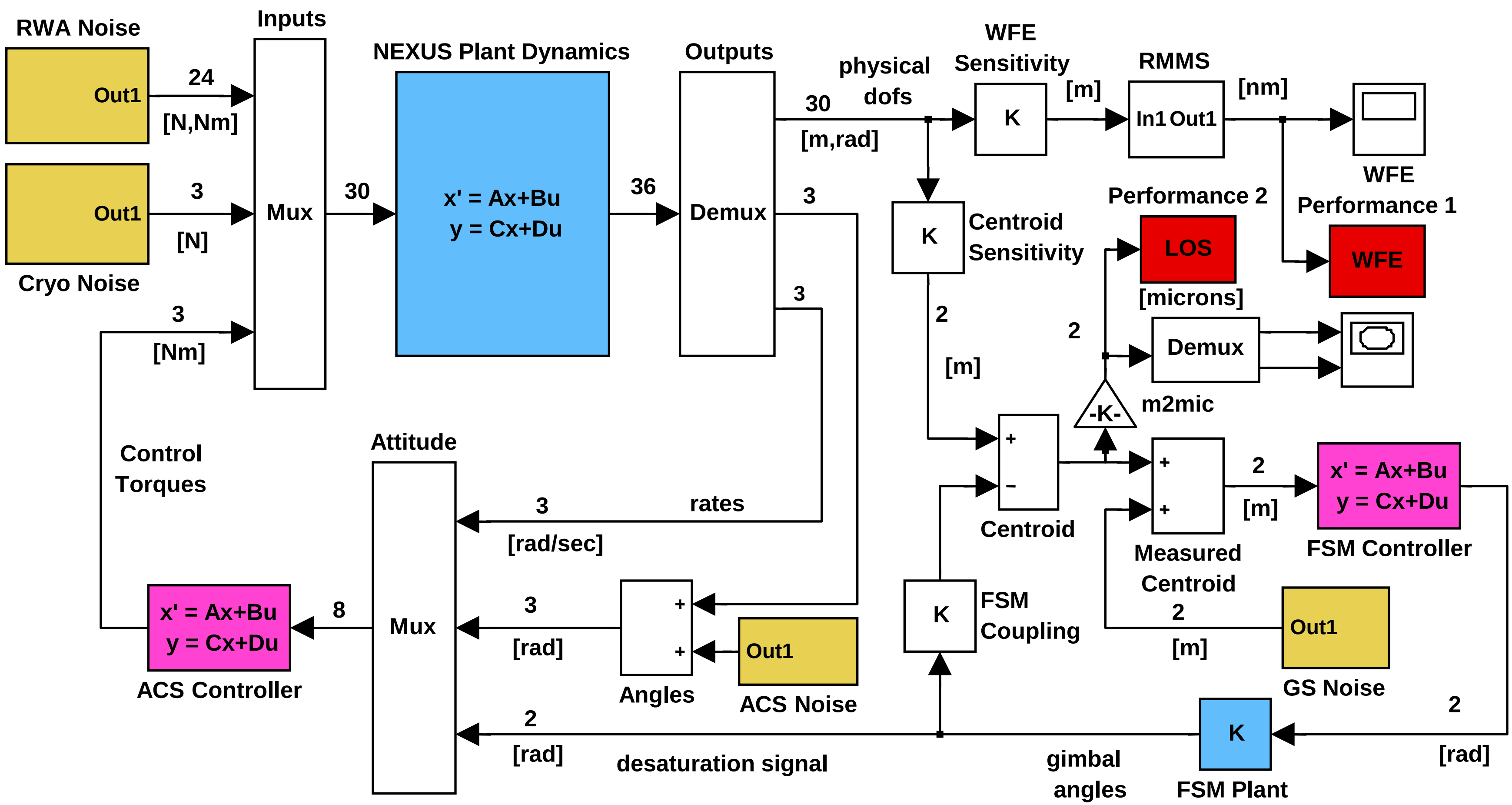
Nexus Block Diagram

Number of performances: $n_z=2$
Number of design parameters: $n_p=25$

Number of states $n_s= 320$
Number of disturbance sources: $n_d=4$



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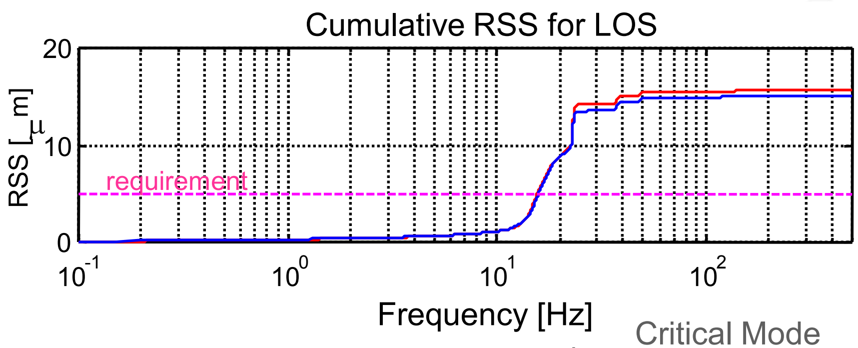
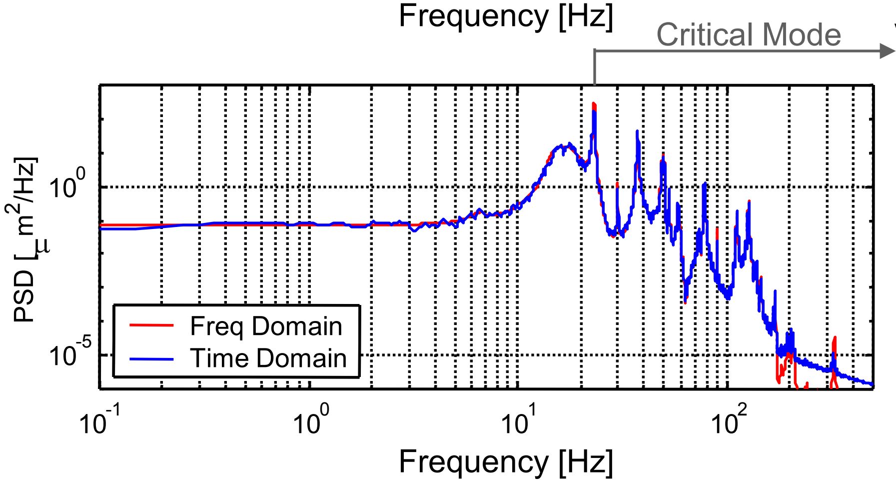
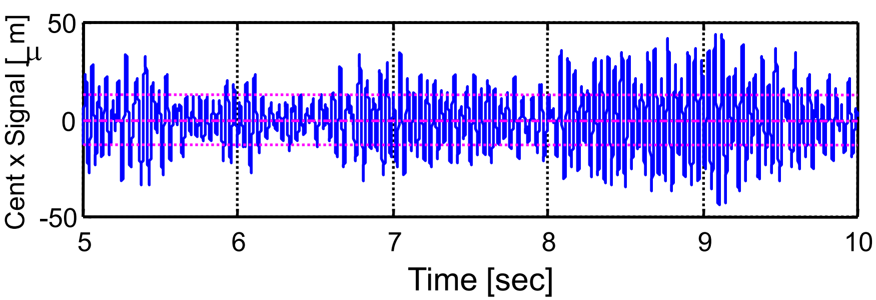
Initial Performance Assessment $J_z(p^o)$

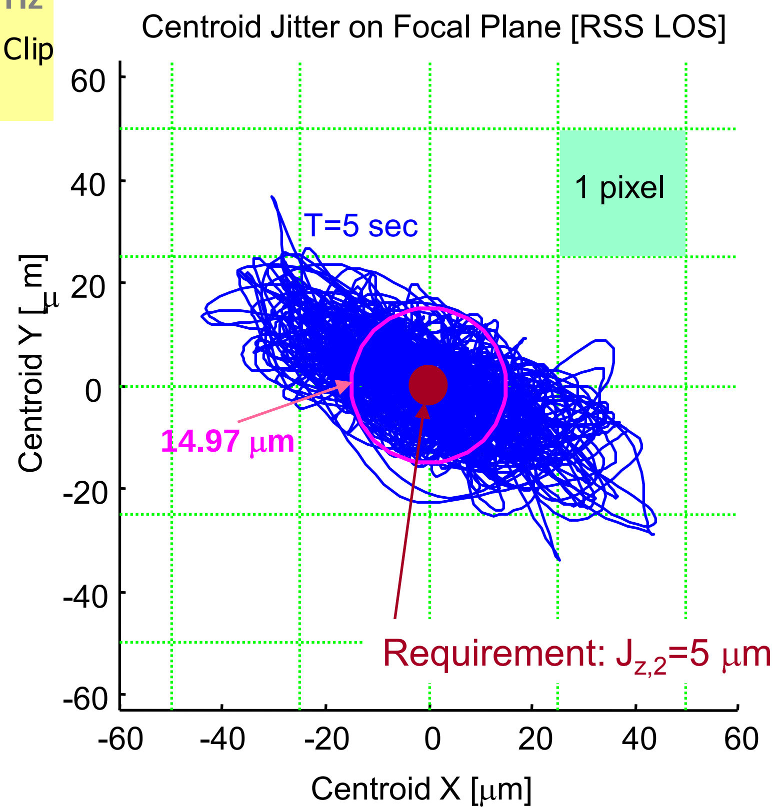
Results	Lyap/Freq	Time	
$J_{z,1}$ (RMMS WFE)	**25.61**	**19.51**	[nm]
$J_{z,2}$ (RSS LOS)	**15.51**	**14.97**	[μm]





23.1 Hz
Video Clip





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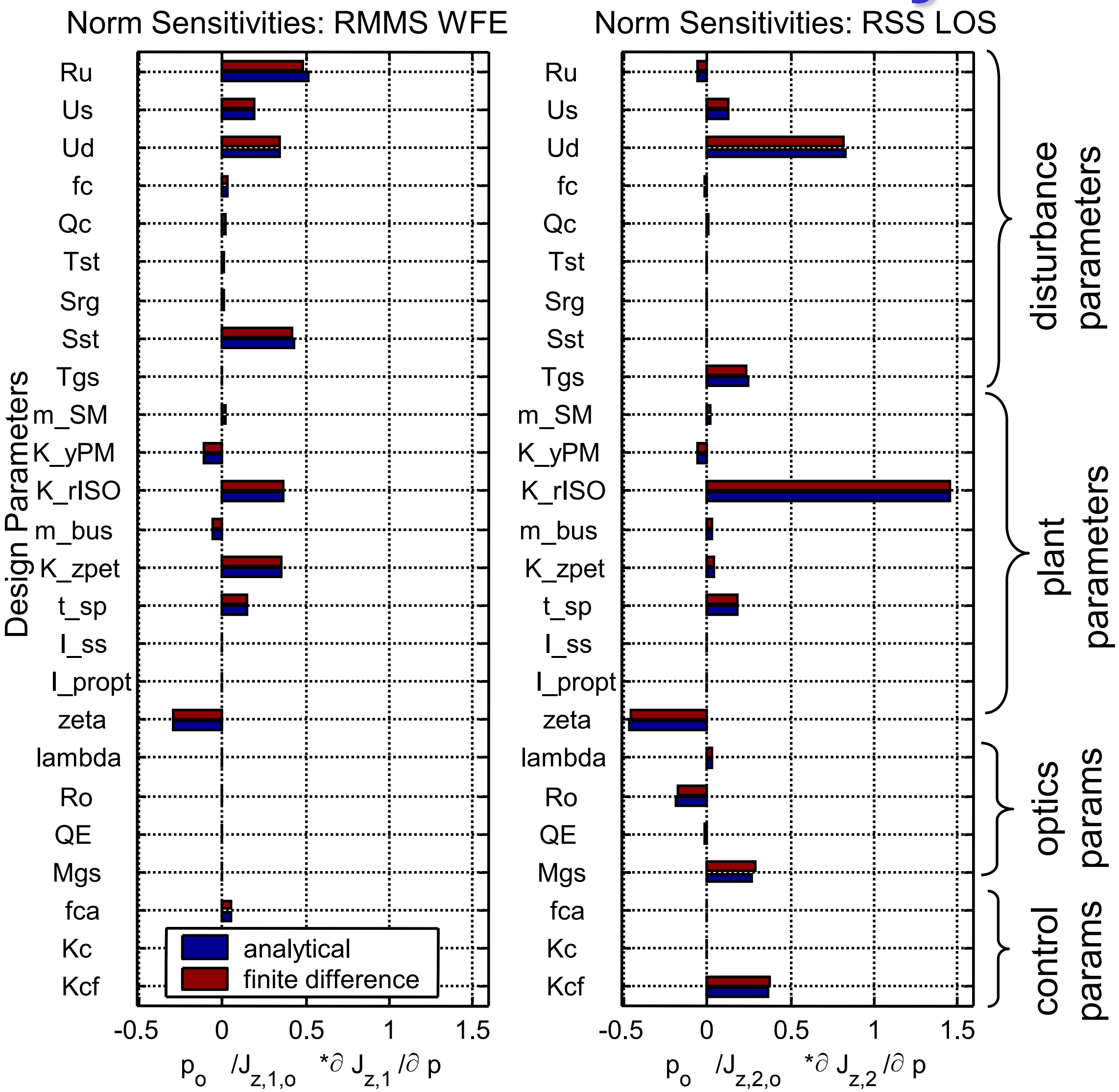
Nexus Sensitivity Analysis



Graphical Representation of Jacobian evaluated at design p_o, normalized for comparison.

$$\bar{\nabla} J_z = \frac{p_o}{J_{z,o}} \begin{bmatrix} \frac{\partial J_{z,1}}{\partial R_u} & \frac{\partial J_{z,2}}{\partial R_u} \\ \cdots & \cdots \\ \frac{\partial J_{z,1}}{\partial K_{cf}} & \frac{\partial J_{z,2}}{\partial K_{cf}} \end{bmatrix}$$

RMMS WFE most sensitive to:

Ru - upper op wheel speed [RPM]
Sst - star track noise 1σ [asec]
K_rISO - isolator joint stiffness [Nm/rad]
K_zpet - deploy petal stiffness [N/m]

RSS LOS most sensitive to:

Ud - dynamic wheel imbalance [gcm^2]
K_rISO - isolator joint stiffness [Nm/rad]
zeta - proportional damping ratio [-]
Mgs - guide star magnitude [mag]
Kcf - FSM controller gain [-]

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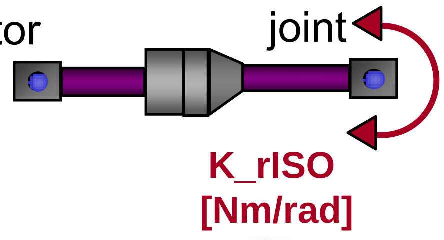
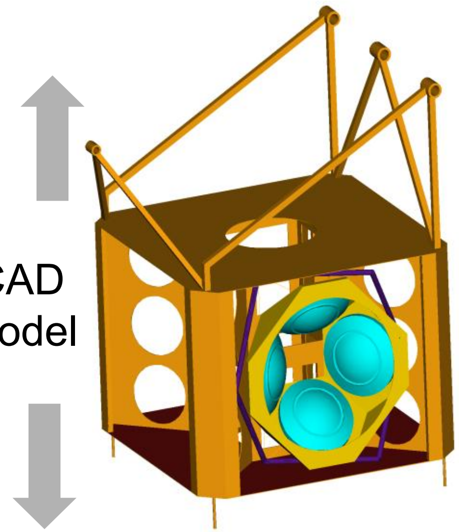
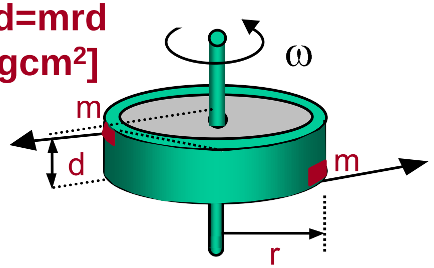
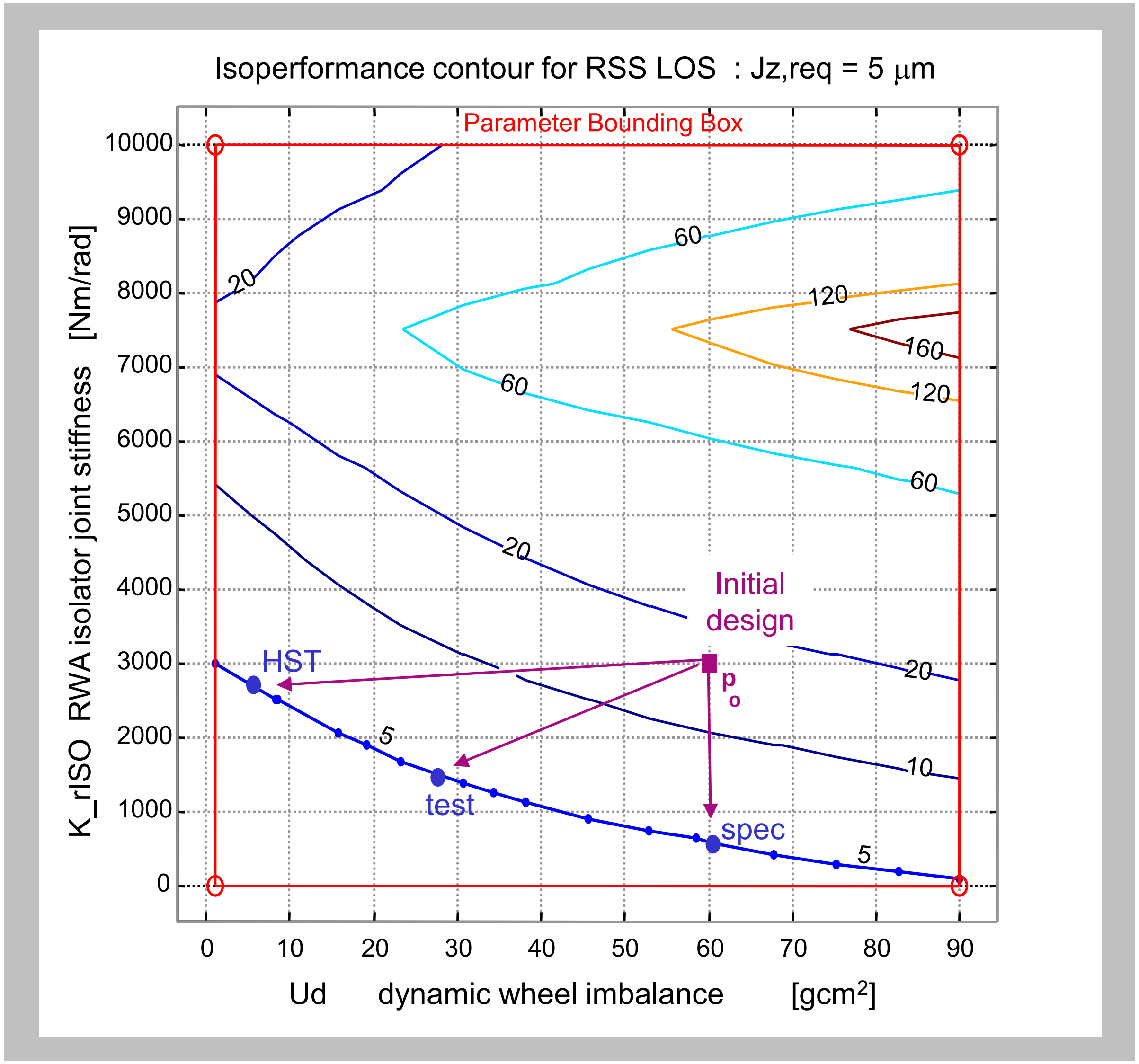
2D-Isoperformance Analysis









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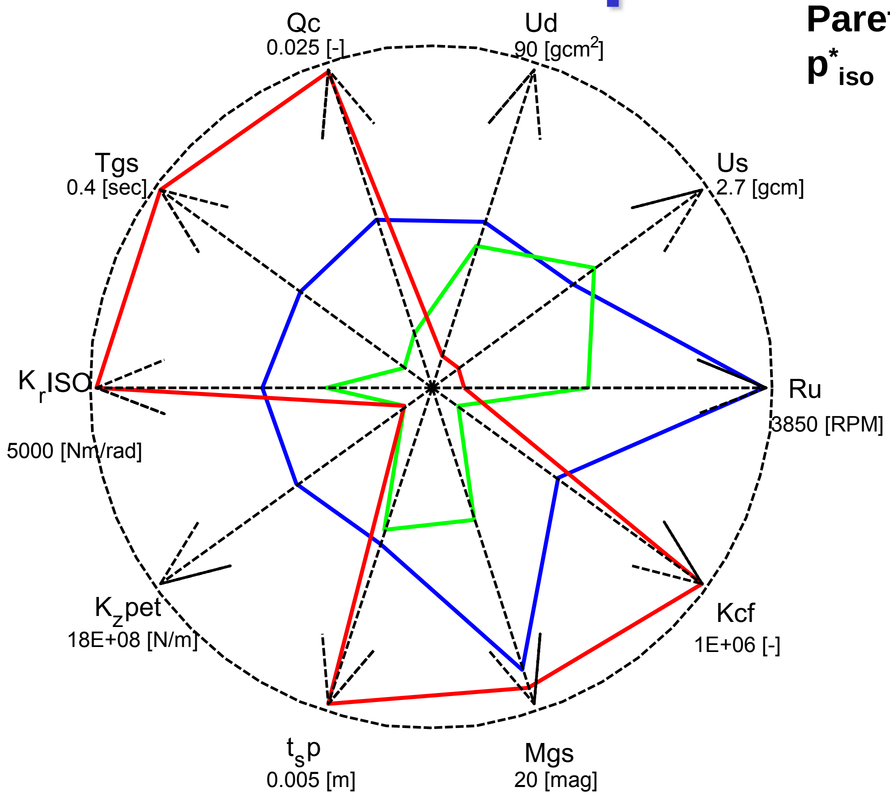
Nexus Multivariable Isoperformance $n_p=10$



Pareto-Optimal Designs

p^*_{iso}

Design A

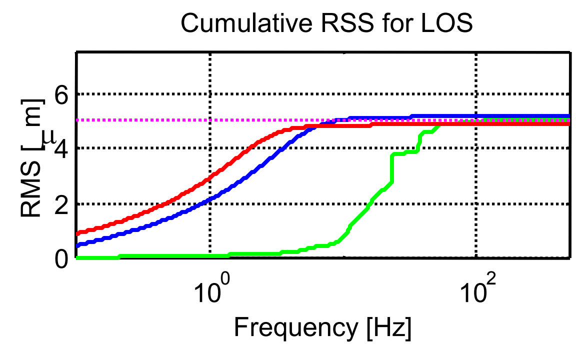
Best "mid-range" compromise

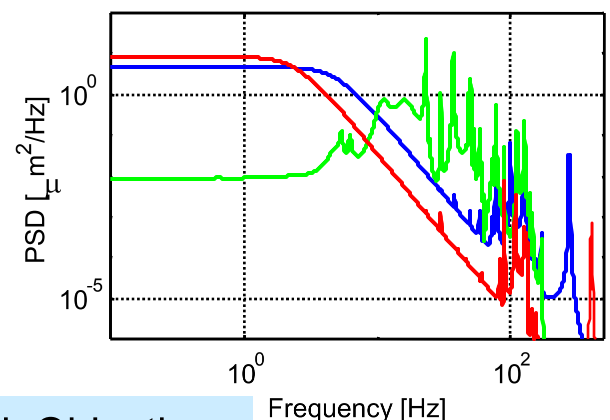
Design B

Smallest FSM control gain

Design C

Smallest performance uncertainty





	Performance		Cost and Risk Objectives		
	$J_{z,1}$	$J_{z,2}$	$J_{c,1}$	$J_{c,2}$	$J_{r,1}$
Design A	20.0000	5.2013	0.6324	0.4668	+/- 14.3218 %
Design B	20.0012	5.0253	0.8960	0.0017	+/- 8.7883 %
Design C	20.0001	4.8559	1.5627	1.0000	+/- 5.3067 %

A: min(J_{c1})
B: min(J_{c2})
C: min(J_{r1})

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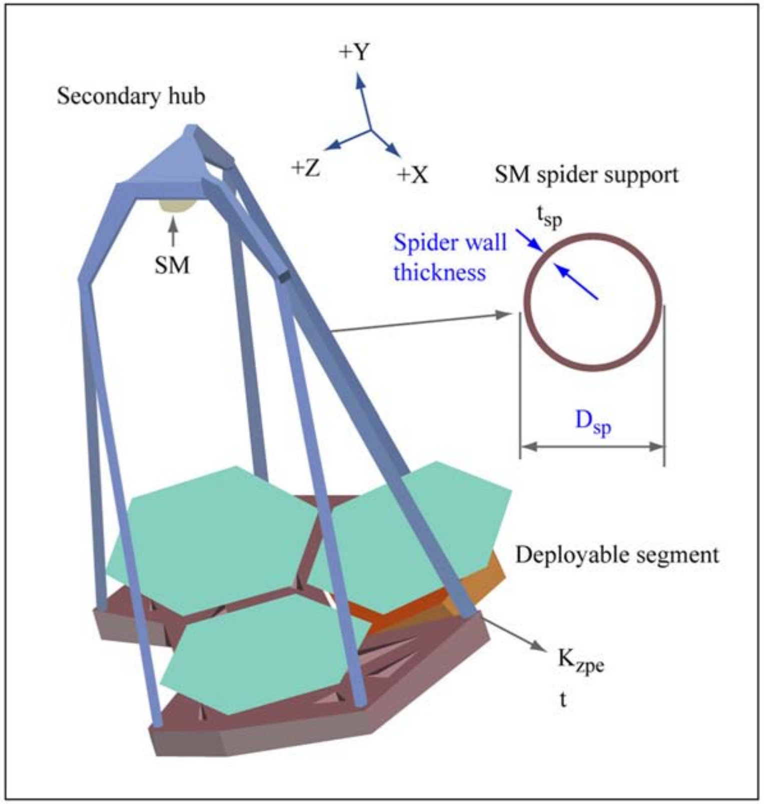
Nexus Initial p° vs. Final Design p**$_{iso}$



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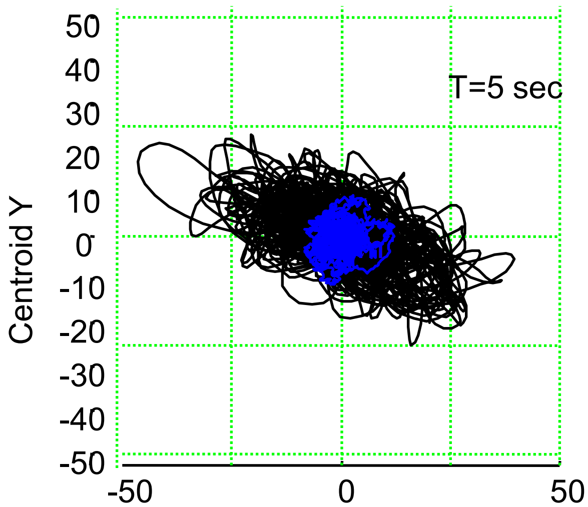
Parameters	Initial	Final	
Ru	3000	3845	[RPM]
Us	1.8	1.45	[gcm]
Ud	60	47.2	[gcm^2]
Qc	0.005	0.014	[-]
Tgs	0.040	0.196	[sec]
KrISO	3000	2546	[Nm/rad]
Kzpet	0.9E+8	8.9E+8	[N/m]
tsp	0.003	0.003	[m]
Mgs	15	18.6	[Mag]
Kcf	2E+3	4.7E+5	[-]



Centroid Jitter on Focal Plane [RSS LOS]

Initial: 14.97

Final: 5.155

Centroid X

Improvements are achieved by a well balanced mix of changes in the disturbance parameters, structural redesign and increase in control gain of the FSM fine pointing loop.

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Isoperformance with Stochastic Data

Example: Baseball season is starting soon !

What determines success of a team ?

Pitching ⇩ ERA *"Earned Run Average"*

Batting ⇩ RBI *"Runs Batted In"*

How is success of team measured ?

FS= Wins/Decisions

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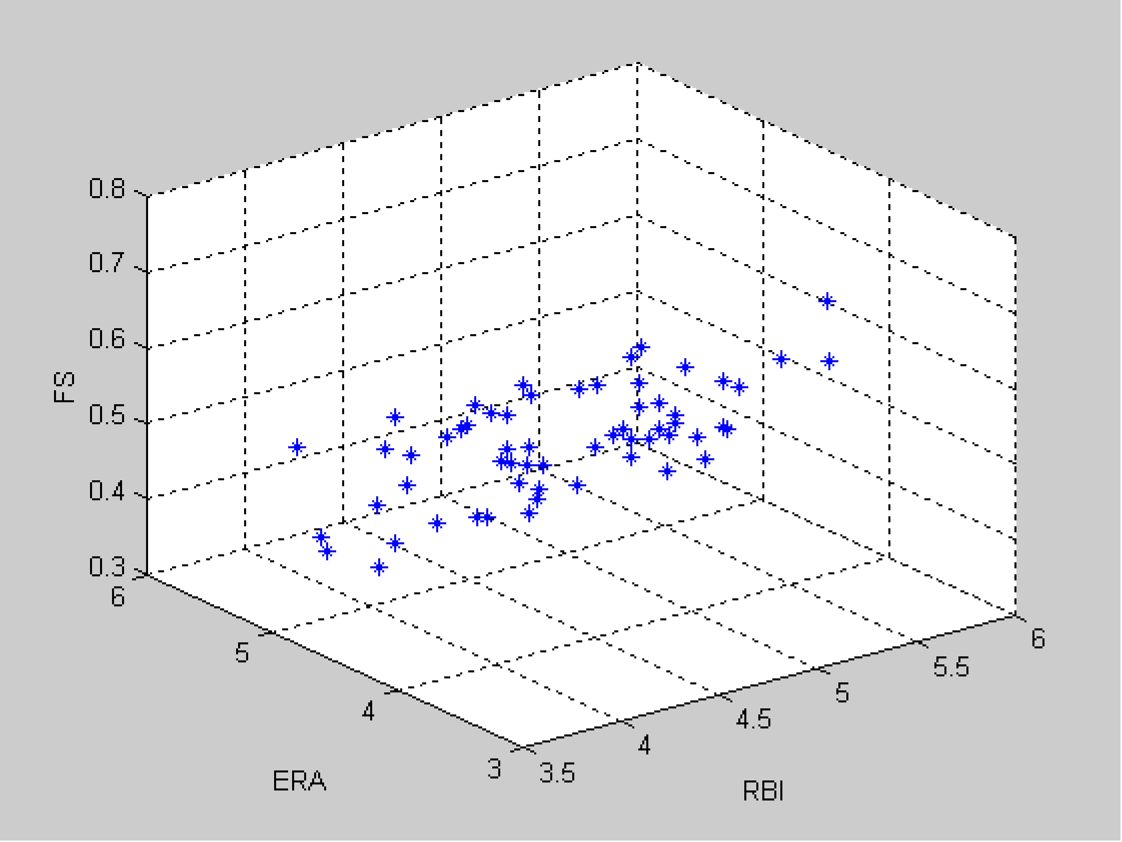
Raw Data

Team results for 2000, 2001 seasons: RBI,ERA,FS



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Stochastic Isoperformance (I)

Step-by-step process for obtaining (bivariate) isoperformance curves given statistical data:

Starting point, need:

- Model - derived from empirical data set
- (Performance) Criterion
- Desired Confidence Level

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Model

Step 1: Obtain an expression from model for expected performance of a "system" for individual design i as a function of design variables $x_{1,I}$ and $x_{2,i}$

1.1 assumed model

$$E\ \ J_i\ \ = a_0 + a_1\ \ x_{1,i}\ \ + a_2\ \ x_{2,i}\ \ + a_{12}\ \ x_{1,i} - \overline{x_1}\ \ \ x_{2,i} - \overline{x_2} \qquad (1)$$

1.2 model fitting

mean $$a_o = \frac{1}{N}\sum_{j=1}^{N} J_j$$

E.g. use MATLAB fminunc.m for optimal surface fit

Baseball: Obtain an expression for expected final standings (FS_i) of individual Team *i* as a function of RBI_i and ERA_i

$$E\ \ FS_i\ \ = m + a\ \ RBI_i\ \ + b\ \ ERA_i\ \ + c\ \ RBI_i - \overline{RBI}\ \ \ ERA_i - \overline{ERA}$$

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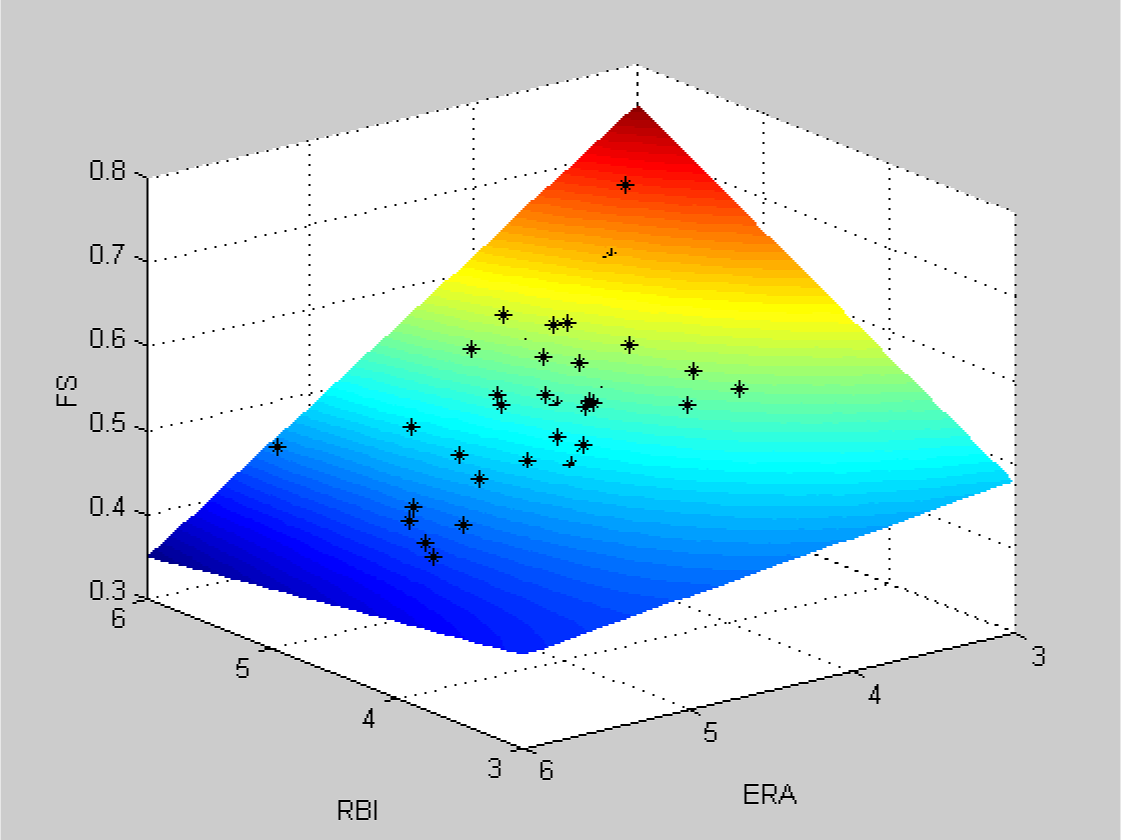
Fitted Model

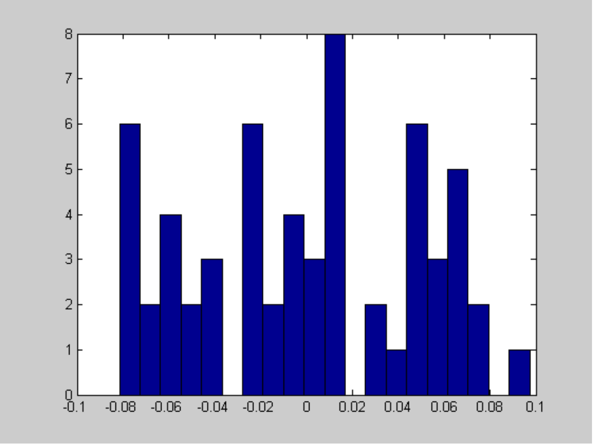


Coefficients:

ao=0.7450
a1=0.0321
a2=-0.0869
a12= -0.0369

RMSE:
Error
σ_e= 0.0493

Error
Distribution



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Expected Performance

Step 2: Determine expected level of performance for design i such that the probability of adequate performance is equal to specified confidence level

$$E\ J_i = J_{req} + z\sigma_\varepsilon \qquad (2)$$

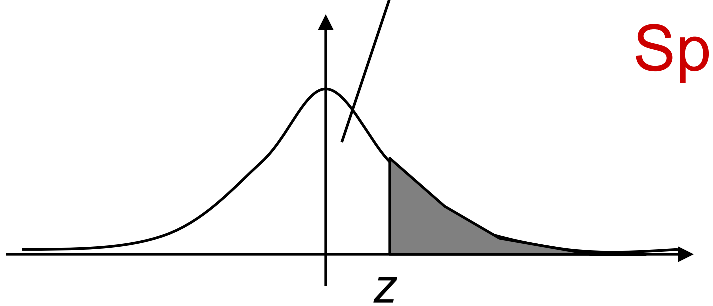
Required performance level

Confidence level normal variable z (Lookup Table)

Error Term (total variance)

$\Phi\ z$

Specify



$$\Phi\ z = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{z^2}{2}} dz$$

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Expected Performance

Baseball:

Performance criterion

- User specifies a final desired standing of FS_i=0.550

Confidence Level

- User specifies a .80 confidence level that this is achieved

Spec is met if for Team *i*:

From normal table lookup

Error term from data

$$E\ \ FS_i\ = .550 + z\sigma_r = .550 + 0.84\ \ 0.0493\ = 0.5914$$

If the final standing of team *i* is to equal or exceed .550 with a probability of .80, then the expected final standing for Team i must equal 0.5914

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Get Isoperformance Curve

Step 3: Put equations (1) and (2) together

$$J_{req} + z\sigma_r = E\;\; J_i \;\; = a_0 + a_1 \;\; x_{1,i} \;\; + a_2 \;\; x_{2,i} \;\; + a_{12} \;\; x_{1,i} - \overline{x}_1 \;\;\; x_{2,i} - \overline{x}_2 \qquad (3)$$

- Four constant parameters: a_o, a_1, a_2, a_{12}
- Two sample statistics: $\overline{x}_1, \overline{x}_2$
- Two design variables: $x_{1,i}$ and $x_{2,i}$

Then rearrange: $x_{2,i} = f \;\; x_{1,i}$

Baseball:

$$RBI_i = \frac{.5914 - m - bERA_i + c\overline{RBI} \;\; ERA_i - \overline{ERA}}{a + c \;\; ERA_i - \overline{ERA}}$$

Equation for isoperformance curve

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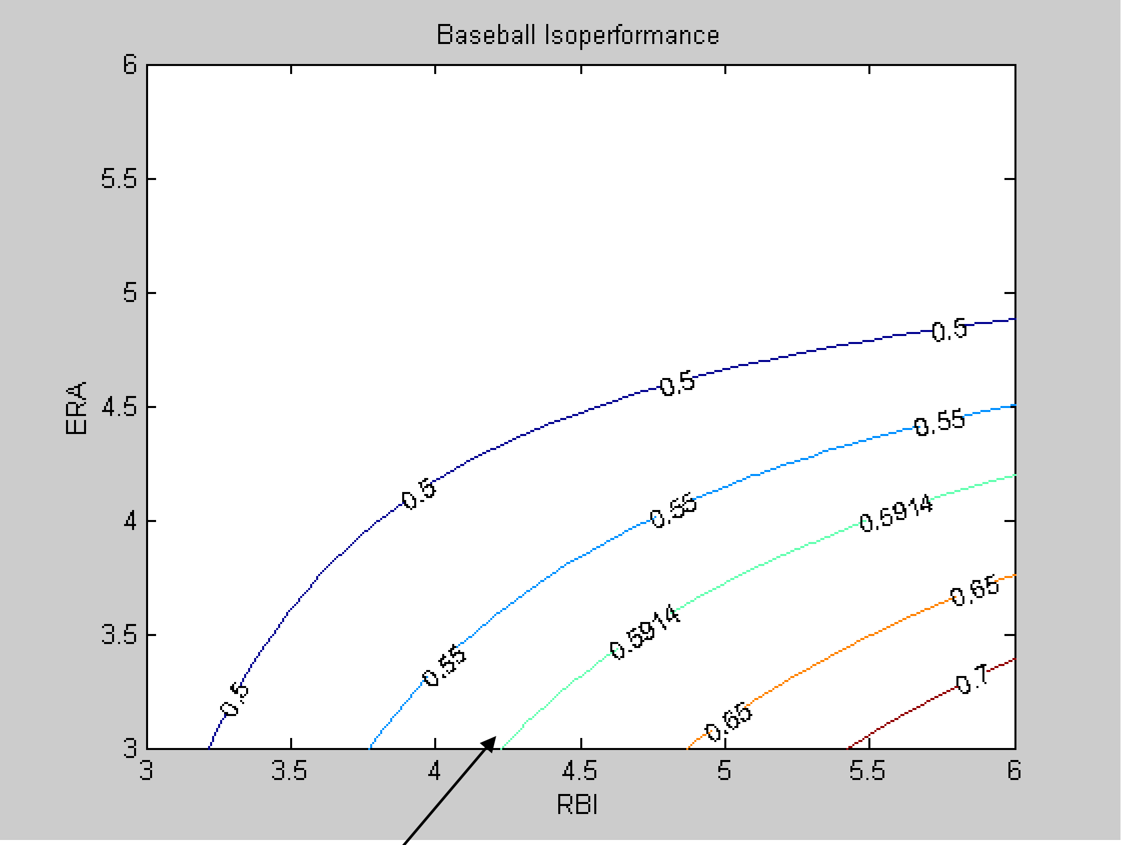
Stochastic Isoperformance



This is our desired tradeoff curve

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Lecture Summary

- Traditional process goes from design space **x** → objective space **J** (forward process)
- Many systems are designed to meet “targets”
  - Performance, Cost, Stability Margins, Mass …
- Methodological Options
  - Formulate optimization problem with equality constraints given by targets
  - Goal Programming minimizes the “distance” between a desired “target” state and the achievable design
  - Isoperformance finds a set of (non-unique) performance invariant solutions → multiple solutions
- Isoperformance works backwards from objective space **J** → design space **x** (reverse process)
  - Deterministically
  - Stochastically

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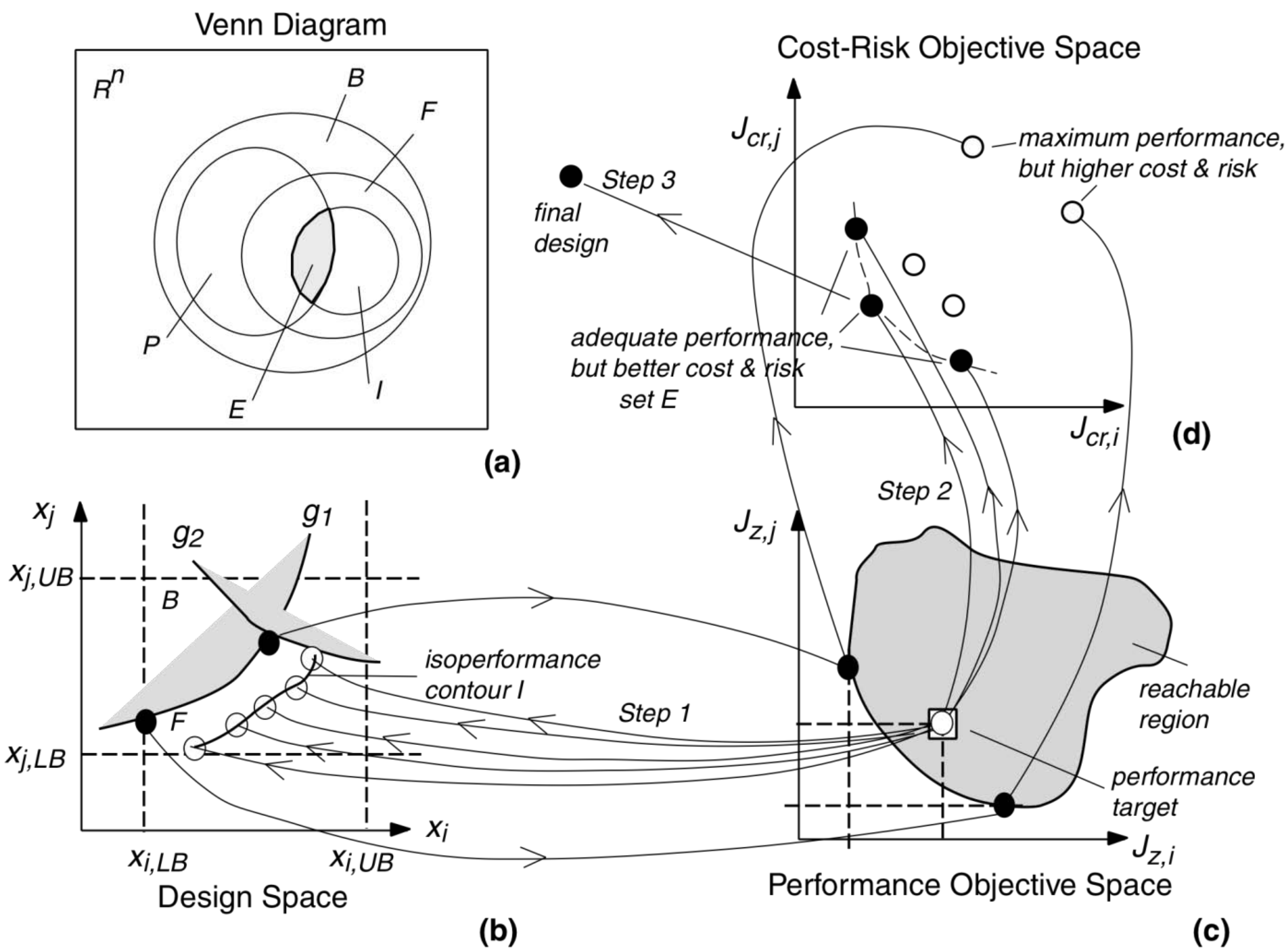
Visual Summary



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