16.888/ESD.77J Multidisciplinary System Design Optimization (MSDO) Spring 2010

Assignment 5

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Issued:	Lecture 18
Due:	Lecture 22

You are expected to solve **Part (a)** individually and **Part (b)** in your project team. Each person must submit their own Part (a) but you should submit Part (b) as a group. Please indicate the name(s) of your teammate(s).

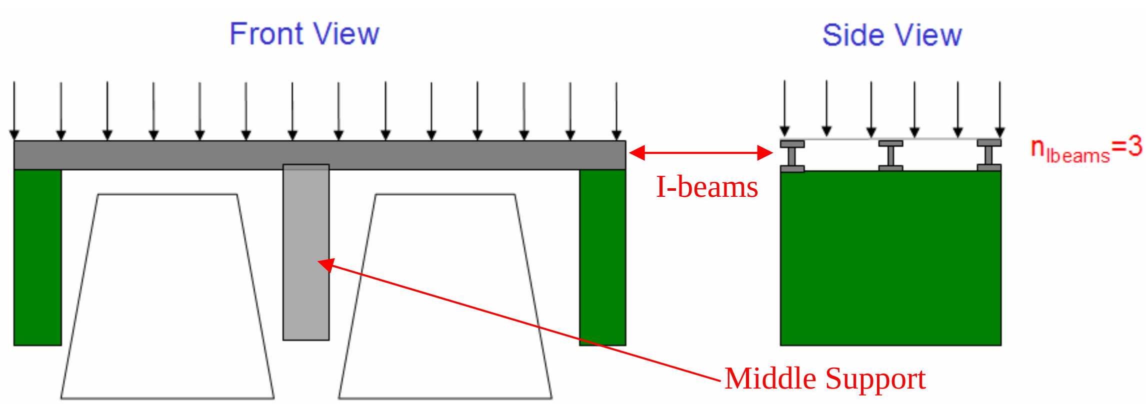
Topics: Isoperformance, Multi-Objective Optimization

Part (a)

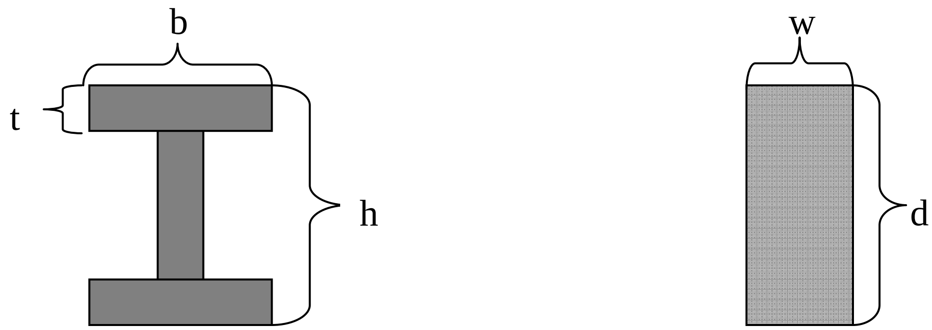
Revisit of Bridge Design from A4

Part A1

This problem will revisit the bridge design problem from A4. However, this time you are going to propose a set of designs to the department of transportation so that they can choose a bridge based on the load it will carry and the price they have to pay for the bridge. In other words they will select a bridge based on the applied load it can carry, F=qL, where q is a load per unit length (N/m), L is the length of the bridge, 30m, and the cost, C in dollars.



The bridge span will be supported by between one and four I-beams. In the figure above, the I-beams would be parallel to each other going into the page, for example it could be one I-beam in the middle of the bridge, or one I-beam on both sides of the bridge, etc. and this will be represented by the design variable, n_{Ibeams}. The shape of the I-beams will be represented by three continuous design variables, the height, h, flange width, b, and thickness, t. The middle support will be rectangular (when viewed from above), and will have two design variables, the width, w, and depth, d.



Where, ρ_{Ibeams}, is the density of the material used for the I-beams, the mass of the I-beams can be computed using,

$$M_{Ibeams} = \left[2bt + (h-2t)t\right]L\rho_{Ibeams}n_{Ibeams},$$

and where, $\rho_{Support}$, is the density of the material used for the support, and H=5m is the height of the bridge above the ground, the mass of the middle support can be computed using,

$$M_{Support} = wdH\rho_{Support}.$$

There is a constraint that the stress the I-beam is less than the material failure stress for the I-beam, $\sigma_{Failure\text{-}Ibeams}$. Note: g is the gravitational constant (9.81 m/s^2).

$$\sigma_{Ibeams} = \frac{q\left(\frac{L}{2}\right)^2 + M_{Ibeams}\left(\frac{L}{4}\right)g}{8I_{Ibeam}n_{Ibeams}}\left(\frac{h}{2}\right) \le \sigma_{Failure-Ibeams}$$

Where, I_{Ibeam}, is the moment of inertia for the I-beam given by,

$$I_{Ibeam} = \frac{(h-2t)^3 t}{12} + 2\left[\frac{t^3 b}{12} + tb\left(\frac{h}{2} - \frac{t}{2}\right)^2\right].$$

In addition that the shear stress in the I-beams is less than the material failure stress:

$$\tau_{Ibeams} = \frac{M_{Ibeams}g + qL}{4\left[2bt + (h-2t)t\right]n_{Ibeams}} \le \sigma_{Failure-Ibeams}.$$

For the middle-support there are two constraints, the column cannot buckle and the stress must be less than the material failure stress. Buckling is based on a requirement that the applied load is less than a critical load,

$$P_{Applied} = \frac{M_{Ibeams}g + qL}{2} \le P_{Crit}$$

Where the critical load is a function of the lowest moment of inertia of the support and the modulus of elasticity of the support material, $E_{support}$,

$$P_{Crit} = \frac{\pi^2 E_{Support} \min\left\{ \frac{w^3 d}{12}, \frac{wd^3}{12} \right\}}{4H^2}.$$

The stress requirement is that the applied stress is less than the support material failure stress,

$$\sigma_{Support} = \frac{P_{Applied}}{wd} \leq \sigma_{Failure-Support}$$

The bridge span (I-beams) can be made from Al 6061, A36 Steel, A514 Steel, or Titanium; however, the support can be made from Al 6061, A36 Steel, A514 Steel, or Concrete. The reason for the difference is that concrete cannot be loaded in tension. The material properties and prices are listed in the Table:

Material	Density (kg/m^3)	Modulus of Elasticity ($GPa=10^9 N/m^2$)	Failure Stress ($MPa=10^6 N/m^2$)	Cost ($/kg)
Al 6061	2700	70	270	2.05
A36 Steel	7850	210	250	0.62
A514 Steel	7900	210	700	0.90
Titanium	4500	120	760	16.00
Concrete	2400	31	70	0.04

Your objective is to find the dimensions of the I-beams, number of I-beams, and material type for the I-beams, as well as the dimensions of the support and material type for the support to minimize cost of the bridge. Where c_{Ibeams}, and $c_{Support}$ are the cost per kilogram of the materials used for the I-beams and Support, the total bridge cost is:

$$C = c_{Ibeams} M_{Ibeams} + M_{Support} c_{Support}$$

a) For the optimal cost solution from A4, what is the Jacobian of the cost and applied load? (An estimate is fine.)
b) Find at least two substantially different bridge designs that each has a cost 10% higher than the optimal you have found, while satisfying the same load condition.
c) Find the Pareto front of cost and applied load with respect to the continuous variables using the optimal values of the discrete variables from A4. Please describe what algorithm you used and why?
d) Using your Pareto front estimate, what is the trade between the cost of the bridge and the load it can carry at the minimum cost design you found?
e) Is this Pareto front convex?

Bonus (Optional):

Find the Pareto front for cost and load for all of the variables.

Part (b)

(b1) Multiobjective Optimization

Select two objective functions for your project (e.g. range [km] and cruise speed [m/s] or vehicle acceleration [0-60 mph time] and cost [$], etc….). Optimize your system for both objectives simultaneously using any multiobjective method of your choice. Answer the following questions:

(b1.1) Are your objectives mutually supportive or opposing? How do you know?
(b1.2) What is an estimate of the Pareto front? Show this graphically.
(b1.3) How sensitive is your answer to the scale factors or weights you have chosen (if you use a method that requires weights as in the weighted sum method)?

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