

**16.888/ESD.77J Multidisciplinary System Design Optimization (MSDO)  
Spring 2010**

**Assignment 4**

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<b>Issued:</b>	Lecture 13
<b>Due:</b>	Lecture 17

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You are expected to solve **Part (a)** individually and **Part (b)** in your project team. Each person must submit their own Part (a) but you should submit Part (b) as a group. Please indicate the name(s) of your teammate(s).

**Topics:** genetic algorithms, mixed-integer optimization, scaling

## **Part (a)**

### **Part A1**

The following questions refer to a Genetic Algorithm as defined in Lecture 10:

- a) Which of the following will most increase population diversity?
  - a. Increasing mutation rate
  - b. Changing the crossover location
  - c. Increasing the amount of elitism
  - d. All of the above
  - e. None of the above
  
- b) In a binary GA, how many bits are needed to represent numbers between 1 and 10 with a resolution of 0.0001?
  - a. 3
  - b. 4
  - c. 16
  - d. 17
  - e. 18
  
- c) In a binary GA, what is the minimum number of bits required to represent a discrete design variable that can have values, 1, 2, 3 or 4?

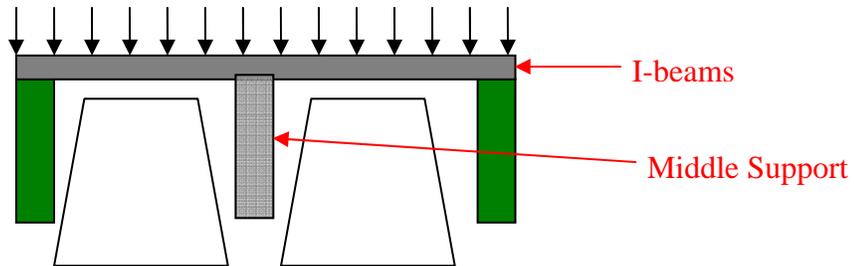
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- a. 1
  - b. 2
  - c. 3
  - d. 4
- d) For a discrete variable that can have the values, 1, 2, 3, or 4, in the binary representation of this variable with the minimum number of bits, what would the representation of 3 be?
- a. 1
  - b. 01
  - c. 10
  - d. 111
  - e. 010
  - f. 0110
- e) Given the parents 01001101 and 01100100, which of the following are possible single point crossovers?
- A: 11001101 B: 01000100 C: 01001100 D: 01111101
- a. A & B
  - b. B & C
  - c. A & D
  - d. A & C
  - e. A, B, C & D
- f) In a population, the fitness function values of each member are:  $F_1=100$ ,  $F_2=800$ ,  $F_3=1$ ,  $F_4=90$ ,  $F_5=9$ . Using roulette wheel selection, what is the approximate probability the member with  $F_1$  will be chosen as a parent?
- a. 0.01
  - b. 0.10
  - c. 0.80
  - d. 0.90
  - e. 0.99
- g) In a population, the fitness function values of each member are:  $F_1=100$ ,  $F_2=800$ ,  $F_3=1$ ,  $F_4=90$ ,  $F_5=9$ . Now, using the fraction of the population a member dominates as the selection criterion, what is the approximate probability the member with  $F_1$  will be chosen as a parent?
- a. 1
  - b. 0.90
  - c. 0.80
  - d. 0.50
  - e. 0.10

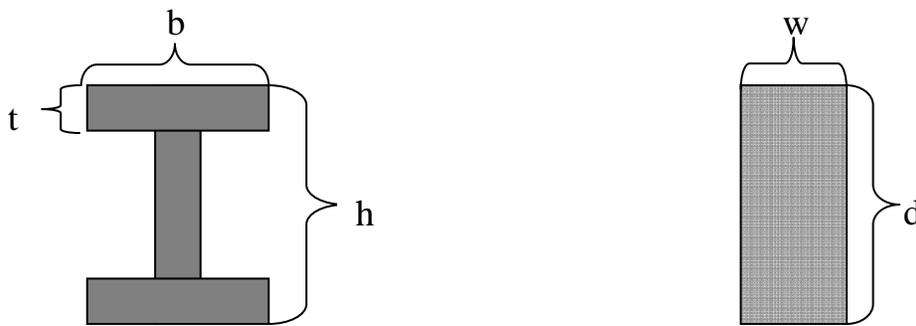
- h) In a binary encoded GA with two design variables, one with 4 bits, and one with 16 bits, what is the total number of possible population members?
- $2^{18}$
  - $2^{20}$
  - $2^{24}$
  - $2^{32}$

**Part A2**

Your objective is to design the cheapest possible bridge to span a highway. The total span of the bridge (across both halves of the highway) must be  $L=30$  meters, and it must support its own weight and a load  $q=33 \times 10^4$  N/m along its span (a total load of about  $1 \times 10^6$  N plus its own weight).



The bridge span will be supported by between one and four I-beams. In the figure above, the I-beams would be parallel to each other going into the page, for example it could be one I-beam in the middle of the bridge, or one I-beam on both sides of the bridge, etc. and this will be represented by the design variable,  $n_{Ibeams}$ . The shape of the I-beams will be represented by three continuous design variables, the height,  $h$ , flange width,  $b$ , and thickness,  $t$ . The middle support will be rectangular (when viewed from above), and will have two design variables, the width,  $w$ , and depth,  $d$ .



Where,  $\rho_{Ibeams}$ , is the density of the material used for the I-beams, the mass of the I-beams can be computed using,

$$M_{Ibeams} = [2bt + (h - 2t)t]L\rho_{Ibeams}n_{Ibeams} ,$$

and where,  $\rho_{Support}$ , is the density of the material used for the support, and  $H=5$ m is the height of the bridge above the ground, the mass of the middle support can be computed using,

$$M_{Support} = wdH\rho_{Support} .$$

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There is a constraint that the stress the I-beam is less than the material failure stress for the I-beam,  $\sigma_{Failure-Ibeams}$ . Note:  $g$  is the gravitational constant ( $9.81 \text{ m/s}^2$ ).

$$\sigma_{Ibeams} = \frac{q\left(\frac{L}{2}\right)^2 + M_{Ibeams}\left(\frac{L}{4}\right)g}{8I_{Ibeam}n_{Ibeams}}\left(\frac{h}{2}\right) \leq \sigma_{Failure-Ibeams}$$

Where,  $I_{Ibeam}$ , is the moment of inertia for the I-beam given by,

$$I_{Ibeam} = \frac{(h-2t)^3 t}{12} + 2\left[\frac{t^3 b}{12} + tb\left(\frac{h-t}{2} - \frac{t}{2}\right)^2\right].$$

In addition that the shear stress in the I-beams is less than the material failure stress:

$$\tau_{Ibeams} = \frac{M_{Ibeams}g + qL}{4[2bt + (h-2t)t]n_{Ibeams}} \leq \sigma_{Failure-Ibeams}.$$

For the middle-support there are two constraints, the column cannot buckle and the stress must be less than the material failure stress. Buckling is based on a requirement that the applied load is less than a critical load,

$$P_{Applied} = \frac{M_{Ibeams}g + qL}{2} \leq P_{Crit}$$

Where the critical load is a function of the lowest moment of inertia of the support and the modulus of elasticity of the support material,  $E_{Support}$ ,

$$P_{Crit} = \frac{\pi^2 E_{Support} \min\left\{\frac{w^3 d}{12}, \frac{wd^3}{12}\right\}}{4H^2}.$$

The stress requirement is that the applied stress is less than the support material failure stress,

$$\sigma_{Support} = \frac{P_{Applied}}{wd} \leq \sigma_{Failure-Support}$$

The bridge span (I-beams) can be made from Al 6061, A36 Steel, A514 Steel, or Titanium; however, the support can be made from Al 6061, A36 Steel, A514 Steel, or Concrete. The reason for the difference is that concrete cannot be loaded in tension. The material properties and prices are listed in the Table:

Material	Density ( $\text{kg/m}^3$ )	Modulus of Elasticity ( $\text{GPa}=10^9\text{N/m}^2$ )	Failure Stress ( $\text{MPa}=10^6\text{N/m}^2$ )	Cost (\$/kg)
Al 6061	2700	70	270	2.05
A36 Steel	7850	210	250	0.62
A514 Steel	7900	210	700	0.90
Titanium	4500	120	760	16.00
Concrete	2400	31	70	0.04

Your objective is to find the dimensions of the I-beams, number of I-beams, and material type for the I-beams, as well as the dimensions of the support and material type for the support to minimize cost of the bridge. Where  $c_{Ibeams}$ , and  $c_{Support}$  are the cost per kilogram of the materials used for the I-beams and Support, the total bridge cost is:

$$C = c_{Ibeams}M_{Ibeams} + M_{Support}c_{Support}$$

- a) Please explain what optimization algorithm you chose to find the cheapest possible bridge and why.
- b) What are the design variables and cost for the cheapest bridge?

**Part A3**

Consider the 8-dimensional function:

$$f(\mathbf{x}) = \frac{1}{2} \left( 0.0001x_1^2 + 0.001x_2^2 + 0.01x_3^2 + 0.1x_4^2 + x_5^2 + 10x_6^2 + 100x_7^2 + 1000x_8^2 \right)$$

Note: for this question you may either create your own optimization algorithms or use any optimization package you like that has the required capabilities.

- a) Minimize this function using Newton's method from 10 random starting points on the interval  $x_i \in [-5, 5]$  and consider convergence when  $\|\nabla f(\mathbf{x})\| \leq 10^{-8}$ . What is the average number of iterations required for the optimization to converge?
- b) Repeat the optimization in part (a) this time using either a conjugate gradient method or a quasi-Newton method. If you use a quasi-Newton method ensure the initial Hessian estimate is the identity matrix. On average how many iterations are required for the optimization to converge?
- c) Repeat the optimization in part (a) this time using steepest descent. On average how many iterations are required for the optimization to converge?
- d) What is the condition number of the Hessian?
- e) Find a non-singular transformation  $\mathbf{x} = \mathbf{L}\mathbf{y}$  such that the condition number of the Hessian is 1.
- f) Repeat parts a, b, and c on the scaled problem. What are the average number of iterations each method requires to minimize  $f(\mathbf{x})$ ?
- g) What can you say about the sensitivity of each of these optimization algorithms to scaling?

## Part (b)

### (b1) Scaling

Use a gradient-based algorithm as in A3 for the question below (pick one algorithm). The current optimal solution  $\mathbf{x}^*$  refers to the “optimal” solution in your project found using this algorithm in A3.

**(b1.1)** Compute the  $n$  diagonal entries of the Hessian at your current optimal solution,  $\mathbf{H}(\mathbf{x}^*)$ . Use finite differencing to evaluate these entries. (Note: if you have a large number of design variables, you may do these steps for any 10 design variables.)

**(b1.2)** If any of the entries computed in **(b1.1)** are greater than  $10^2$  or less than  $10^{-2}$ , then the corresponding design variable should be scaled. Note that  $H_{ii} \sim 1/x_i^2$ , so if  $H_{ii} \sim 10^{-4}$  the appropriate scaling is  $X_i = 10^{-2}x_i$ . Compute the scaling required for each design variable to make  $H_{ii} \sim O(1)$ . Consider only scalings of the form  $10^{-2}, 10^{-1}, 10^1, 10^2$  etc. (i.e. worry about magnitude only).

**(b1.3)** Redefine your design variables using the scalings computed in **(b1.2)** and re-run the optimizer, starting from the previous “optimal” solution  $\mathbf{x}^*$ . Do you see any change in the optimal solution?

**Please note:** If you do not have any continuous design variables in your problem, please instead analyze the scaling of your constraints. Study whether scaling certain constraints in your problem affects convergence or convergence rates.

### (b2) Heuristic Optimization

In A3 you used a gradient search technique to optimize the system for your design project. In this assignment we want you to apply a heuristic technique (SA, GA, PSO or Tabu Search) to your design project.

**(b2.1)** Describe which technique you chose and why.

**(b2.2)** Attempt to optimize the system and compare the answers with the answers you received with the gradient search technique.

**(b2.3)** Tune the parameters of the algorithm and observe the differences in behavior. What appears to be the best algorithm tuning parameter settings for your project?

**(b2.4)** How confident are you that you have found the true global optimum?

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