

**16.888/ESD.77J Multidisciplinary System Design Optimization (MSDO)  
Spring 2010**

**Assignment 2**

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<b>Issued:</b>	Lecture 5
<b>Due:</b>	Lecture 10

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You are expected to solve **Part (a)** individually and **Part (b)** in your project team. Each person must submit their own Part (a) but you should submit Part (b) as a group. Please indicate the name(s) of your teammate(s).

**Topics:** Design of Experiments, Gradient-Based Optimization, Sensitivity, Simulation Code Implementation

## **Part (a)**

### **(a1) Design of Experiments**

Recall the airplane design experiment we did in Lecture 5. Results from this experiment are available in the file "Airplane Design Results (unavailable on OCW)."

- Calculate the mean and variance for each experiment (9 experiments).
- Calculate the effect of each design variable setting (12 effects).
- What are the design variable settings of the predicted optimal airplane?
- Assuming that the effects can be added linearly, estimate the range of the predicted optimal airplane.
- Download the airplane template (unavailable on OCW), build the predicted optimal airplane, and fly it 5 times. Report your results and discuss similarities/differences between the predicted and actual performance.
- If we were now to perform a parameter study using the predicted optimal airplane as the baseline, what would the experimental matrix be? Comment on what, if any, new information this new experiment might bring.
- In ~1 paragraph, discuss the variance results computed in part a). Explain the possible significance of these results and how they might be used to inform an actual design process.

**(a2) Gradient-Based Optimization**

a) Consider the function from Assignment 1:

$$f(x_1, x_2) = x_1^4 - x_1^2 x_2 + x_2^2 + \frac{1}{2} x_1^2$$

- (i) Perform one iteration of steepest descent by hand starting from  $(x_1, x_2) = (2, 2)$ .
- (ii) Perform one iteration of Newton's method by hand starting from  $(x_1, x_2) = (2, 2)$ .

b) Consider the optimization problem:

$$\begin{aligned} \min_{x_1, x_2} \quad & x_1^2 + x_2^2 \\ \text{s.t.} \quad & 2 \leq x_1 \end{aligned}$$

- (i) Formulate the Lagrangian and solve.
- (ii) Solve using a logarithmic barrier function. Where  $r_p$  is the coefficient of the logarithm, write an explicit solution for  $x^*$  as a function of  $r_p$ . (Discussed in L8.)
- (iii) Perform one iteration of Newton's method (by hand) on the Lagrangian formulation in part (i). (Hint: Newton's method is a method of finding zero's in nonlinear systems of equations, in optimization it is typically used to find a zero of the derivative.)

c) Consider the problem

$$\begin{aligned} \min_{x_1, x_2} \quad & f(x_1, x_2) = 4x_1^2 + 12x_2^2, \\ \text{s.t.} \quad & g(x_1, x_2) = x_2^2 - (x_1 - 1)^3 = 0 \end{aligned}$$

- (i) Formulate the Lagrangian function and derive the optimality conditions. Can you solve the resulting system of equations to determine the optimal solution? Explain why this method fails for this problem.
- (ii) Solve this problem using the exterior penalty method discussed in L8, using a quadratic penalty function. (Hint: derive an expression for the solution as a function of the penalty parameter  $\rho$ , as then take the limit as  $\rho \rightarrow \infty$ .)
- (iii) Plot the contours of objective and the equality constraint. Explain how the plot relates to your answers in (i) and (ii).

## Part (b)

We will begin the numerical implementation of your design project.

**Note:** For Assignment 1, many of you may have described large, complex systems with many disciplines, objective functions and design variables. For this class, you should focus in on a portion of this problem for which you can realistically hope to get numerical results. It does not matter if your initial analysis is not physically comprehensive – you can always increase the complexity of your problem later on. For example, you may want to turn certain design variables into prescribed parameters (especially discrete variables) and start by exploring just a subset of the full design space.

**(b1) Simulation Implementation**

Numerically implement your system model with all of its disciplinary modules. Demonstrate that the system can be executed in analysis mode, *i.e.* provide a design vector  $\mathbf{x}$  and show that your simulation returns the correct values of the objective functions  $\mathbf{J}$  and constraints  $\mathbf{g}$ ,  $\mathbf{h}$ . Describe how you validated your analysis routines.

**(b2) Feasibility**

Describe the initial design vector that you have available. Is this initial guess feasible?

**(b3) DOE**

Carry out an initial exploration of the design space using a DOE technique of your choice, recall lecture L5. Describe your choice of factors, levels and experiments. What design variable (factor) and level shows the largest effect? Based on this analysis what initial start point  $\mathbf{x}_0$  do you recommend for numerical optimization? You don't have to actually carry out this optimization (yet).

**Note:** This assignment is aimed at getting you to complete the problem description and implement the simulation code. Do not panic if your first attempt at optimization is a dismal failure – in practice, MDO codes seldom get things right first time. Throughout the semester you will have a chance to improve your formulation and implementation.

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