Engineering Risk Benefit Analysis

1.155, 2.943, 3.577, 6.938, 10.816, 13.621, 16.862, 22.82, ESD.72, ESD.721

RPRA 5. Data Analysis

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Statistical Inference

Theoretical Model

Failure distribution,
e.g., $f(t) = \lambda e^{-\lambda t}$

Evidence

Sample, e.g.,
$\{t_1, \ldots, t_n\}$

- **How do we estimate** λ **from the evidence?**
- **How confident are we in this estimate?**
- **Two methods:**
  - ***Classical (frequentist) statistics***
  - ***Bayesian statistics***

Random Samples

- **The observed values are independent and the underlying distribution is constant.**

- **Sample mean:**

$$\bar{t} = \frac{1}{n}\sum_{1}^{n} t_i$$

- **Sample variance:**

$$s^2 = \frac{1}{(n-1)}\sum_{1}^{n}(t_i - \bar{t})^2$$

The Method of Moments: Exponential Distribution

- **Set the theoretical moments equal to the sample moments and determine the values of the parameters of the theoretical distribution.**

- ***Exponential distribution:*** $\frac{1}{\lambda} = \bar{t}$

Sample: {10.2, 54.0, 23.3, 41.2, 73.2, 28.0} hrs

$$\bar{t} = \frac{1}{6}(10.2 + 54 + 23.3 + 41.2 + 73.2 + 28) = \frac{229.9}{6} = 38.32$$

$$\text{MTTF} = 38.32 \quad \text{hrs;} \qquad \lambda = \frac{1}{38.32} = 0.026 \quad \text{hr}^{-1}$$

The Method of Moments: Normal Distribution

- **Sample: {5.5, 4.7, 6.7, 5.6, 5.7}**

$$\bar{x} = \frac{(5.7 + 4.7 + 6.7 + 5.6 + 5.7)}{5} = \frac{28.4}{5} = 5.68 = \mu$$

$$\sum_{1}^{5} (x_i - \bar{x})^2 = (5.5 - 5.68)^2 + \ldots + (5.7 - 5.68)^2 = 2.032$$

$$s^2 = \frac{2.032}{(5-1)} = 0.508$$

$$s = 0.713 = \sigma$$

The Method of Moments: Poisson Distribution

- **Sample: {r events in t}**

- **Average number of events: r**

$$\lambda t = r \quad \Rightarrow \quad \lambda = \frac{r}{t}$$

- **{3 eqs in 7 years}** $\quad \Rightarrow \quad \lambda = \frac{3}{7} = 0.43 \quad yr^{-1}$

The Method of Moments: Binomial Distribution

- **Sample:** **{k 1s in n trials}**
- **Average number of 1s: k**
- **qn = k** $\Rightarrow \quad q = \frac{k}{n}$
- **{3 failures to start in 17 tests}** $q = \frac{3}{17} = 0.176$

Censored Samples and the Exponential Distribution

- ***Complete sample:*** **All n components fail.**
- ***Censored sample:*** **Sampling is terminated at time t_0 (with k failures observed) or when the r^{th} failure occurs.**
- **Define the *total operational time* as:**

$$T = \sum_{1}^{k} t_i + (n-k)t_0 \qquad\qquad T = \sum_{1}^{r} t_i + (n-r)t_r$$

- **It can be shown that:** $\lambda = \frac{k}{T}$ **or** $\lambda = \frac{r}{T}$
- **Valid for the exponential distribution <u>only</u> (no memory).**

Example

- Sample: 15 components are tested and the test is terminated when the 6th failure occurs.
- The observed failure times are:

{10.2, 23.3, 28.0, 41.2, 54.0, 73.2} hrs

- The total operational time is:

$$T = 10.2 + 23.3 + 28 + 41.2 + 54 + 73.2 + (15 - 6)73.2 = 888.7$$

- Therefore

$$\lambda = \frac{6}{888.7} = 6.75x10^{-3} \quad hr^{-1}$$

Bayesian Methods

- **Recall Bayes' Theorem (slide 16, RPRA 2):**

Likelihood of the Evidence

Prior Probability

$$P(H_i/E) = \frac{P(E/H_i)P(H_i)}{\sum_{1}^{N} P(E/H_i)P(H_i)}$$

Posterior Probability

- **Prior information can be utilized via the prior distribution.**
- **Evidence other than statistical can be accommodated via the likelihood function.**

The Model of the World

- **Deterministic, e.g., a mechanistic computer code**
- **Probabilistic (Aleatory), e.g., $R(t/\lambda) = \exp(-\lambda t)$**
- **The MOW deals with <u>observable</u> quantities.**
- **Both deterministic and aleatory models of the world have assumptions and parameters.**
- **How confident are we about the validity of these assumptions and the numerical values of the parameters?**

The Epistemic Model

- **Uncertainties in assumptions are not handled routinely. If necessary, sensitivity studies are performed.**

- **The epistemic model deals with non-observable quantities.**

- **Parameter uncertainties are reflected on appropriate probability distributions.**

- **For the failure rate:** $\pi(\lambda)\, d\lambda$ **= Pr(the failure rate has a value in dλ about λ)**

Unconditional (predictive) probability

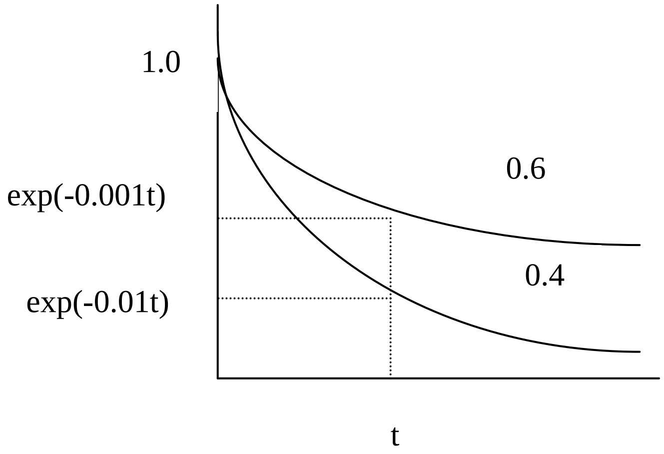
$$\mathbf{R(t) = \int R(t/\lambda)\pi(\lambda)d\lambda}$$

Communication of Epistemic Uncertainties: The discrete case

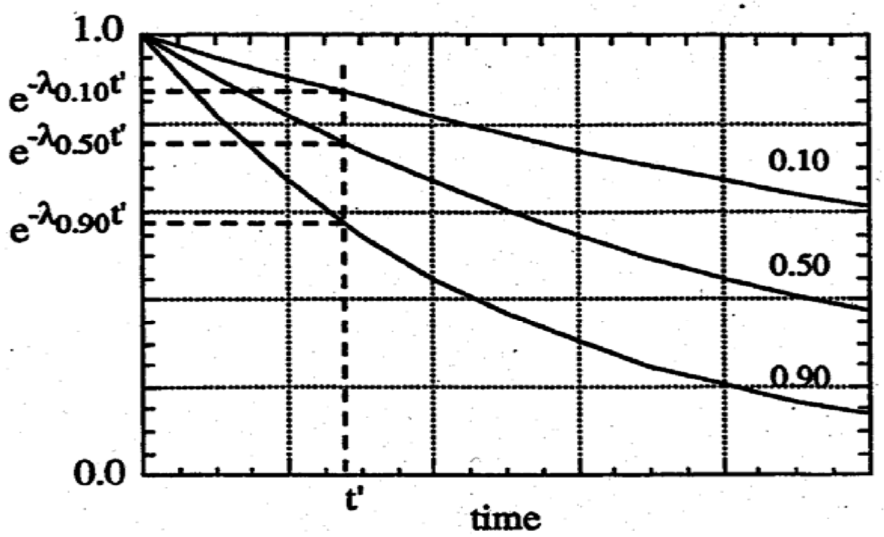
Suppose that $P(\lambda = 10^{-2}) = 0.4$ and $P(\lambda = 10^{-3}) = 0.6$

Then, $P(e^{-0.001t}) = 0.6$ and $P(e^{-0.01t}) = 0.4$

$R(t) = 0.6\, e^{-0.001t} + 0.4\, e^{-0.01t}$



Communication of Epistemic Uncertainties: The continuous case



Risk Curves

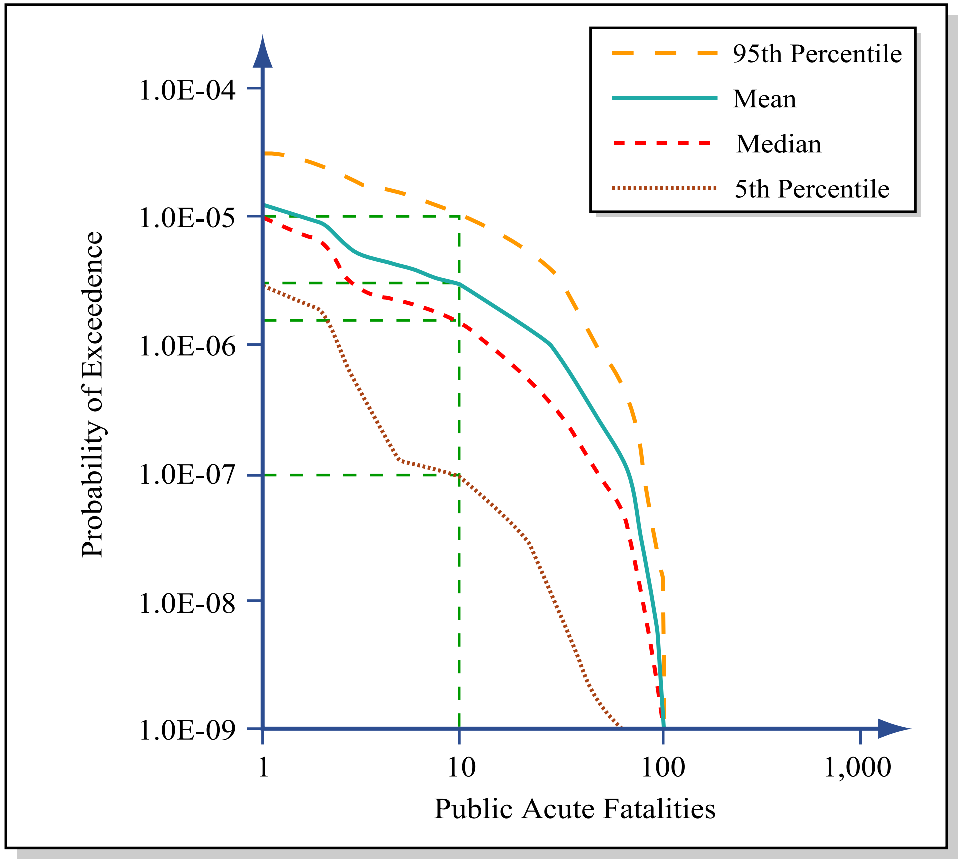


Figure by MIT OCW.

The Quantification of Judgment

- Where does the epistemic distribution $\pi(\lambda)$ come from?
- Both substantive and normative "goodness" are required.
- Direct assessments of parameters like failure rates should be avoided.
- A reasonable measure of central tendency to estimate is the median.
- Upper and lower percentiles can also be estimated.

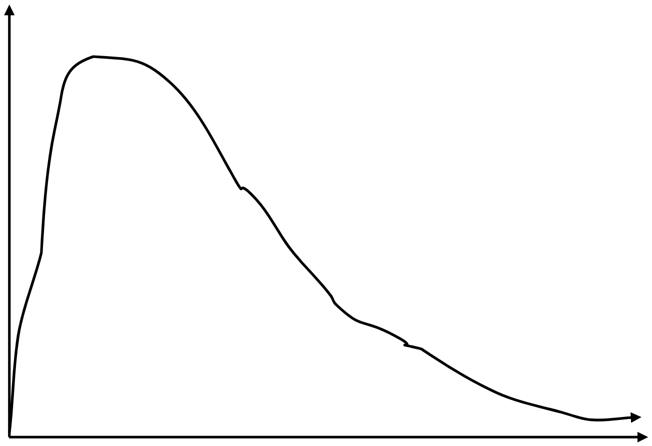
The lognormal distribution

- It is very common to use the lognormal distribution as the epistemic distribution of failure rates.

$$\pi(\lambda) = \frac{1}{\sqrt{2\pi}\sigma\lambda} \exp\left[-\frac{(\ln\lambda - \mu)^2}{2\sigma^2} \right]$$

UB $= \lambda_{95} = \exp(\mu + 1.645\sigma)$

LB $= \lambda_{05} = \exp(\mu - 1.645\sigma)$



Mechanical Hardware

Component/Primary Failure Modes	Assessed Values Lower Bound	Assessed Values Upper Bound
Pumps		
Failure to start, Q_d:	3×10^{-4}/d	3×10^{-3}/d
Failure to run, λ_o: (Normal Environments)	3×10^{-6}/hr	3×10^{-4}/hr
Valves		
Motor Operated		
Failure to operate, Q_d:	3×10^{-4}/d	3×10^{-3}/d
Plug, Q_d:	3×10^{-5}/d	3×10^{-4}/d
Solenoid Operated		
Failure to operate, Q_d:	3×10^{-4}/d	3×10^{-3}/d
Plug, Q_d:	3×10^{-5}/d	3×10^{-4}/d
Air Operated		
Failure to operate, Q_d:	1×10^{-4}/d	1×10^{-3}/d
Plug, Q_d:	3×10^{-5}/d	3×10^{-4}/d
Check		
Failure to open, Q_d:	3×10^{-5}/d	3×10^{-4}/d
Relief		
Failure to open, Q_d:	3×10^{-6}/d	3×10^{-5}/d
Manual		
Plug, Q_d:	3×10^{-5}/d	3×10^{-4}/d
Pipe		
Plug/rupture		
< 3" diameter, λ_o:	3×10^{-11}/hr	3×10^{-8}/hr
> 3" diameter, λ_o:	3×10^{-12}/hr	3×10^{-9}/hr
Clutches		
Mechanical		
Failure to engage/disengage	1×10^{-4}/d	1×10^{-3}/d

Table by MIT OCW.

Adapted from Rasmussen, et al. "The Reactor Safety Study." WASH-1400, US Nuclear Regulatory Commission, 1975.

Electrical Hardware

Component/Primary Failure Modes	Assessed Values Lower Bound	Assessed Values Upper Bound
Electrical Clutches		
Failure to operate, Q_d:	1×10^{-4}/d	1×10^{-3}/d
Motors		
Failure to start, Q_d:	1×10^{-4}/d	1×10^{-3}/d
Failure to run (Normal Environments), λ_o:	3×10^{-6}/hr	3×10^{-5}/hr
Transformers		
Open/shorts, λ_o:	3×10^{-7}/hr	3×10^{-6}/hr
Relays		
Failure to energize, Q_d:	3×10^{-5}/d	3×10^{-4}/d
Circuit Breaker		
Failure to transfer, Q_d:	3×10^{-4}/d	3×10^{-3}/d
Limit Switches		
Failure to operate, Q_d:	1×10^{-4}/d	1×10^{-3}/d
Torque Switches		
Failure to operate, Q_d:	3×10^{-5}/d	3×10^{-4}/d
Pressure Switches		
Failure to operate, Q_d:	3×10^{-5}/d	3×10^{-4}/d
Manual Switches		
Failure to operate, Q_d:	3×10^{-6}/d	3×10^{-5}/d
Battery Power Supplies		
Failure to provide proper output, λ_s:	1×10^{-6}/hr	1×10^{-5}/hr
Solid State Devices		
Failure to function, λ_o:	3×10^{-7}/hr	3×10^{-5}/hr
Diesels (complete plant)		
Failure to start, Q_d:	1×10^{-2}/d	1×10^{-1}/d
Failure to run, λ_o:	3×10^{-4}/hr	3×10^{-2}/hr
Instrumentation		
Failure to operate, λ_o:	1×10^{-7}/hr	1×10^{-5}/hr

a. All values are rounded to the nearest half order of magnitude on the exponent.

b. Derived from averaged data on pumps, combining standby and operate time.

c. Approximated from plugging that was detected.

d. Derived from combined standby and operate data.

e. Derived from standby test on batteries, which does not include load.

Table by MIT OCW.

Adapted from Rasmussen, et al. "The Reactor Safety Study." WASH-1400, US Nuclear Regulatory Commission, 1975.

Example

Lognormal prior distribution with median and 95th percentile given as:

$$\lambda_{50} = \exp(\mu) = 3\text{x}10^{-3}\,\text{hr}^{-1}$$

$$\lambda_{95} = \exp(\mu + 1.645\sigma) = 3\text{x}10^{-2}\,\text{hr}^{-1}$$

Then $\mu = -5.81, \quad \sigma = 1.40$

$$E[\lambda] = \exp(\mu + \frac{\sigma^2}{2}) = 8\text{x}10^{-3}\,\text{hr}^{-1}$$

$$\lambda_{05} = \exp(\mu - 1.645\sigma) = 3\text{x}10^{-4}\,\text{hr}^{-1}$$

Updating Epistemic Distributions

- **Bayes' Theorem allows us to incorporate new evidence into the epistemic distribution.**

$$\pi'(\lambda / E) = \frac{L(E/\lambda)\pi(\lambda)}{\int L(E/\lambda)\pi(\lambda)d\lambda}$$

Example of Bayesian updating of epistemic distributions

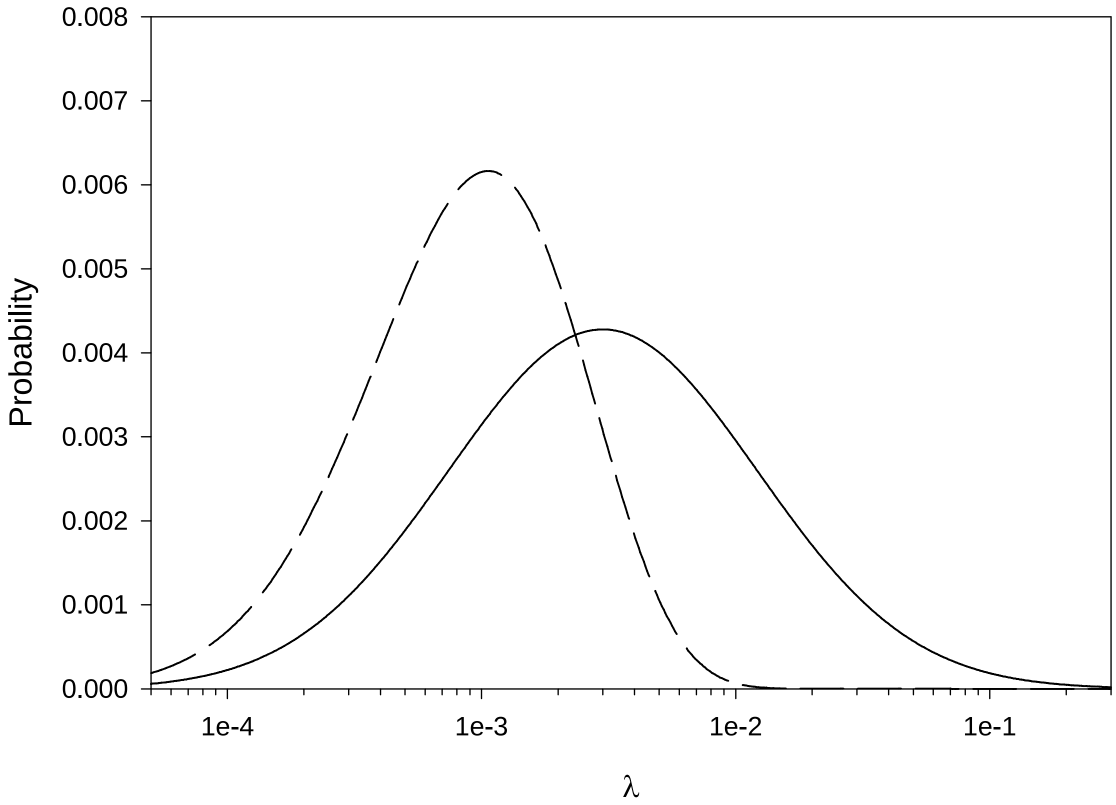
- ***Five components were tested for 100 hours each and no failures were observed.***

- **Since the reliability of each component is exp(-100 λ), the likelihood function is:**

- **L(E/ λ) = P(comp. 1 did not fail <u>AND</u> comp. 2 did not fail <u>AND</u>… comp. 5 did not fail) = exp(-100λ) x exp(-100 λ) x…x exp(-100 λ) = exp(-500 λ)**

- **L(E/ λ) = exp(-500 λ)**
  - ***<u>Note</u>: The classical statistics point estimate is zero since no failures were observed.***

Prior (—) and posterior (-------) distributions



Impact of the evidence

	Mean (hr^{-1})	95^{th} (hr^{-1})	Median (hr^{-1})	5^{th} (hr^{-1})
Prior distr.	$8.0x10^{-3}$	$3.0x10^{-2}$	$3x10^{-3}$	$3.0x10^{-3}$
Posterior distr.	$1.3x10^{-3}$	$3.7x10^{-3}$	$9x10^{-4}$	$1.5x10^{-4}$

Selected References

- ***Proceedings of Workshop on Model Uncertainty: Its Characterization and Quantification*, A. Mosleh, N. Siu, C. Smidts, and C. Lui, Eds., Center for Reliability Engineering, University of Maryland, College Park, MD, 1995.**
- ***Reliability Engineering and System Safety*, Special Issue on the Treatment of Aleatory and Epistemic Uncertainty, J.C. Helton and D.E. Burmaster, Guest Editors., vol. 54, Nos. 2-3, Elsevier Science, 1996.**
- **Apostolakis, G., "The Distinction between Aleatory and Epistemic Uncertainties is Important: An Example from the Inclusion of Aging Effects into PSA," *Proceedings of PSA '99, International Topical Meeting on Probabilistic Safety Assessment*, pp. 135-142, Washington, DC, August 22 - 26, 1999, American Nuclear Society, La Grange Park, Illinois.**