Engineering Risk Benefit Analysis

1.155, 2.943, 3.577, 6.938, 10.816, 13.621, 16.862, 22.82, ESD.72, ESD.721

DA 6. Multiattribute Utility Theory

George E. Apostolakis

Massachusetts Institute of Technology

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Consequences

- **Not all the consequences are monetary.**
- **In risk management problems, for example, they may include the impact on health and safety of groups of stakeholders.**
- **In general, the consequences are described by a vector $\underline{x} \equiv (x_1, \ldots, x_N)$.**

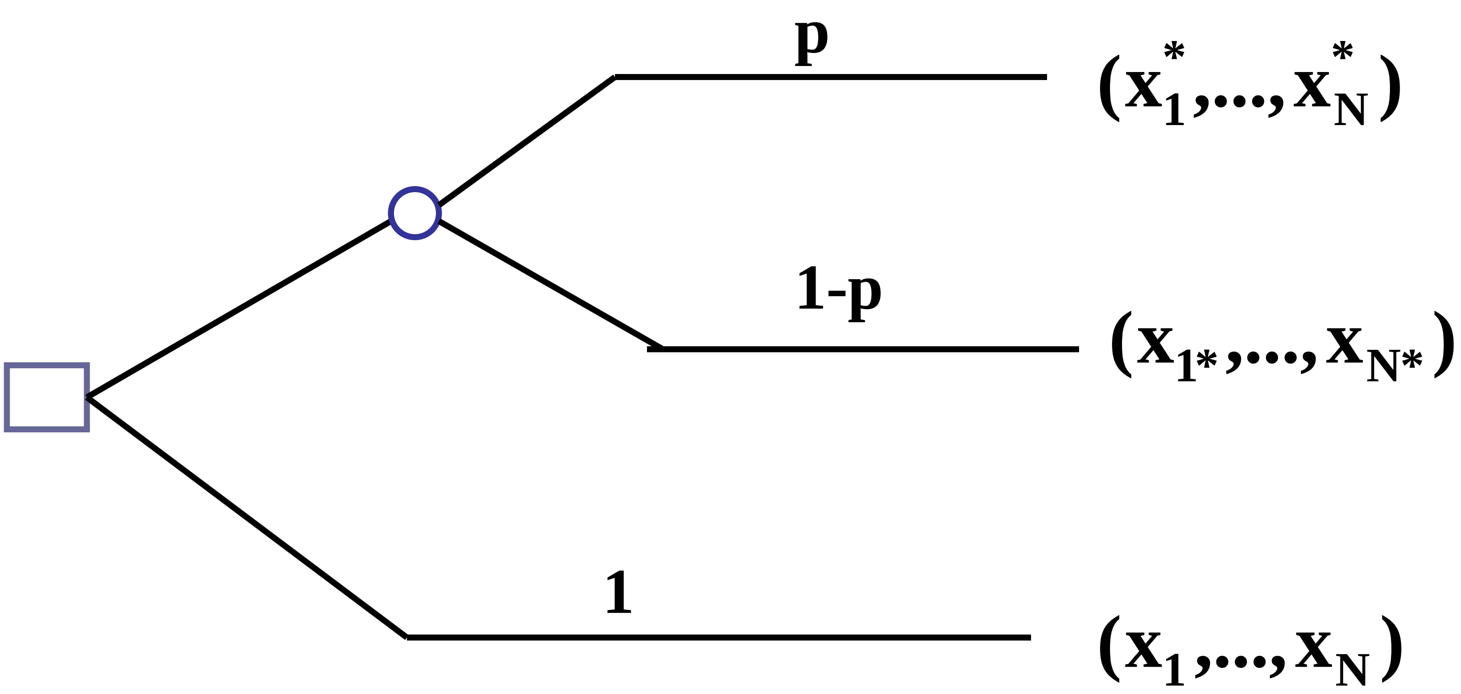
Multiattribute Utility

- $u(x_1, \ldots, x_N)$
- **Decision alternative A_i is preferred over alternative A_k if and only if its expected utility is greater, i.e.,**

$$E_i[u] > E_k[u] \quad \Leftrightarrow \quad A_i \succ A_k$$

Finding $u(x_1, \ldots, x_N)$

Use the certainty-equivalent approach:



Independence Assumptions

- Finding the multiattribute utility function using the preceding method is very burdensome.
- Can we find a function f such that

$$u(x_1, \ldots, x_N) = f[u_1(x_1), \ldots , u_N(x_N)]$$

where $u_i(x_i)$ is the utility function of attribute x_i?

- The answer is "yes," if we can establish "independence" among the attributes.

Mutual Preferential Independence

- **Attribute Y is preferentially independent of attribute Z, if preferences for y levels do not depend on the level of z, i.e.,**

$$(y, z^0) \succ (y', z^0)$$

implies

$$(y, z) \succ (y', z) \qquad \forall z$$

where y and y' are two levels of y.

Mutual Preferential Independence: Example

- **Y: Departure time (morning, afternoon)**
- **Z: Ticket cost ($300, $500)**

- **If you prefer "afternoon" to "morning" departure regardless of the price of the ticket, and you prefer $300 to $500 regardless of the departure time, then Y and Z are mutually preferentially independent.**

- **If you prefer "afternoon" to "morning" departure regardless of the price of the ticket, but the price depends on when you leave, then Y is preferentially independent of Z, but they are not mutually preferentially independent.**

Utility Independence (1)

- It is similar to preferential independence, except that the assessments are made with uncertainty present. It is a stronger assumption.
- Y is utility independent of Z if preferences over lotteries involving different levels of Y do not depend on a fixed level of Z.
- For the previous example: The preference value of the lottery L(morning, afternoon; 0.5, 0.5) is independent of the price of the ticket.
- The CEs of lotteries on Y levels are independent of z.

Utility Independence (2)

- **A form of the utility function for attributes X_1 and X_2 that are utility independent, is**

$$u(x_1, x_2) =$$
$$= k_1 u_1(x_1) + k_2 u_2(x_2) + (1 - k_1 - k_2)\, u_1(x_1)\, u_2(x_2)$$

with $0 \le k_i \le 1 \qquad i = 1,2$

$$0 \le u_i(x_i) \le 1 \qquad i = 1,2$$

Utility Independence (3)

- **Fix the level of X_2 at x_2', then**

$u(x_1, x_2') =$

$$= k_1 u_1(x_1) + k_2 u_2(x_2') + (1 - k_1 - k_2)\, u_1(x_1)\, u_2(x_2')$$

$$= [k_1 + (1 - k_1 - k_2)\, u_2(x_2')]\, u_1(x_1) + k_2 u_2(x_2')$$

- **This is a linear transformation of $u_1(x_1)$, therefore, the preferences over levels of X_1 are independent of the level of X_2.**
- **For another level of X_2, we will get another linear transformation of $u_1(x_1)$. $\Rightarrow$**
- **Lotteries on X_1 are independent of the level of X_2.**

Utility Independence (4)

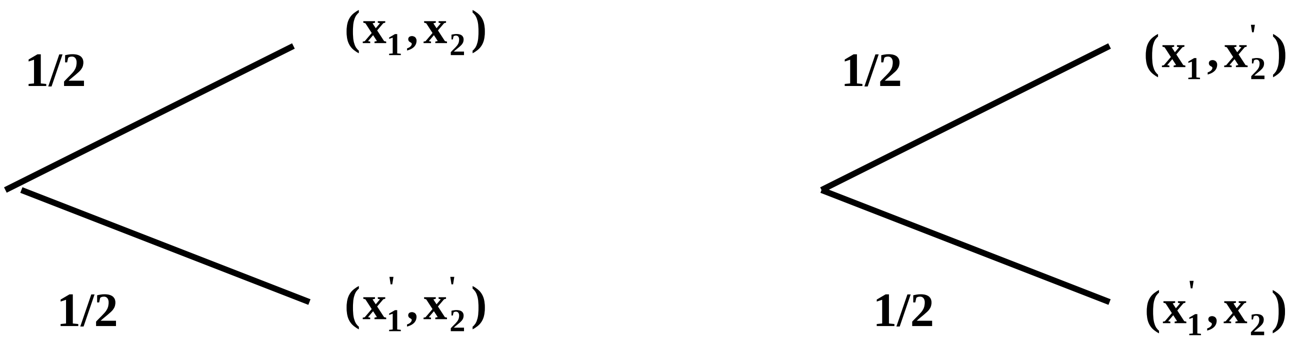
- **When X_1 and X_2 are utility independent of each other, they are *mutually* utility independent.**

- **$u(x_1, x_2) = g(x_1) + h(x_1)u_2(x_2)$**

- **X_2 is utility independent of X_1 but not vice versa.**

Additive Independence

- A stronger assumption than utility independence.
- For two attributes, we must be indifferent between



x_1' and x_1 are different levels of x_1

We can get any pair of consequences with probability 0.5; the only difference is how the levels are combined.

Additive Utility Function

$$u(\underline{x}) = \sum_{1}^{N} k_i u_i(x_i)$$

where

$$0 \leq k_i \leq 1$$

$$\sum_{1}^{N} k_i = 1$$

$$0 \leq u_i(x_i) \leq 1$$

Two attributes: $u(x_1, x_2) = k\, u_1(x_1) + (1-k)\, u_2(x_2)$

Additive Independence: Implications

- **When we assess the utility of one attribute, it should not matter what the other attribute's level is.**
- **Interaction among the attributes is not allowed.**
- **For cases with no or little uncertainty, additive independence represents reasonably well people's utilities.**
- **For complex problems, it could be a useful first-cut approximation.**