Engineering Risk Benefit Analysis

1.155, 2.943, 3.577, 6.938, 10.816, 13.621, 16.862, 22.82, ESD.72, ESD.721

DA 2. The Value of Perfect Information

George E. Apostolakis

Massachusetts Institute of Technology

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Recall the evaluation of the survey results (Slide 14, DA 1):

Strong

$P(s/L_2) = 0.8$ $P(s/L_3) = 0.2$ $P(s/L_4) = 0.0$

Mild

$P(m/L_2) = 0.2$ $P(m/L_3) = 0.6$ $P(m/L_4) = 0.3$

Weak

$P(w/L_2) = 0.0$ $P(w/L_3) = 0.2$ $P(w/L_4) = 0.7$

1.0 1.0 1.0

Perfect Information (Clairvoyant)

- **A clairvoyant, CV, is <u>always correct</u>, i.e.,**

  P[CV says L_2 /L_2 materializes] = 1.0 = P[s/L_2]
  P[CV says L_3 /L_2 materializes] = 0.0 = P[m/L_2]
  P[CV says L_4 /L_2 materializes] = 0.0 = P[w/L_2]

- **Receiving the CV's report removes all uncertainty.**

Calculations for "survey result is s" or "survey says L_2" (Slide 18, DA 1)

Payoff	Prior Prob.	Likelihood	Product	Posterior Probability
L_2	0.3	$P(s/ L_2)=0.8$	0.24	$P(L_2/s)=0.706$
L_3	0.5	$P(s/ L_3)=0.2$	0.10	$P(L_3/s)=0.294$
L_4	0.2	$P(s/ L_4)=0.0$	0.00	$P(L_4/s)=0.000$
	1.0		0.34	1.000

$P[L_2$ materializes/survey says $L_2] = 0.706$, because the survey is not perfect.

Bayes' Theorem for the Clairvoyant

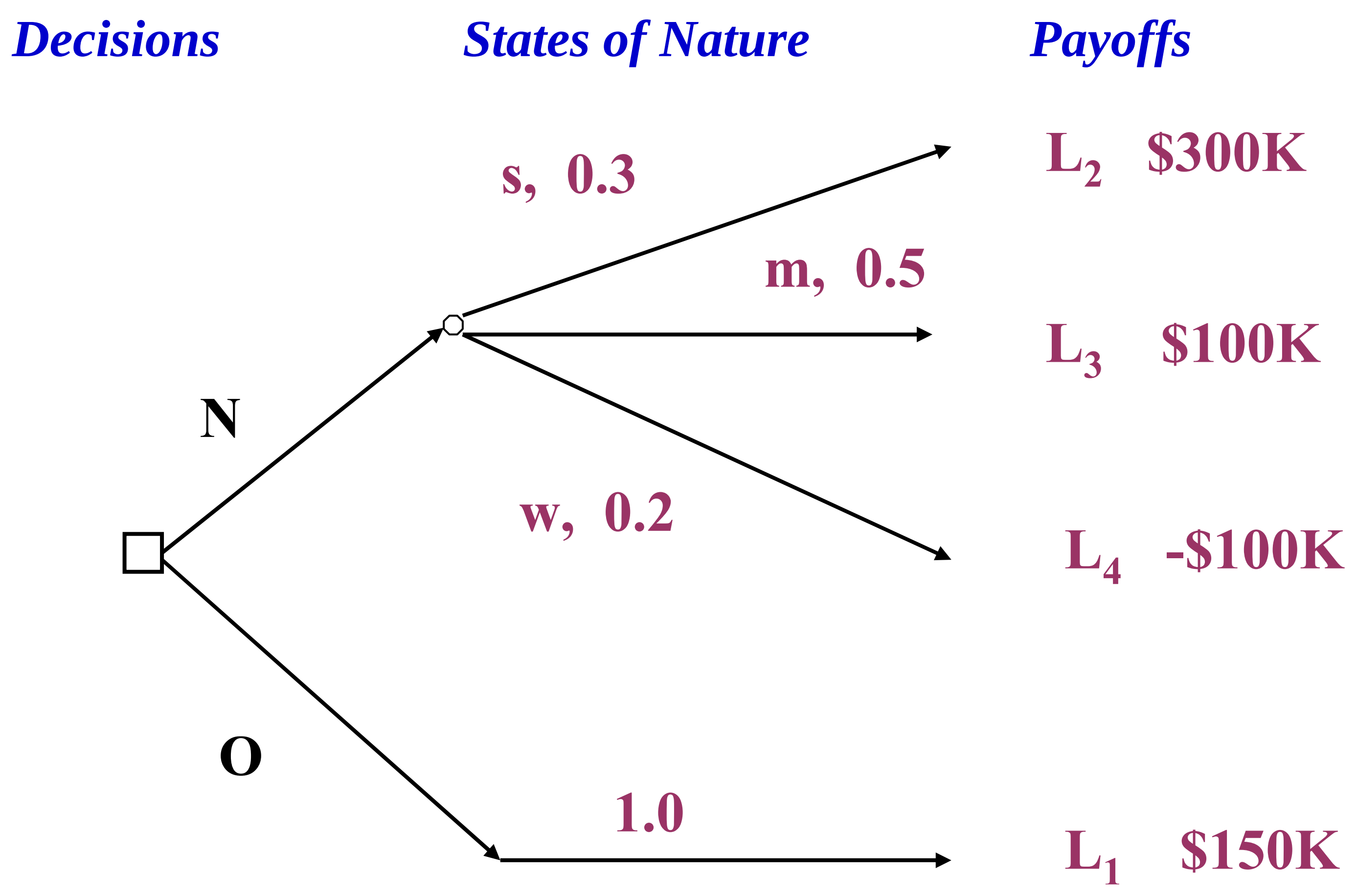
$$P[L_2 \text{ materializes/ CV says } L_2] =$$

$$= \frac{P(\text{CVsays}L_2 / L_2\text{materializes}) x P(L_2\text{materializes})}{\sum_2^4 P(\text{CVsays}L_2 / L_i\text{materializes}) x P(L_i\text{materializes})} =$$

$$= \frac{1xP(L_2\text{materializes})}{1xP(L_2\text{materializes}) + 0 + 0} = 1$$

$P[L_2$ materializes/ CV says $L_2] = 1$ regardless of the prior probability, because the CV is perfect.

The original decision tree

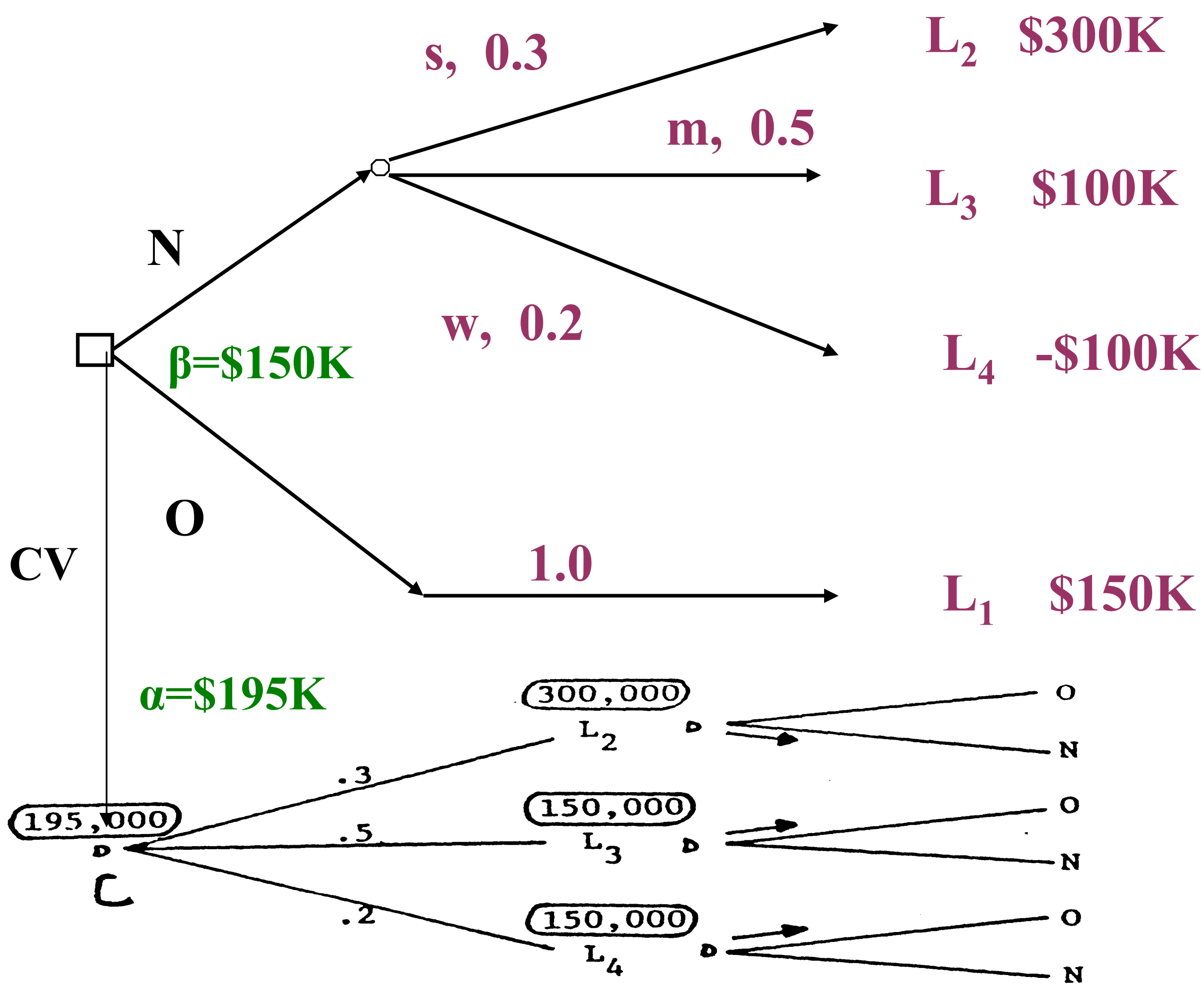


Modifications

In a decision tree, the order of the nodes is chronological.

- With perfect information, the uncertainty is resolved before the decision is made (a chance node is followed by a decision node).
- The evaluation is done a priori (before the CV is hired).
- Therefore, the DM believes that the CV will predict L_2 with probability 0.3, L_3 with probability 0.5, and L_4 with probability 0.2.

Decision tree with a clairvoyant



The value of alpha

α: EMV, if the terminal decision is to be made with perfect information at no cost.

$$\alpha = 0.3x300 + 0.5x150 + 0.2x150 = \$195K$$

The value of beta

- **What is the EMV without any information?**
- **We solved this problem in DA 1 (original decision tree).**

EMV[no information] = \$150K ≡ β

β: EMV, if the terminal decision is to be made without any opportunity to obtain additional information.

<u>Note:</u>The chance node follows the decision node.

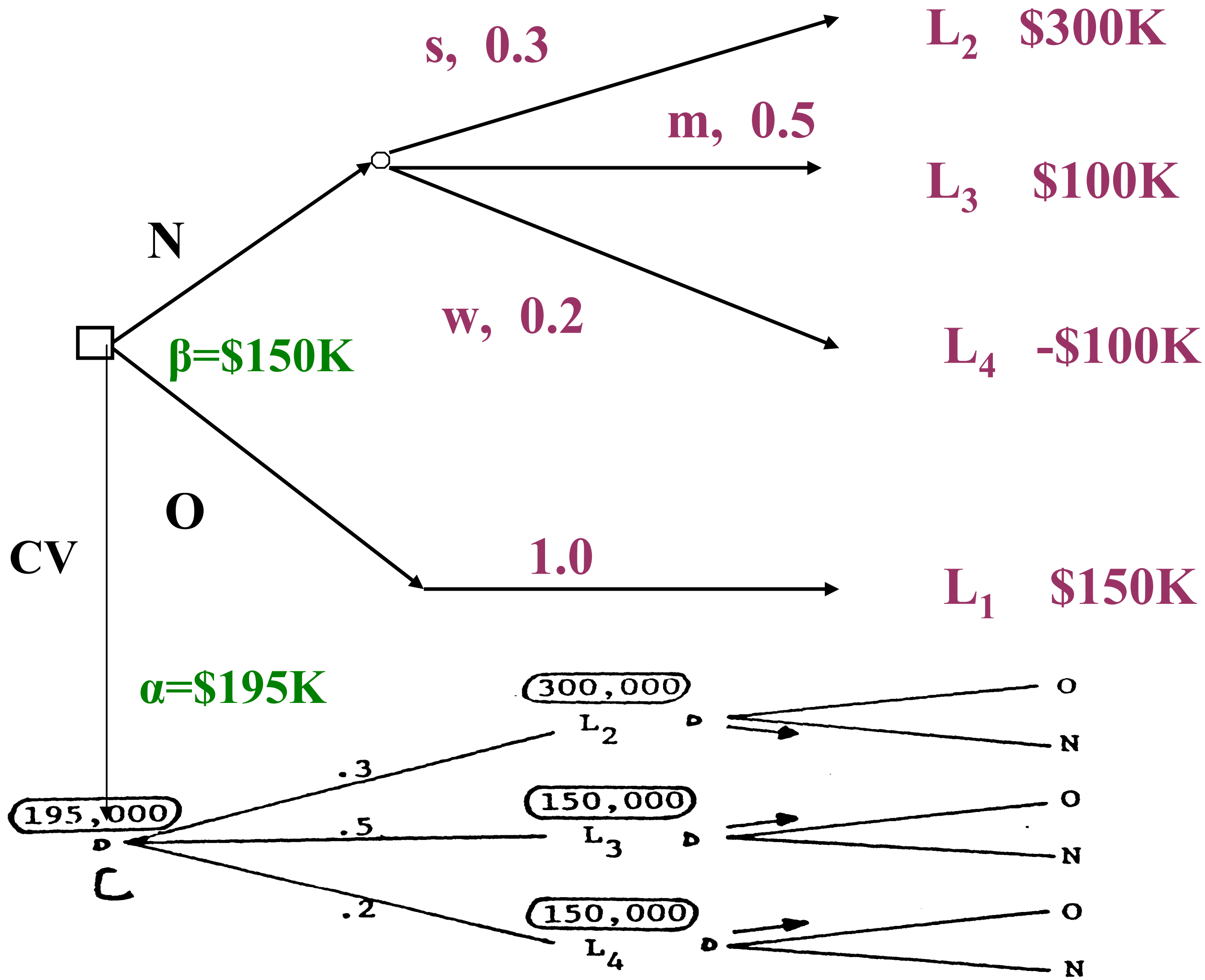
Expected Value of Perfect Information (EVPI)

$$EVPI \equiv \alpha - \beta = \$195 - \$150 = \$45K$$

- The EVPI is an upper bound on the amount the DM would be willing to pay for additional information.

- The expected value of any information source must be between zero and the EVPI. In DA 1, the cost of the survey was $20K < EVPI.

Decision tree with a clairvoyant



General Tree

- **If the DM faces uncertainty in a decision (uncertainty nodes after the decision node), the impact of perfect information will be evaluated by *redrawing* the tree and reordering the decision and chance nodes.**

- **The evaluation of perfect information is done a priori. The DM has not yet consulted the clairvoyant. The DM is considering whether to actually do it.**

Summary and Observations

- **We have developed single-attribute, multi-stage sequential Decision Trees.**
- **The model is useful to a *single* decision maker.**
- **Decision Criterion: Maximize the EMV.**
- **Maximizing the EMV is *not* the best decision criterion.**