Engineering Risk Benefit Analysis

1.155, 2.943, 3.577, 6.938, 10.816, 13.621, 16.862, 22.82, ESD.72, ESD.721

DA 1. The Multistage Decision Model

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Why decision analysis?

A structured way for ranking decision options by:

1. **Enumerating the immediate and later choices available to the DM**
2. **Characterizing the relevant uncertainties**
3. **Quantifying the relative desirability of outcomes**
4. **Providing rules for ranking the decision options, thus helping the DM to select the "best" one.**

Value of formal analysis

- Provides a systematic way to process large amounts of information
- Decision making process is explicit and enhances communication
- Provides formal rules for quantifying preferences

Limitations of DA

- The theory is for an individual decision maker. This reduces considerably its applicability in practice. (But, great normative tool.) In most cases there is no satisfactory way to combine the utility function of several people
- As with all formal analysis, the results are no better than the quality of the model and its supporting assessments
- The required inputs may not be easily obtainable

Manufacturing Example

- **<u>Decision:</u>** **To continue producing old product (O) or convert to a new product (N).**

The payoffs depend on the market conditions:

s: strong market for the new product

m: mild market for the new product

w: weak market for the new product

Manufacturing Example Payoffs

- **Earnings (payoffs):**

L_1: $150,000, old product, $P[L_1/O] = 1.0$

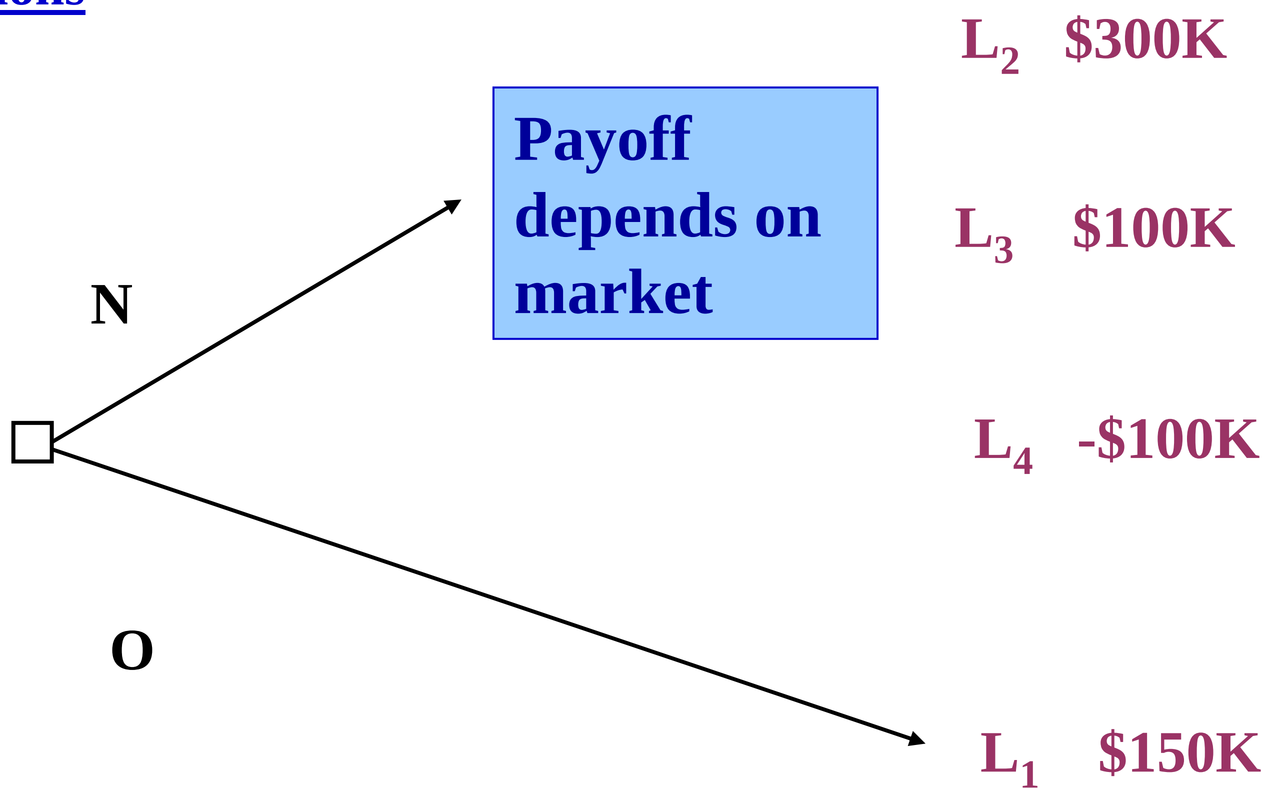
L_2: $300,000, new product and the market is strong, $P[s] = P[L_2/N] = 0.3$

L_3: $100,000, new product and the market is mild, $P[m] = P[L_3/N] = 0.5$

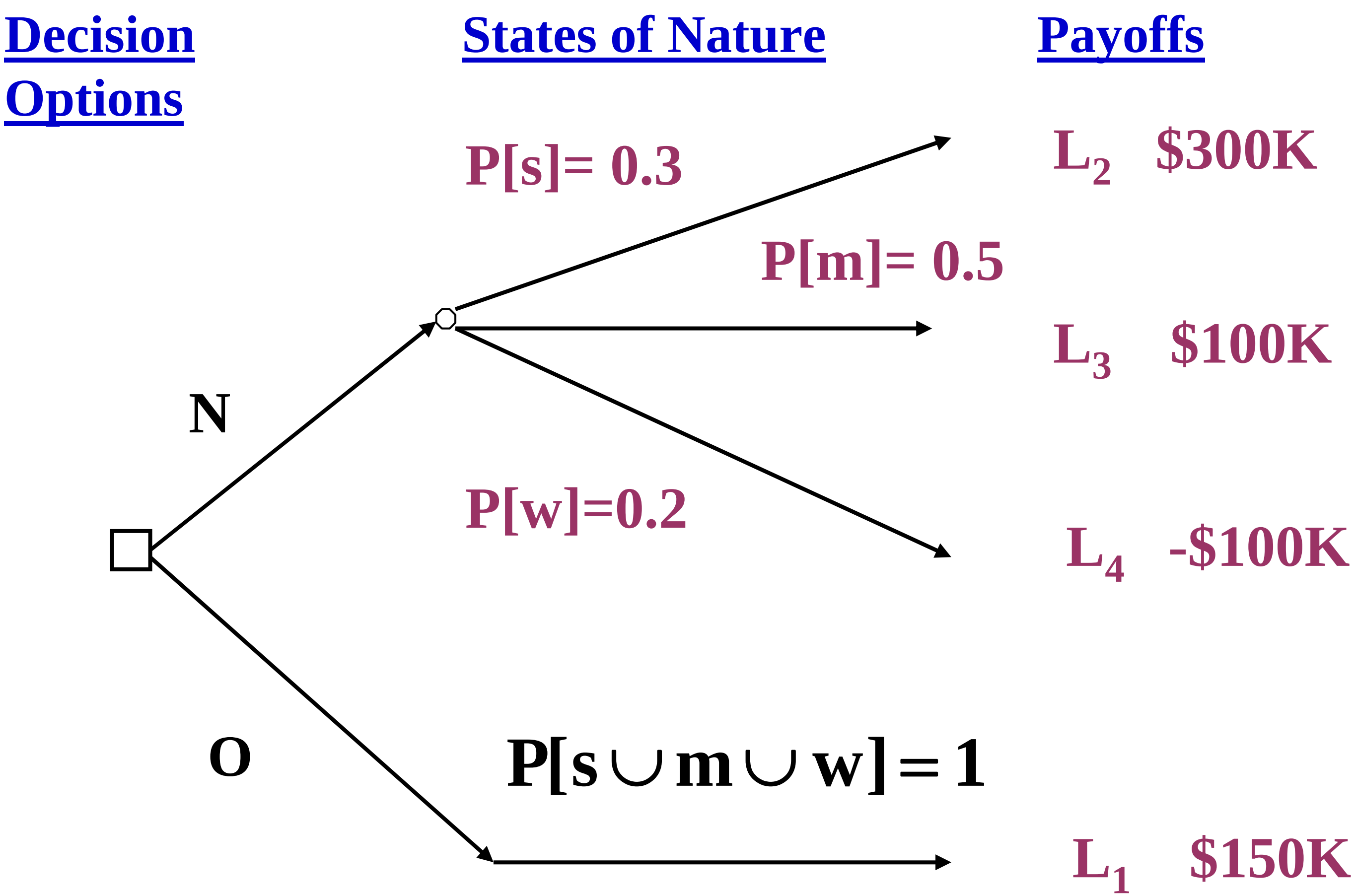
L_4: -$100,000, new product and the market is weak, $P[w] = P[L_4/N] = 0.2$

Building the decision tree

Decision Options | **Payoffs**



Decision Tree



$$P[s \cup m \cup w] = 1$$

Non-Probabilistic Decision Rules

- **Maximin Rule:** **Choose option with the largest smallest payoff (*risk averse* DM).**

  N: -$100 O: $150

  Choose O

- **Maximax Rule:** **Choose option with the largest payoff (*risk taker*).**

  N: $300 O: $150

  Choose N

Probabilistic Decision Rule

- **The maximin and maximax rules are incomplete because they ignore uncertainties.**
- **We include probabilities by taking expected values of the payoffs (slide 31, RPRA 2).**
- **Decision Rule: Maximize the *expected monetary value (EMV)* of the earnings (payoffs).**
- **In the decision tree, work from right to left and compute expectations.**

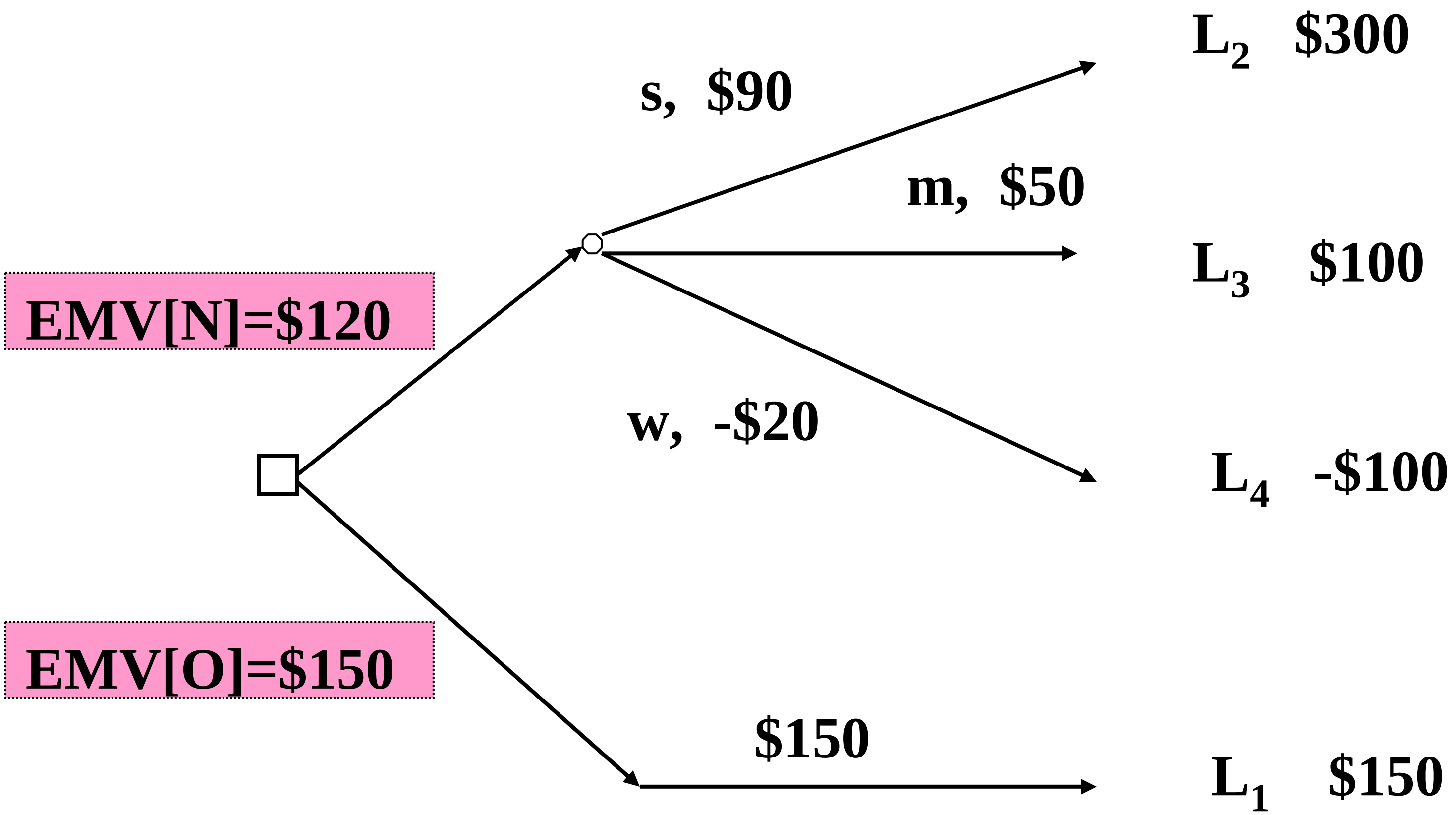
Calculation of the EMV

EMV[N] = 0.3x300 + 0.5x100 + 0.2x(-100) = $120K

EMV[O] = 1.0x150 = $150K

Option O has the largest EMV, therefore it should be chosen.

Calculation of the EMV (cont'd)



Best Decision: O

A New Decision

- **The DM considers the possibility of commissioning a survey to be able to better judge the future market.**
- **The survey costs $20,000.**
- **There are now *two* decisions (*multistage model)*:**
  - **The <u>initial</u> decision of whether to buy the survey**
  - **The <u>terminal</u> decision of whether to market the new product.**

The survey results can be:

Strong

$P(s/L_2) = 0.8$ $P(s/L_3) = 0.2$ $P(s/L_4) = 0.0$

Mild

$P(m/L_2) = 0.2$ $P(m/L_3) = 0.6$ $P(m/L_4) = 0.3$

Weak

$P(w/L_2) = 0.0$ $P(w/L_3) = 0.2$ $P(w/L_4) = 0.7$

1.0 1.0 1.0

The new decision tree

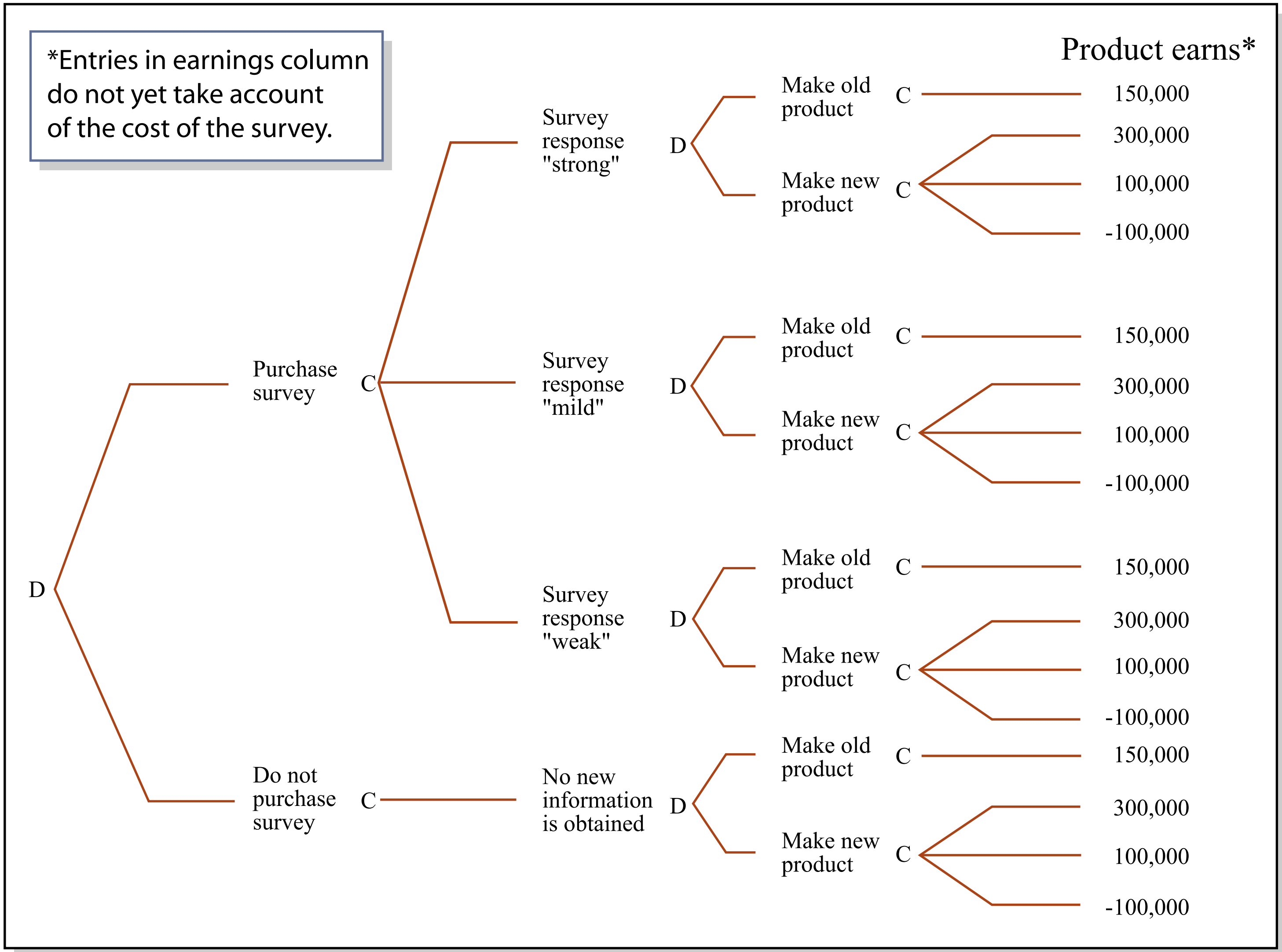


Figure by MIT OCW.

New inputs

- **The earnings must be reduced by the survey cost of \$20K:** $L_1 = \$130K$, $L_2 = \$280K$, $L_3 = \$80K$, $L_4 = -\$120K$

- **The probabilities of the states of nature (the probabilities of earnings) must also be updated to reflect the survey findings.**

Bayes' Theorem (Slide 16, RPRA 2)

- **The mutually exclusive and exhaustive states are: L_2, L_3, and L_4**

- **<u>Evidence</u>: "Survey result is strong"**

$$P(L_j / s) = \frac{P(s / L_j)P(L_j)}{\sum_{2}^{4} P(s / L_j)P(L_j)}$$

$$j = 2,3,4$$

Calculations for "survey result is s"

Payoff	Prior Prob.	Likelihood	Product	Posterior Probability
L_2	0.3	$P(s/\ L_2)=0.8$	0.24	$P(L_2/s)=0.706$
L_3	0.5	$P(s/\ L_3)=0.2$	0.10	$P(L_3/s)=0.294$
L_4	0.2	$P(s/\ L_4)=0.0$	0.00	$P(L_4/s)=0.000$
	1.0		0.34	1.000

Results for m and w

$P(L_2/m) = 0.143$

$P(L_3/m) = 0.714$

$P(L_4/m) = \underline{0.143}$

1.000

$P(L_2/w) = 0.000$

$P(L_3/w) = 0.417$

$P(L_4/w) = \underline{0.583}$

1.000

The updated decision tree

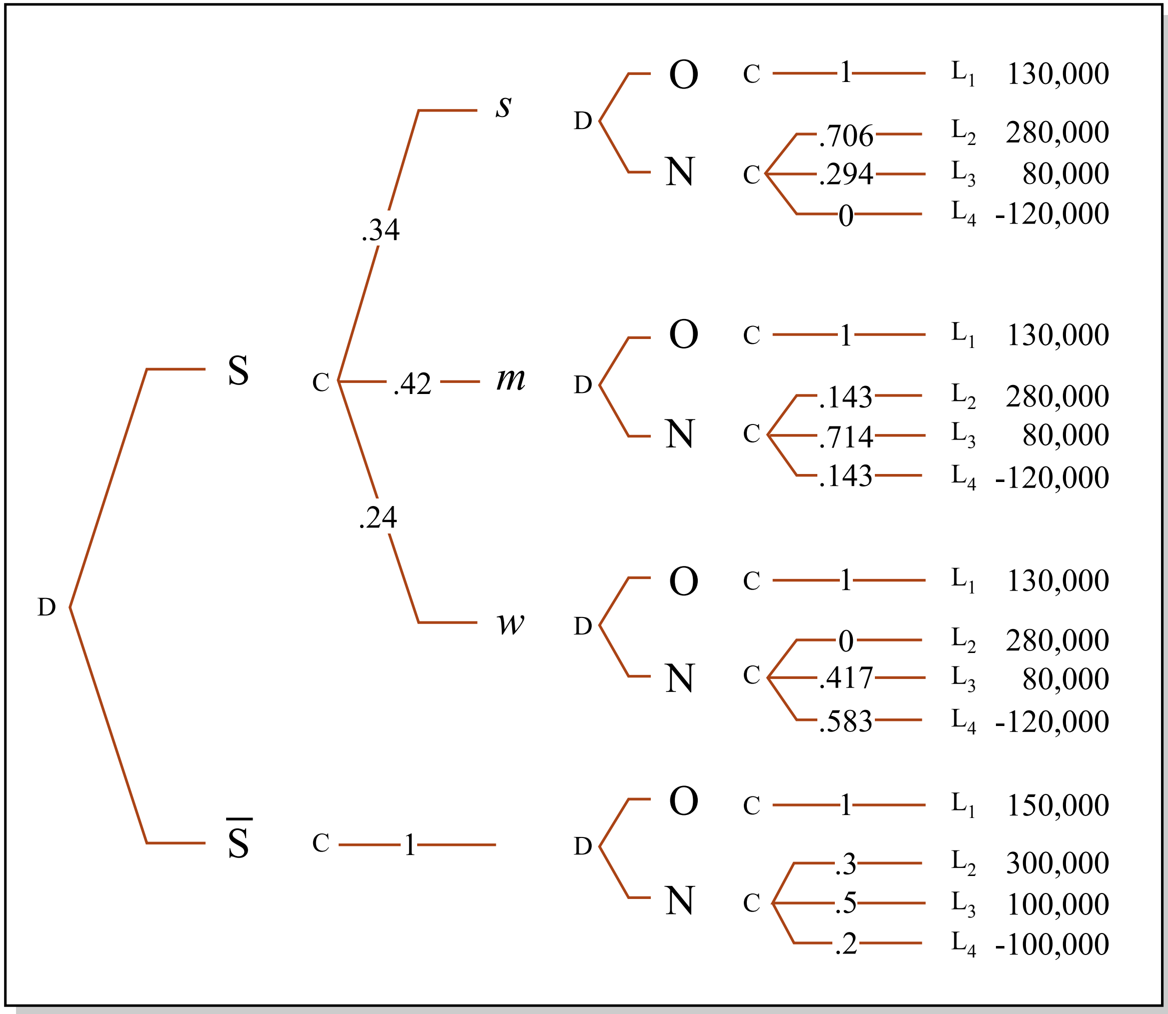


Figure by MIT OCW.

Optimal terminal decisions

1. **Solve "backwards in time."**

2. **Determine the best solution at every terminal node, conditional on the DM's being there.**

3. **Find the EMV for each terminal node and the decision option that maximizes the EMV.**

4. **An arrow indicates the best decision for each terminal node.**

Decision tree solution

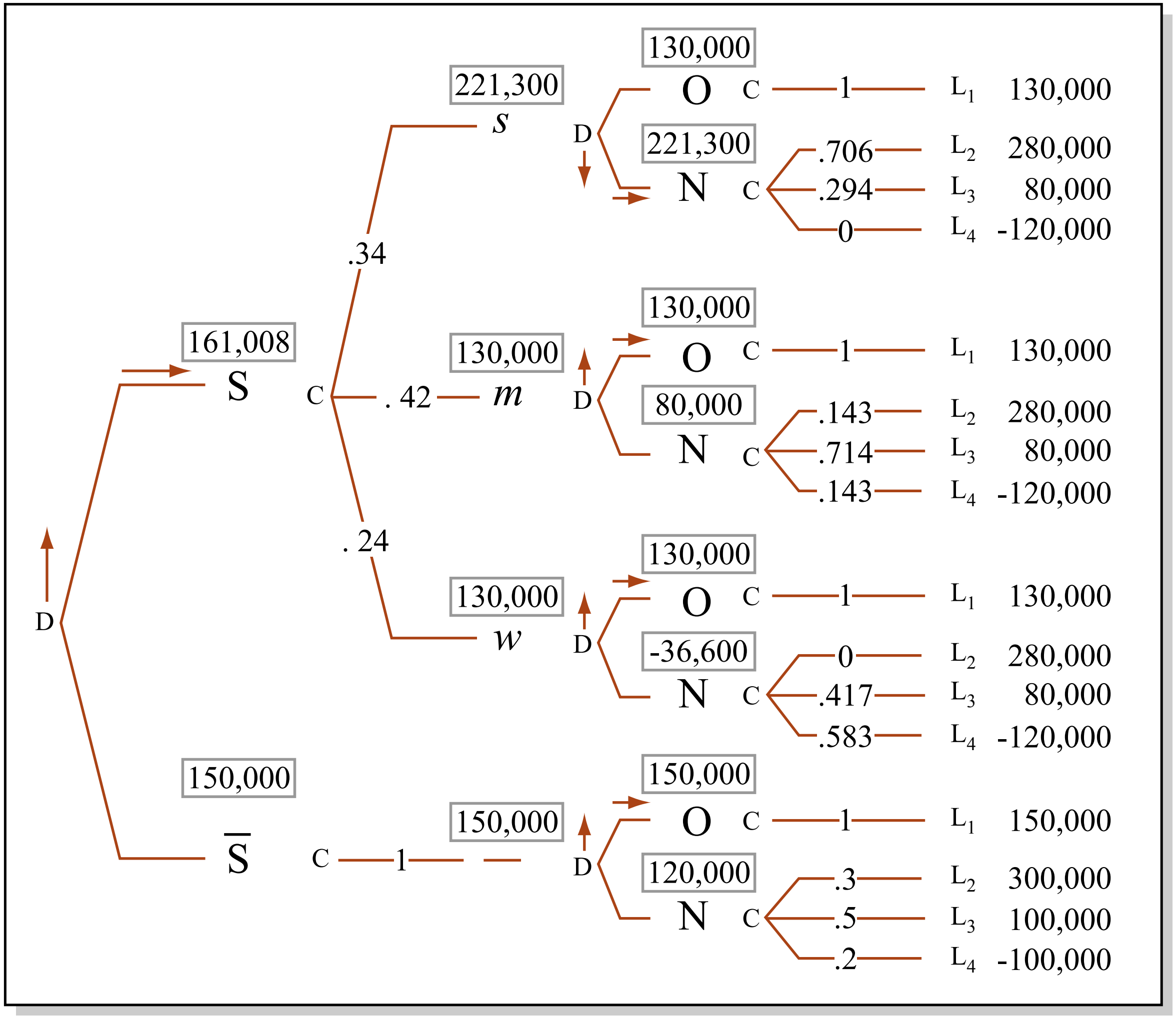


Figure by MIT OCW.

Best decision

1. **Assume that the DM makes the best decision at each terminal node, if it is reached.**
2. **Find the EMV for the initial node.**
3. **Buy survey: EMV = $161,008**

   Do not buy survey: EMV = $150,000
4. **Best initial decision: Buy survey**

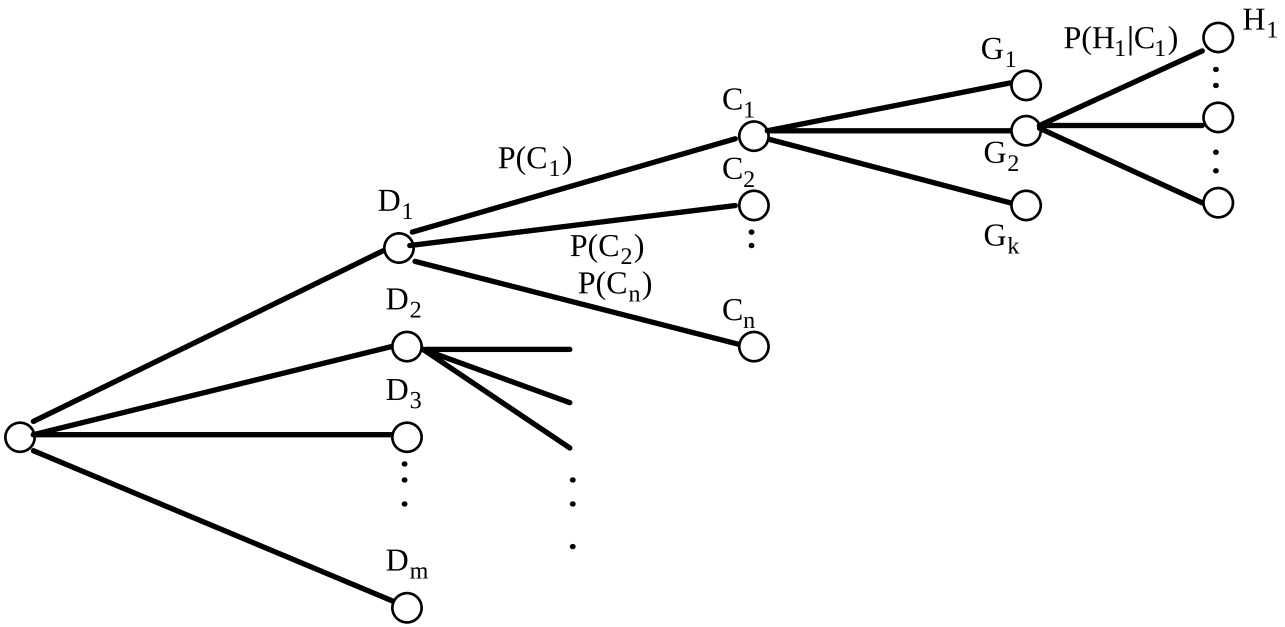
Best terminal decision

If survey result is "strong" ⇒ Market new product (EMV = $221,200)

If survey result is "mild" ⇒ Market old product (EMV = $130,000)

If survey result is "weak" ⇒ Market old product (EMV = $130,000)

General form of a decision tree



GENERAL FORM OF A DECISION TREE

The multistage decision model

1. **Each stage consists of a decision node followed by a chance node for each of the decision options available in this stage.**

2. **The DM must select one of the initial acts A_j.**

3. **The A_j may be viewed as "learning experiments" providing, at specified costs, opportunities for obtaining partial or complete information about present uncertainties.**

4. **Following the probabilistic results a_1, a_2, …, of the initial decision, the DM must select the next decision B_1, B_2, …**

The multistage decision model (cont'd)

5. Finally, a terminal chance node, b_1, b_2,…, occurs.

6. As a result of the initial decision A_w, its chance outcome a_x, the terminal decision B_y, and its chance outcome b_z, the total consequence $C(A_w, a_x, B_y, b_z)$ is obtained.

7. The solution proceeds sequentially backwards in time, i.e., we first identify the best decision at each terminal node, then the best decision one stage earlier assuming that, whatever chance event results from this stage, it will be followed by the best of the available terminal decisions.