ESD.71 / 1.146 / 3.56 / 16.861 Engineering Systems Analysis for Design
Fall 2008

ESD 71 Assignment **Option Valuation by Lattice Assignment**

Summary: This exercise demonstrates the mechanics of evaluating options through a lattice.

Note: This exercise stresses the mechanics of working through the lattice. It does not invoke the concepts of "arbitrage enforced pricing", or of "risk neutral probabilities" -- these are developed later in the course (and in the sequence of assignments).

Learning Objective: to help you develop confidence in the mechanics of evaluating an option using a lattice model of the evolution of a system.

Setting: Imagine that you are working with a company that has the opportunity to open a copper mine. Assume that the

- current price of copper is $2500/ton, and
- its price will rise, in real terms net of inflation, by 4%/year.

Assume that the mine has these characteristics:

- capacity of 5000 tons/year, and
- an operating cost of $ 6 million/year in real terms. This cost is "fixed", in that it does not change as a function of the amount. It is "variable" in that it occurs every year. Thus it can be stopped if the plant is closed.

Tasks:

1. **Baseline Analysis**
   Calculate the NPV of this investment assuming that you will open the mine, operate it fully for 6 years, and sell the product at each year's current price (adjusting the starting price for projected annual increases). Use the discount rate, r = 12%/year.

2. **Modeling Uncertainty**
   Now bring recognition of uncertainty into the analysis. Assume that the annual standard deviation is 15% (i.e., the volatility). On this basis, calibrate the binomial model for this analysis, and present the range of future copper prices. On the same graph, plot the probability density function of copper prices at all times from year 0 to year 6.

3. **Analysis recognizing Uncertainty**
   Use the above results to calculate the NPV of the mine on the assumption that you operate the mine regardless of the evolution of the price of copper. Does the NPV calculated this way agree with the NPV you calculated using the DCF approach?

4. **Valuation of Option to close**
   Your analysis should now recognize that you have the option to close the mine *permanently* if it is optimal to do so. What is the value of the mine on this basis? Therefore, what is the value of the option on this basis (i.e., what is the additional value of the flexibility to close down)?

Assuming you did the analysis on a spreadsheet similar to the one shown in the lecture and recitation, it should be easy to repeat it for *current price of copper at (1) $4000/ton and (2) $1000/ton.* How significant is the flexibility to close down the mine in each case?