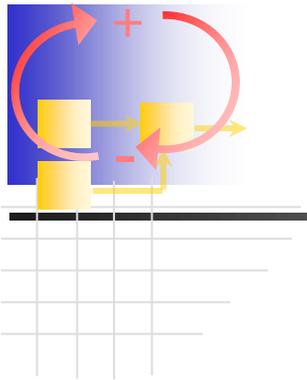


ESD.36 System Project Management

Lecture 9



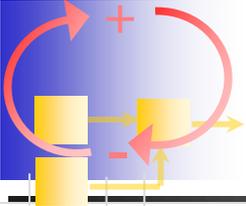
Probabilistic Scheduling

Instructor(s)

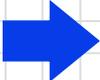
Prof. Olivier de Weck

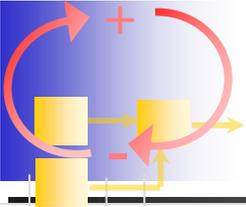
Dr. James Lyneis

October 4, 2012



Today's Agenda

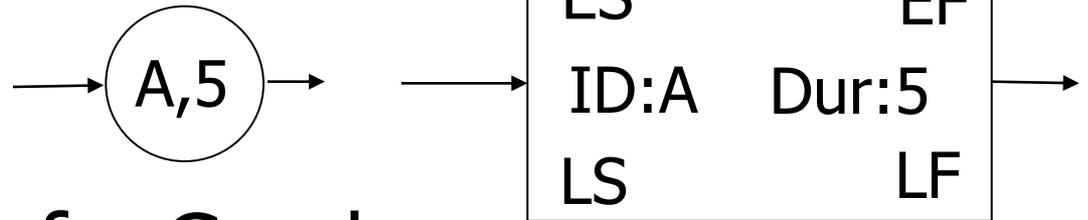
- 
- Probabilistic Task Times
 - PERT (Program Evaluation and Review Technique)
 - Monte Carlo Simulation
 - Signal Flow Graph Method
 - System Dynamics



Task Representations

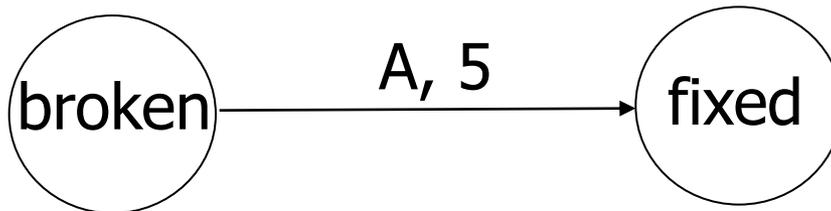
- Tasks as Nodes of a Graph

- Circles
- Boxes

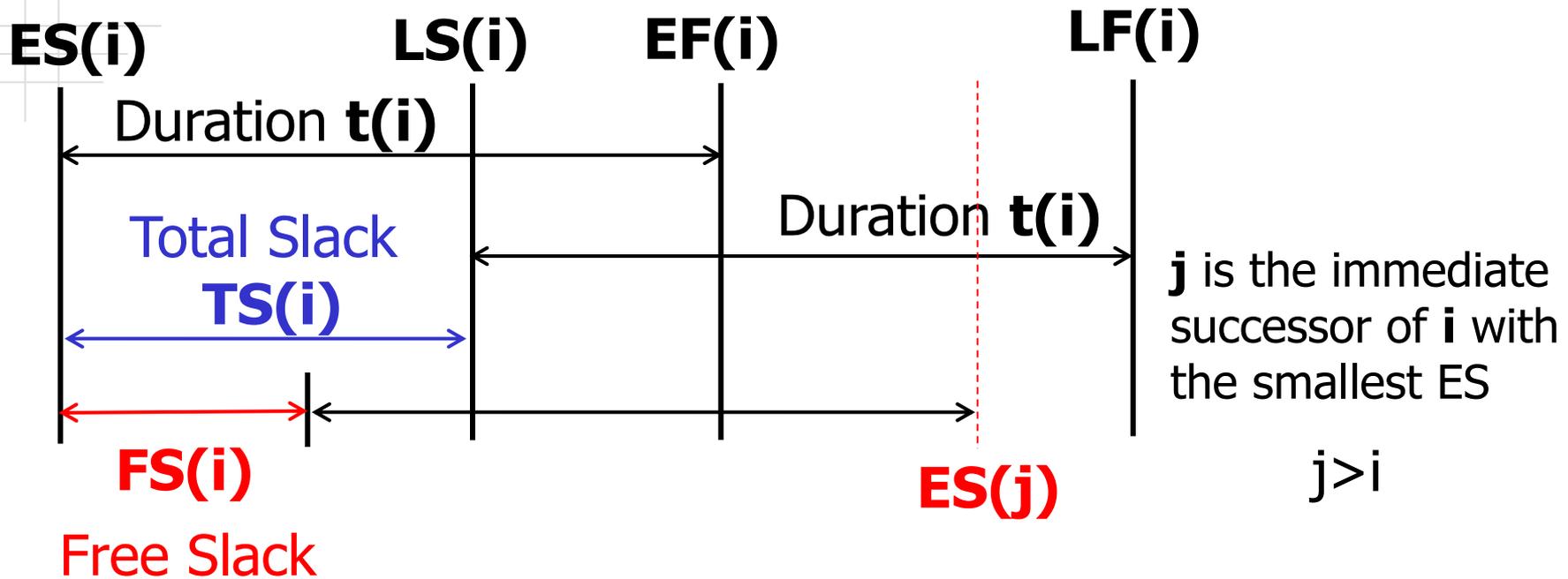


- Tasks as Arcs of a Graph

- Tasks are uni-directional arrows
- Nodes now represent “states” of a project
- Kelley-Walker form

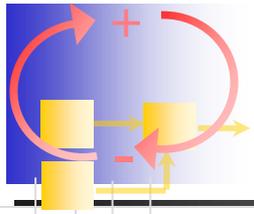


Task Times Detail - Task i



- Free Slack (**FS**) is the amount a job can be delayed without delaying the Early Start (ES) of any other job.

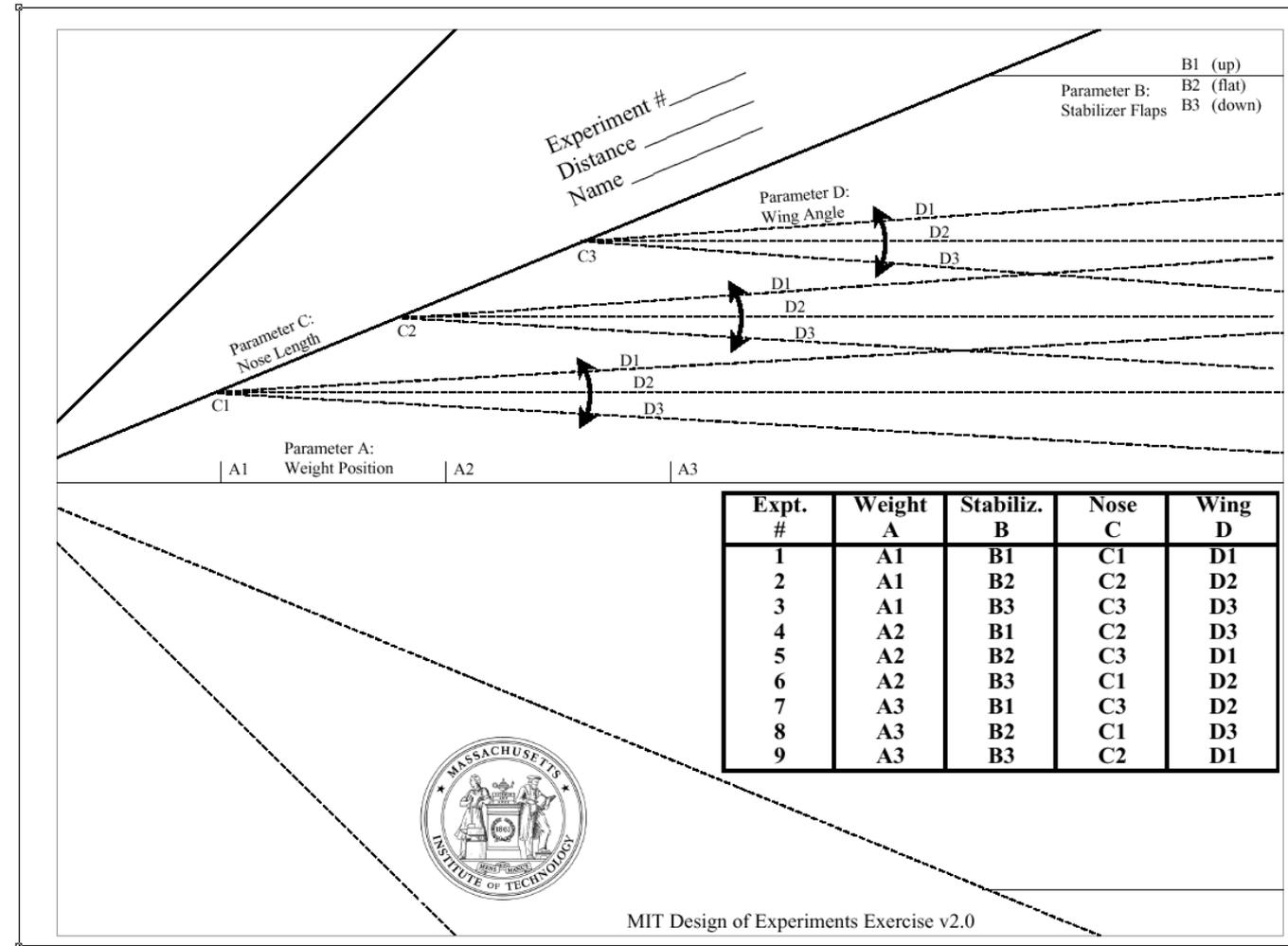
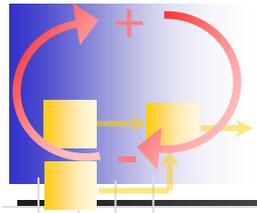
$FS \leq TS$ always



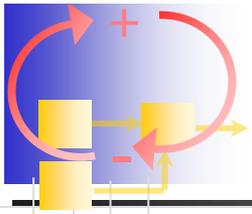
How long does a task take?

- Conduct a small in-class experiment
- Fold MIT paper airplane
 - Have sheet & paper clip ready in front of you
 - Paper airplane type will be announced, e.g. A1-B1-C1-D1
 - Build plane, focus on quality rather than speed
 - Note the completion time in seconds +/- 5 [sec]
- Plot results for class and discuss
 - Submit your task time online , e.g. 120 sec
 - We will build a histogram and show results

MIT Paper Airplane



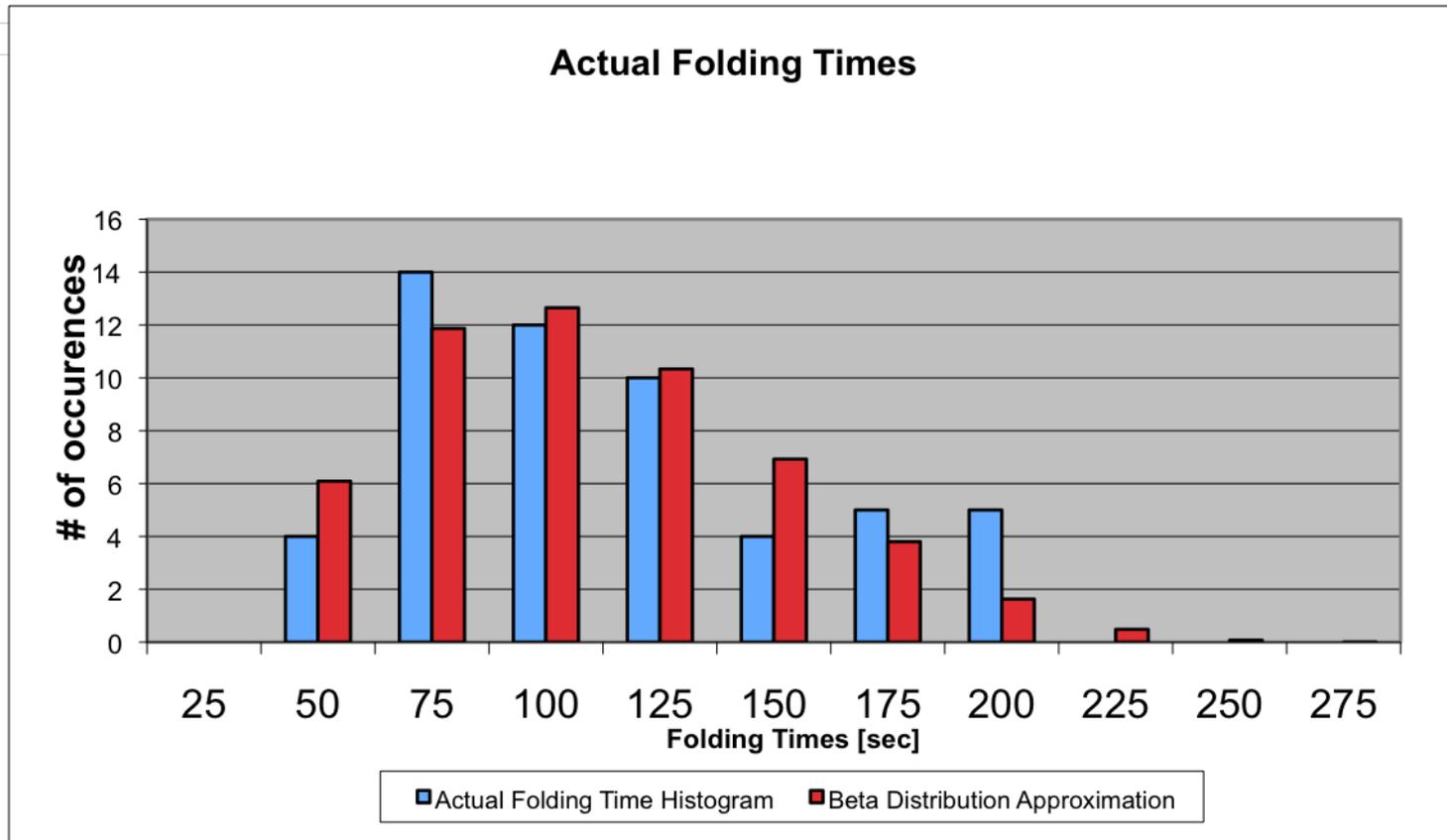
Courtesy of Steven D. Eppinger. Used with permission.



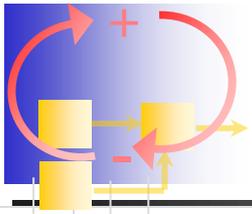
Concept Question 1

- How long did it take you to complete your paper airplane (round up or down)?
 - 25 sec
 - 50 sec
 - 75 sec
 - 100 sec
 - 125 sec
 - 150 sec
 - 175 sec
 - 200 sec
 - 225 sec
 - > 225 sec

MIT Class Results (2008)

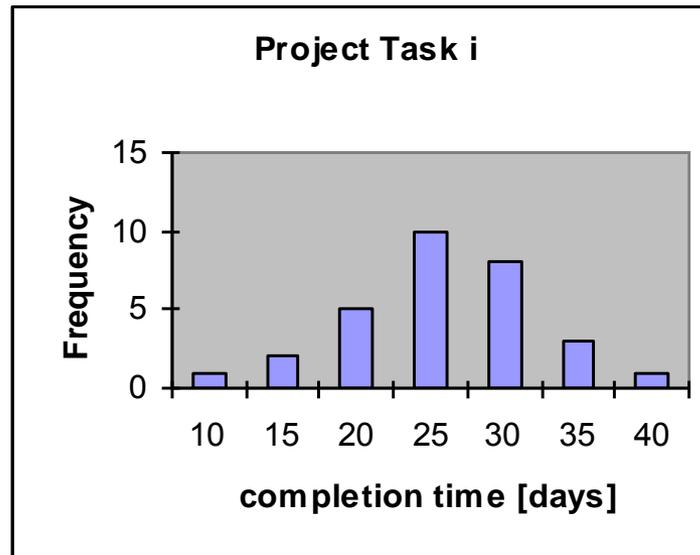


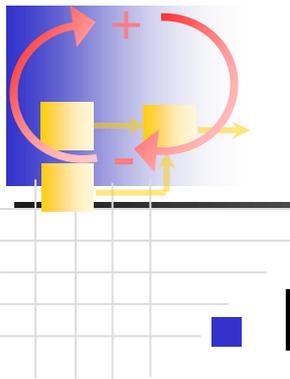
Beta Distribution parameters: $\alpha=2.27$, $\beta=5.26$



Discussion Point 1

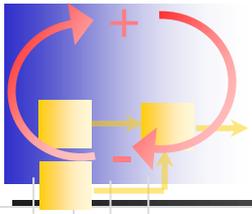
- Job task durations are stochastic in reality
- Actual duration affected by
 - Individual skills
 - Learning curves ... what else?
 - Why is the distribution not symmetric (Gaussian)?





Today's Agenda

- Probabilistic Task Times
- ➔ ■ PERT (Program Evaluation and Review Technique)
- Monte Carlo Simulation
 - Signal Flow Graph Method
 - System Dynamics



PERT

- PERT invented in 1958 for U.S Navy Polaris Project (BAH)
- Similar to CPM
- Treats task times probabilistically

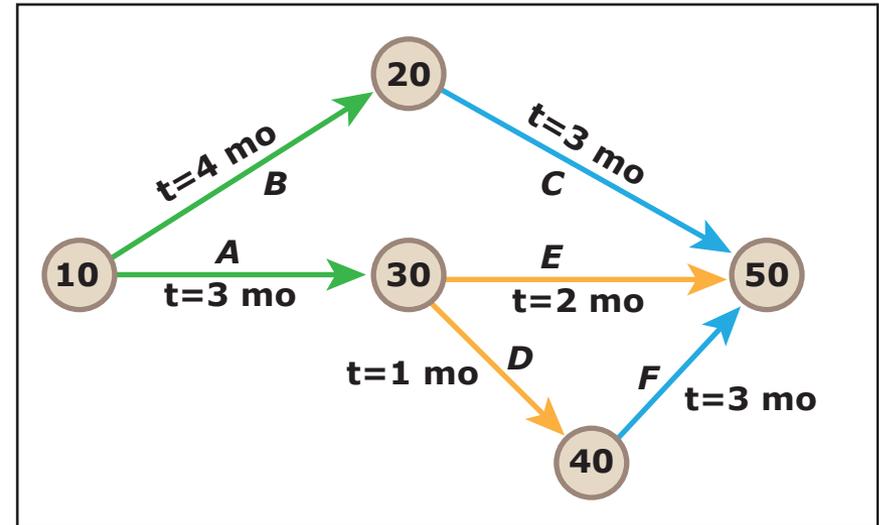
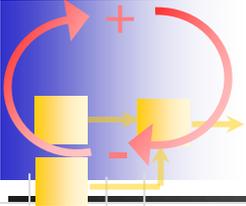


Image by MIT OpenCourseWare.

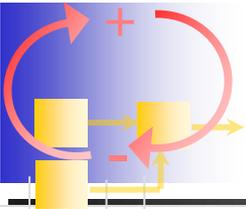
Original PERT chart used "activity-on-arc" convention



CPM vs PERT

Difference how “task duration” is treated:

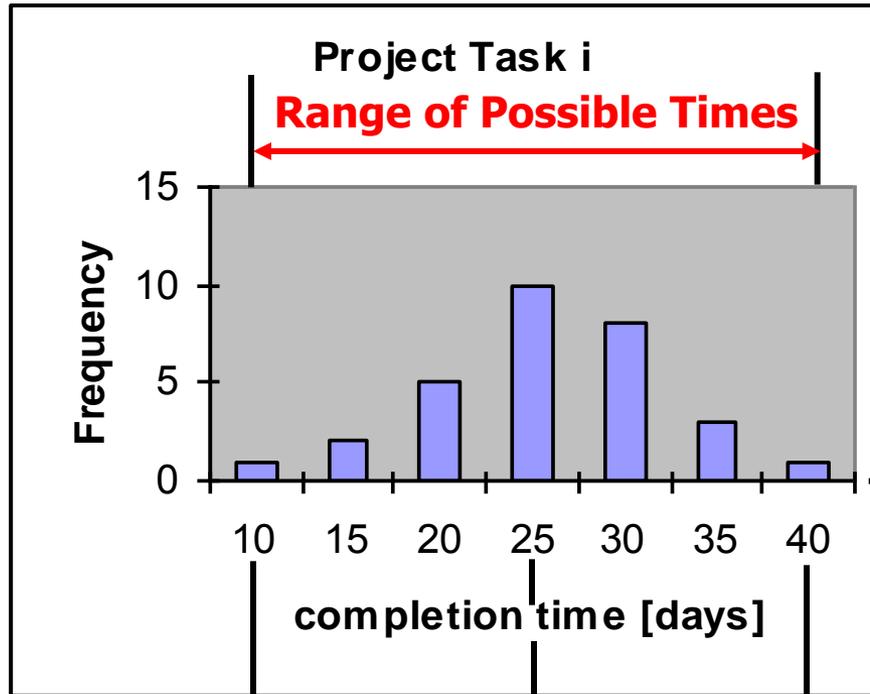
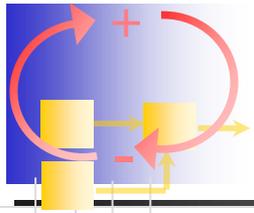
- CPM assumes time estimates are deterministic
 - Obtain task duration from previous projects
 - Suitable for “implementation”-type projects
- PERT treats durations as probabilistic
 - $PERT = CPM + \text{probabilistic task times}$
 - Better for “uncertain” and new projects
 - Limited previous data to estimate time durations
 - Captures schedule (and implicitly some cost) risk



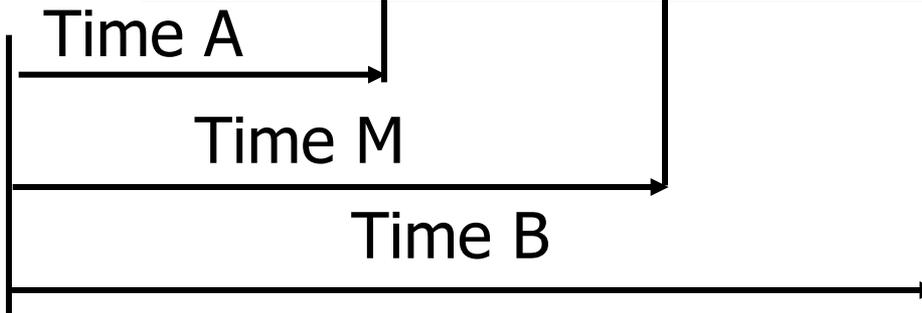
PERT -- Task time durations are treated as uncertain

- **A** - optimistic time estimate
 - minimum time in which the task could be completed
 - everything has to go right
- **M** - most likely task duration
 - task duration under “normal” working conditions
 - most frequent task duration based on past experience
- **B** - pessimistic time estimate
 - time required under particularly “bad” circumstances
 - most difficult to estimate, includes unexpected delays
 - should be exceeded no more than 1% of the time

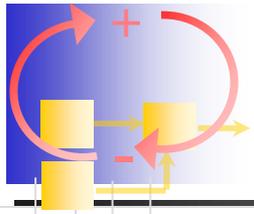
A-M-B Time Estimates



times

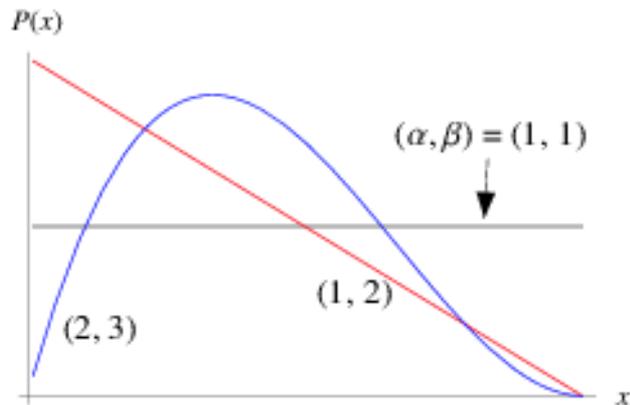


Assume a
Beta-distribution



Beta-Distribution

- All values are enclosed within interval $t \in [A, B]$
- As classes get finer - arrive at β -distribution
- Statistical distribution



$$x \in [0, 1]$$

pdf:

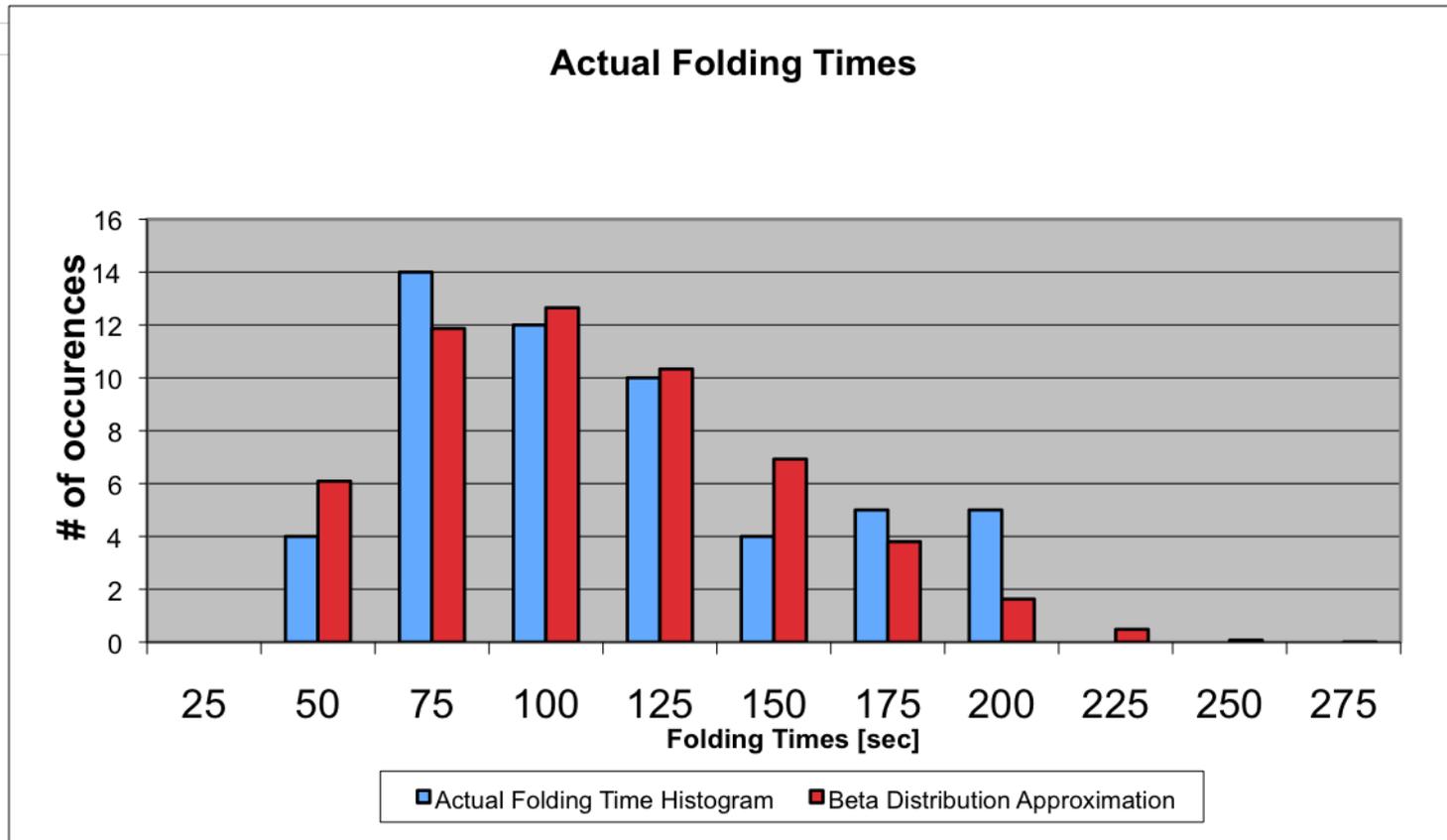
$$P(x) = \frac{(1-x)^{\beta-1} x^{\alpha-1}}{B(\alpha, \beta)}$$

$$= \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} (1-x)^{\beta-1} x^{\alpha-1}$$

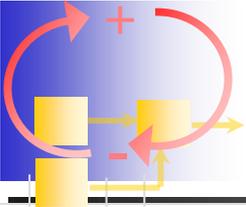
Beta function:

$$B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} = \frac{(p-1)!(q-1)!}{(p+q-1)!}$$

MIT Class Results (2008)

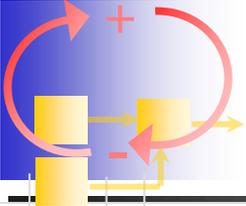


Beta Distribution parameters: $\alpha=2.27$, $\beta=5.26$



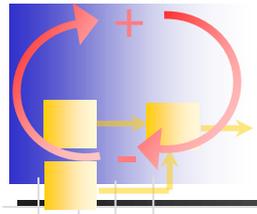
Expected Time & Variance Estimated Based on A, M & B

- Mean expected Time (**TE**) $TE = \frac{A + 4M + B}{6}$
- Time Variance (**TV**) $TV = \sigma_t^2 = \left(\frac{B - A}{6} \right)^2$
- Early Finish (**EF**) and Late Finish (**LF**) computed as for CPM with **TE**
- Set **T=F** for the end of the project
- Example: A=3 weeks, B=7 weeks, M=5 weeks --> then **TE**=5 weeks



What Can We Do With This?

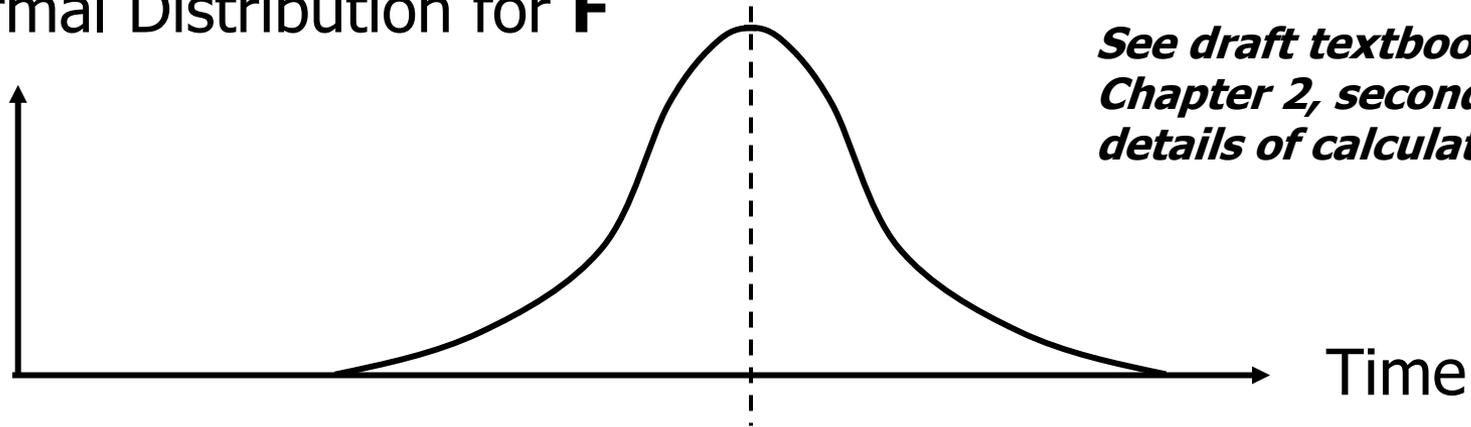
- Compute probability distribution for project finish
- Determine likelihood of making a specific target date
- Identify paths for buffers and reserves



Probability Distribution for Finish Date

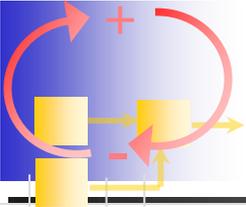
- PERT treats task times as probabilistic
 - Individual task durations are β -distributed
 - Simplify by estimating A, B and C times
 - Sums of multiple tasks are normally distributed

Normal Distribution for **F**



*See draft textbook
Chapter 2, second half, for
details of calculations.*

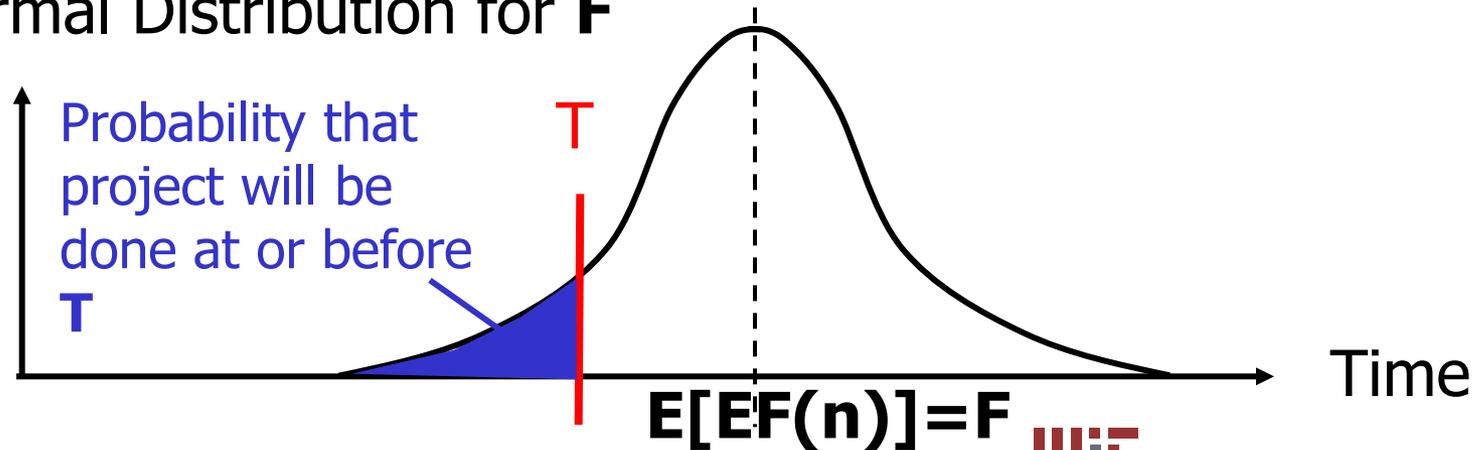
$$E[EF(n)] = F$$

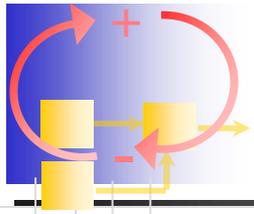


Probability of meeting target ?

- Many Projects have target completion dates, **T**
 - Interplanetary mission launch windows 3-4 days
 - Contractual delivery dates involving financial incentives or penalties
 - Timed product releases (e.g. Holiday season)
 - Finish construction projects before winter starts
- Analyze expected Finish **F** relative to **T**

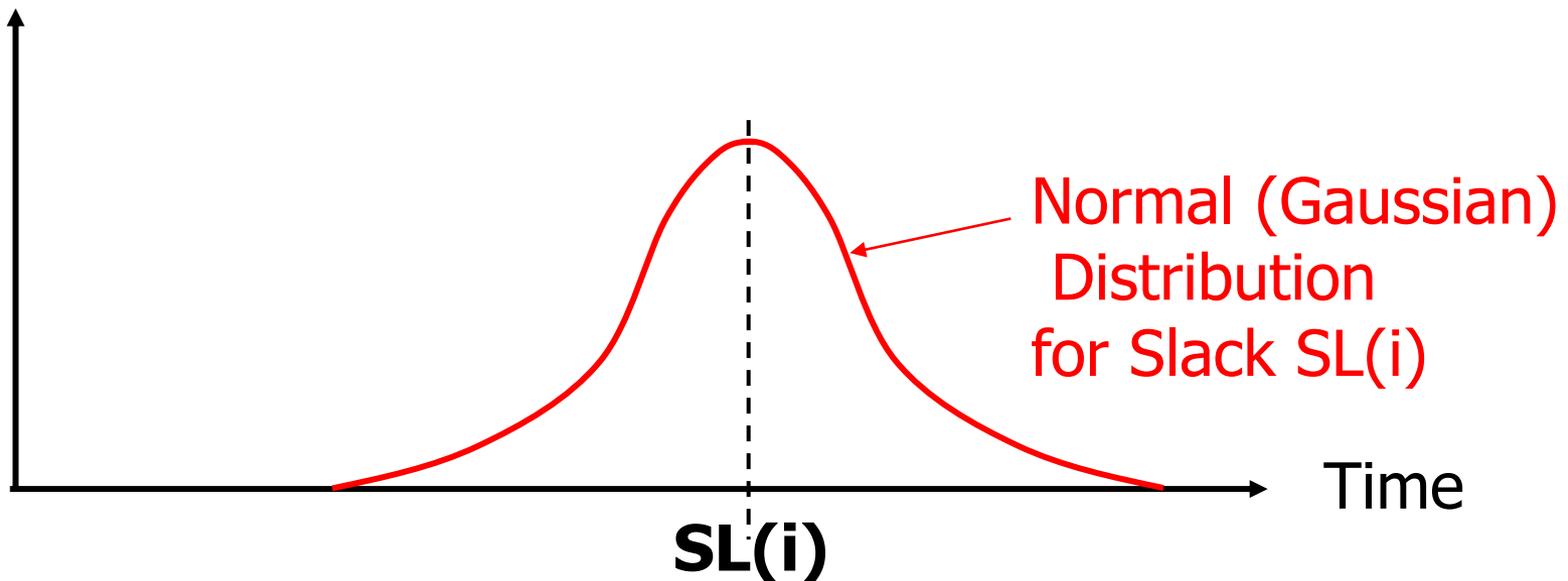
Normal Distribution for **F**

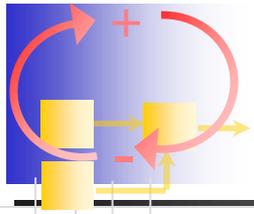




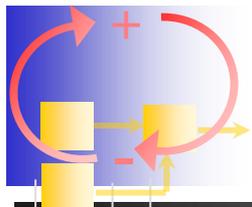
Probabilistic Slack

- Target date for a task is not met when **$SL(i) < 0$** , i.e. negative slack occurs
- Put buffers in paths with high probability of negative slack



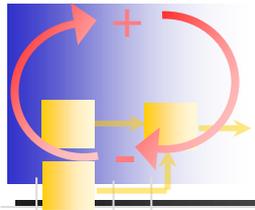


Experiences with PERT?



Today's Agenda

- Probabilistic Task Times
- PERT (Program Evaluation and Review Technique)
- ➔ ■ Monte Carlo Simulation
 - Signal Flow Graph Method
 - System Dynamics



Project Simulation (from Lecture 5)

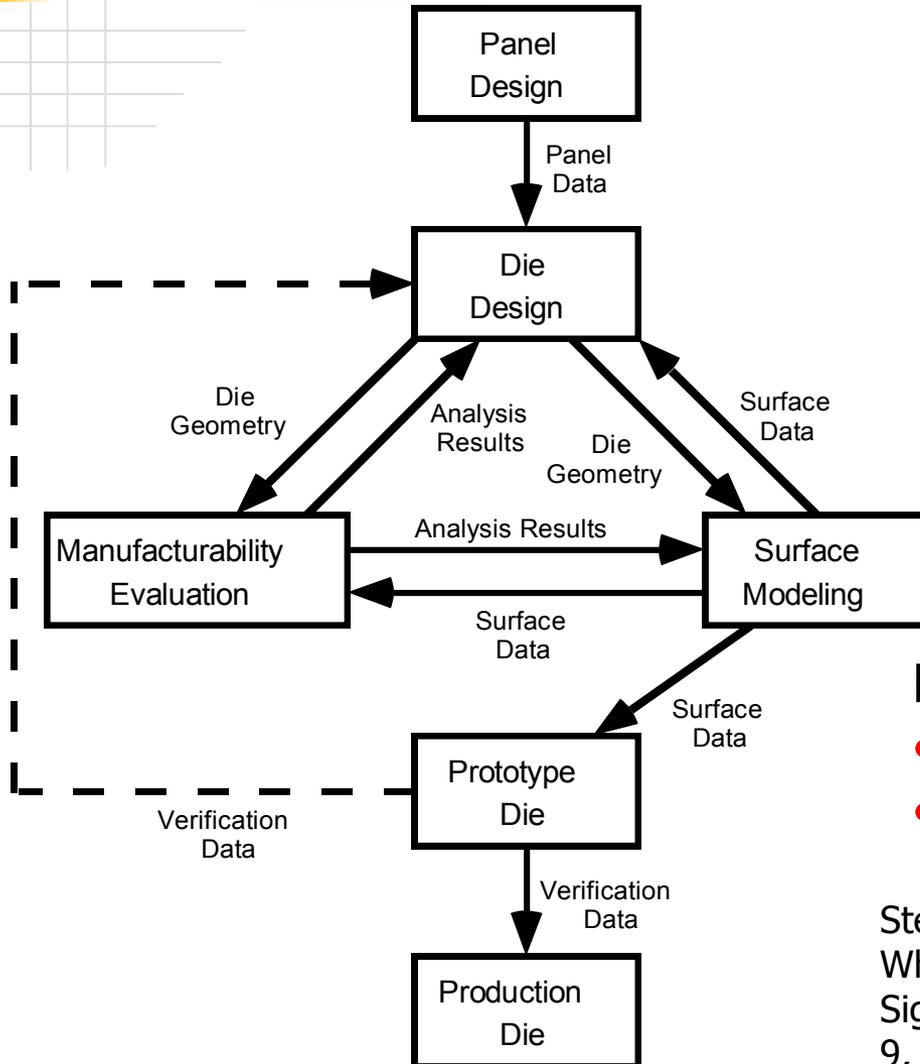


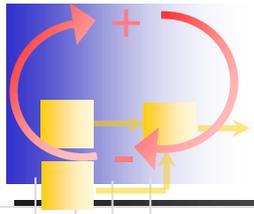
Image removed due to copyright restrictions.

Die Design and Manufacturing

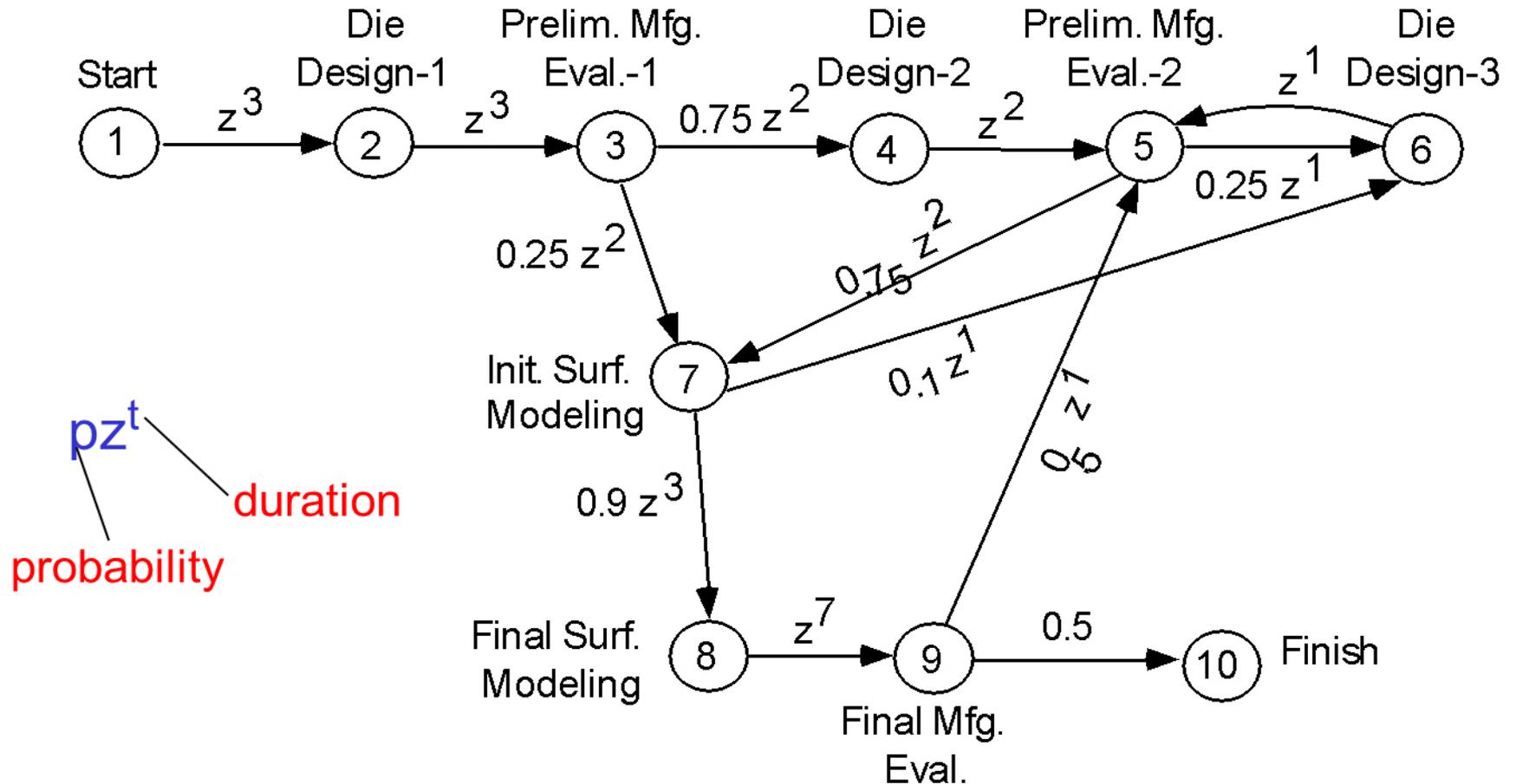
Highly Iterative Process

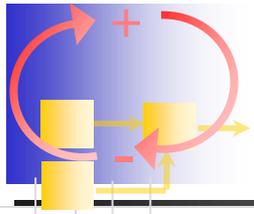
- how often is each task carried out ?
- how long to complete?

Steven D. Eppinger, Murthy V. Nukala, and Daniel E. Whitney. "Generalised Models of Design Iteration Using Signal Flow Graphs", Research in Engineering Design. vol. 9, no. 2, pp. 112-123, 1997.

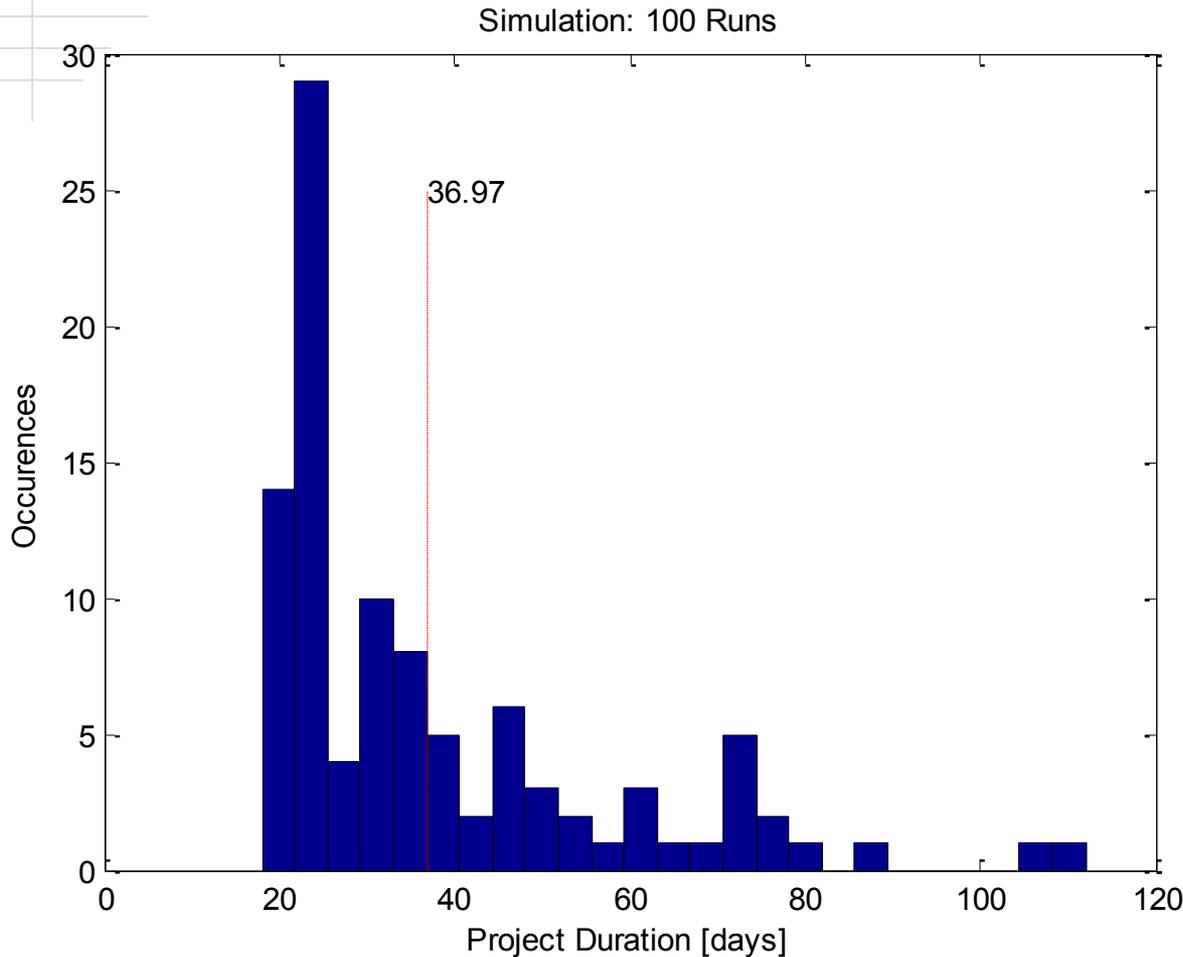


Signal Flow Graph Model: Stamping Die Development





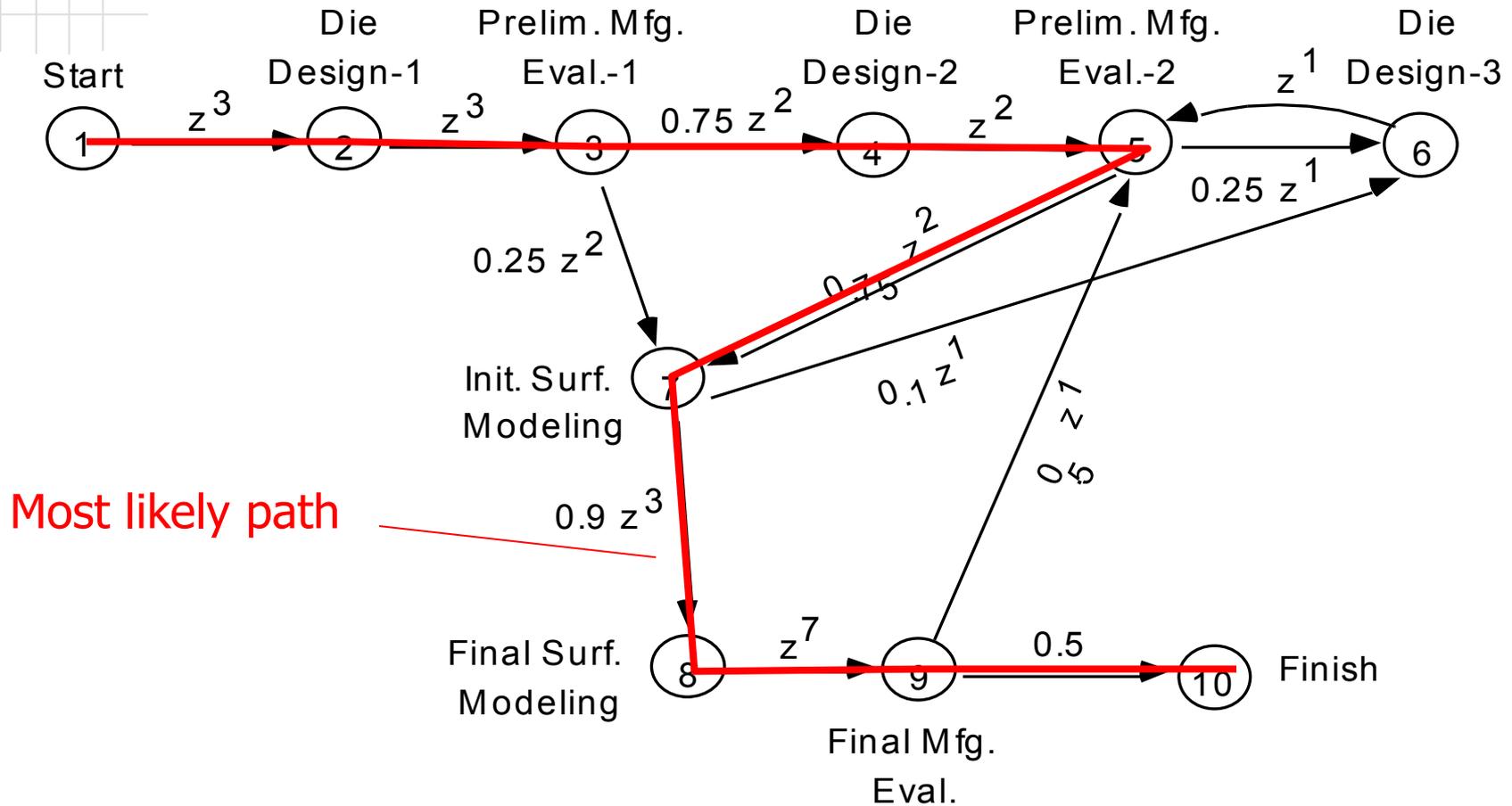
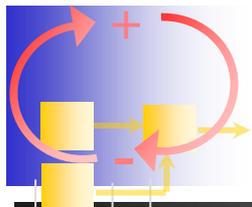
Computed Distribution of Die Development Timing

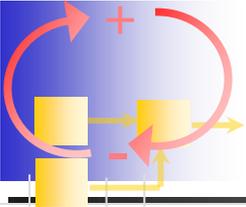


Estimate likely completion time

What else can we do with the simulation?

Process Redesign/Refinement

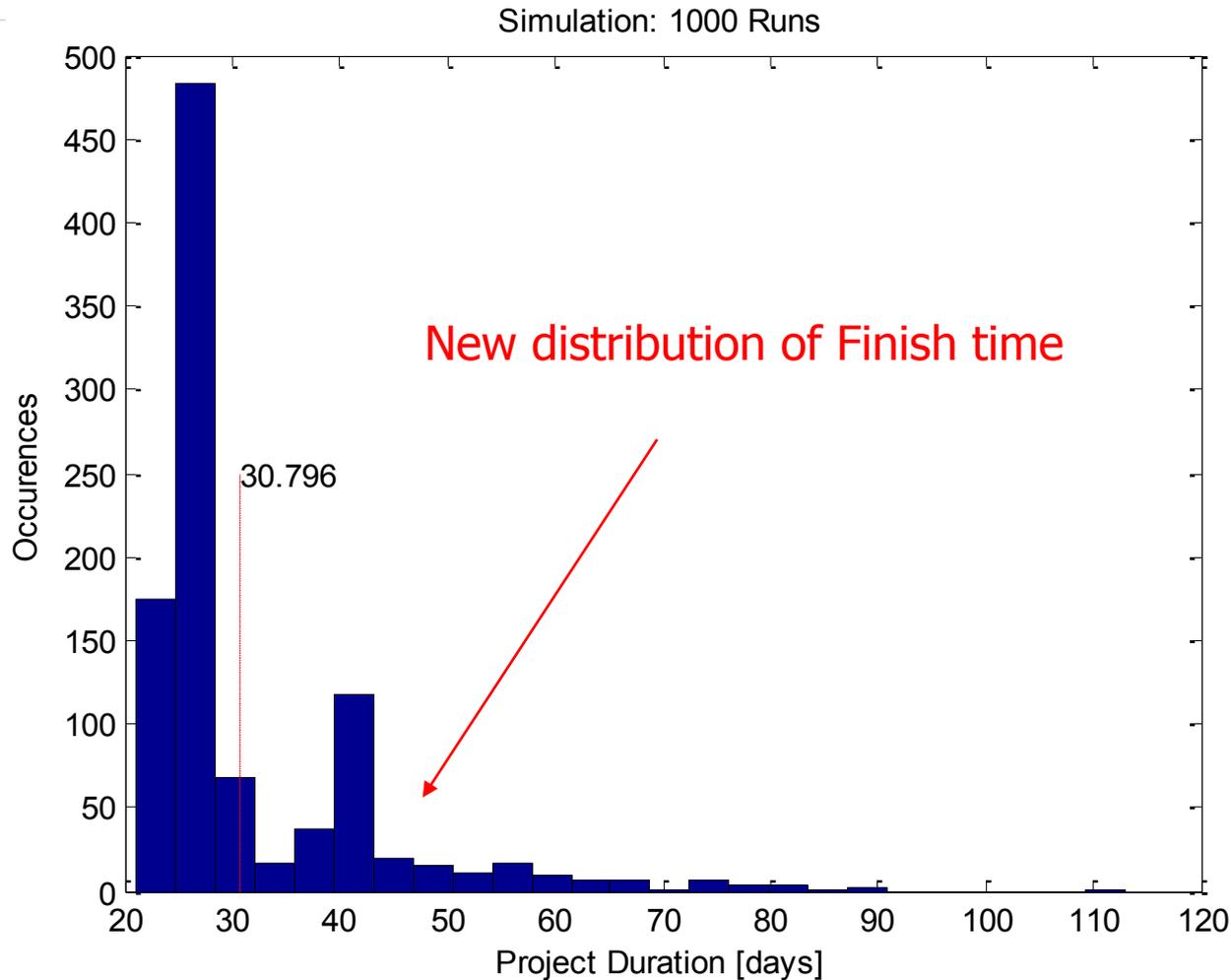
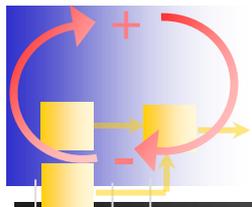




What-if analysis

- Spend more time on die design (1):
 - Increase time spent on initial die design (1) from 3 to 6 days
 - Increase likelihood of going to Initial Surface Modeling (7) from 0.25 to 0.75
 - Is this worthwhile doing?
 - Original $E[F]=37$ days
 - New $E[F]= 37$ days – no real effect ! **Why?**
- Spend more time on final surface modeling (8):
 - Increase time for that task from 7 to 10 days
 - Increase likelihood of Finishing from 0.5 to 0.75
 - New $E[F] = 30.8$ days
 - Why is this happening?

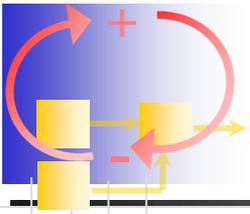
New Project Duration



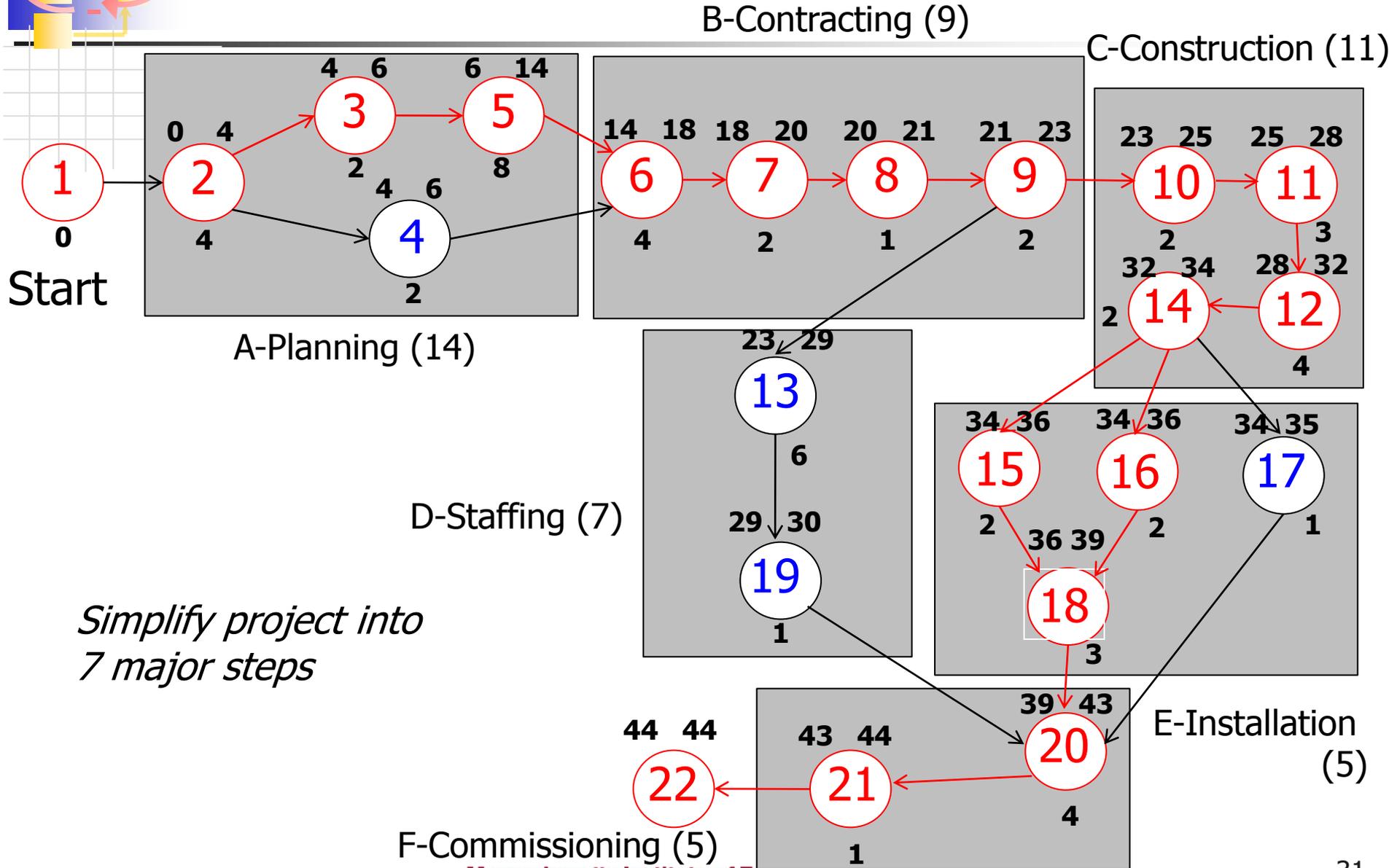
Applying Project Simulation to HumLog Distribution Center Project (From Lecture 4)

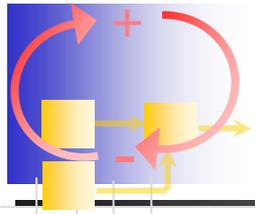


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Simulation Application to HumLog DC

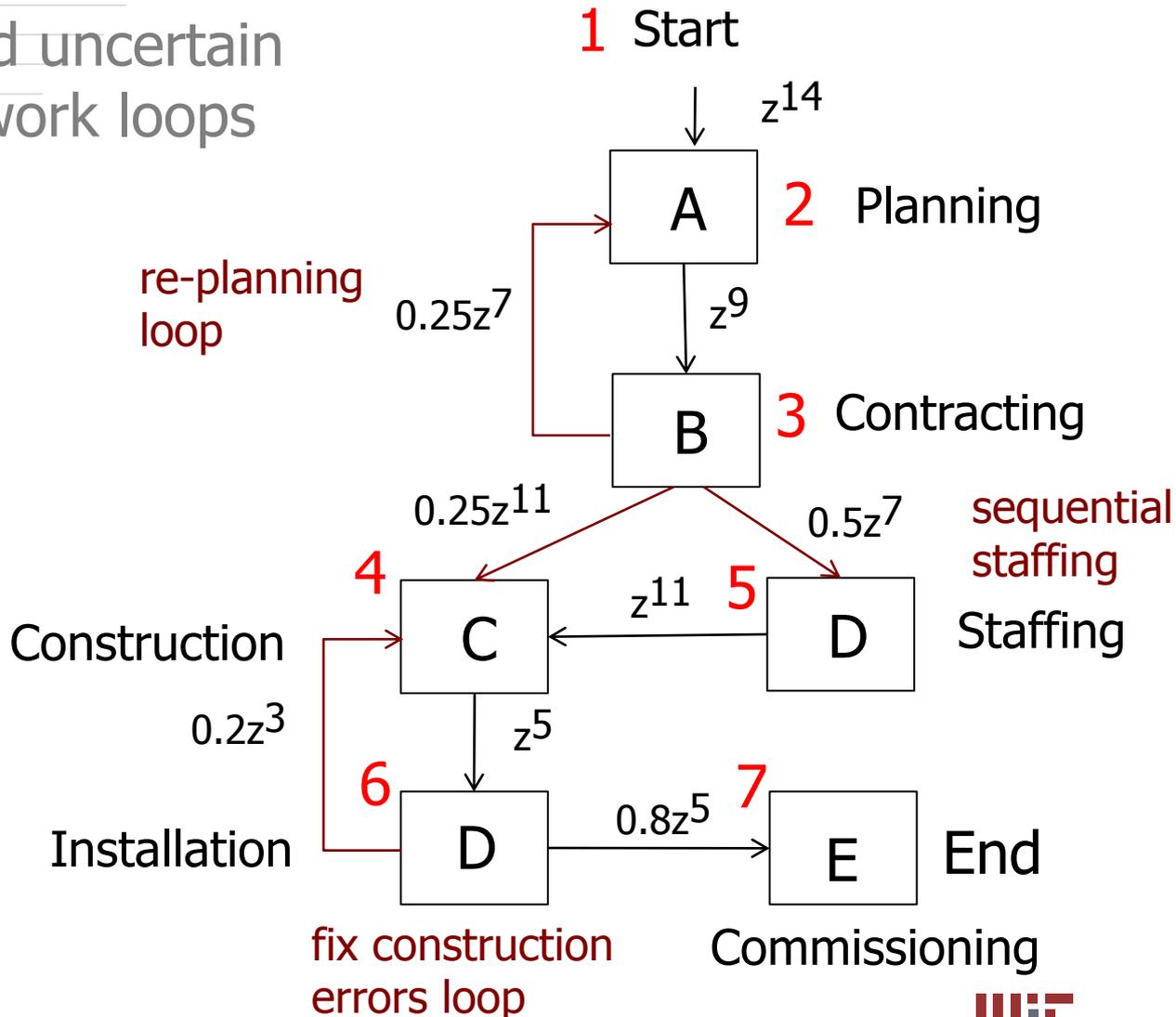




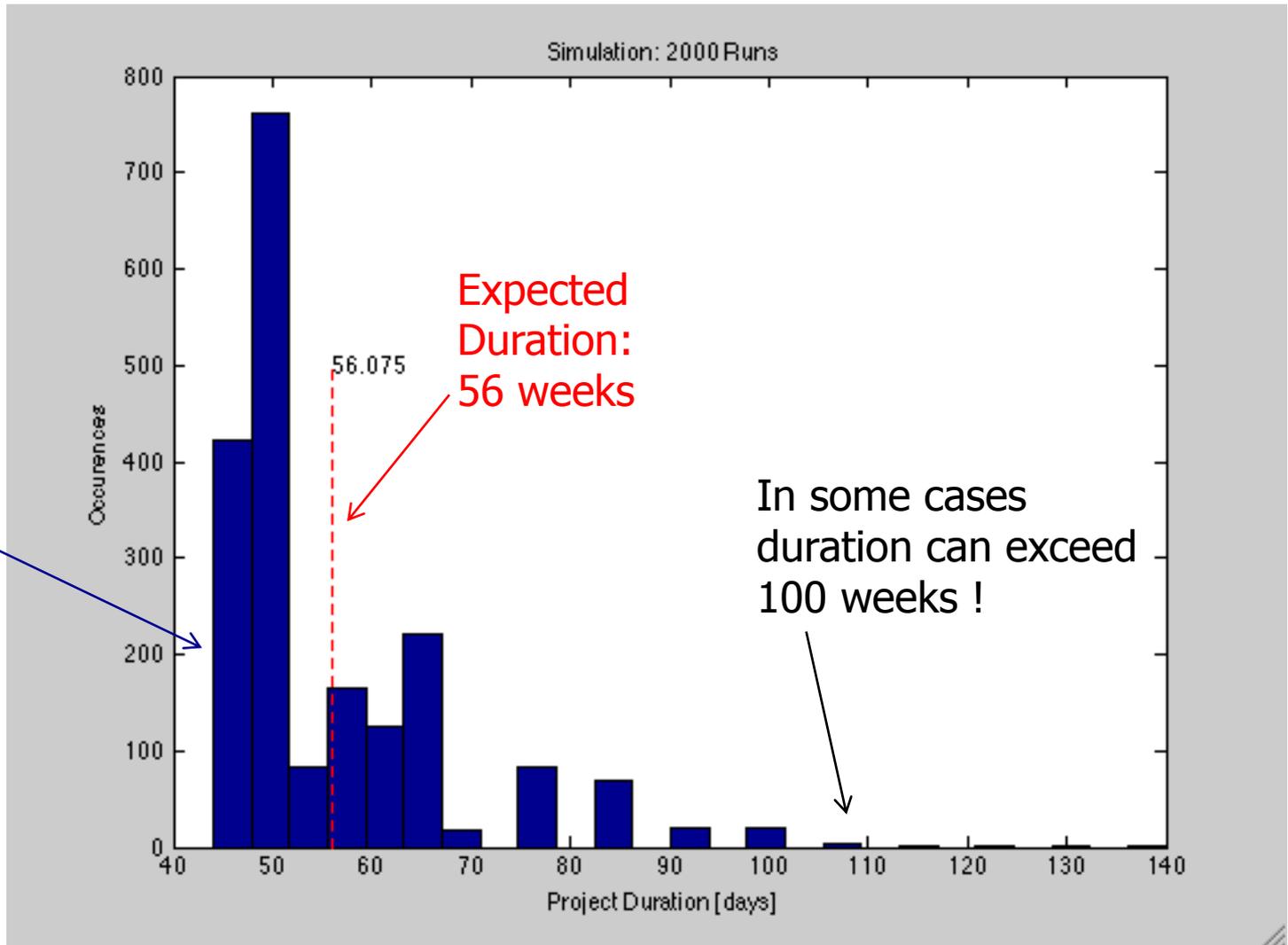
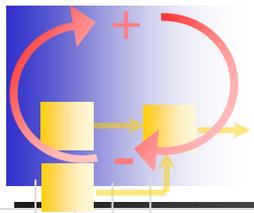
Signal Flow Graph (with iterations)

Add uncertain
rework loops

state in red



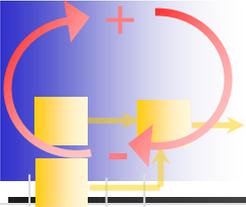
HumLog Simulation Results



Shortest
Duration:
44 weeks

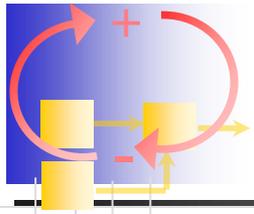
Expected
Duration:
56 weeks

In some cases
duration can exceed
100 weeks !



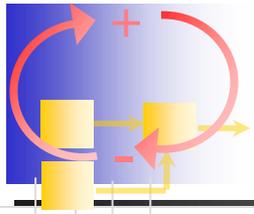
Concept Question 2

- What is your opinion of these “long tails” in project schedule distributions?
 - I don't think they are real. This is a simulation artifact.
 - Yes, they exist. I have experienced this.
 - This is only relevant for projects that deal with very new products or extreme environments.
 - I'm not sure.
 - I don't care.



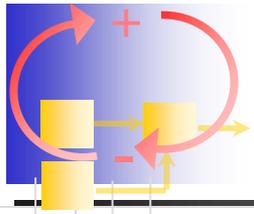
System Dynamics Simulation

- Can carry out Monte-Carlo Simulations with System Dynamics
 - Vary key model parameters such as fraction correct and complete, productivity, rework discovery fraction, number of staff etc...
 - Assume distributions for these parameters
- Obtain insights into potential distribution of
 - Time to completion, required staffing levels, error rates, project cost ...
- Improve planning and adaptation of projects, and confidence in “claim” numbers



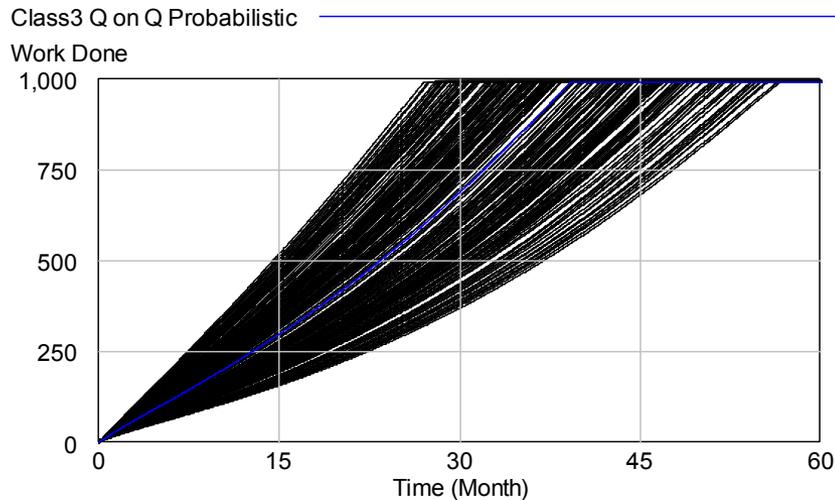
Example

- Model without project control from last class and HW#3
- Probabilistic Simulations:
 - Normal Productivity uniform distribution (0.9 – 1.1)
 - Normal Fraction Correct and Complete uniform distribution (0.75 – 0.95)

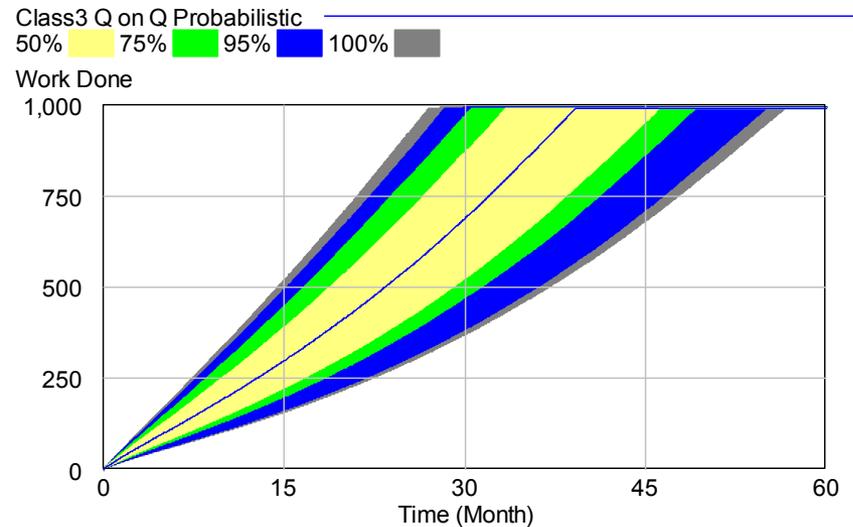


Results for project finish ...

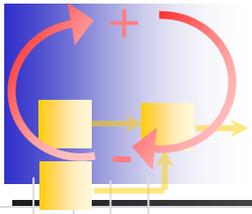
Individual simulations



Confidence bounds

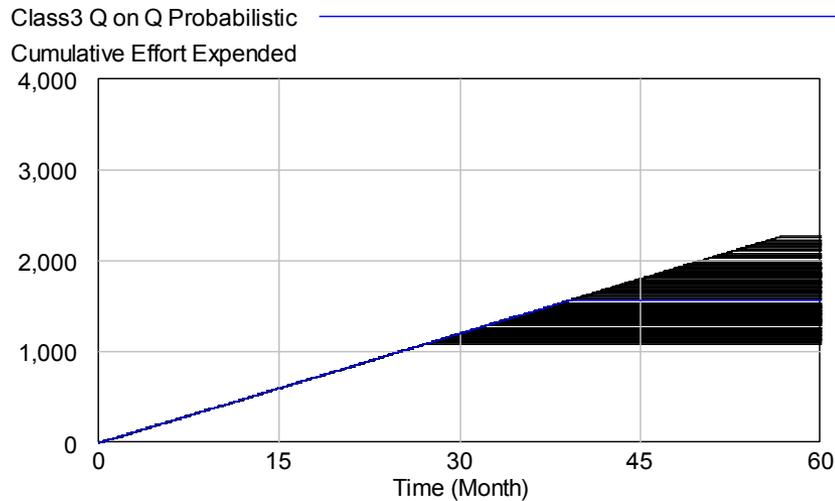


Distribution skewed to later finish

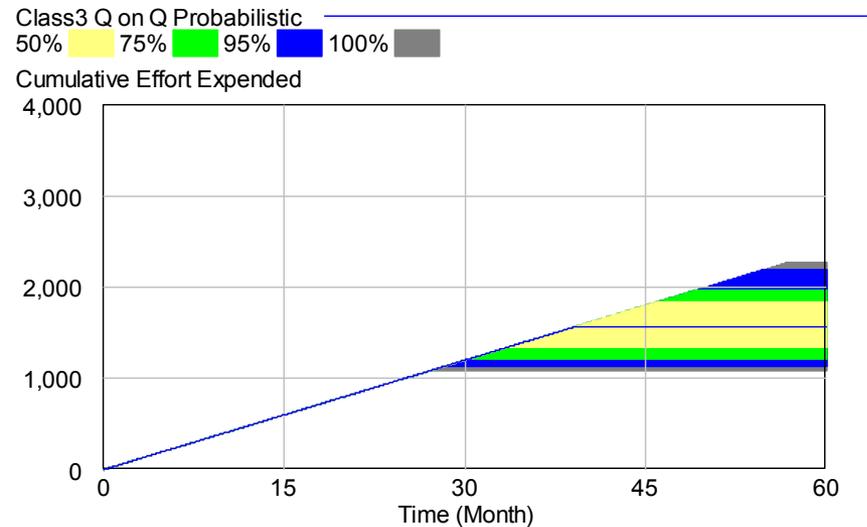


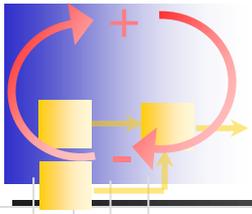
Results for project cost ...

Individual simulations



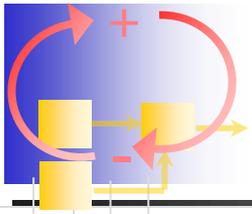
Confidence bounds





Monte Carlo and SD

- Use in planning and adaptation:
 - Size and timing (fraction complete) of buffers
 - Timing of project milestones
- Important use in specifying confidence bands in a “claim” situation for distribution of cost overrun due to client (owner)
 - “Fit” constrained – only simulations which fit the data can be accepted
 - Graham AK, Choi CY, Mullen TW. 2002. Using fit-constrained Monte Carlo trials to quantify confidence in simulation model outcomes. *Proceedings of the 35th Annual Hawaii Conference on Systems Sciences*. IEEE: Los Alamitos, Calif. (Available on request)



Usefulness of PERT and Simulation

- Account for task duration uncertainties
 - Optimistic Schedule
 - Expected Schedule
 - Pessimistic Schedule
- Helps set time reserves (buffers)
- Compute probability of meeting target dates when talking to management, donors
- Identify and carefully manage critical parts of the schedule

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Fall 2012

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