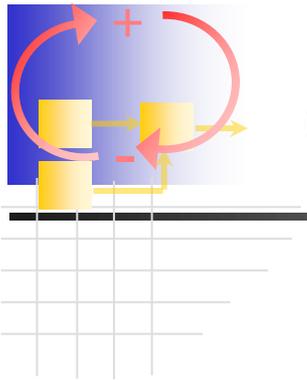


ESD.36 System & Project Management

Lecture 2

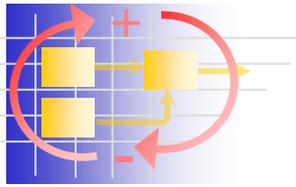


# Critical Path Method (CPM)

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Instructor(s)

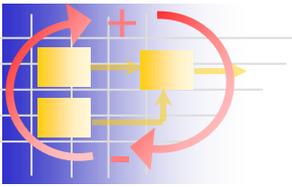
Prof. Olivier de Weck



# Today's Agenda

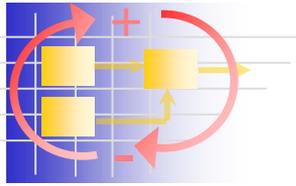
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- Overview of PM methods and tools
- CPM 101
- Critical Paths, Slack
- Task “Crashing” and Cost
- Conclusions and Class Discussions
- Introduce HW3



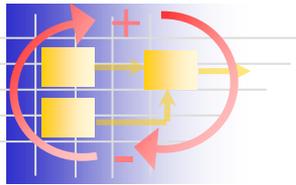
# History of Project Management

- Big Projects since antiquity
  - Pyramids (Egypt), Great Wall (China)
  - Enormous workforce, but little documented evidence of formal project management
- Formal Project Management
  - Henry Gantt (1861-1919) → bar chart 1910
  - 1957 Sputnik Crisis → revival of “scientific management”
  - Polaris (1958) → Project Evaluation and Review Technique (PERT)
  - DuPont Company (1960) → Critical Path Method (CPM)
  - 1960’s NASA projects: Mercury, Gemini, Apollo
    - Work Breakdown Structures (WBS)
    - Cost and Schedule Tracking, Configuration Management



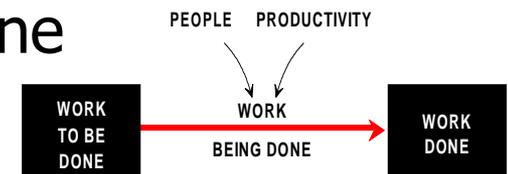
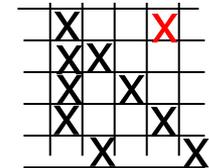
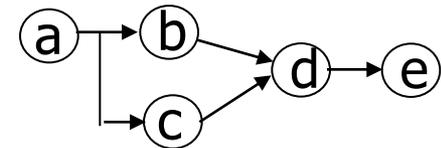
# Comments about early PM

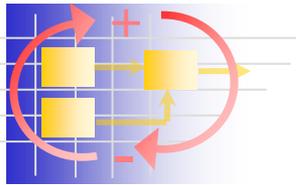
- Project decomposition necessary due to complexity
- Resource allocation and workload smoothing
- Schedule urgency .."before the decade is out" *JFK*
- Circumstances
  - Complex Relations between Government and Contractors
  - "Shielded" from Society, Competition, Regulations
  - Cold War Pressures for Nuclear Power, Space Race ..
- Other Innovations
  - Project Manager as a central figure
  - Beginnings of Matrix Organization
  - "Earned Value" – adopted by USAF (1963)
- Professionalization since 1969
  - Diffusion into other industries: computers, automotive ...
  - Project Management Institute (PMI) founded – PMBOK
  - ISO 10006:2003 Quality in Project Management
  - Recent criticism about PM standards as "bureaucratic"



# Fundamental Approaches

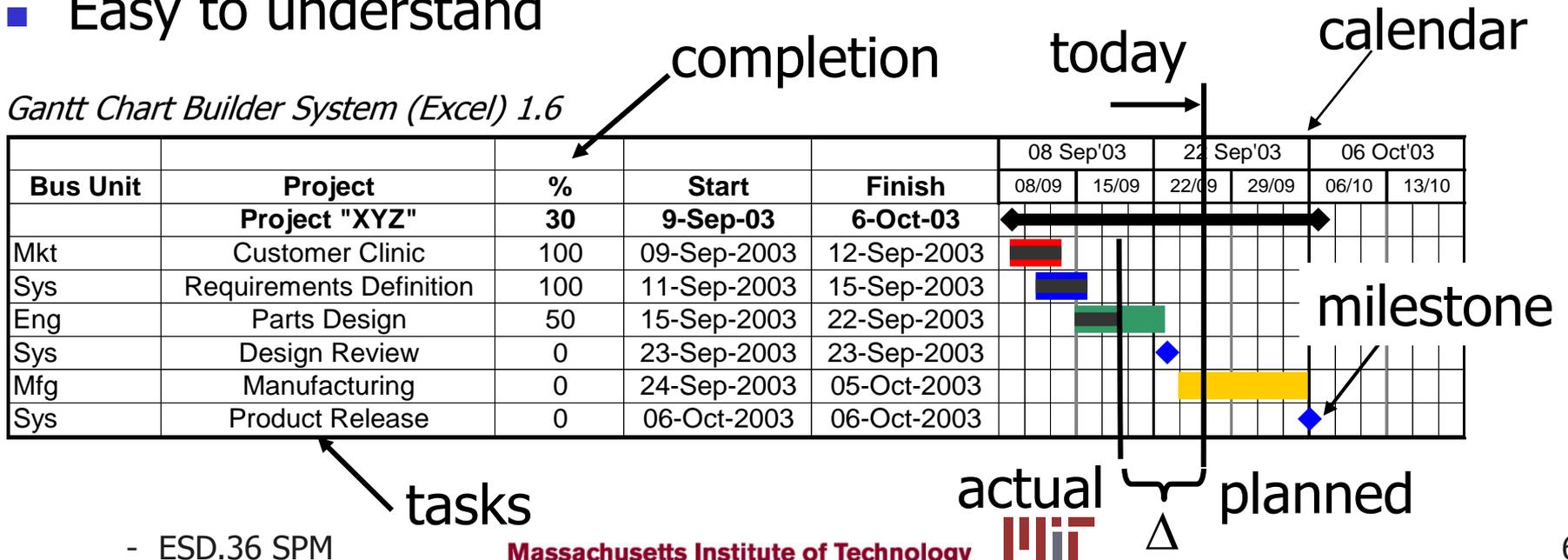
- How to represent task relationships?
- Network-based (graph theory) methods
  - CPM, PERT, ....
  - Task is a node or an arc
- Matrix-based methods
  - DSM - Tasks are columns and rows
  - Interrelationships are off-diagonal entries
- System Dynamics
  - Feedback loops, causal relationships
  - Stocks and flows simulation
  - Tasks that are done or waiting to be done are stocks – “amount of work”
  - Doing project work causes a “flow”

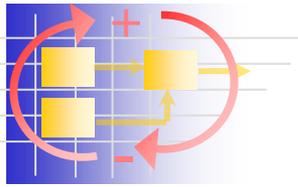




# Gantt Charts

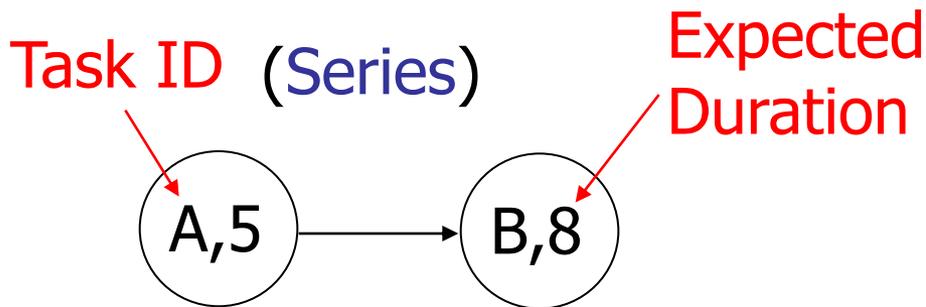
- Attributed to Henry Gantt – most popular PM tool (80%)
- Used to plan big shipbuilding projects (cargo ships WWI)
- Graphical way of showing task durations, project schedule
- Does not explicitly show relationships between tasks
- Limited use for project tracking
- Easy to understand



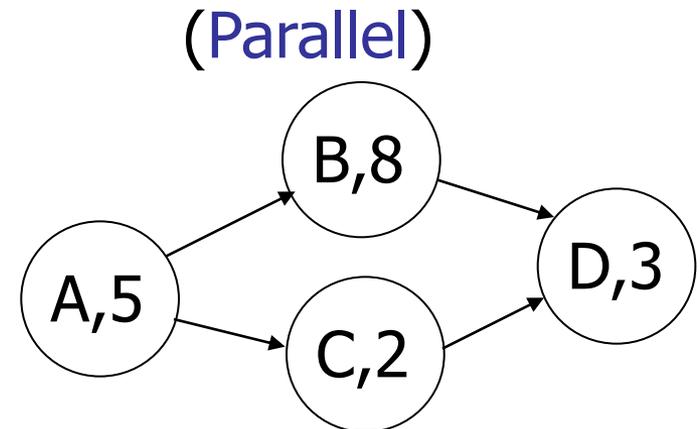


# CPM 101

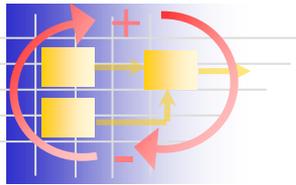
- Represent a project (set of task) as a network using graph theory
  - Capture task durations
  - Capture task logic (dependencies)



"B can only start after A is completed"

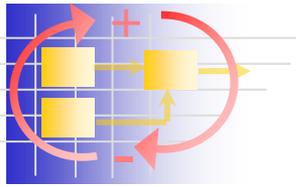


"B and C do not depend on each other"



# CPM Assumptions

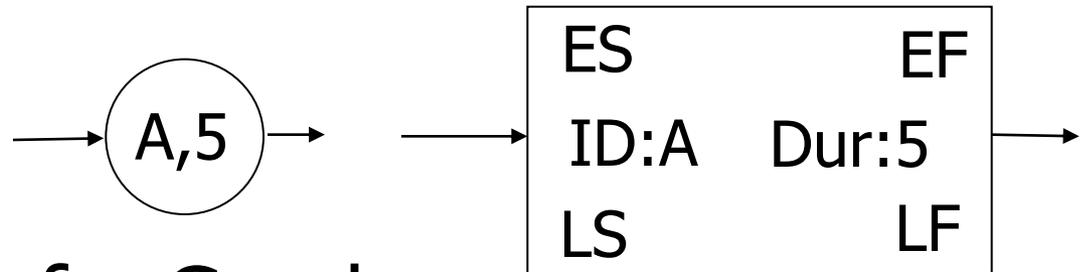
- Project consists of a collection of well defined tasks (jobs)
- Project ends when all jobs completed
- Jobs may be started and stopped independently of each other within a given sequence (no “continuous-flow” processes)
- Jobs are ordered → “technological sequence”



# Task Representations

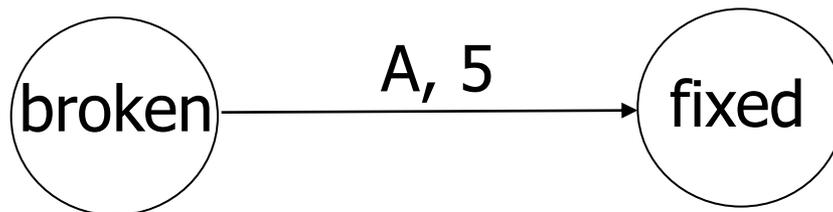
- Tasks as Nodes of a Graph

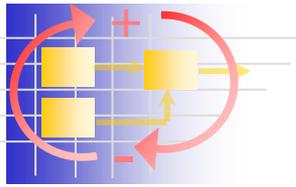
- Circles
- Boxes



- Tasks as Arcs of a Graph

- Tasks are uni-directional arrows
- Nodes now represent “states” of a project
- Kelley-Walker form

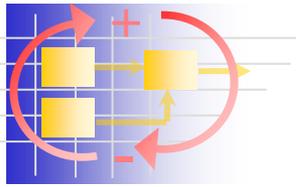




# Concept Question 1

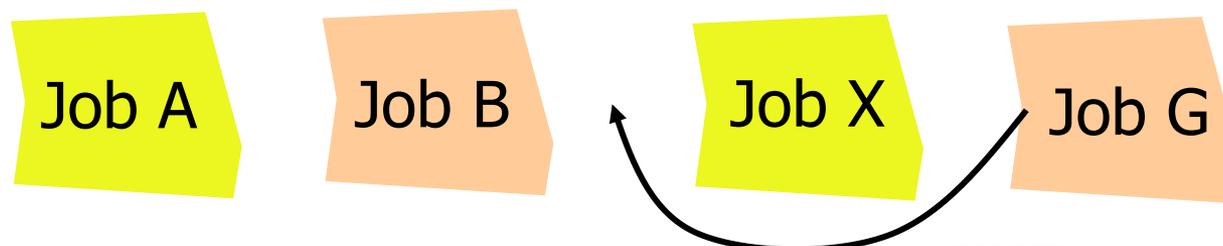
## What in project management is named after Henry Gantt (1861-1919)?

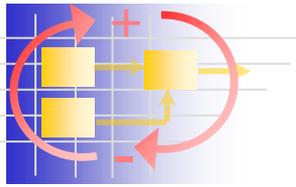
- A new method for managing budgets in shipyards
- A matrix representation of the task relationships known as the Gantt-Matrix
- A horizontal bar chart known as the Gantt-Chart
- Gantt-teams, a new form of organizational structure
- Gantt was a mechanical engineer and did not care about projects



# Work Breakdown Structure

- Used to create the task (job) list
- Tree-decomposition of project tasks
- WBS identifies “terminal elements”
- The key starting point for project planning
- Required by US Govt as part of SOW
- Can be activity-oriented or deliverable- oriented
- Use “sticky-notes” method early on
- Carl L. Pritchard. *Nuts and Bolts Series 1: How to Build a Work Breakdown Structure*. [ISBN 1890367125](#)





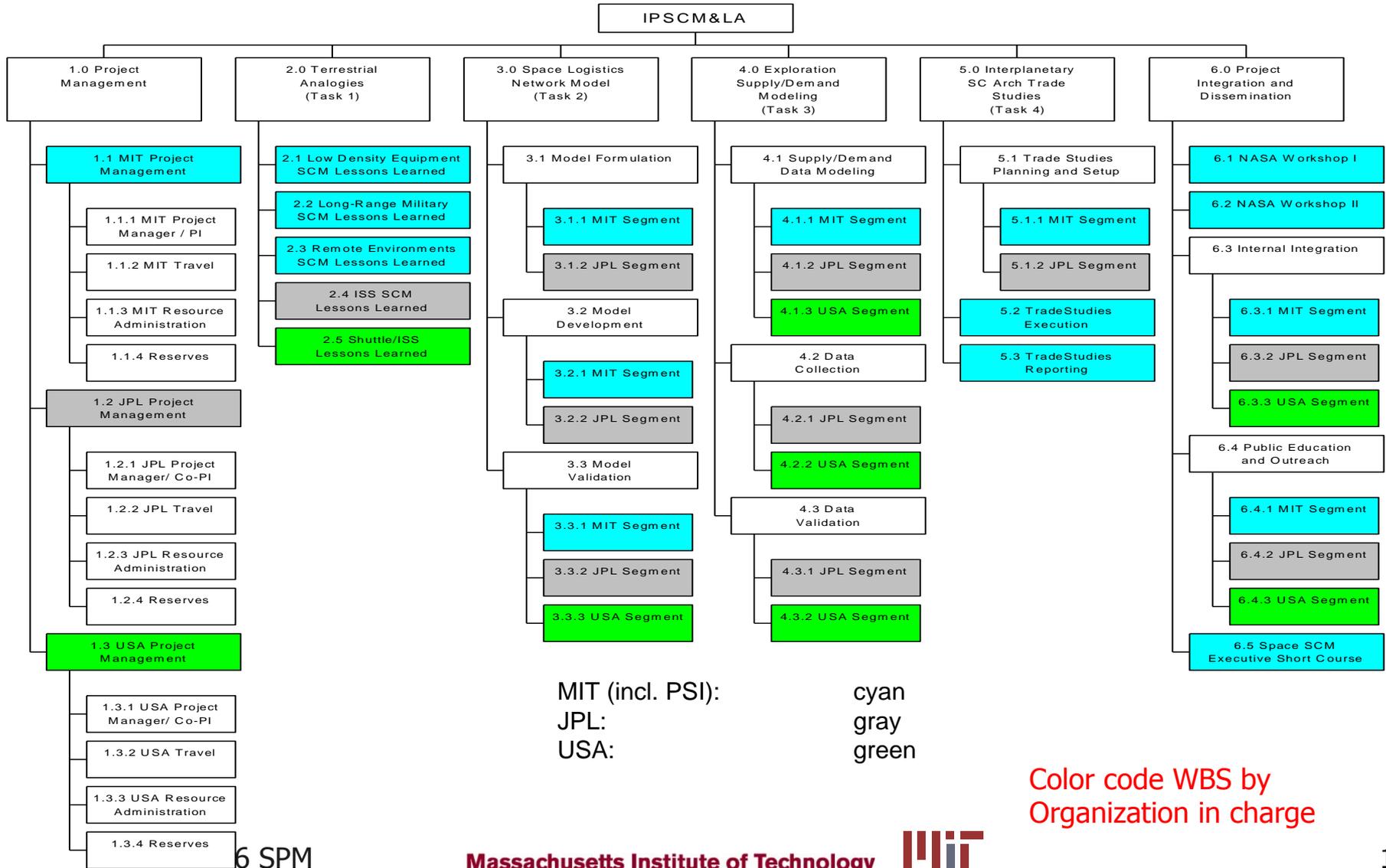
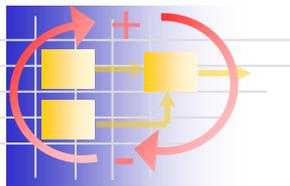
# WBS – Painting a Room

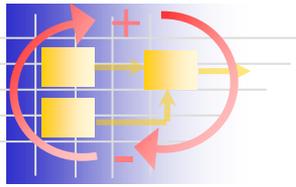
- 1 Prepare materials
  - 1.1 Buy paint
  - 1.2 Buy brushes/rollers
  - 1.3 Buy wallpaper remover
- 2. Prepare room
  - 2.1 Remove old wallpaper
  - 2.2 Remove detachable decorations
  - 2.3 Cover floor with old newspapers
  - 2.4 Cover electrical outlets/switches with tape
  - 2.5 Cover furniture with sheets
- 3. Paint the room
- 4. Clean up the room
  - 4.1 Dispose or store left over paint
  - 4.2 Clean brushes/rollers
  - 4.3 Dispose of old newspapers
  - 4.4 Remove covers

Source: <http://www.wikipedia.org>



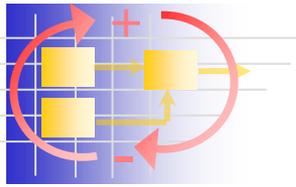
# WBS of MIT/NASA Space Logistics Project





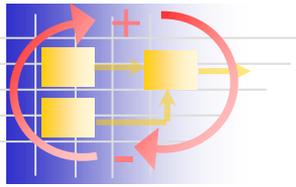
# Discussion Point 1

- Why is it difficult to come up with a good WBS (task list, task structure) in a complex project?
  - Not all tasks known ahead of time if completely new product/system
  - Others...?



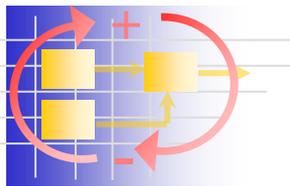
# WBS Guidelines

- No more than 100-200 terminal elements, if more → use subprojects
- Can be up to 3-4 Levels deep
- Not more than 5-9 jobs at one level
  - Human cognitive “bandwidth” only 3 bits= $2^3=8$
  - Short term memory for most people 5-9 items
  - Poorer planning if “too-fine grained” – dilution of attention
  - The more tasks there are, the more intricate dependencies there will be to keep track of
- Jobs should be of similar size/complexity
- Manageable chunks → sense of progress
- Level of graininess → very difficult to answer



# Task List

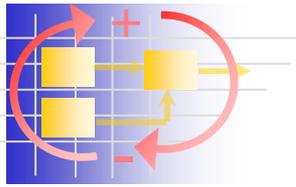
- List all tasks in a table with
  - Identifying symbol (tag, ID number)
  - Task description
  - Immediate prerequisite jobs
  - Expected task duration
- Arrange jobs in “technological order”
  - No job appears in the list until all its predecessors have been listed
  - Iterations are NOT allowed → “cycle error”
  - Job **a** precedes **b** precedes **c** precedes **a**
  - We will discuss iterations a lot in this class !!!



# Simple Example: Job List

- Two Parts X and Y: Manufacture and Assembly

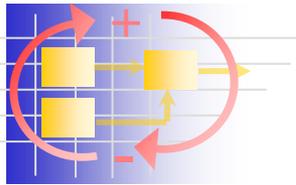
<b>Job #</b>	<b>Description</b>	<b>Immediate Predecessors</b>	<b>Time [min]</b>
<b>A</b>	<b>Start</b>		<b>0</b>
<b>B</b>	<b>Get materials for X</b>	<b>A</b>	<b>10</b>
<b>C</b>	<b>Get materials for Y</b>	<b>A</b>	<b>20</b>
<b>D</b>	<b>Turn X on lathe</b>	<b>B,C</b>	<b>30</b>
<b>E</b>	<b>Turn Y on lathe</b>	<b>B,C</b>	<b>20</b>
<b>F</b>	<b>Polish Y</b>	<b>E</b>	<b>40</b>
<b>G</b>	<b>Assemble X and Y</b>	<b>D,F</b>	<b>20</b>
<b>H</b>	<b>Finish</b>	<b>G</b>	<b>0</b>



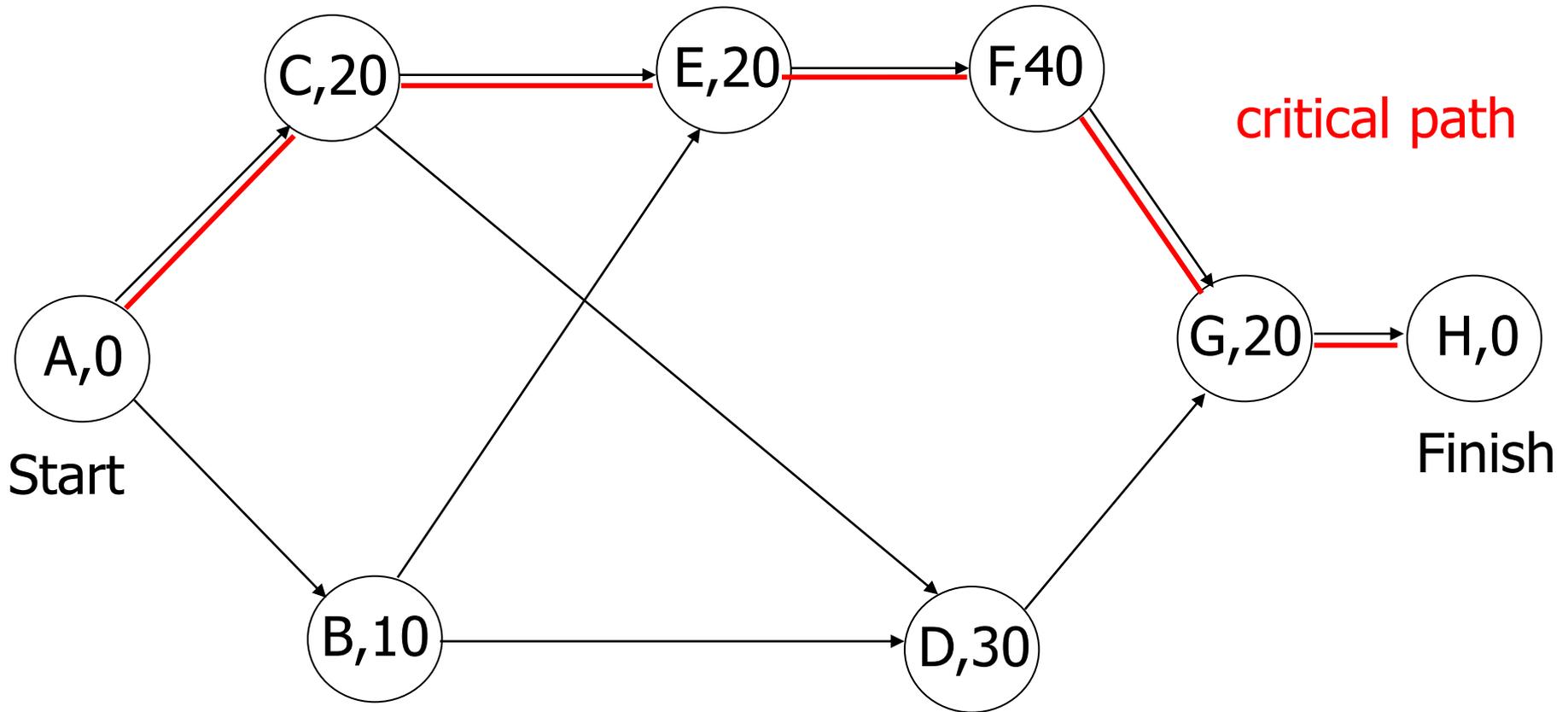
# Project Graph

- Each job is drawn on a graph as a circle\*
- Connect each job with immediate predecessor(s) – unidirectional arrows “→”
- Jobs with no predecessor connect to “Start”
- Jobs with no successors connect to “Finish”
- “Start” and “Finish” are pseudo-jobs of length 0
- A finite number of “arrow paths” from “Start” to “Finish” will be the result
- Total time of each path is the sum of job times
- Path with the longest total time → **“critical path”**
- There can be multiple critical paths → **minimum time to complete project**

\* or other symbol, see before

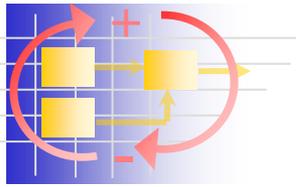


# Project Graph



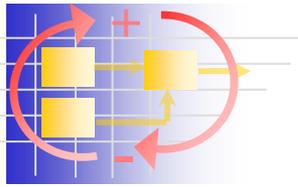
4 unique paths: A,C,E,F,G,H; A,C,D,G,H; A,B,D,G,H; A,B,E,F,G,H  
100 70 60 90





# Critical Path

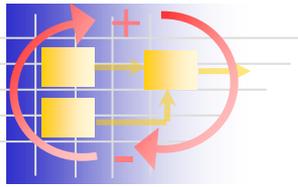
- CP is the “bottleneck route”
- Shortening or lengthening tasks on the critical path directly affects project finish
- Duration of “non-critical” tasks is irrelevant
- “Crashing” all jobs is ineffective, focus on the few % of jobs that are on the CP
- “Crashing” tasks can shift the CP to a different task
- Shortening tasks – technical and economical challenge
  - How can it be done?
- Previously non-critical tasks can become critical
- Lengthening of non-critical tasks can also shift the critical path (see HW1).



# Discussion Point 2

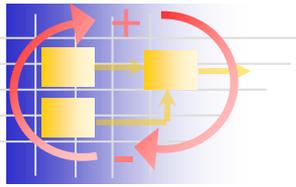
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- What is the usefulness of knowing the CP in a project?
  - Tells which task to shorten to finish project earlier.
  - Others ...?



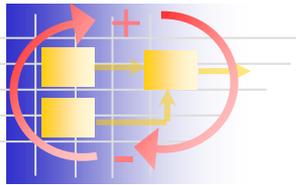
# Critical Path Algorithm

- For large projects there are many paths
- Need an algorithm to identify the CP efficiently
- Develop information about each task in context of the overall project
- Times
  - Start time (S)
  - For each job: Earliest Start (ES)
    - Earliest start time of a job if all its predecessors start at ES
  - Job duration:  $t$
  - Earliest Finish (EF) = (ES) +  $t$
- Finish time (F) – earliest finish time of the overall project
- Show algorithm using project graph

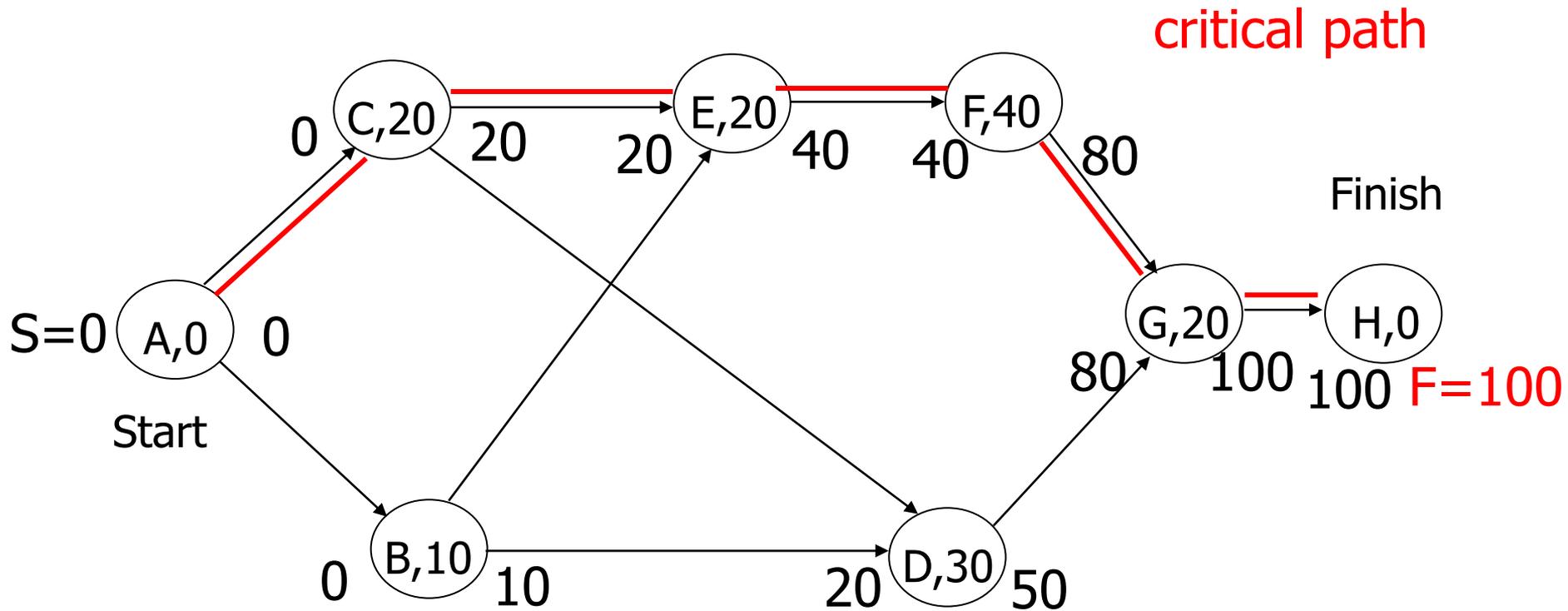


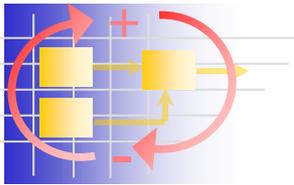
# CP Algorithm

1. Mark the value of **S** to left and right of Start
2. Consider any new unmarked job, all of whose predecessors have been marked. Mark to the left of the new job the largest number to the right of its immediate predecessors: (**ES**)
3. Add to **ES** the job time **t** and mark result to the right (**EF**)
4. Stop when **Finish** has been reached



# CP Algorithm - Graphical





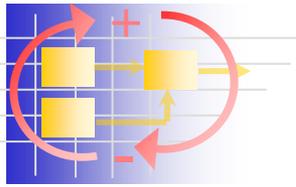
## Concept Question 2

A project starts with (A,5). Task (B,10) can start after A is completed. This is also true for task (E,5). Task (C,8) depends only on (B,10), while task (F,10) depends on both (B,10) and (E,5). Task (D,5) is the last task in the project and it can start once (C,8) and (F,10) have been finished.

The Earliest Finish (EF) time for the whole project is:

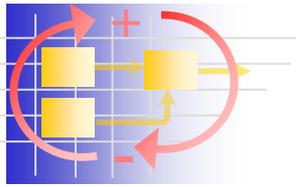
Possible Answers

- 20
- 22
- 25
- 27
- 30
- 35
- 40



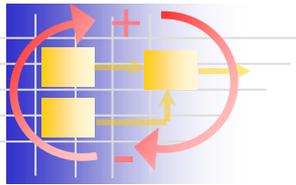
# Latest Start and Finish Times

- Set target finish time for project:  $T \geq F$
- Usually target is a specific calendar date, e.g. October 1, 2007
- When is the latest date the project can be started?
- Late Finish (**LF**) - latest time a job can be finished, without delaying the project beyond its target time (**T**)
- Late Start: **LS = LF - t**

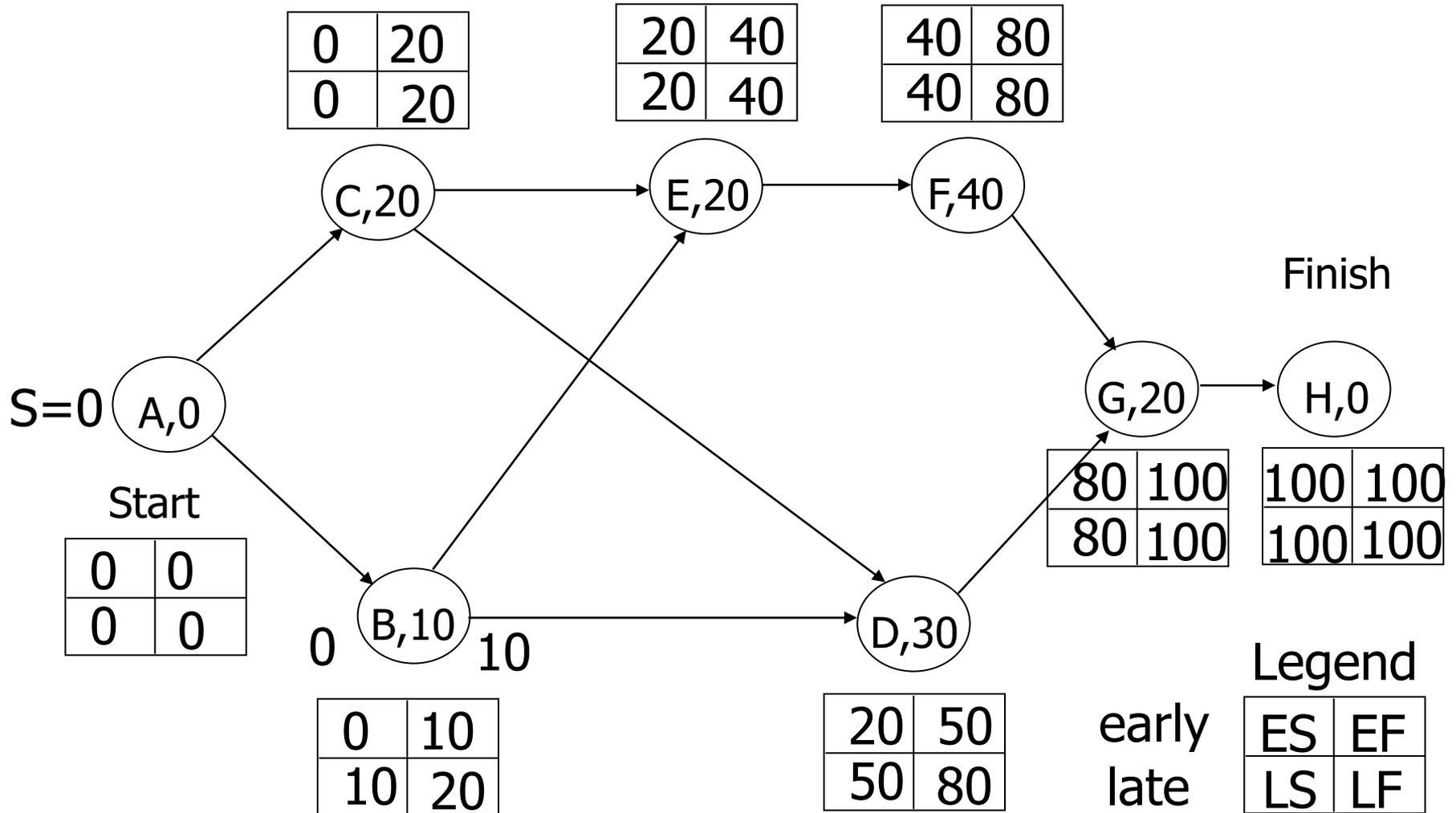


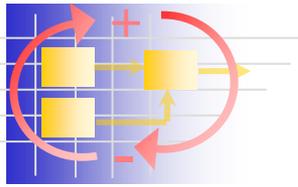
# Determine LF and LS

- Work from the end of the project: **T**
  1. Mark value of **T** to left and right of Finish
  2. Consider any new unmarked job, all of whose successors have been marked - mark to the right the smallest **LS** time marked to the left of any of its immediate successors
  3. Subtract from this number, **LF**, the job time **t** and mark result to the left of the job: **LS**
  4. Continue upstream until Start has been reached, then stop



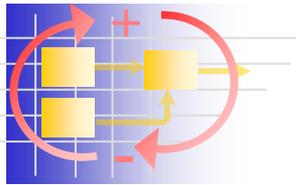
# LS and LF : Project Graph



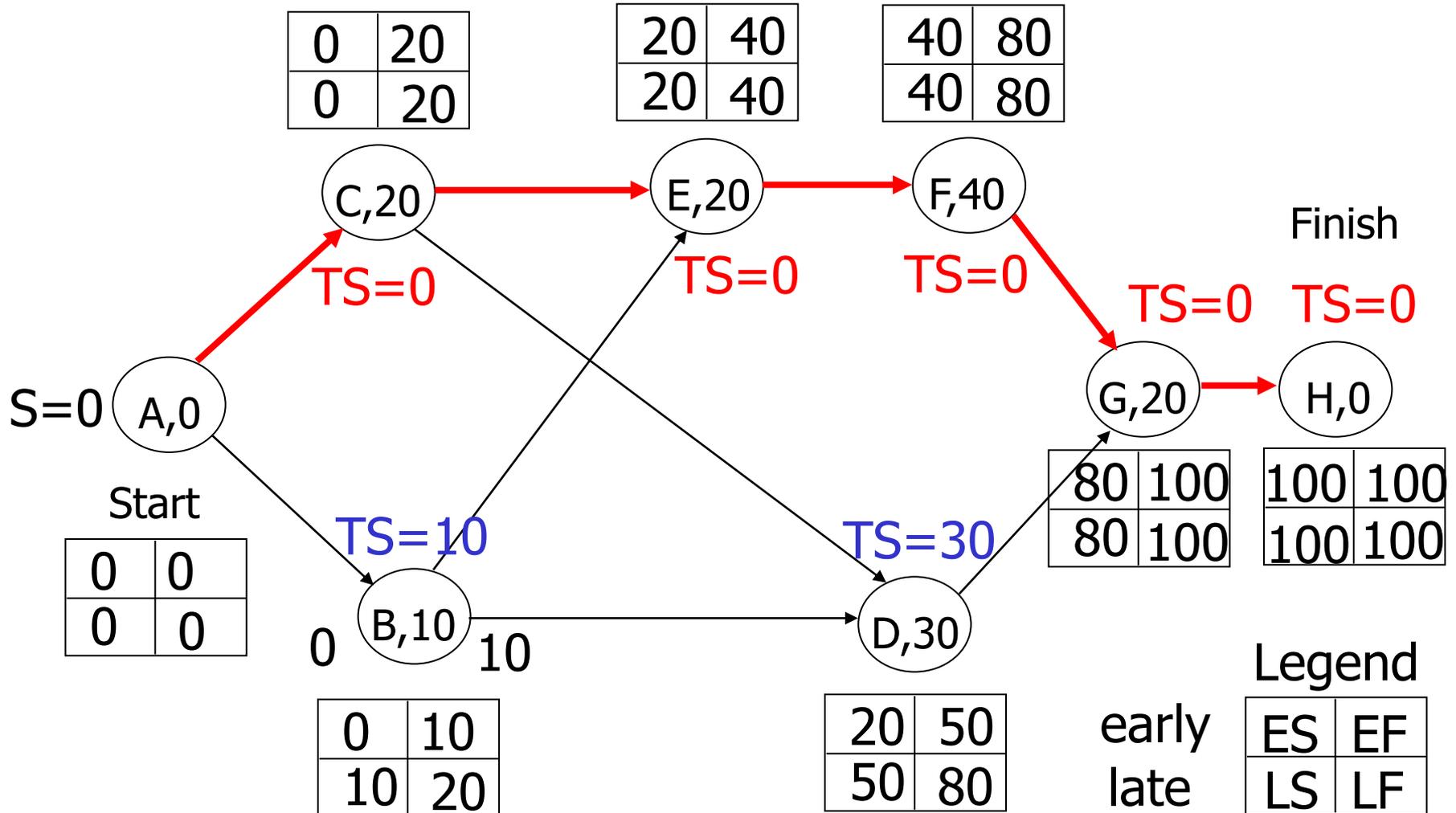


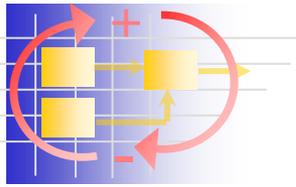
# Slack

- Some tasks have **ES=LS** --> no slack
- Total Slack of a task **TS=LS-ES**
- **Maximum amount of time a task may be delayed beyond its early start without delaying project completion**
- Slack time is precious ... managerial freedom, don't squander it unnecessarily
  - e.g. resource, work load smoothing
- When **T=F** then all critical tasks have **TS=0**
- At least one path from Start->Finish with critical jobs only
- When **T>F**, then all critical jobs have **TS=T-F**

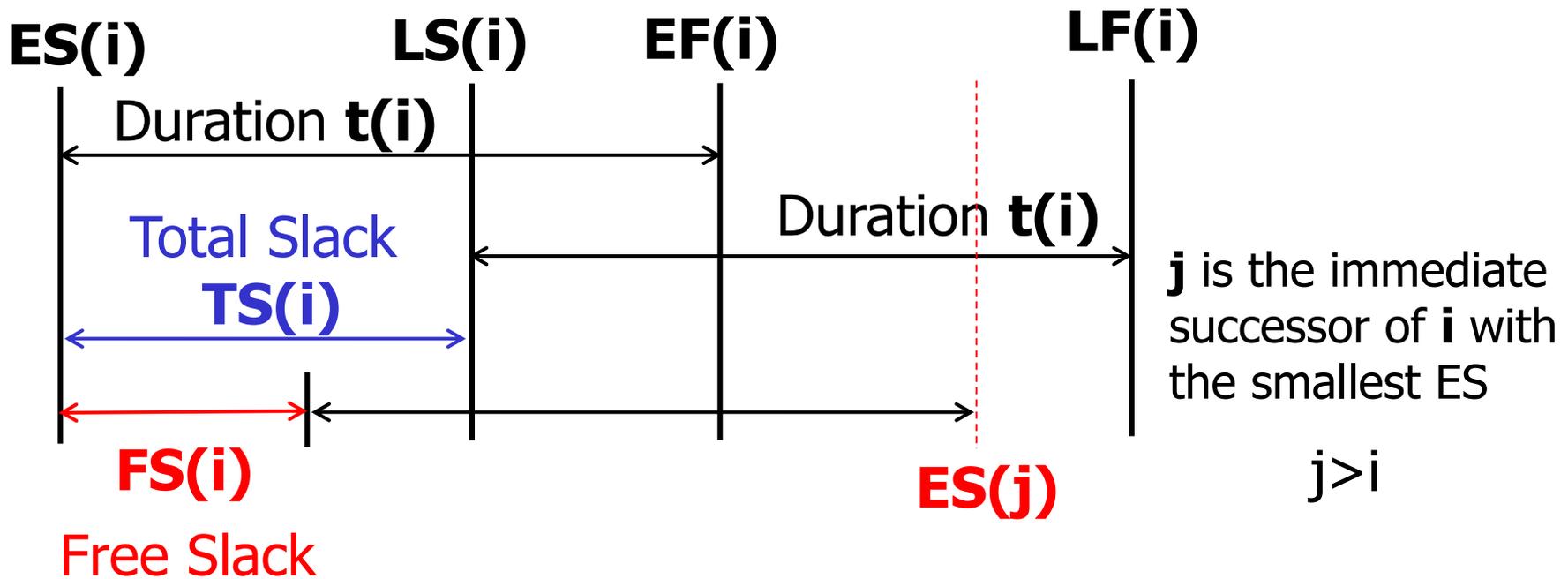


# Project Graph - Slack



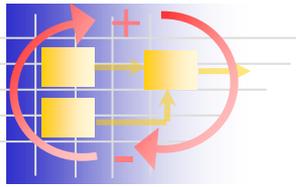


# Task Times Detail - Task i



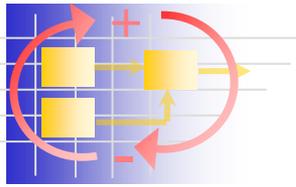
- Free Slack (**FS**) is the amount a job can be delayed without delaying the Early Start (ES) of any other job.

**FS ≤ TS always**



# Main CPM Errors

- Estimated job times are wrong
- Predecessor relationships may contain cycles → “cycle error”
- List of prerequisites contains more than the immediate predecessors, e.g. **a→b**, **b→c** and **a,b→c**
- Overlooked some predecessor relationships
- Some predecessor relationships may be listed that are spurious
- and .... Some tasks/jobs may be missing !!!



# Gradual Refinement of CPM

## ■ Job Times

- Given rough time estimates construct CPM chart
- Re-estimate times for **CP** and those with very small **TS**
- Iterate until the critical path is stable
- Focus attention on a subset of tasks

## ■ Predecessor Relationships

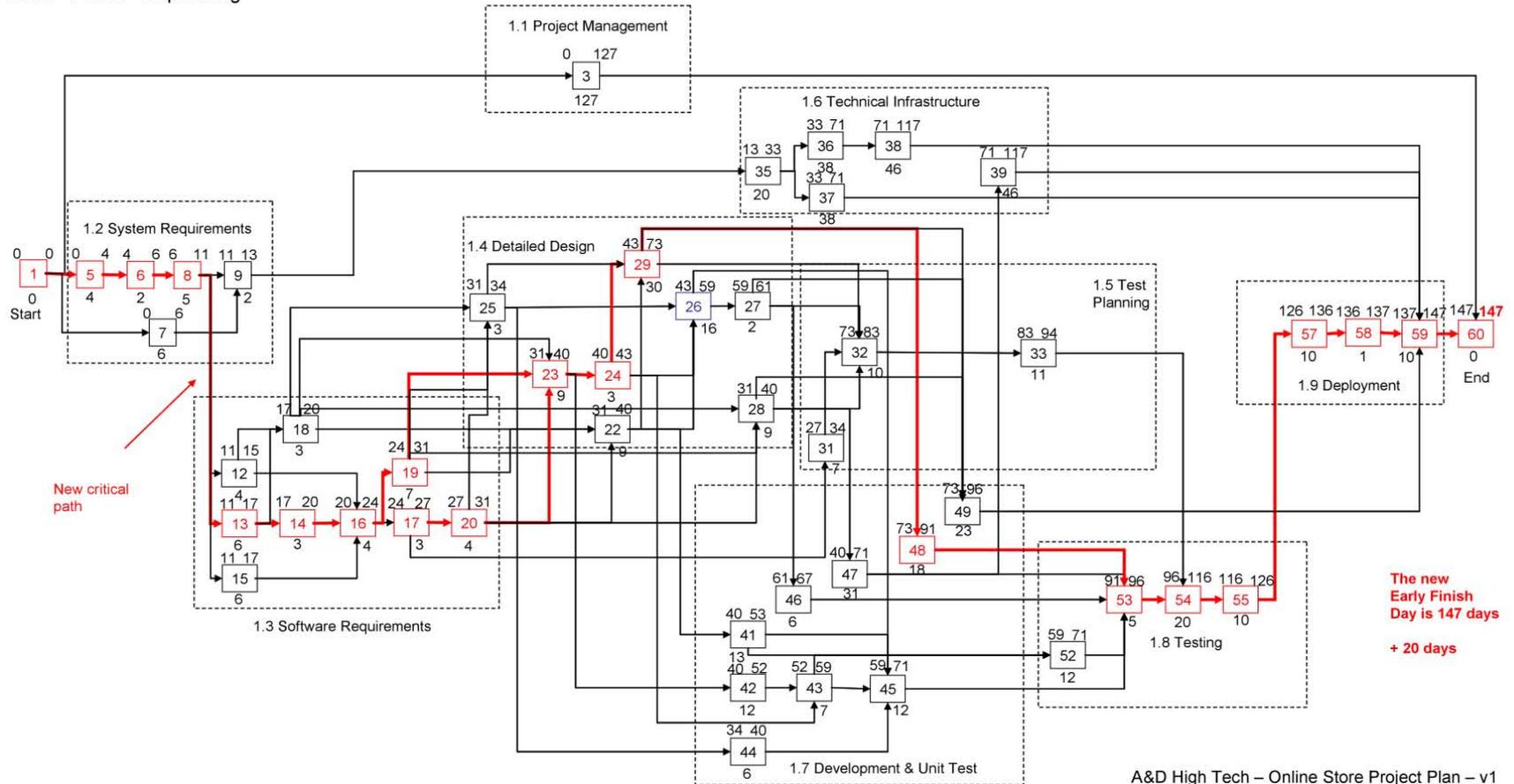
- Check algorithmically for cycle errors and pre-predecessor errors
- Cancel all except immediate predecessor relationships

## ■ Wrong or Missing Facts

- Cannot be detected by computers!

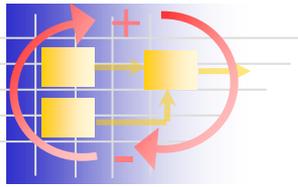
# Sample CPM (2007)

HW2 – Part 8 - Replanning



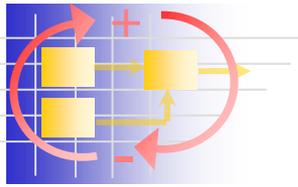
A&D High Tech – Online Store Project Plan – v1

*A&D High Tech Case, Online Store Project, 60 tasks, HW2, 2007*



# Crashing Tasks

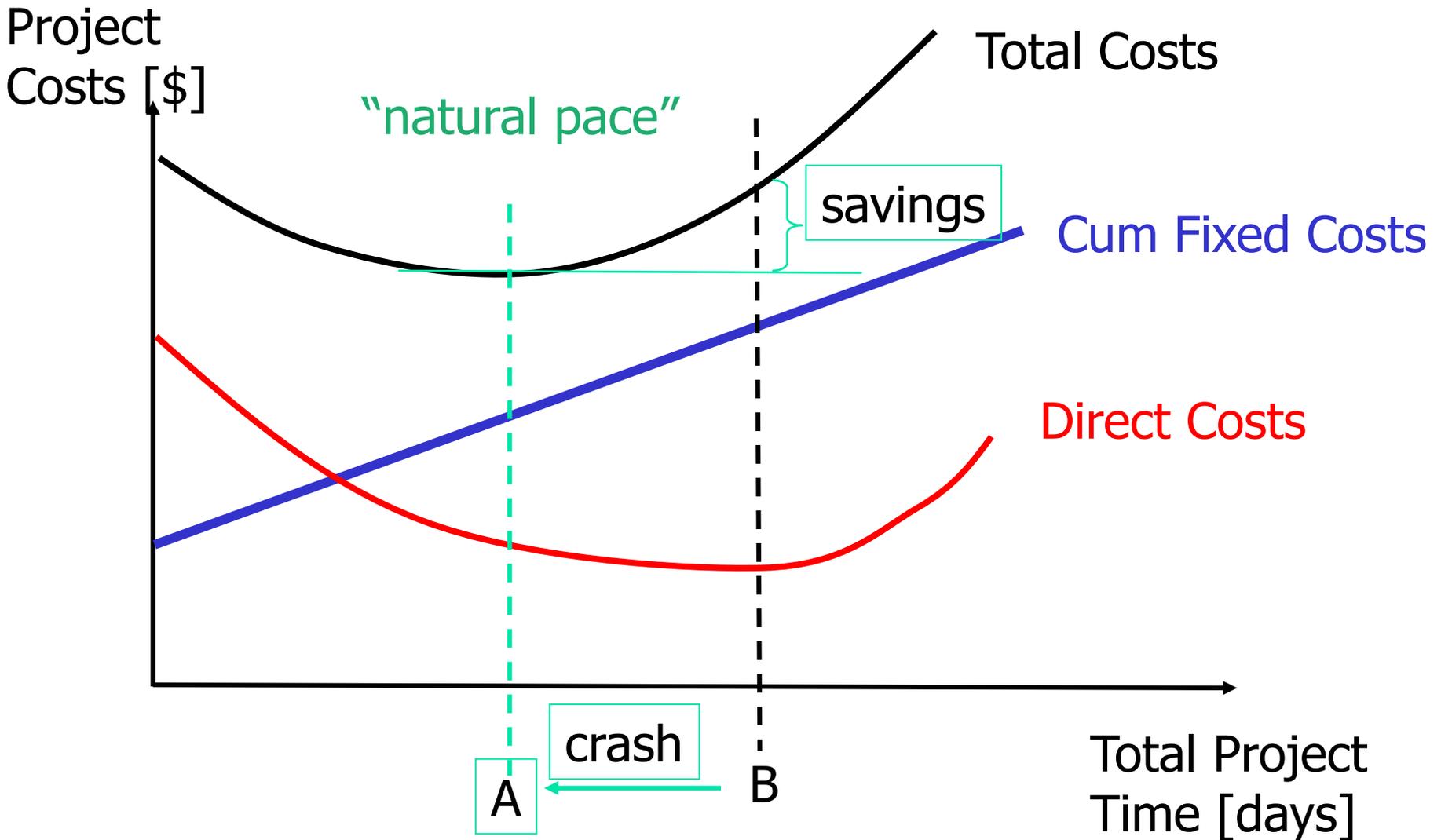
- What is we want to speed up project completion?
- Options
  - Work overtime
  - Put more resources on the critical tasks
  - Parallelize tasks that are really serial (later in class)
- Cost of speedup?
- Is there a net savings resulting from reduction in overall project time?

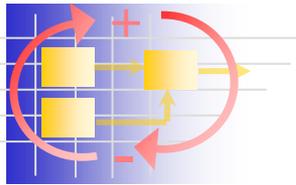


# Cost Calculations

- Can compute project costs if cost of each job is included in the task data
- (Potentially) shorten crew jobs by adding personnel or working overtime with existing personnel
- Speedup carries price tag: “normal time”, “crash time”
- Assign some critical jobs to their “crash time”
- Direct costs will increase as we “crash” critical tasks
- Indirect (fixed, overhead) costs will decrease as the overall project duration decreases – “standing army phenomenon”
- Minimize the sum of fixed and direct costs

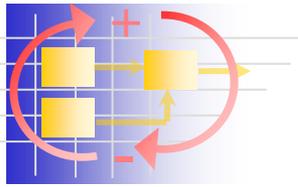
# Typical Cost Pattern





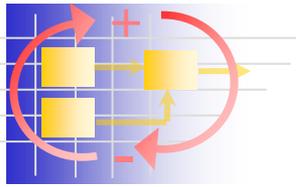
# CPM Judgment

- +
  - Focuses attention on a subset of critical tasks
  - Determine effect of shortening/lengthening tasks
  - Evaluate costs of a "crash" program
- -
  - Doesn't capture task iterations, in fact ...
  - Prohibits iterations = "cycle error"
  - Treats task durations as deterministic



# Summary

- CPM is useful, despite criticism, to identify the critical path - focus on a subset of the project
- Slack (TS and FS) is precious
  - apply flexibility to smooth resource/schedules
- PERT treats task times as probabilistic
  - Next lecture
- Selective “crashing” of critical tasks can reduce (or increase) total project cost
- CPM does not allow for task iterations

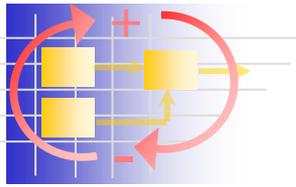


# Class Frustrations

- Poor examples set by project managers
- Perception of PMs as bureaucratic “box-checkers” and “ankle-biters”

## Why?

- Traditional Project Management ...
  - Doesn't acknowledge the existence of iterations
  - Is inflexible, “changing the plan” considered a failure
  - Does not think of projects in a probabilistic sense
  - “Hostage” to existing project management software
  - In a reactive mode – no “early warning” systems



# HW1 Introduction

- You are Project Manager for the new CityCar development project
- Plan the project
  - Task list
  - Create project graph
  - Critical path
  - Slack times
  - Re-planning after change
  - “managerial”-type questions

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