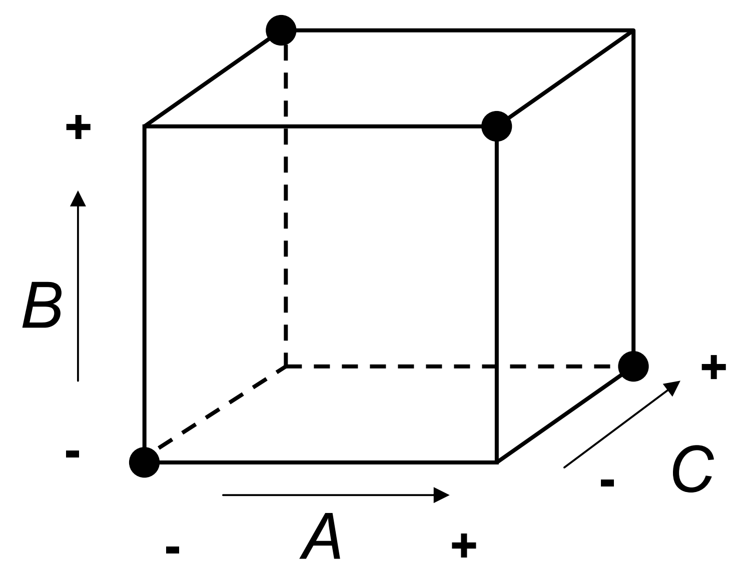
ESD.33 -- Systems Engineering

Session #11

Supplementary Session on

Design of Experiments



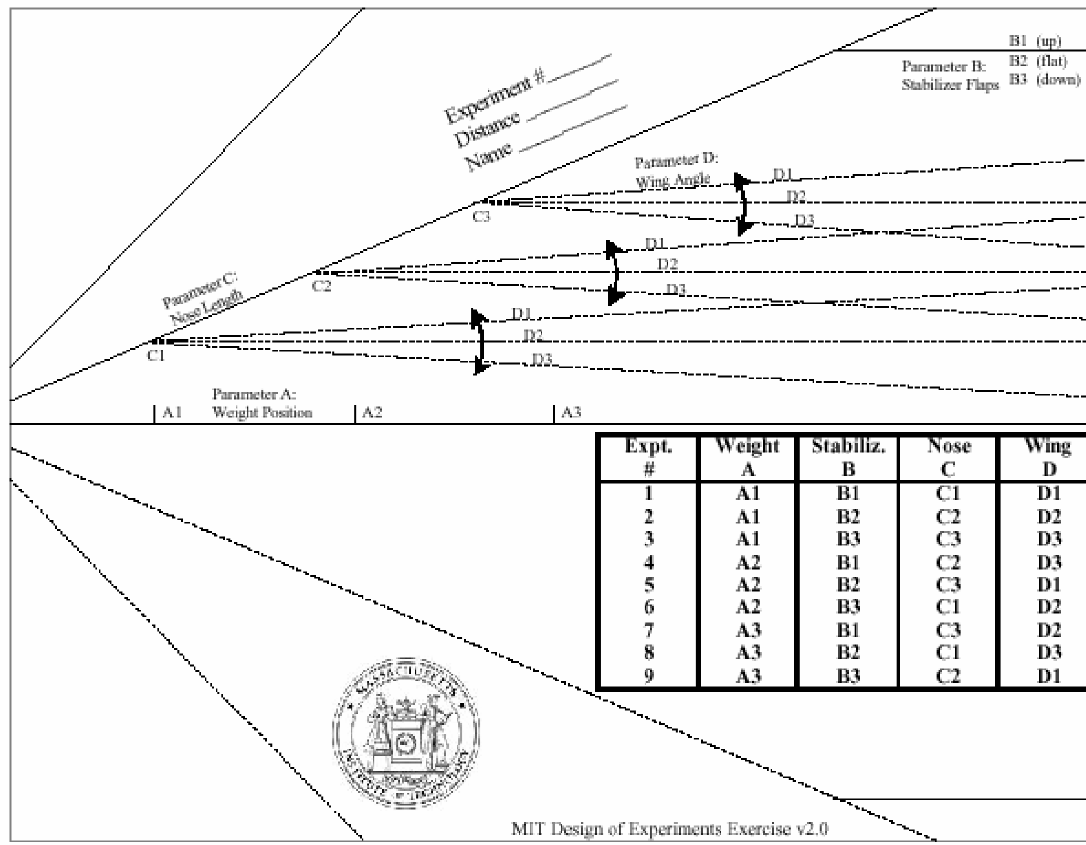
Dan Frey

Plan for the Session

- de Weck on Isoperformance
- Assignment #6
- Review of Statistical Preliminaries
- Review of Design of Experiments
- Frey – A role for one factor at a time?
- Next steps

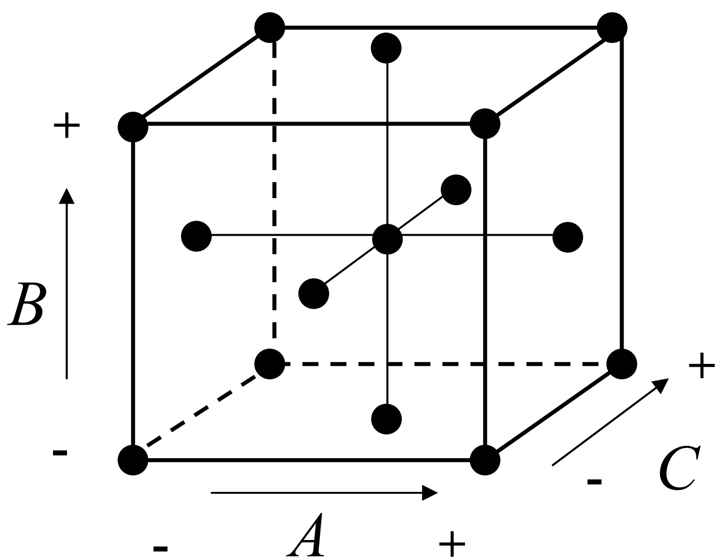
Assignment #6

1) Short answers
2) Regression
3) DOE



Expt. #	Weight A	Stabiliz. B	Nose C	Wing D
1	A1	B1	C1	D1
2	A1	B2	C2	D2
3	A1	B3	C3	D3
4	A2	B1	C2	D3
5	A2	B2	C3	D1
6	A2	B3	C1	D2
7	A3	B1	C3	D2
8	A3	B2	C1	D3
9	A3	B3	C2	D1

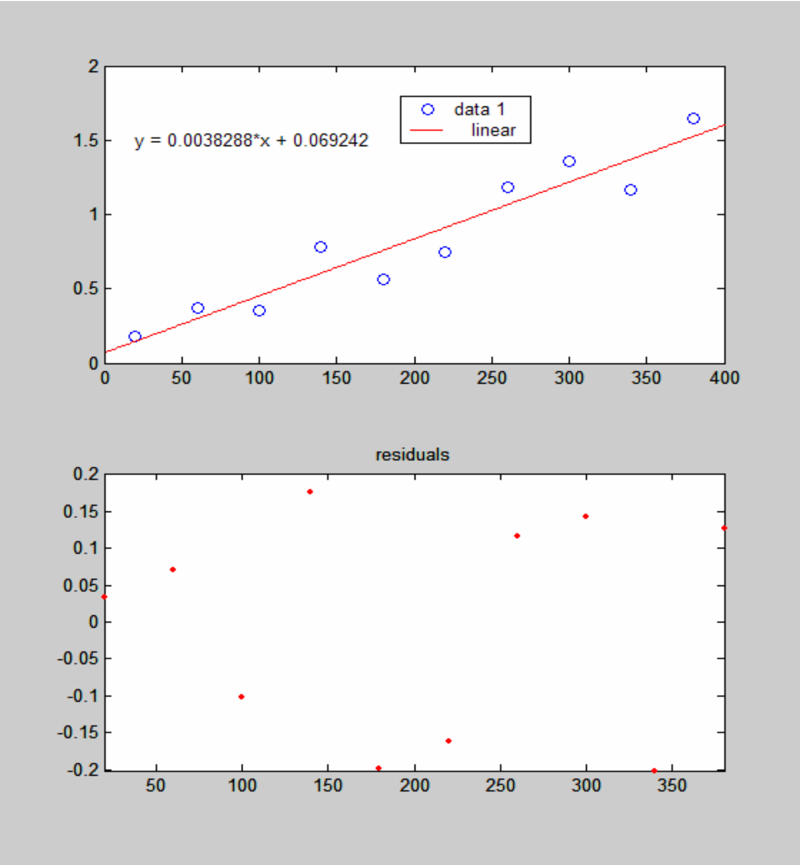
Central Composite Design



2^3 with center points and axial runs

Regression

- Fit a linear model to data & answer certain statistical questions

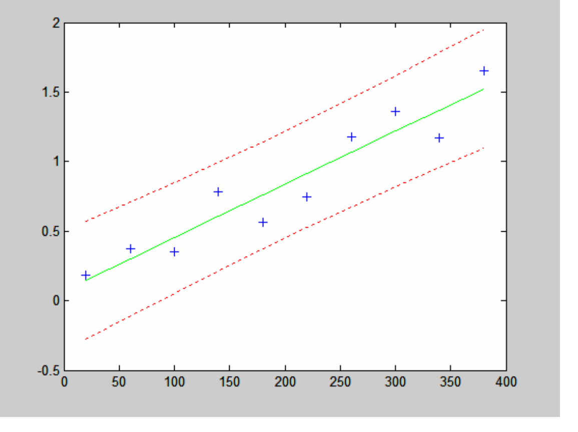


Air vel (cm/sec)	Evap coeff. (mm^2/sec)
20	0.18
60	0.37
100	0.35
140	0.78
180	0.56
220	0.75
260	1.18
300	1.36
340	1.17
380	1.65

Evaporation vs Air Velocity Confidence Intervals for Prediction

Air vel (cm/sec)	Evap coeff. (mm^2/sec)
20	0.18
60	0.37
100	0.35
140	0.78
180	0.56
220	0.75
260	1.18
300	1.36
340	1.17
380	1.65

```
[p,S] = polyfit(x,y,1);
alpha=0.05;
[y_hat,del]=polyconf(p,x,S,alpha);
plot(x,y,'+',x,y_hat,'g')
hold on
plot(x,y_hat+del,'r:')
plot(x,y_hat-del,'r:')
```



Evaporation vs Air Velocity Hypothesis Tests

Air vel (cm/sec)	Evap coeff. (mm^2/sec)
20	0.18
60	0.37
100	0.35
140	0.78
180	0.56
220	0.75
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SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.934165
R Square	0.872665
Adjusted R Square	0.854474
Standard Error	0.159551
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*ignificance F*
Regression	1	1.221227	1.221227	47.97306	0.000226
Residual	7	0.178196	0.025457		
Total	8	1.399422			

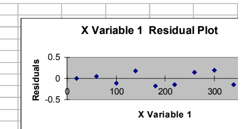
	Coefficients	*tandard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*Ipper 95.0%*
Intercept	0.102444	0.106865	0.958637	0.369673	-0.15025	0.355139	-0.15025	0.355139
X Variable 1	0.003567	0.000515	6.926259	0.000226	0.002349	0.004784	0.002349	0.004784

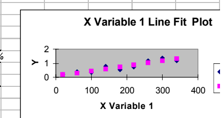
RESIDUAL OUTPUT

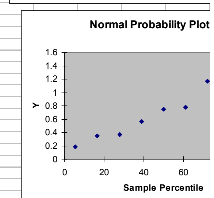
Observation	*Predicted Y*	*Residuals*	*dard Residuals*
1	0.173778	0.006222	0.041691
2	0.316444	0.053556	0.35884
3	0.459111	-0.10911	-0.73108
4	0.601778	0.178222	1.194149
5	0.744444	-0.18444	-1.23584
6	0.887111	-0.13711	-0.91869
7	1.029778	0.150222	1.006539
8	1.172444	0.187556	1.256685
9	1.315111	-0.14511	-0.97229

PROBABILITY OUTPUT

Percentile	*Y*
5.555556	0.18
16.66667	0.35
27.77778	0.37
38.88889	0.56
50	0.75
61.11111	0.78
72.22222	1.17
83.33333	1.18
94.44444	1.36







Fractional Factorial Experiments

Two Levels

Trial	A	B	C	D	E	F	G		FG=-A
1	-1	-1	-1	-1	-1	-1	-1		+1
2	-1	-1	-1	+1	+1	+1	+1		+1
3	-1	+1	+1	-1	-1	+1	+1		+1
4	-1	+1	+1	+1	+1	-1	-1		+1
5	+1	-1	+1	-1	+1	-1	+1		-1
6	+1	-1	+1	+1	-1	+1	-1		-1
7	+1	+1	-1	-1	+1	+1	-1		-1
8	+1	+1	-1	+1	-1	-1	+1		-1

2^{7-4} Design (aka “orthogonal array”)

Every factor is at each level an equal number of times (balance).

High replication numbers provide precision in effect estimation.

Resolution III.

Fractional Factorial Experiments

Three Levels

The design below is also fractional factorial design.

Plackett Burman $(P\text{-}B)_{3,9}$ Taguchi $OA_9(3^4)$

Control Factors			
A	B	C	D
1	1	1	1
1	2	2	2
1	3	3	3
2	1	2	3
2	2	3	1
2	3	1	2
3	1	3	2
3	2	1	3
3	3	2	1

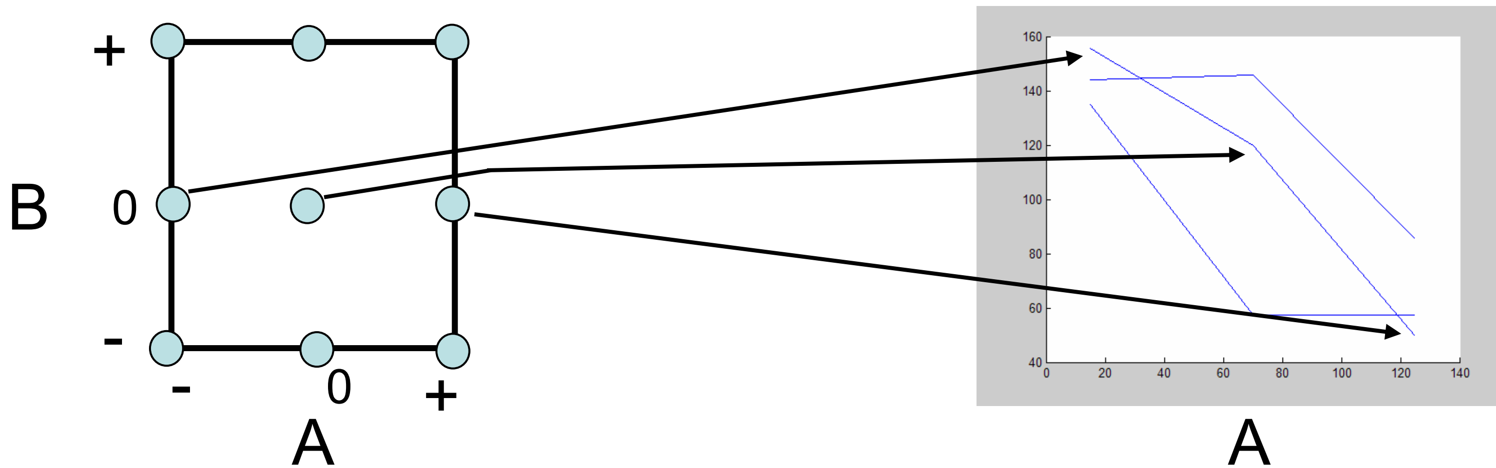
requires only

$$k(p-1)+1=9$$

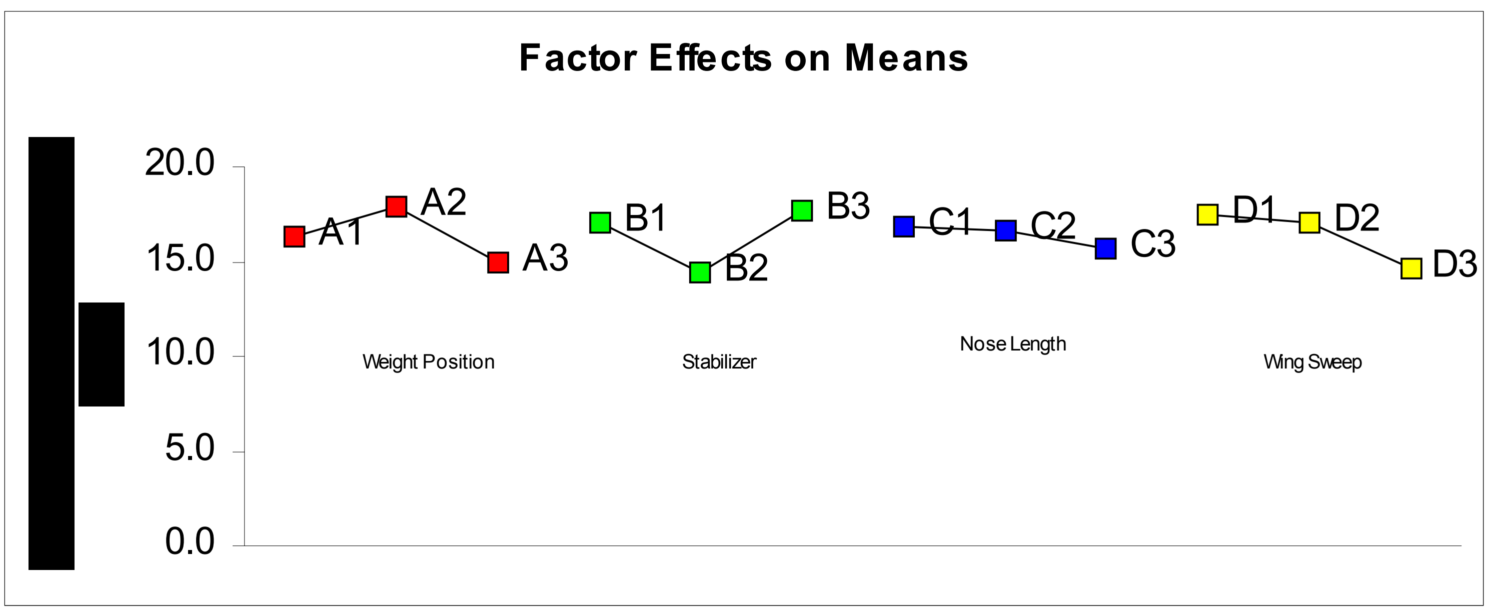
experiments

But it is only Resolution III and also has complex confounding patterns.

Factor Effect Plots



Full Factorial 3^2



Fractional Factorial L9

Plan for the Session

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- Review of Statistical Preliminaries
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- Frey – A role for one factor at a time?
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*One way of thinking of the great advances of the science of experimentation in this century is as **the final demise of the "one factor at a time" method**, although it should be said that there are still organizations which have never heard of factorial experimentation and use up many man hours wandering a crooked path.*

– N. Logothetis and H. P. Wynn

"The factorial design is ideally suited for experiments whose purpose is to map a function in a pre-assigned range."

"…however, the factorial design has certain deficiencies … It devotes observations to exploring regions that may be of no interest."

"…These deficiencies of the factorial design suggest that an efficient design for the present purpose ought to be sequential; that is, ought to adjust the experimental program at each stage in light of the results of prior stages."

Friedman, Milton, and L. J. Savage, 1947, "Planning Experiments Seeking Maxima", in *Techniques of Statistical Analysis*, pp. 365-372.

"Some scientists do their experimental work in single steps. They hope to learn something from each run … they see and react to data more rapidly …"

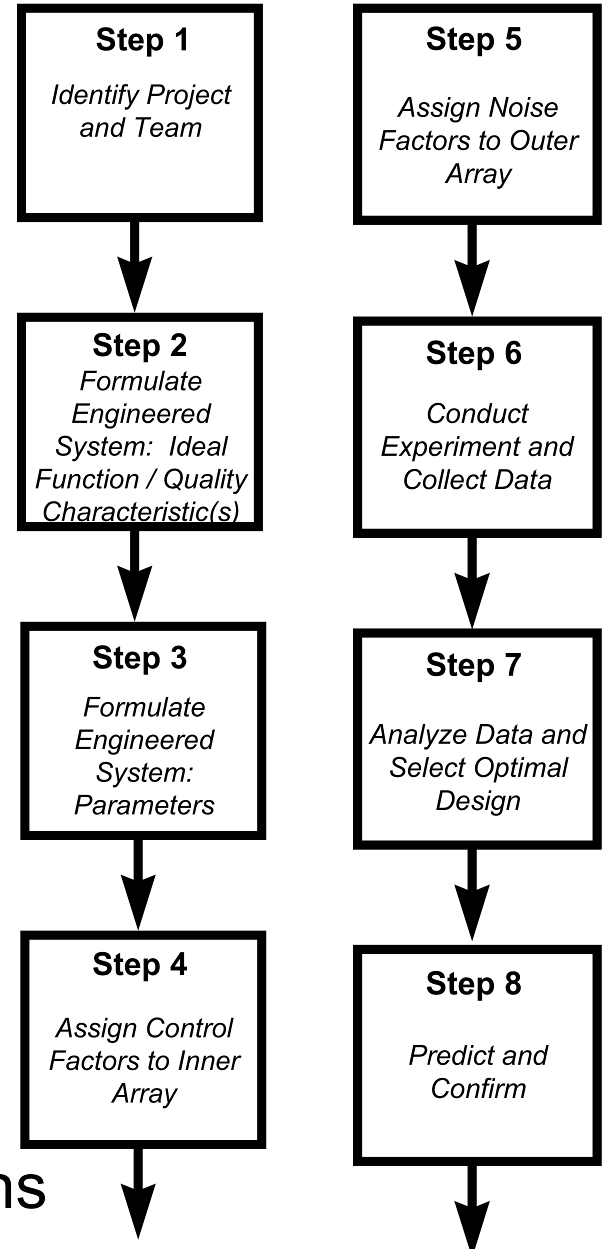
"…Such experiments are economical"

"…May give biased estimates"

"If he has in fact found out a good deal by his methods, it must be true that the effects are at least three or four times his average random error per trial."

Cuthbert Daniel, 1973, "One-at-a-Time Plans", *Journal of the American Statistical Association*, vol. 68, no. 342, pp. 353-360.

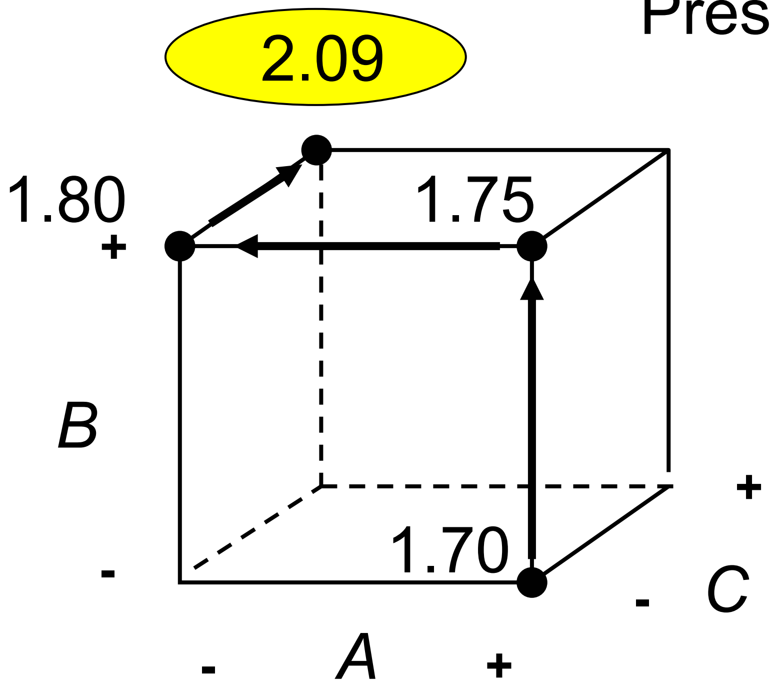
Ford Motor Company, “Module 18: Robust System Design Application,” FAO Reliablitiy Guide, Tools and Methods Modules.



Step 4 Summary:

- Determine control factor levels
- Calculate the DOF
- Determine if there are any interactions
- Select the appropriate orthogonal array

One at a Time Strategy

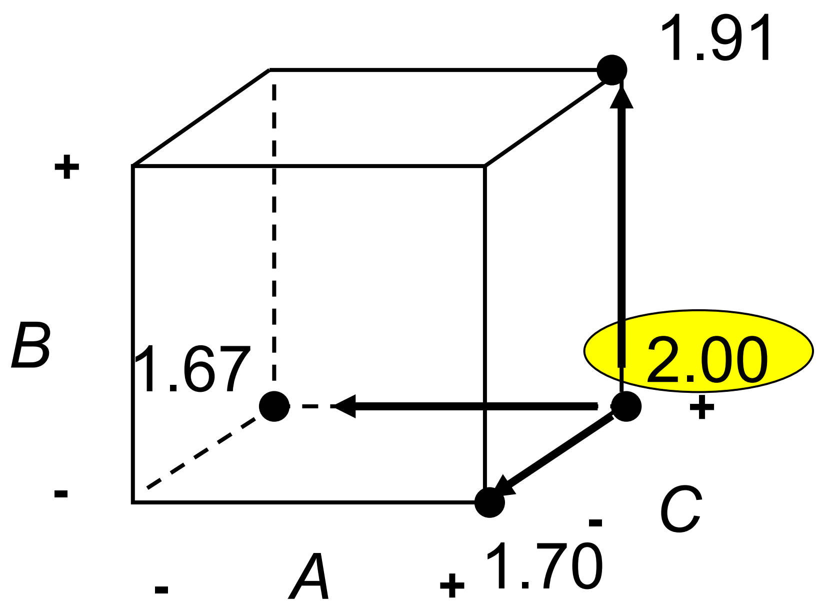


Pressure Temperature Time

A	B	C	Transverse stiffness [GPa]
-	-	-	1.30
-	-	+	1.67
-	+	-	1.80
-	+	+	2.09
+	-	-	1.70
+	-	+	2.00
+	+	-	1.75
+	+	+	1.91

Bogoeva-Gaceva, G., E. Mader, and H. Queck (2000) Properties of glass fiber polypropylene composites produced from split-warp-knit textile preforms, *Journal of Thermoplastic Composite Materials* 13: 363-377.

One at a Time Strategy



Pressure A	Temperature B	Time C	Transverse stiffness [GPa]
-	-	-	1.30
-	-	+	1.67
-	+	-	1.80
-	+	+	2.09
+	-	-	1.70
+	-	+	2.00
+	+	-	1.75
+	+	+	1.91

One at a Time Strategy

Starting point			Order in which factors were varied					
A	B	C	ABC	ACB	BAC	BCA	CAB	CBA
-	-	-	2.09	2.00	2.09	2.09	2.00	2.09
-	-	+	2.00	2.00	2.09	2.09	2.00	2.09
-	+	-	2.09	2.09	2.09	2.09	2.09	2.09
-	+	+	2.09	2.09	2.09	2.09	2.09	2.09
+	-	-	2.09	2.00	2.09	2.09	2.00	2.00
+	-	+	2.00	2.00	2.00	2.00	2.00	2.00
+	+	-	2.09	2.09	2.09	2.09	2.09	2.00
+	+	+	2.09	2.09	2.00	2.00	2.09	2.00

1/2 of the time -- the optimum level setting 2.09GPa.
1/2 of the time – a sub-optimum of 2.00GPa
Mean outcome is 2.04GPa.

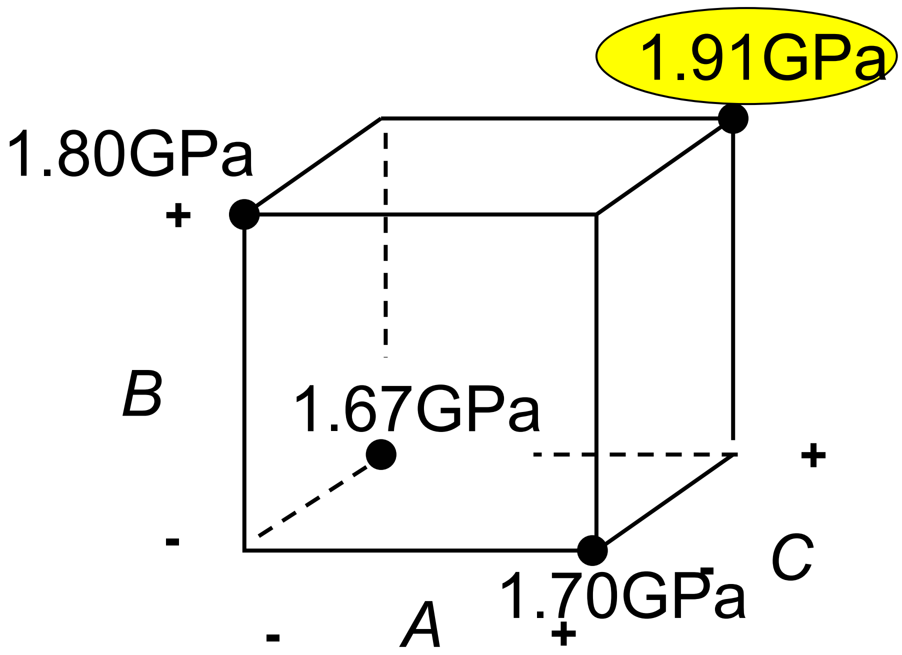
Main Effects and Interactions

Effect	Transverse stiffness [GPa]
μ	1.778
A	0.063
B	0.110
C	0.140
AB	-0.120
AC	-0.025
BC	-0.027
ABC	-0.008

A	B	C	Transverse stiffness [GPa]
-	-	-	1.30
-	-	+	1.67
-	+	-	1.80
-	+	+	2.09
+	-	-	1.70
+	-	+	2.00
+	+	-	1.75
+	+	+	1.91

The approach always *exploited* the two largest effe
including an interaction although the experiment
cannot *resolve* interactions

Fractional Factorial



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+	+	-	1.75
+	+	+	1.91

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μ	1.778
A	0.063
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C	0.140
AB	-0.120
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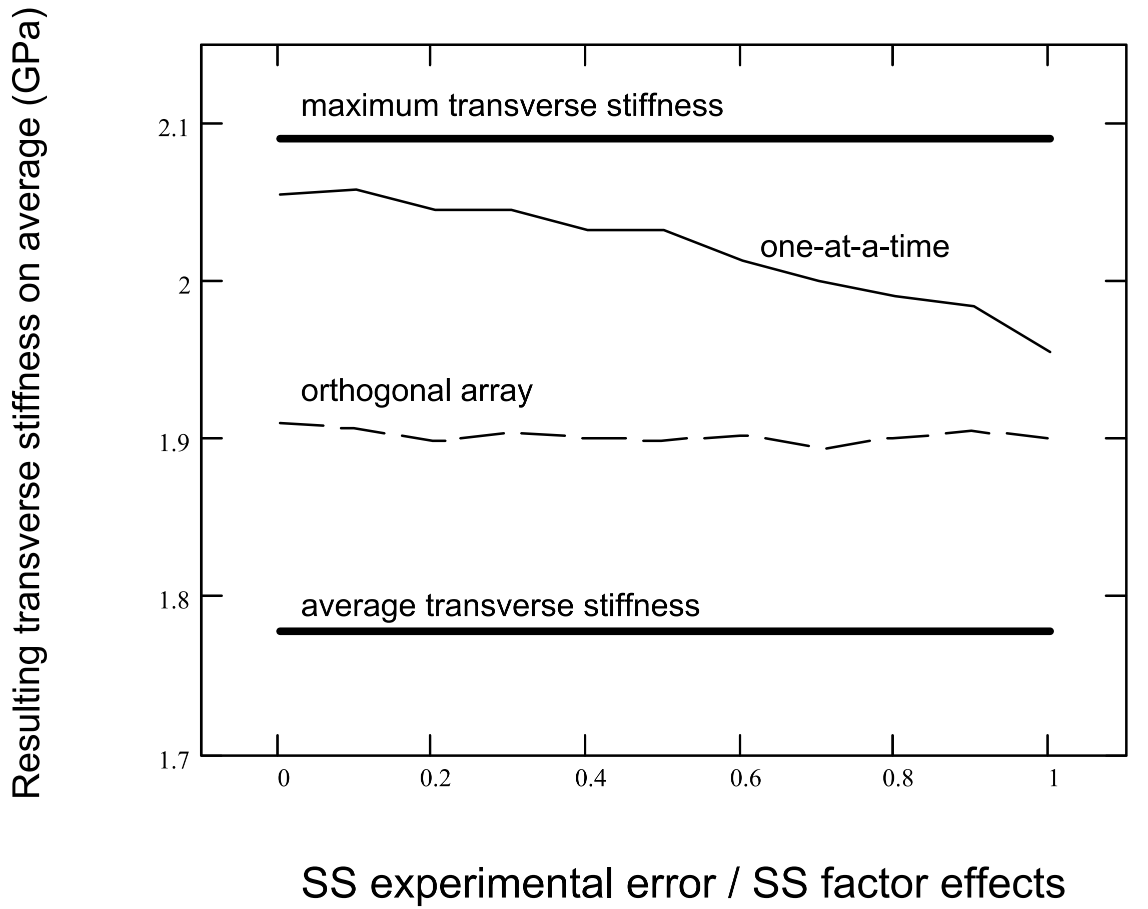
BC	-0.027
ABC	-0.008

} Factorial design correctly estimates main effects BUT

← AB interaction is larger than main effects of factor A or B and is anti-synergistic

Factorial design worked as advertised but missed the optimum

Effect of Experimental Error



Results from a Meta-Study

- 66 responses from journals and textbooks
- Classified according to interaction strength

Interaction Strength	Strength of Experimental Error: 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Mild	100/99	99/98	98/98	96/96	94/94	89/92	86/88	81/86	77/82	73/79	69/75
Moderate	96/90	95/90	93/89	90/88	86/86	83/84	80/81	76/81	72/77	69/74	64/70
Strong	86/67	85/64	82/62	79/63	77/63	72/64	71/63	67/61	64/58	62/55	56/50
Dominant	80/39	79/36	77/34	75/37	72/37	70/35	69/35	64/34	63/31	61/35	59/35

OAT/OA

% of possible improvement with the indicated approach

Conclusions

- Factorial design of experiments may not be best for all engineering scenarios
- Adaptive one-factor-at-a-time may provide more improvement
  - When you must use very few experiments AND
  - EITHER Interactions are >25% of factorial effects OR
  - Pure experimental error is 40% or less of factorial effects
- One-at-a-time designs exploit some interactions (on average) even though it can't resolve them
- There may be human factors to consider too

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Next Steps

- You can download HW #6 DOE
  - Due 8:30AM Tues 20 July
- See you at Thursday's session
  - On the topic "Use of physics-based models in SE"
  - 8:30AM Thursday, 15 July
- Reading assignment for Thursday
  - Senin_Wallace_Distributed Modeling.pdf
  - Hazelrigg_Role and Use of Models.pdf