Analysis of Inventory Models with Limited Demand Information

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Sources

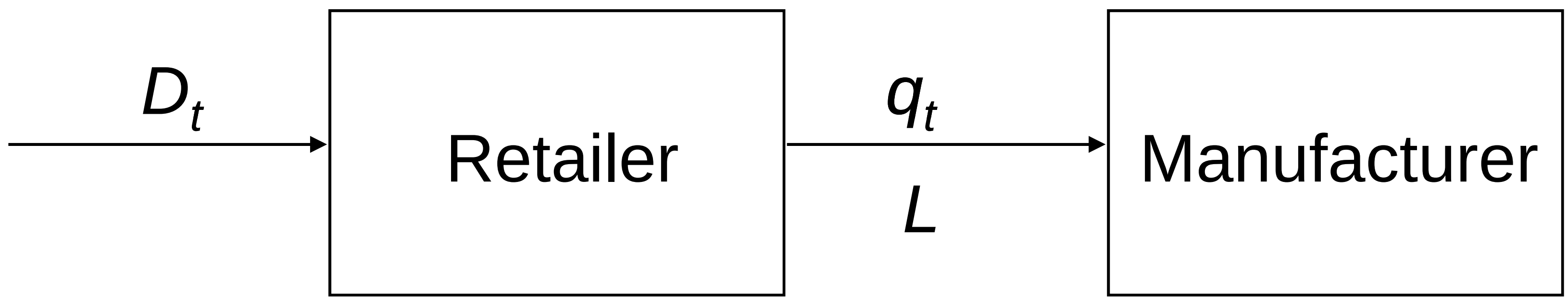
- The Bullwhip Effect
  - ☞Drezner, Ryan, Simchi-Levi, “Quantifying the Bullwhip Effect: The Impact of Forecasting, Lead Times and Information.”
  - ☞Chen, Ryan, Simchi-Levi, “The Impact of Exponential Smoothing Forecasts on the Bullwhip Effect.”
- Minimax Inventory Models
  - ☞Gallego, Ryan, Simchi-Levi, “Minimax Analysis for Finite Horizon Inventory Models.”

Quantifying the Bullwhip Effect

- The Impact of Forecasting Methods
  - ☞ Moving Average
  - ☞ Exponential Smoothing
- Multi-Stage Supply Chains
  - ☞ Centralized Information
  - ☞ Decentralized Information

A Simple Supply Chain

- Single retailer, single manufacturer.
  - ☞ Retailer observes customer demand, D_t.
  - ☞ Retailer orders q_t from manufacturer.
  - ☞ Lead time + $1 = L$.



A Simple Supply Chain: Order of Events

- At end of period t:
  - ☞Retailer updates forecast based on D_t.
  - ☞Calculates order-up-to point, y_{t+1}.
  - ☞Places order q_{t+1}.
  - ☞Order arrives at start of period $t+L$.
  - ☞Demand is observed and filled.
  - ☞Unfilled demand is backlogged.

A Simple Supply Chain: Inventory Policy

- Retailer follows order-up-to policy based on inventory position.
  - ☞ Approximates the optimal policy under the assumption of normal demand.
  - ☞ A policy used frequently in practice.

$$y_t = L\hat{\mu}_t + z\sqrt{L}S_t$$

A Simple Supply Chain: Moving Average Forecast

- Mean and standard deviation of demand are estimated using a moving average of *p* observations:

$$\hat{\mu}_t = \frac{\sum_{i=1}^{p} D_{t-i}}{p} \qquad S_t = \sqrt{\frac{\sum_{i=1}^{p} (D_{t-i} - \hat{\mu}_t)^2}{p-1}}$$

Moving Average Forecasting: Order Quantity

- Excess demand is returned at no cost.
- The order quantity for period *t* is:

$$q_t = y_t - y_{t-1} + D_{t-1}$$

$$= \left(1 + \frac{L}{p}\right) D_{t-1} - \left(\frac{L}{p}\right) D_{t-p-1} + z\sqrt{L}\left(S_t - S_{t-1}\right)$$

Moving Average Forecasting: The Variability of Orders

- Determine the variance of *q* relative to the variance of demand, σ^2:

$$Var(q_t) = \left(1 + \frac{2L}{p} + \frac{2L^2}{p^2}\right)\sigma^2 + z^2 L Var\left(S_t - S_{t-1}\right)$$

$$+2z\sqrt{L}\left(1 + \frac{2L}{p}\right)Cov\left(D_{t-1}, S_t\right).$$

Moving Average Forecasting: Symmetric Demand

Lemma: *Let D_i, i=1,...,p, be i.i.d. observations from a symmetric distribution with variance σ^2. Then*

$$Cov(D_i, S) = 0.$$

Corollary: *The increase in variability from the retailer to the manufacturer is:*

$$\frac{Var(q_t)}{\sigma^2} \geq 1 + \frac{2L}{p} + \frac{2L^2}{p^2}.$$

Moving Average Forecasting: Normal Demand

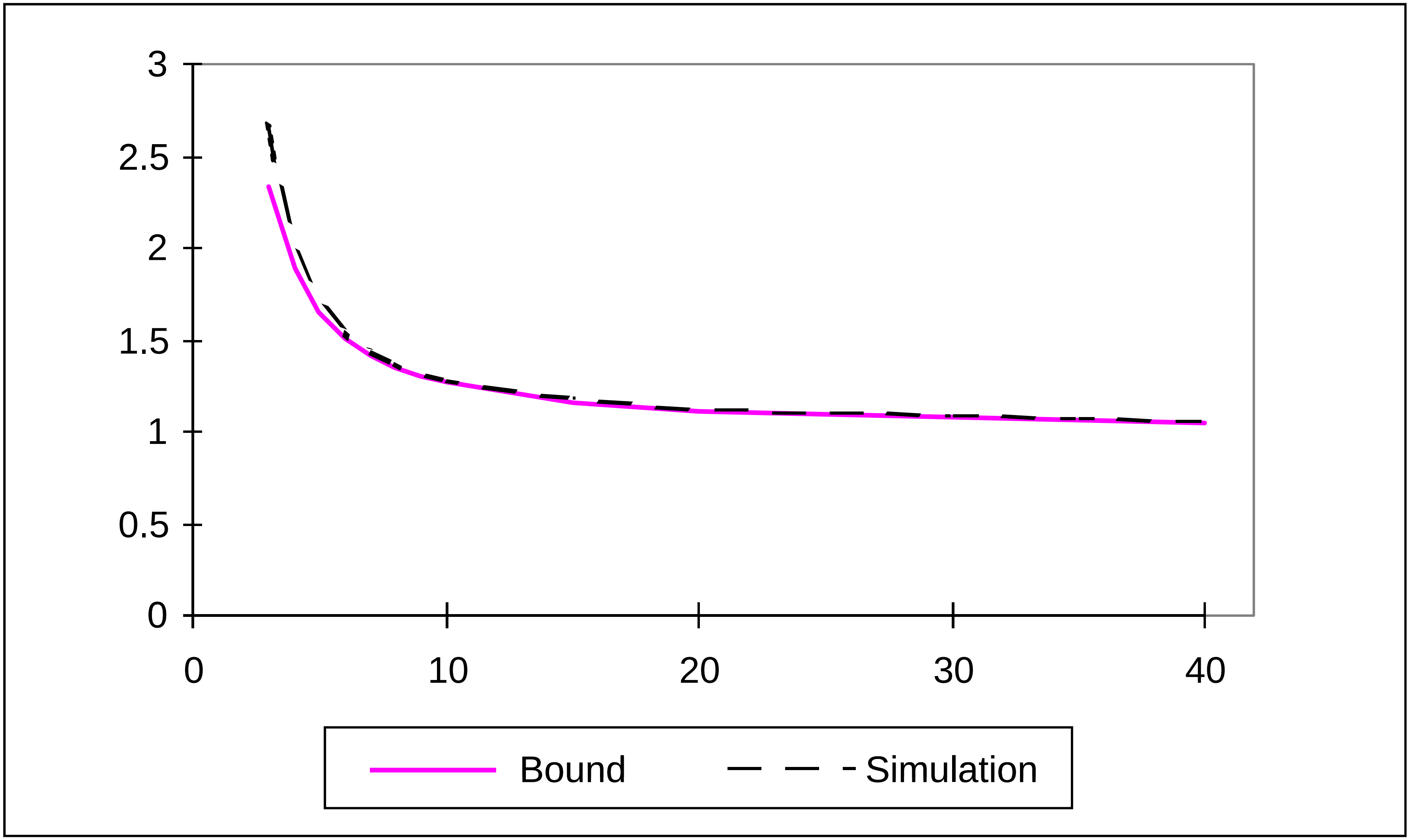
Lemma: *Let D_i, $i=1,\ldots,p$, be i.i.d. observations from a normal distribution with variance σ^2.*

$$\frac{Var(S_t - S_{t-1})}{\sigma^2} \geq {}^{1}\!/_{p^2}\,.$$

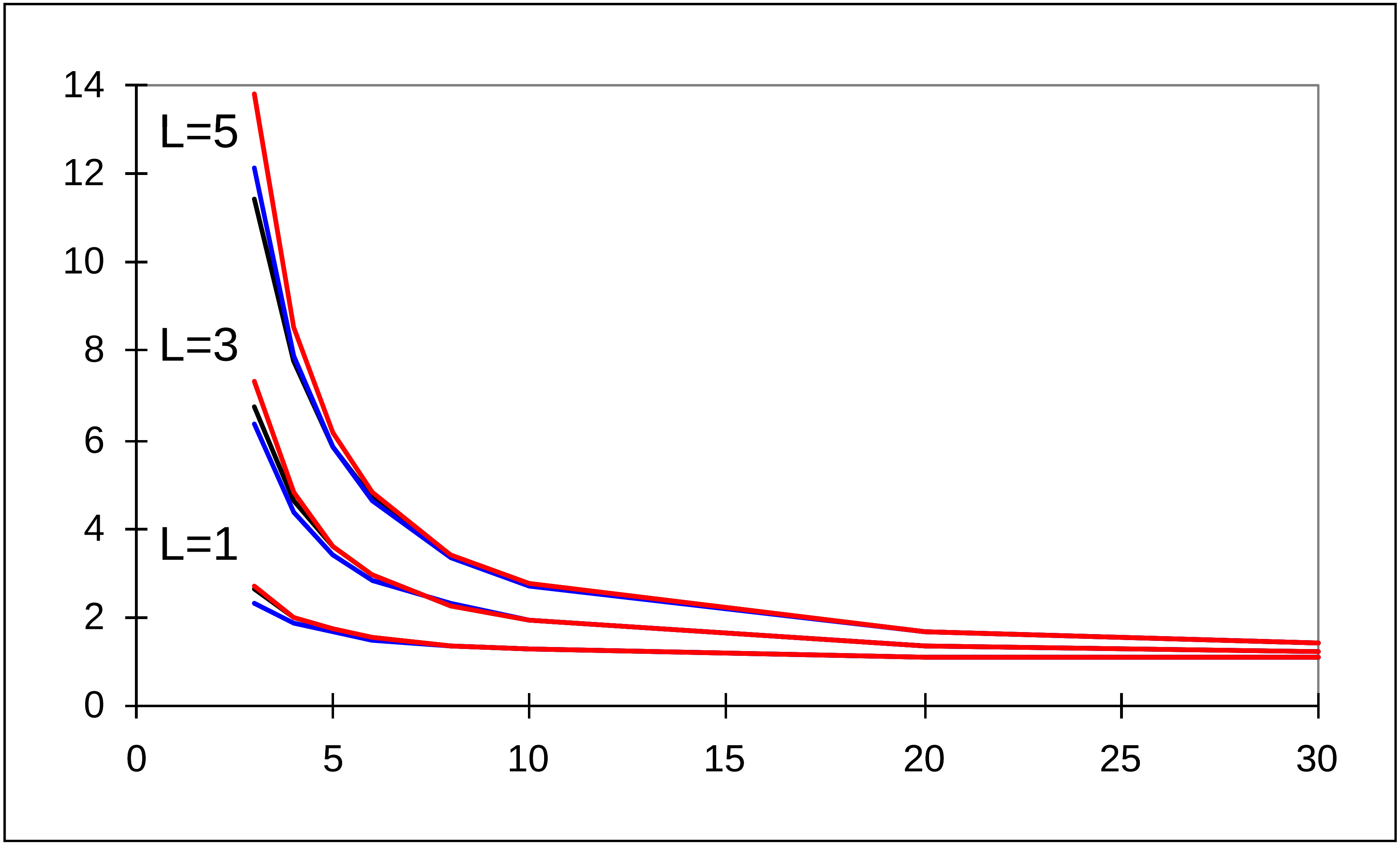
Corollary: *The increase in variability from the retailer to the manufacturer is:*

$$\frac{Var(q_t)}{\sigma^2} \geq 1 + \frac{2L}{p} + \frac{2L^2 + z^2 L}{p^2}\,.$$

$Var(q)/\sigma^2$: Lower Bound vs. Simulation



Var(q)/σ^2: For Various Lead Times



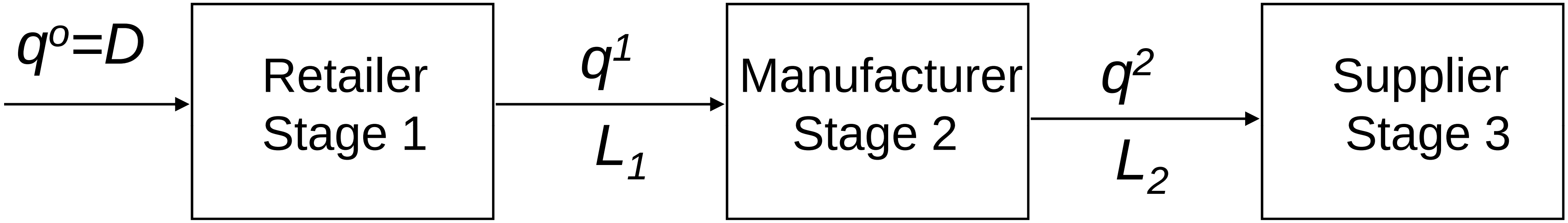
The Bullwhip Effect: Managerial Insights

- **Exists, in part, due to the retailer's need to estimate the mean and variance of demand.**
- **The increase in variability is an increasing function of the lead time and the smoothing parameter.**
  - ☞ **With longer lead time need more demand data to reduce the bullwhip effect.**

Multi-Stage Supply Chains

Consider an N stage supply chain:

- ☞ Stage i places order q^i to stage $i+1$.
- ☞ l_i is lead time between stage i and $i+1$.



Multi-Stage Supply Chain: Centralized Information

At the end of period t-1, stage i:

- ☞Receives order q^{i-1}_t.
- ☞Receives updated forecast from retailer.
- ☞Calculates the order-up-to point, y^i_t.
- ☞Orders q^i_t.

Multi-Stage Supply Chains: Centralized Information

Each stage in the supply chain uses:

- ☞The estimate of lead time demand received from the retailer.
- ☞An order-up-to inventory policy:

$$y_t^i = L_i \hat{\mu}_t$$

where $\hat{\mu}_t$ is received from the retailer.

Multi-Stage Supply Chains: Centralized Information

Lemma: *When the retailer uses a moving average with p observations, the increase in variability at stage k is:*

$$\frac{Var\left(q^k\right)}{\sigma^2} = 1 + \frac{2\left[\sum_{i=1}^{k} L_i\right]}{p} + \frac{2\left[\sum_{i=1}^{k} L_i\right]^2}{p}$$

Multi-Stage Supply Chains: Decentralized Information

- The retailer does not provide upstream stages with customer demand data.
  - ☞ Stage *i* estimates the mean demand from the orders it receives from stage *i-1*, q_t^{i-1}, $i > 1$.

$$\hat{\mu}_t^i = \frac{\sum_{j=0}^{p-1} q_{t-j}^{i-1}}{p}$$

$$y_t^i = L_i \hat{\mu}_t^i$$

Multi-Stage Supply Chains: Decentralized Information

Lemma: *When the retailer uses a moving average with p observations, the increase in variability at stage k is:*

$$\frac{Var(q^k)}{\sigma^2} \geq \prod_{i=1}^{k}\left[1 + \frac{2L_i}{p} + \frac{2L_i^{\ 2}}{p^2}\right]$$

The Bullwhip Effect: Managerial Insights

- Exists, in part, due to the retailer's need to estimate the mean and variance of demand.
- The increase in variability is an increasing function of the lead time and the smoothing parameter.
  - ☞With longer lead time need more demand data to reduce the bullwhip effect.
- The more complicated the demand models and the forecasting techniques, the greater the increase.
- **Centralized demand information can reduce the bullwhip effect, but will not eliminate it.**

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