Inventory Management

Extensions to EOQ

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Agenda

- Review of Basic EOQ
- Non-instantaneous Leadtime
- Finite Replenishment (EPQ)
- Multiple Locations
- Discounting

Economic Order Quantity (EOQ)

- Finding the order quantity Q (and frequency T) that minimizes the total relevant cost.

$$TRC[Q] = \frac{AD}{Q} + \frac{vrQ}{2}$$

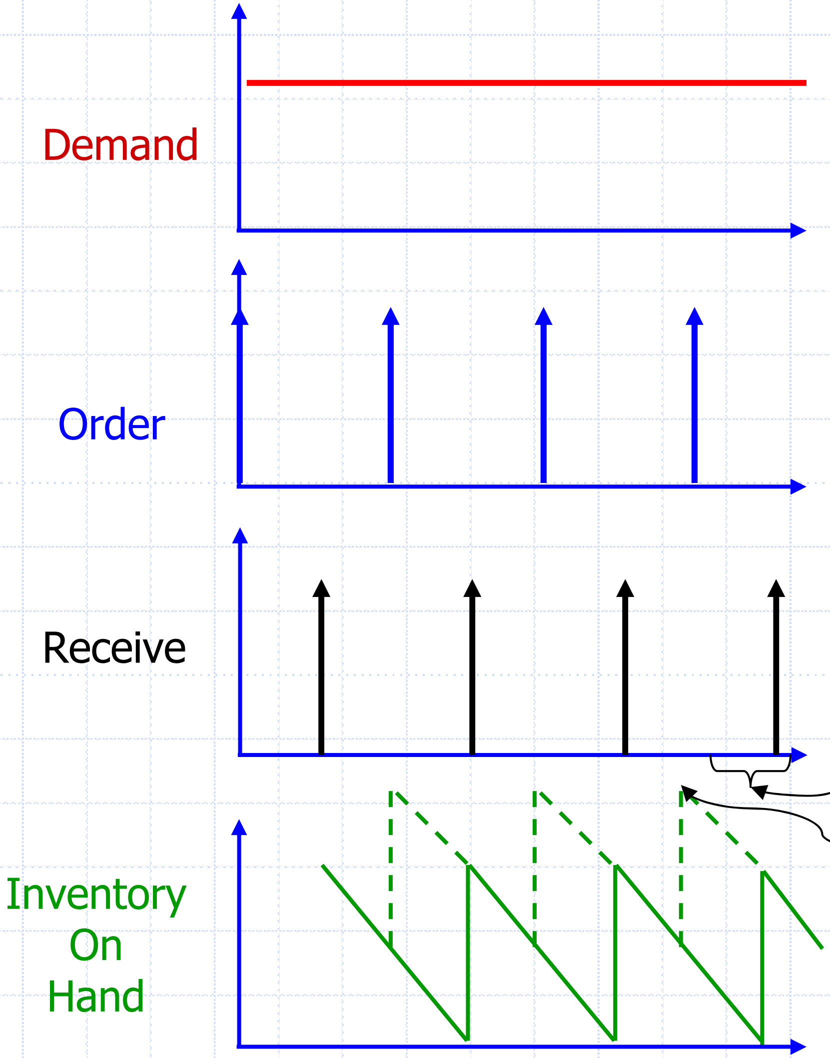
$$Q^* = \sqrt{\frac{2AD}{vr}} \qquad T^* = \sqrt{\frac{2A}{Dvr}}$$

$$TRC^* = \sqrt{2ADvr}$$

Assumptions: Basic EOQ Model

- Demand
  - **Constant** vs Variable
  - **Known** vs Random
  - **Continuous** vs Discrete
- Lead time
  - **Instantaneous**
  - Constant or Variable (deterministic/stochastic)
- Dependence of items
  - **Independent**
  - Correlated
  - Indentured
- Review Time
  - **Continuous**
  - Periodic
- Number of Echelons
  - **One**
  - Multi (>1)
- Capacity / Resources
  - **Unlimited**
  - Limited / Constrained
- Discounts
  - **None**
  - All Units or Incremental
- Excess Demand (Shortages)
  - **None**
  - All orders are backordered
  - Lost orders
  - Substitution
- Perishability
  - **None**
  - Uniform with time
- Planning Horizon
  - Single Period
  - Finite Period
  - **Infinite**
- Number of Items
  - **One**
  - Many
- Form of Product
  - **Single Stage**
  - Multi-Stage

Extensions: Leadtime



- Order Leadtime
  - Positive (nonzero)
  - Deterministic
- Impact
  - Does this change Q*?
  - What is my new policy?
  - What is my new avg IOH?

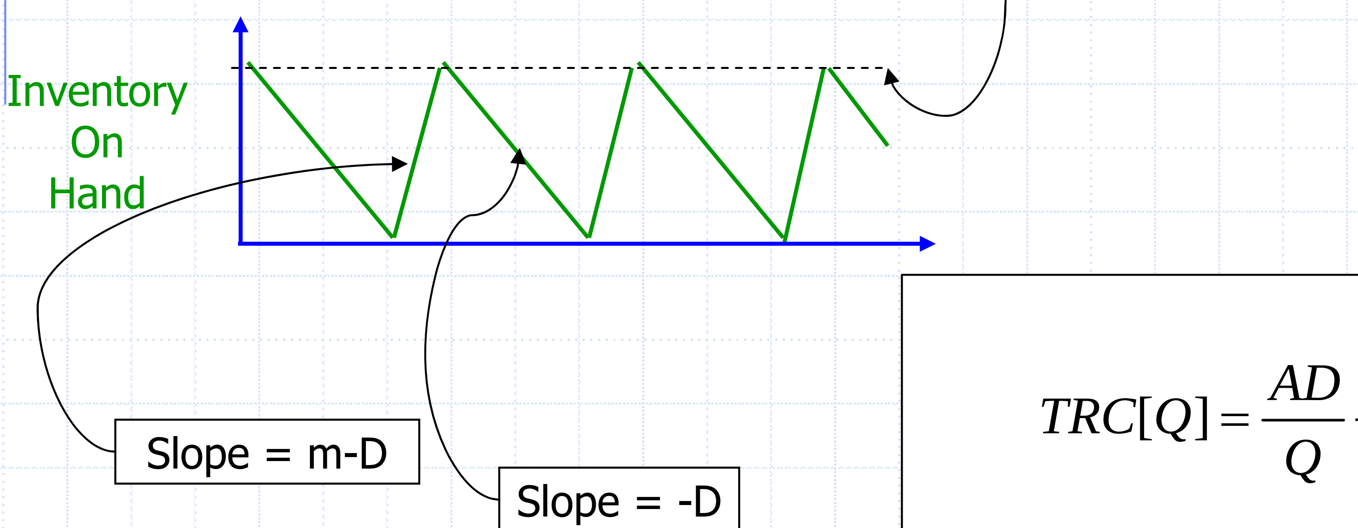
L = Order Leadtime

Inventory On Order

EOQ Inventory Policy:
Order Q* units when IOH = DL

Extensions: Finite Replenishment

- Inventory becomes available at a rate of m units/time rather than all at one time
  - Does this change Q*?
  - What is my new policy?
  - What is my new avg IOH?

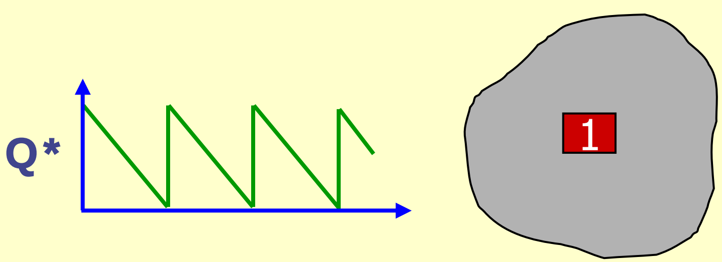


$$TRC[Q] = \frac{AD}{Q} + \frac{Q\left(1 - \frac{D}{m}\right)vr}{2}$$

$$EPQ = \sqrt{\frac{2AD}{vr\left(1 - \frac{D}{m}\right)}} = EOQ\left(\sqrt{\left(1 - \frac{D}{m}\right)}\right)^{-1}$$

Extensions: Multiple Locations

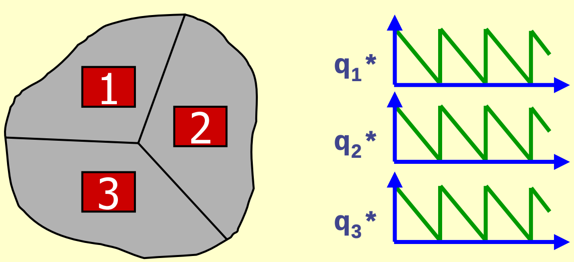
- Suppose that instead of one location satisfying all demand, there are n locations.
  - Each location serves d_i = D/n units of demand
  - Identical (uniform) demand at each location
- Questions
  - What is my new inventory policy?
  - What is my new average Inventory on Hand?
  - How much is this better or worse than a single location?



$$Q^* = \sqrt{\frac{2AD}{vr}}$$

$$\overline{IOH} = \frac{Q^*}{2} \qquad TRC^* = \sqrt{2DAvr}$$

versus



$$q_i^* = \sqrt{\frac{2Ad_i}{vr}} = \sqrt{\frac{2AD}{vrn}} \qquad TRC^* = \sqrt{2nDAvr}$$

$$\overline{IOH} = \sum_{i=1}^{n} \left(\frac{q_i^*}{2} \right) = \sqrt{n} \left(\frac{Q^*}{2} \right)$$

Extensions: Multiple Locations

- What if I reduce number of stocking locations from M to N?

$$\frac{TRC^*[M]}{TRC^*[N]} = \frac{\sqrt{2MDAvr}}{\sqrt{2NDAvr}} = \sqrt{\frac{M}{N}}$$

- What if my sub-regions do not have uniform demand?
- Is this a reduction in safety stock, cycle stock, or both?
- How dependent is this effect on inventory policy at each site?
  - EOQ Policy (order q_{EOQ}^* when IOH_i=0)
  - Fixed Order Size (Always order a full truckload at a time)
  - Days of Supply (Always order a month's supply)

Extensions: Multiple Locations

Fixed Order Size, e.g. only order full truckloads

Days of Supply, e.g. order at start of each month

For a Single Location

Policy	EOQ	FOS	DOS
Order Size	Q*	Q_{FOS}	Q_{DOS}
Average IOH	Q*/2	$Q_{FOS}/2$	$Q_{DOS}/2$
Order Cost	O_{EOQ}	O_{FOS}	O_{DOS}
Holding Cost	H_{EOQ}	H_{FOS}	H_{DOS}
Total Cost	$O_{EOQ} + H_{EOQ}$	$O_{FOS} + H_{FOS}$	$O_{DOS} + H_{DOS}$

For N Locations

Policy	EOQ	FOS	DOS
Order Size	q*	Q_{FOS}	q_{DOS}
Average IOH	$\sqrt{N}(Q^*/2)$	$N(Q_{FOS}/2)$	$Q_{DOS}/2$
Order Cost	$\sqrt{N}(O_{EOQ})$	O_{FOS}	$N(O_{DOS})$
Holding Cost	$\sqrt{N}(H_{EOQ})$	$N(H_{FOS})$	H_{DOS}
Total Cost	$\sqrt{N}(O_{EOQ}+H_{EOQ})$	$O_{FOS}+NH_{FOS}$	$NO_{DOS}+H_{DOS}$

Example

DATA		
A =	500	$/order
D =	2000	Units/year
r =	0.25	$/$/year
v =	50	$/unit
N =	4	locations
Trk Cap =	500	units/shipment
DOS =	0.083	years
	30	days

Single Location

Policy	EOQ	FOS	DOS
Order Size	400	500	167
Average IOH	200	250	83
Order Cost	$ 2,500	$ 2,000	$ 6,000
Holding Cost	$ 2,500	$ 3,125	$ 1,042
Total Cost	$ 5,000	$ 5,125	$ 7,042

4 Locations

Policy	EOQ	FOS	DOS
Order Size	200	500	42
Average IOH	400	1000	21
Order Cost	$ 5,000	$ 2,000	$ 24,000
Holding Cost	$ 5,000	$ 12,500	$ 1,042
Total Cost	$ 10,000	$ 14,500	$ 25,042

Extensions: Discounts

- All Units Discount
  - Discount applies to all units purchased if total amount exceeds the break point quantity
  - Examples?
- Incremental Discount
  - Discount applies only to the quantity purchased that exceeds the break point quantity
  - Examples?
- One Time Only Discount
  - Less common – but not unheard of!
  - A one time only discount applies to all units you order right now (no quantity minimum or limit)

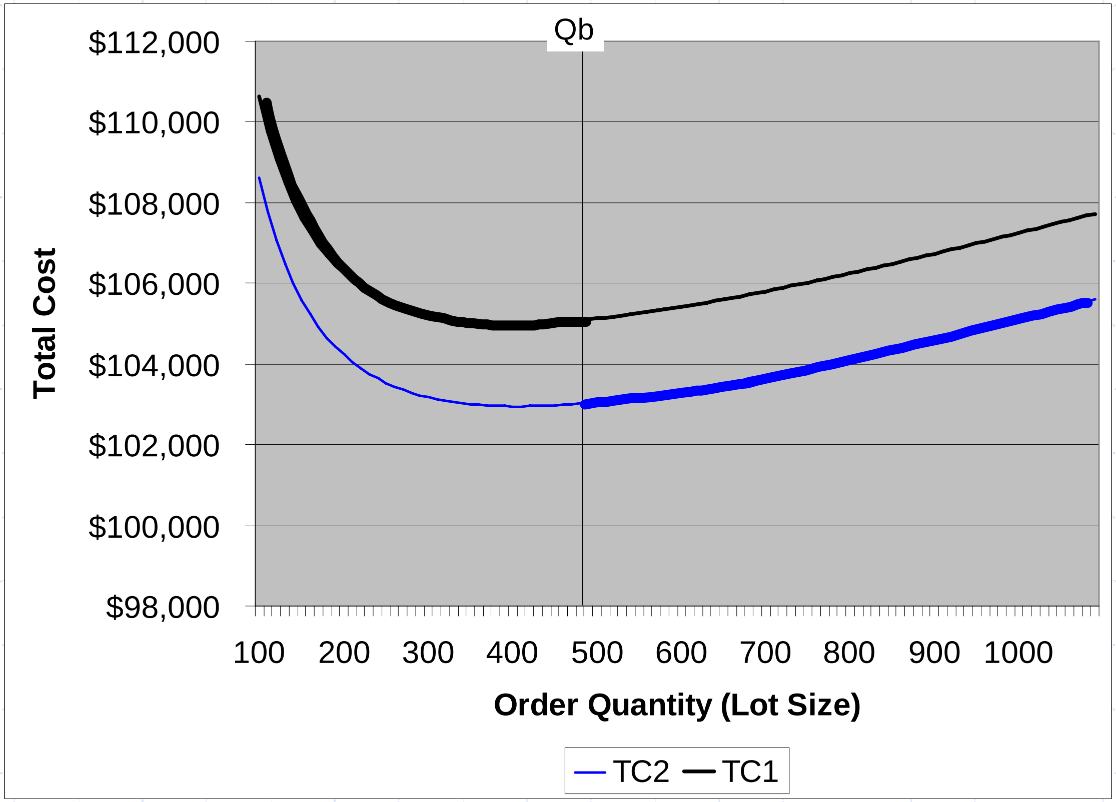
- How will different discounting strategies impact your lot sizing decision?
- What cost elements are relevant?

Extensions: All Units Discounts

Two Cases to Examine . . .

$$v = \begin{cases} v_0 & 0 \le Q \le Q_b \\ v_0(1-d) & Q_b \le Q \end{cases}$$

$$TRC = \begin{cases} Dv_0 + \frac{AD}{Q} + \frac{v_0 rQ}{2} & 0 \le Q \le Q_b \\ Dv_0(1-d) + \frac{AD}{Q} + \frac{v_0(1-d)rQ}{2} & Q_b \le Q \end{cases}$$



Where

- d = Discount
- v_0 = Base unit price
- Q_b = Break quantity

Typically, $Q^* < Q_b$ but what if $Q^* > Q_b$?

Extensions: All Units Discounts

- Simple efficient algorithm
  1. Find EOQ with discount (EOQ_d)
  2. If $EOQ_d \geq Q_b$ then pick EOQ_d
     Otherwise, go to 3
  3. Solve for TRC(Q*) and TRC(Q_b)
     If TRC(Q*) < TRC(Q_b) then pick Q*
     Otherwise, pick Q_b
- Can be extended to more than one break point

Example:

D=2000 Units/yr

r=.25

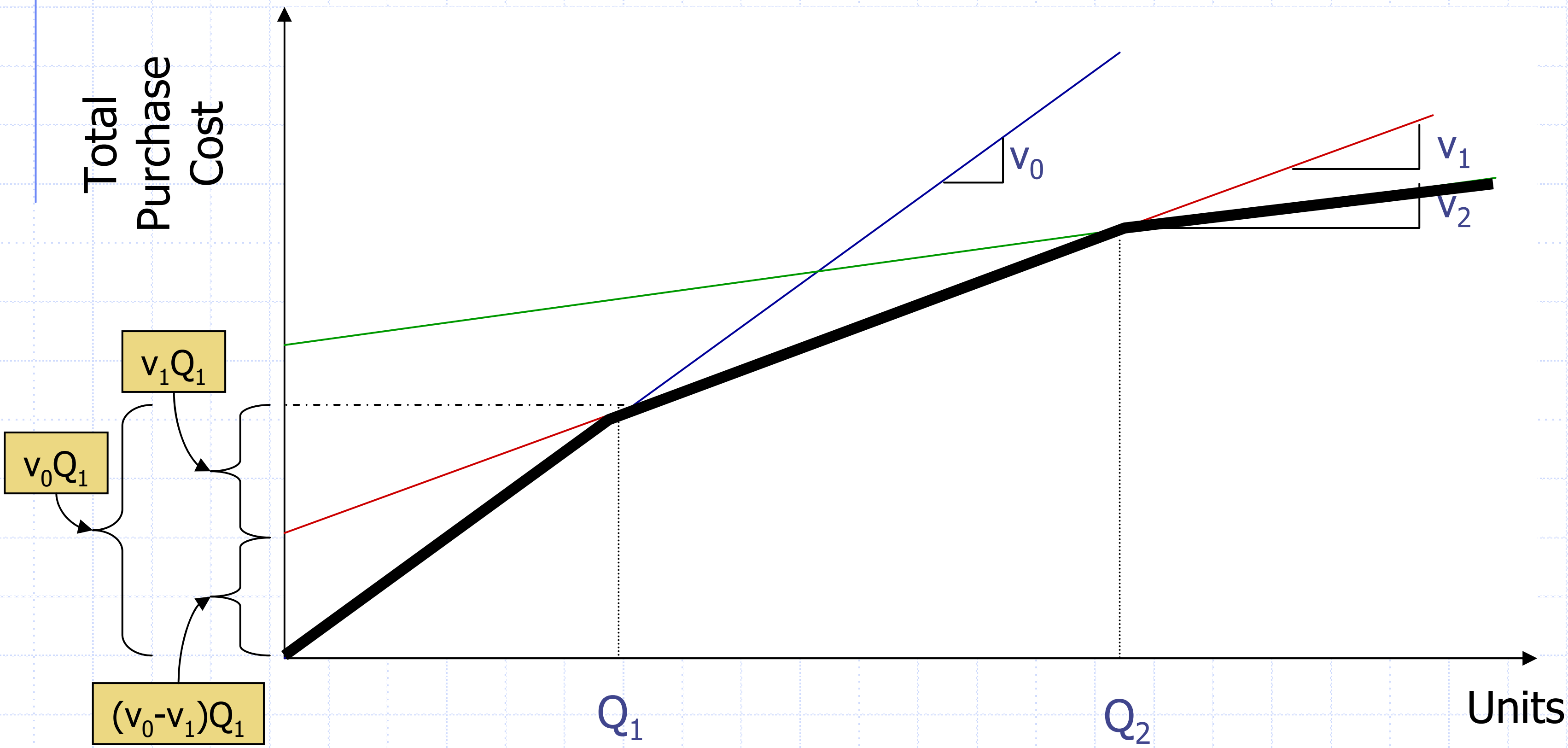
A=$500

v_0 = $50

Discount of 2% off if $Q \geq 500$

Extensions: Incremental Discounts

- Discount only applies to quantity above breakpoint
- Trade-off between lower purchase cost and higher carrying costs
- Cost of units ordered below each breakpoint are essentially ‘fixed’



Extensions: Incremental Discounts

- Efficient algorithm
  1. Find Fixed Cost per breakpoint, F_i, for each break
  2. Find EOQ_i for each range – including the F_i
  3. If EOQ_i is not within allowable range, go to next I
     Otherwise, find TRC_i using effective cost per unit, v_{ei}
  4. Pick EOQ_i with lowest TRC
- Can be extended to more than one break point

$$F_i = F_{i-1} + (v_{i-1} - v_i)Q_i \qquad F_0 = 0$$

$$EOQ_i = \sqrt{\frac{2D(A+F_i)}{rv_i}}$$

$$v_i^e = \frac{v_i EOQ_i + F_i}{EOQ_i}$$

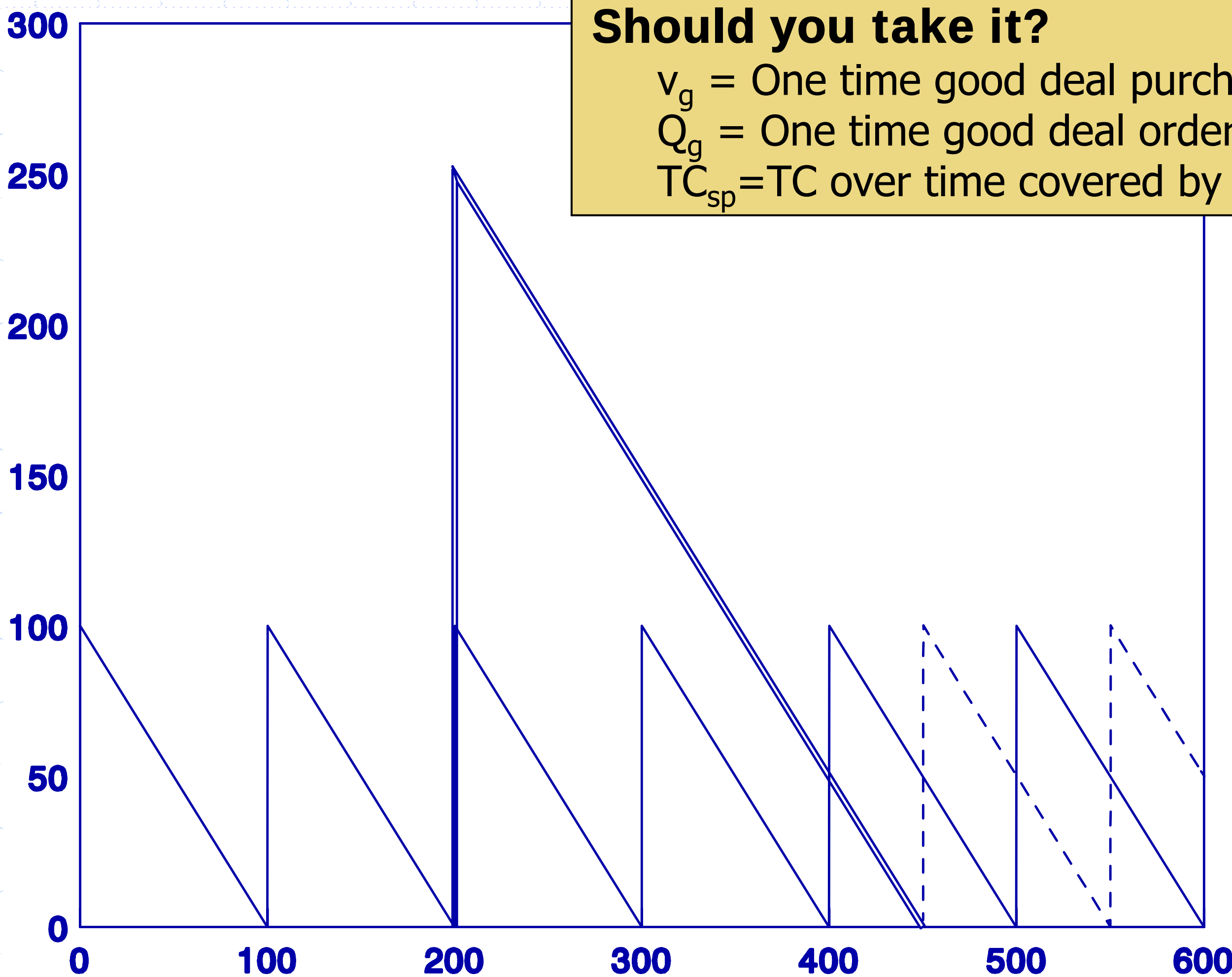
Example: Incremental Discounts

D=2000 Units/yr
r=.25
A=$500
v_0 = $50

Price Breaks:
10% off for 500 to <1000 units
20% off for 1000 or more units

	i=2	i=1	i=0
v_i	$40	$45	$50
Q_{bi}	1,000	500	0
F_i	7,500	2,500	0
EOQ_i	1,789 OK	1,033 X	400 OK
v_{ei}	$44.19		$50
Purch	$ 88,384		$100,000
Order	$ 559		$ 2,500
Hold	$ 9,882		$ 2,500
TRC_i	**$ 98,825**		**$ 105,000**

Extensions: One Time Discount



Suppose you are offered a One Time deal! Should you take it?

v_g = One time good deal purchase price ($/unit)
Q_g = One time good deal order quantity (units)
TC_{sp}=TC over time covered by special purchase ($)

Extensions: One Time Discount

- Compare Options: Not Special Price vs. Special Price
  - Find TC for normal price

$$TC = (CycleTime)(TC^{*} + PurchaseCost)$$

$$TC = \left(\frac{Q_g}{D}\right)\sqrt{2ArvD} + \left(\frac{Q_g}{D}\right)vD$$

  - Find the Savings ($TC-TC_{SP}$)

$$Savings = TC - TC_{SP}$$

$$= \left(\left(\frac{Q_g}{D}\right)\sqrt{2ArvD} + \left(\frac{Q_g}{D}\right)vD\right) - \left(v_g Q_g + rv_g\left(\frac{Q_g}{2}\right)\left(\frac{Q_g}{D}\right) + A\right)$$

Extensions: One Time Discount

- Finding 1st and 2nd order conditions (Maximize Savings)

$$\frac{dS}{d(Q_g)} = 0 = \left(\frac{1}{D}\right)\sqrt{2AvrD} + \left(v - v_g\right) - \left(\frac{2rv_g Q_g}{2D}\right)$$

$$\frac{d^2S}{d^2(Q_g)} = -\left(\frac{2rv_g}{2D}\right) < 0$$

- So that the Optimal Quantity to buy is

$$Q_g^* = \left(\frac{D}{Drv_g}\right)\sqrt{2ArvD} + \frac{D\left(v - v_g\right)}{rv_g}$$

- Cleaning this up gives:

$$Q_g^* = Q^*\left(\frac{v}{v_g}\right) + \frac{D\left(v - v_g\right)}{rv_g}$$

Take-Aways

- EOQ is a good place to start for most analysis
- EOQ can be extended to cover
  - Non-zero leadtimes
  - Finite replenishment systems
  - Multiple locations
    - Square Root law rests on implicit assumptions
    - Distribution of demand and inventory policy will impact results
  - Discounts
    - Purchase price (v) becomes relevant
    - Common in practice (economies of scale)

Questions?
Comments?
Suggestions?