

Inventory Management: Fundamental Concepts & EOO

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Oct 2006

Agenda

- ◆ Wrap up of Demand Forecasting
- ◆ Fundamentals of Inventory Management
- ◆ Economic Order Quantity (EOQ) Model

Wrap Up Of Demand Forecasting

◆ What did we learn?

- 1.
- 2.
- 3.
- 4.

◆ How do you think this will influence our inventory policy?

Why Hold Inventory?



Three Levels of Inventory Decisions

- ◆ Supply Chain Decisions (strategic)
 - What are the potential alternatives to inventory?
 - How should the product be designed?
- ◆ Deployment Decisions (strategic)
 - What items should be carried as inventory?
 - In what form should they be maintained?
 - How much of each should be held and where?
- ◆ Replenishment Decisions (tactical/operational)
 - How often should inventory status be determined?
 - When should a replenishment decision be made?
 - How large should the replenishment be?

Total Relevant Costs (TRC)

$$TC = \text{Purchase} + \text{Order} + \text{Holding} + \text{Shortage}$$

- ◆ What makes a cost relevant?
- ◆ Four Standard Cost Components
 - Purchase (Unit Value) Cost
 - Ordering (Set Up) Cost
 - Holding (Carrying) Cost
 - Shortage Cost

Relevant Costs

◆ Purchase (Unit Value) Costs

V

- Units?



- What does it contain?



- How do we determine this number?



- When is it relevant?



Relevant Costs

◆ Ordering (Set Up) Costs A

- Units?



- What does it contain?



- How do we determine this number?



- When is it relevant?



Relevant Costs

◆ Holding (Carrying) Costs rv

- Units?



- What does it contain?



- How do we determine this number?



- When is it relevant?



Relevant Costs

◆ Shortage (Stock-Out) Costs

$B_{\#}$

- Units?



- What does it contain?



- How do we determine this number?



- When is it relevant?



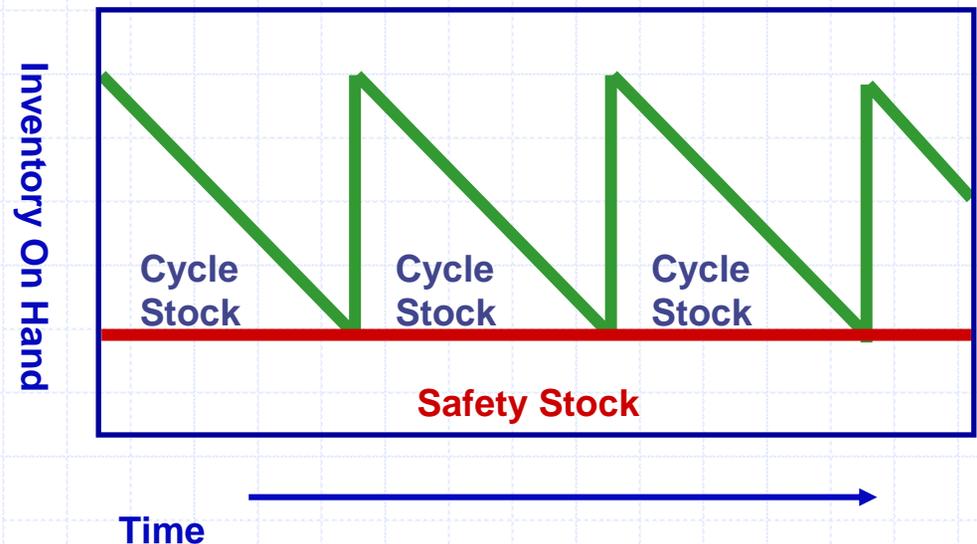
Classification of Inventory

◆ Financial / Accounting Categories

- Raw Materials
- Work in Process (WIP)
- Finished Goods
- Components, Semi-Finished Goods

◆ Functional Classifications

- Cycle Stock
- Safety Stock
- Pipeline Inventory
- Decoupling Stock
- Congestion Stock
- Anticipation Inventory



What factors influence inventory replenishment models?

- ◆ Demand
 - Constant vs Variable
 - Known vs Random
 - Continuous vs Discrete
- ◆ Lead time
 - Instantaneous
 - Constant or Variable (deterministic/stochastic)
- ◆ Dependence of items
 - Independent
 - Correlated
 - Indentured
- ◆ Review Time
 - Continuous
 - Periodic
- ◆ Number of Echelons
 - One
 - Multi (>1)
- ◆ Capacity / Resources
 - Unlimited
 - Limited / Constrained
- ◆ Discounts
 - None
 - All Units or Incremental
- ◆ Excess Demand
 - None
 - All orders are backordered
 - Lost orders
 - Substitution
- ◆ Perishability
 - None
 - Uniform with time
- ◆ Planning Horizon
 - Single Period
 - Finite Period
 - Infinite
- ◆ Number of Items
 - One
 - Many
- ◆ Form of Product
 - Single Stage
 - Multi-Stage

Assumptions: Basic EOQ Model

- ◆ Demand
 - **Constant** vs Variable
 - **Known** vs Random
 - **Continuous** vs Discrete
- ◆ Lead time
 - **Instantaneous**
 - Constant or Variable (deterministic/stochastic)
- ◆ Dependence of items
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Notation

D = Average Demand (units/unit time)

$A = C_o$ = Fixed Ordering Cost (dollar/order)

$r = C_h$ = Carrying or Charge (dollars/dollars held/time)

$v = C_p$ = Variable (Purchase) Cost (dollars/unit)

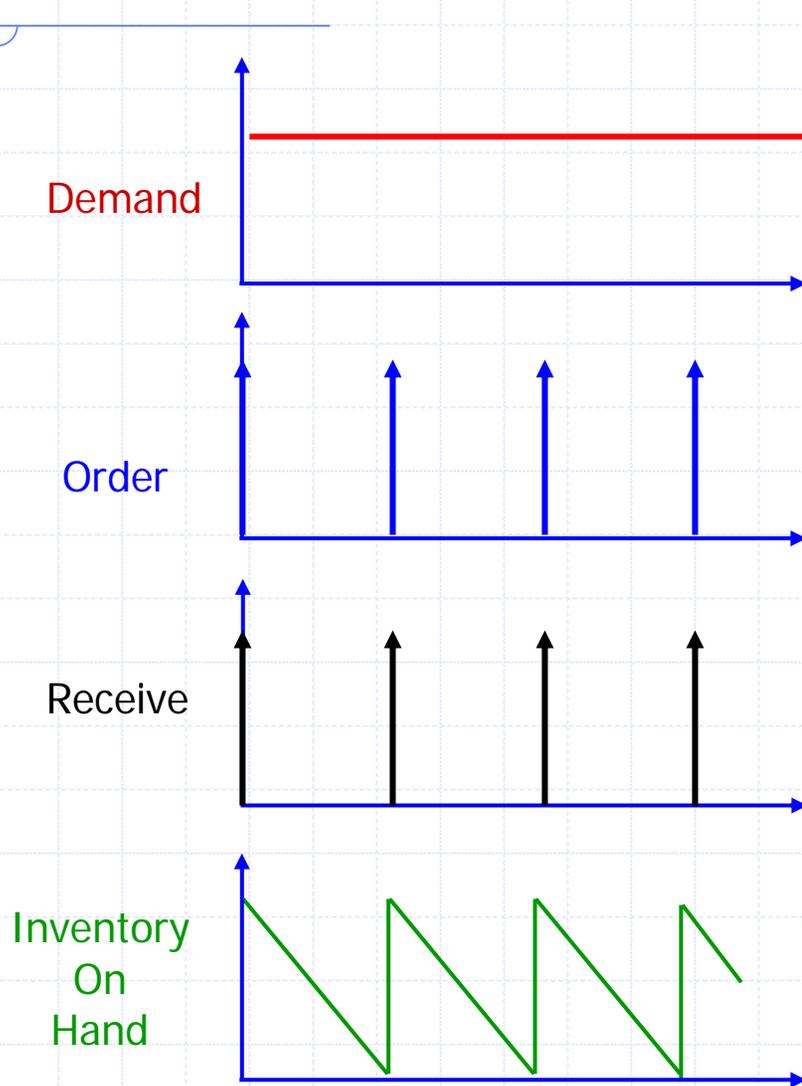
Q = Replenishment Order Quantity (units/order)

T = Order Cycle Time (time/order)

$TRC(Q)$ = Total Relevant Cost (dollar/time)

$TC(Q)$ = Total Cost (dollar/time)

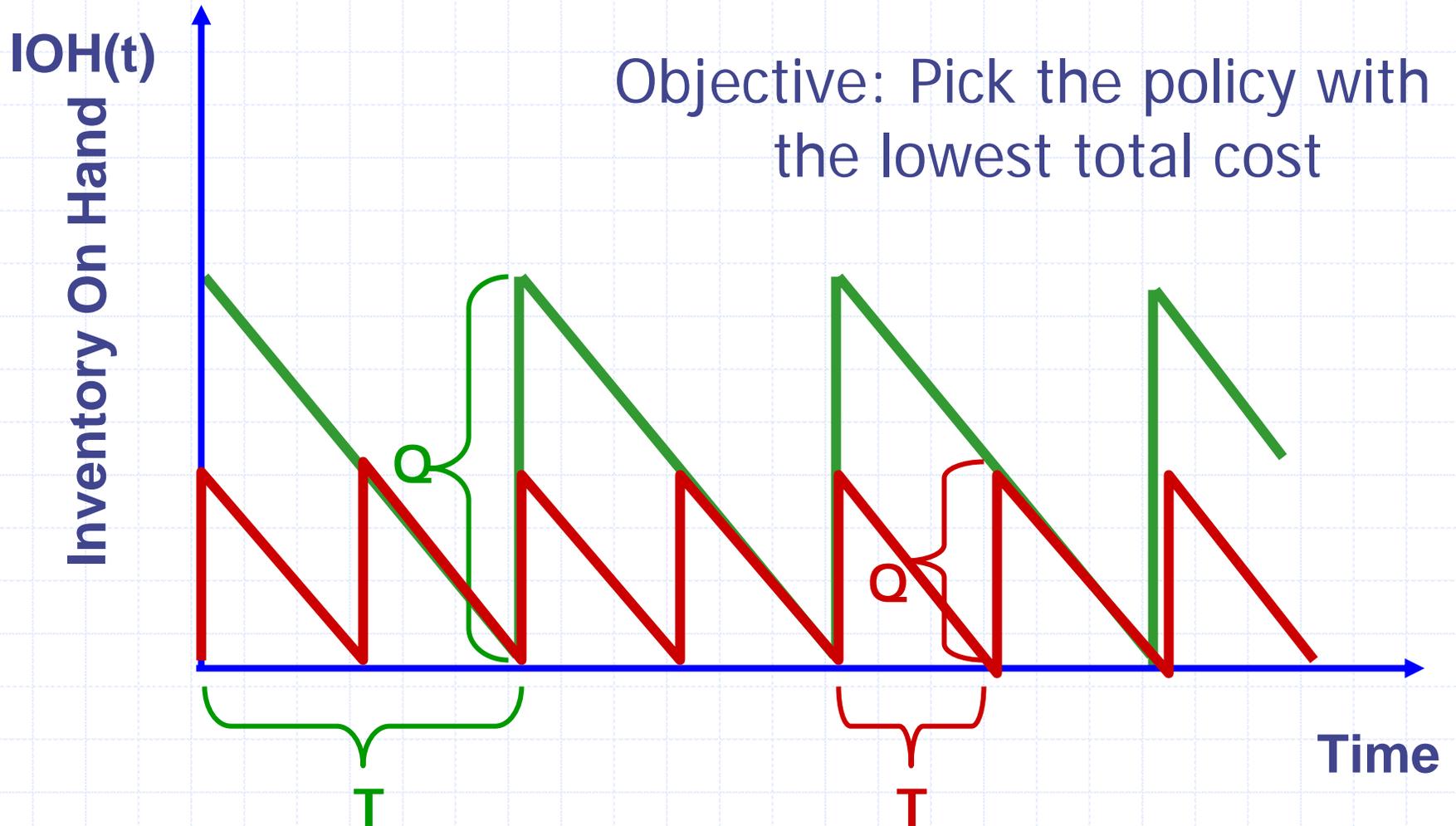
Inventory Charts



Assumptions

- ◆ Demand is uniform and deterministic
- ◆ Leadtime is 0
- ◆ Total amount ordered is received

Lot Sizing: Many Potential Policies



What is the total cost?

TC = Purchase + Order + Holding + Shortage Costs

$$TC(Q) = vD + A\left(\frac{D}{Q}\right) + rv\left(\frac{Q}{2}\right) + B_{Short}D_{Short}$$

Which costs are relevant to the order quantity decision?

- Purchase Costs?
- Ordering Costs?
- Holding Costs?
- Shortage Costs?

$$TRC(Q) = A\left(\frac{D}{Q}\right) + rv\left(\frac{Q}{2}\right)$$

Example

Annual demand of widgets is 2,000. The cost of placing an order is \$500. Widgets are procured for \$50 each and are sold for \$95 each. Holding cost for the company is estimated to be 25%.

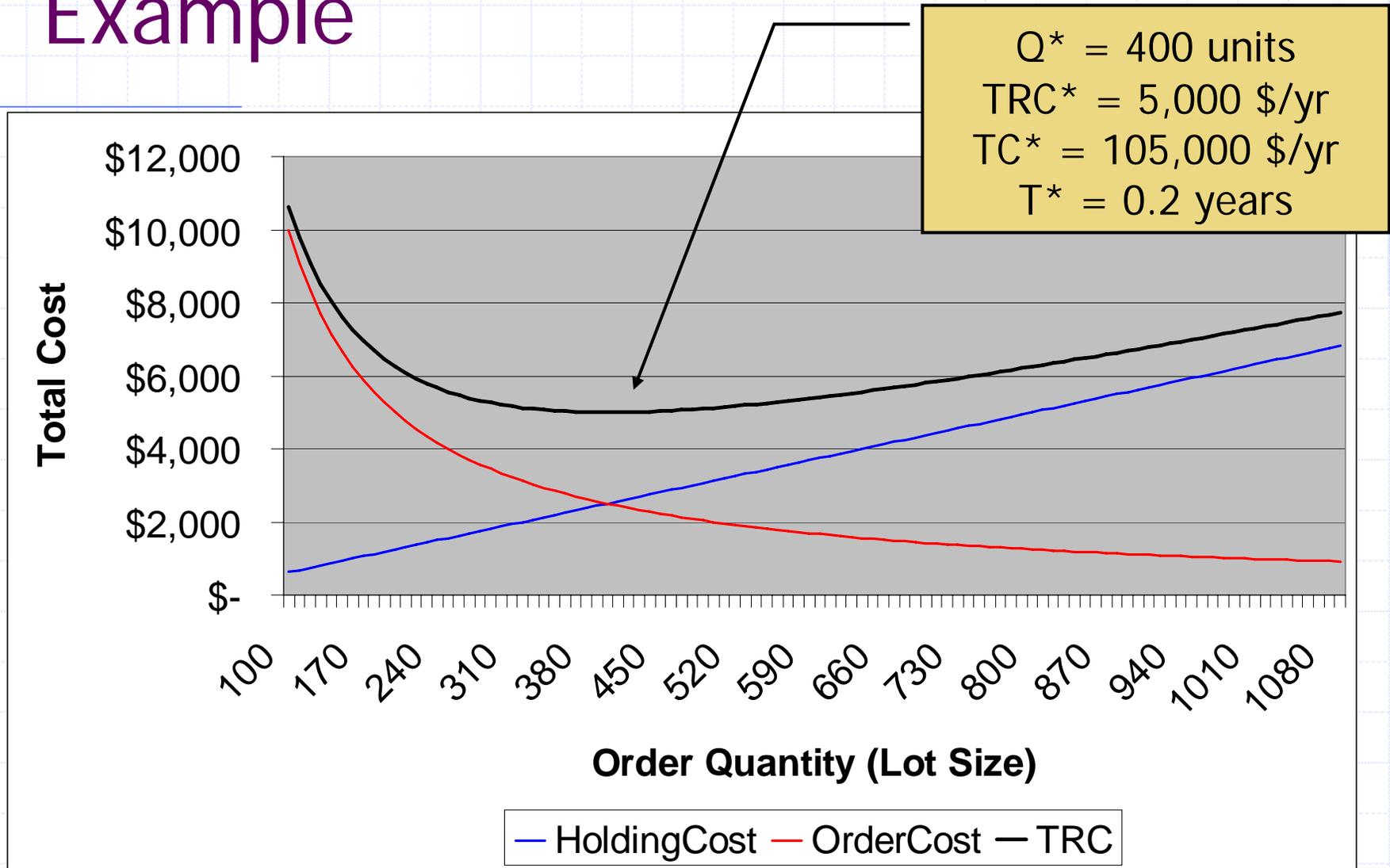
Find the following:

- a) Optimal Order Quantity (EOQ)
- b) Total cost under the EOQ policy? (relevant and total)
- c) Optimal Cycle Time for replenishment under EOQ?

Approaches:

1. Solve for all possible values of Q and pick lowest TC
2. Graph each component and pick the minimum value
3. Solve analytically

Example



EOQ Inventory Policy:
Order Q^* units every T^* time periods (or when IOH=0)

Economic Order Quantity (EOQ)

Find Q that minimizes total relevant cost

Set derivative wrt Q to zero (1st order conditions)

Check that 2nd derivative is >0 (2nd order conditions)

$$TRC[Q] = \frac{AD}{Q} + \frac{vrQ}{2}$$

$$Q^* = \sqrt{\frac{2AD}{vr}} \quad T^* = \sqrt{\frac{2A}{Dvr}}$$

$$TRC^* = \sqrt{2ADvr}$$

Sensitivity Analysis of EOQ

- ◆ How sensitive is my inventory policy to ...
 - Order size (larger or smaller than optimal)?
 - Demand (higher or lower than expected)?
 - Order Cycle Time (shorter or longer than optimal)?

- ◆ We will take an analytical approach to quantify the impact

Sensitivity Analysis of EOQ

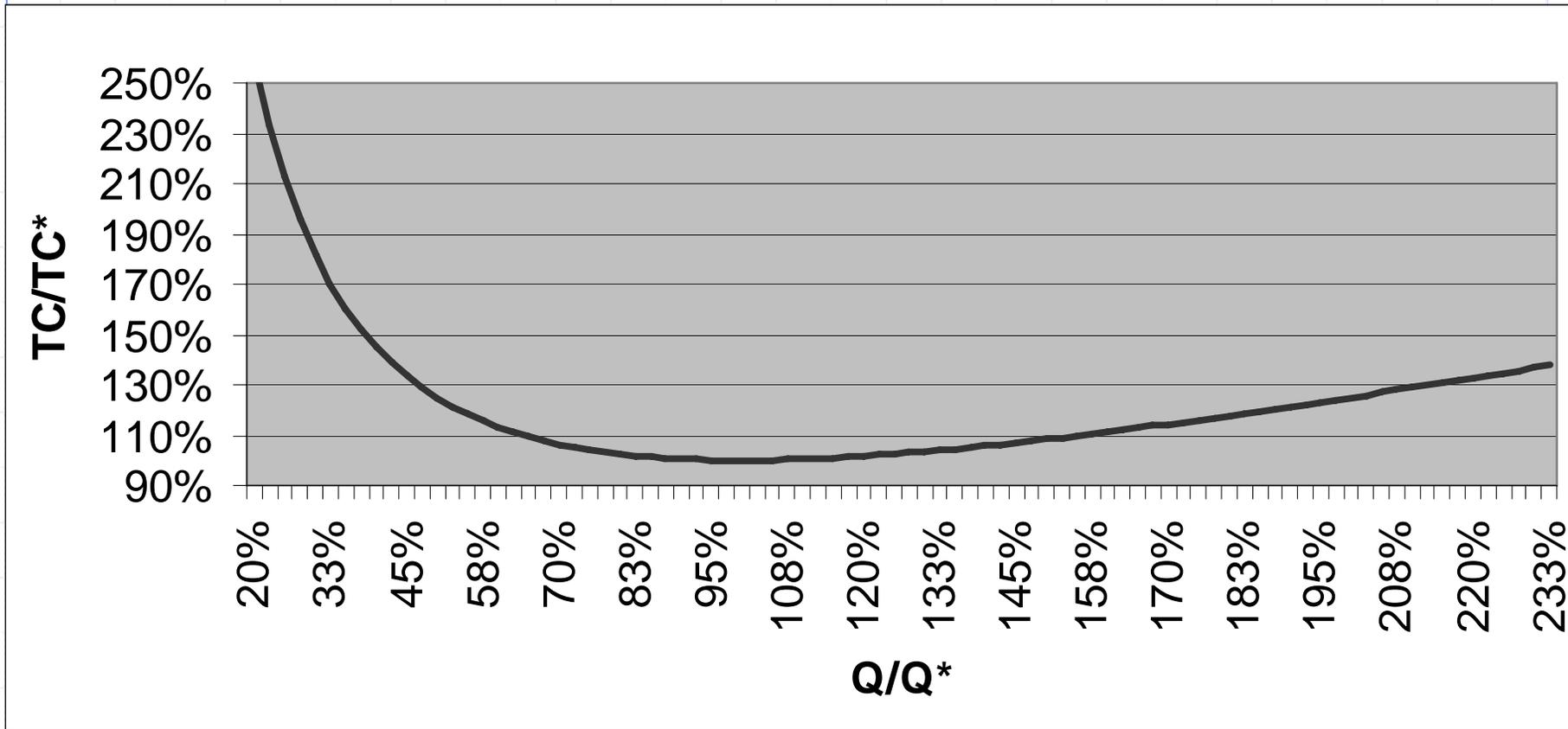
How sensitive is total cost to order quantity?

$$\frac{TRC(Q)}{TRC(Q^*)} = \frac{\frac{AD}{Q} + \frac{vrQ}{2}}{\sqrt{2ADvr}} = \frac{1}{2} \left[\frac{Q^*}{Q} + \frac{Q}{Q^*} \right]$$

Q	Ordering Costs (DA/Q)	Inventory Costs (rvQ/2)	Total Relevant Costs	Q/Q*	TRC/TRC*
2000	\$500	\$12,500	\$13,000	500%	260%
500	\$2,000	\$3,125	\$5,125	125%	102.5%
Q* = 400	\$2,500	\$2,500	\$5,000	--	--
200	\$5,000	\$1,250	\$6,250	50%	125%
20	\$50,000	\$125	\$50,125	5%	1002.5%

Would you rather order $Q > Q^*$ or $Q < Q^*$?

Sensitivity Analysis of EOQ



How much will TC change if I order lot size 50% larger than optimal?
How much will TC change if I order lot size 50% smaller than optimal?

Sensitivity Analysis of EOQ

- ◆ How sensitive is TRC to change in actual demand?
- ◆ That is, after the fact, how well did my policy hold up?
- ◆ Notation:
 - E = Actual Demand / Estimated Demand
 - Q' = Estimated EOQ
 - Q^* = Actual EOQ, given actual demand that occurred

$$Q' = \sqrt{\frac{2D'A}{vr}}$$

$$E = \frac{D}{D'} = \frac{\text{Actual}}{\text{Estimate}}$$

$$\frac{Q^*}{Q'} = \sqrt{E}$$

$$\frac{TRC'}{TRC^*} = \frac{1}{2} \left(\frac{1}{\sqrt{E}} + \frac{\sqrt{E}}{1} \right)$$

Sensitivity Analysis of EOQ

So, for $D' = 2000$ units/year, if the actual demand was ...

D	E	Q^*/Q'	TC'/TC^*
200	0.10	0.32	1.74
1,000	0.50	0.71	1.06
1,500	0.75	0.87	1.01
1,800	0.90	0.95	1.00
2,000	1.00	1.00	1.00
3,000	1.50	1.22	1.02
4,000	2.00	1.41	1.06
20,000	10.00	3.16	1.74

This also indicates the sensitivity to parameters A , v , and r .

Sensitivity Analysis of EOQ

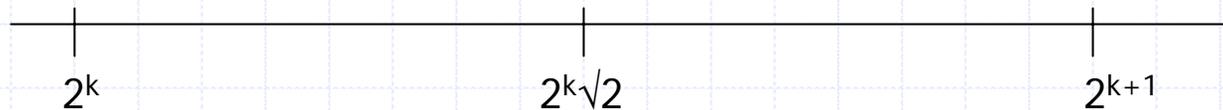
◆ How sensitive is TRC to T^* ?

- Why do we care?
- How do I find the “best” T that is also practical?

$$\frac{TRC(T)}{TRC(T^*)} = \frac{1}{2} \left(\frac{T}{T^*} + \frac{T^*}{T} \right)$$

◆ Power of Two Policies

- Order in intervals of powers of two
- Select a realistic base period, T_{Base} (day, week, month)
- Find the smallest k that satisfies:



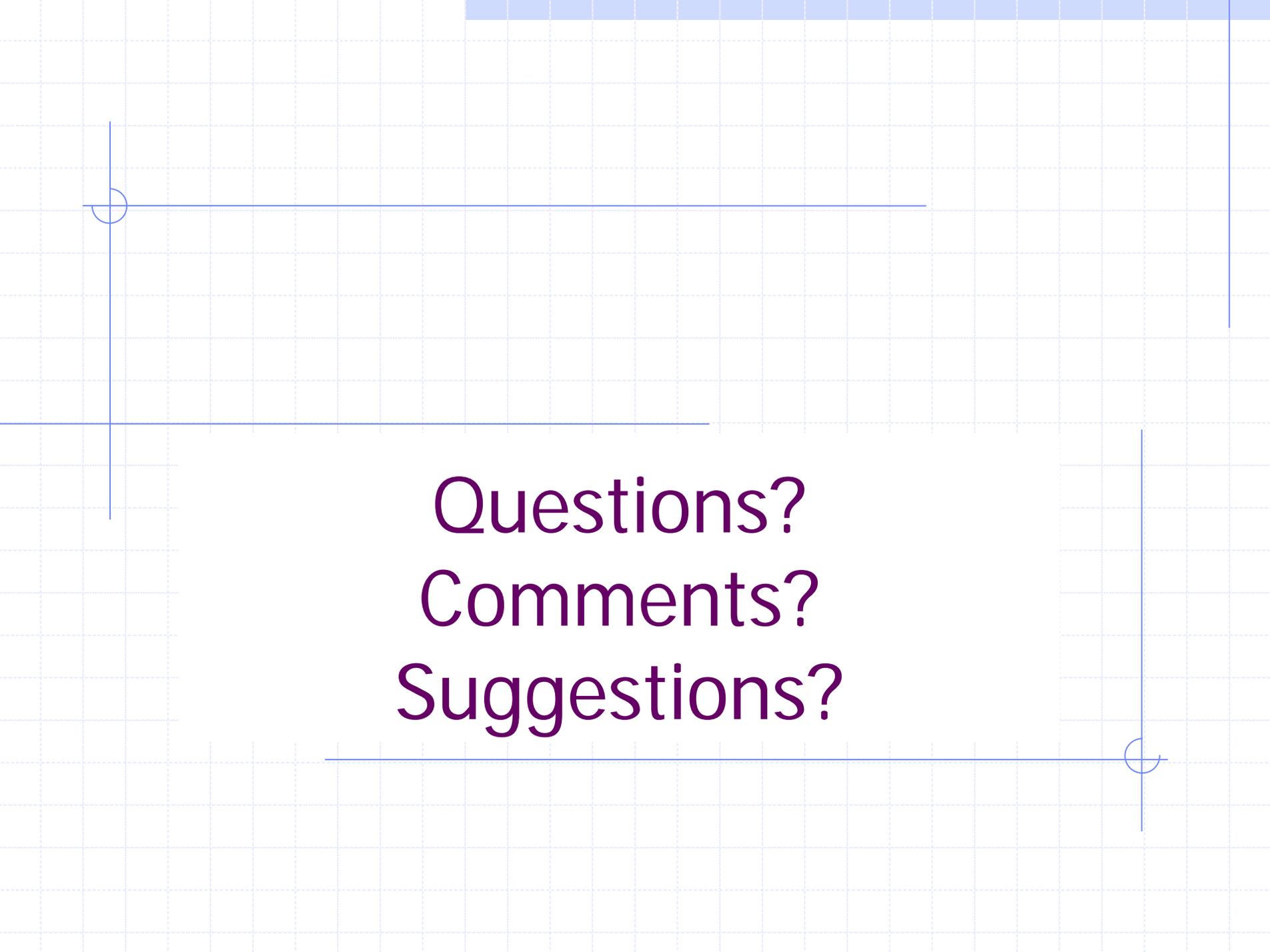
$$\frac{T^*}{\sqrt{2}} \leq T_{Base} 2^k \leq \sqrt{2} T^* \quad \text{for } k \in [0, 1, 2, \dots]$$

$$\frac{TRC(T_{Base} 2^k)}{TRC(T^*)} \leq \frac{1}{2} \left(\frac{1}{\sqrt{2}} + \sqrt{2} \right) \approx 1.06$$

A Power of Two time interval is guaranteed to be within 6% of the costs with the optimal time interval.

Insights from EOQ

- ◆ There is a direct trade off between order size and average inventory
- ◆ Total cost is relatively insensitive to changes in . . .
 - Q – rounding of order quantities
 - D – errors in forecasting
 - A, r, v – errors in cost parameters
- ◆ Thus, EOQ is widely used despite its highly restrictive assumptions
- ◆ EOQ is a good starting point in most inventory systems
- ◆ It also helps to focus management attention on process improvements
 - How do I lower A
 - How do I lower v ?
 - How do I lower r ?

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Questions?
Comments?
Suggestions?