Demand Forecasting I
Time Series Analysis

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Agenda

> “Predictions are usually difficult – especially for the future”
>
> Yogi Berra

- Problem and Background
- Four Fundamental Approaches
- Time Series Methods

Demand Processes

- Demand Forecasting
  - Predict what will happen in the future
  - Typically involves statistical, causal or other model
  - Conducted on a routine basis (monthly, weekly, etc.)
- Demand Planning
  - Develop plans for creating or affecting future demand
  - Results in marketing & sales plans – builds unconstrained forecast
  - Conducted on a routine basis (monthly, quarterly, etc.)
- Demand Management
  - Make decisions in order to balance supply and demand within the forecasting/planning cycle
  - Includes forecasting and planning processes
  - Conducted on an on-going basis as supply and demand changes
  - Includes yield management, real-time demand shifting, forecast consumption tracking, etc.

Four Fundamental Approaches

Subjective

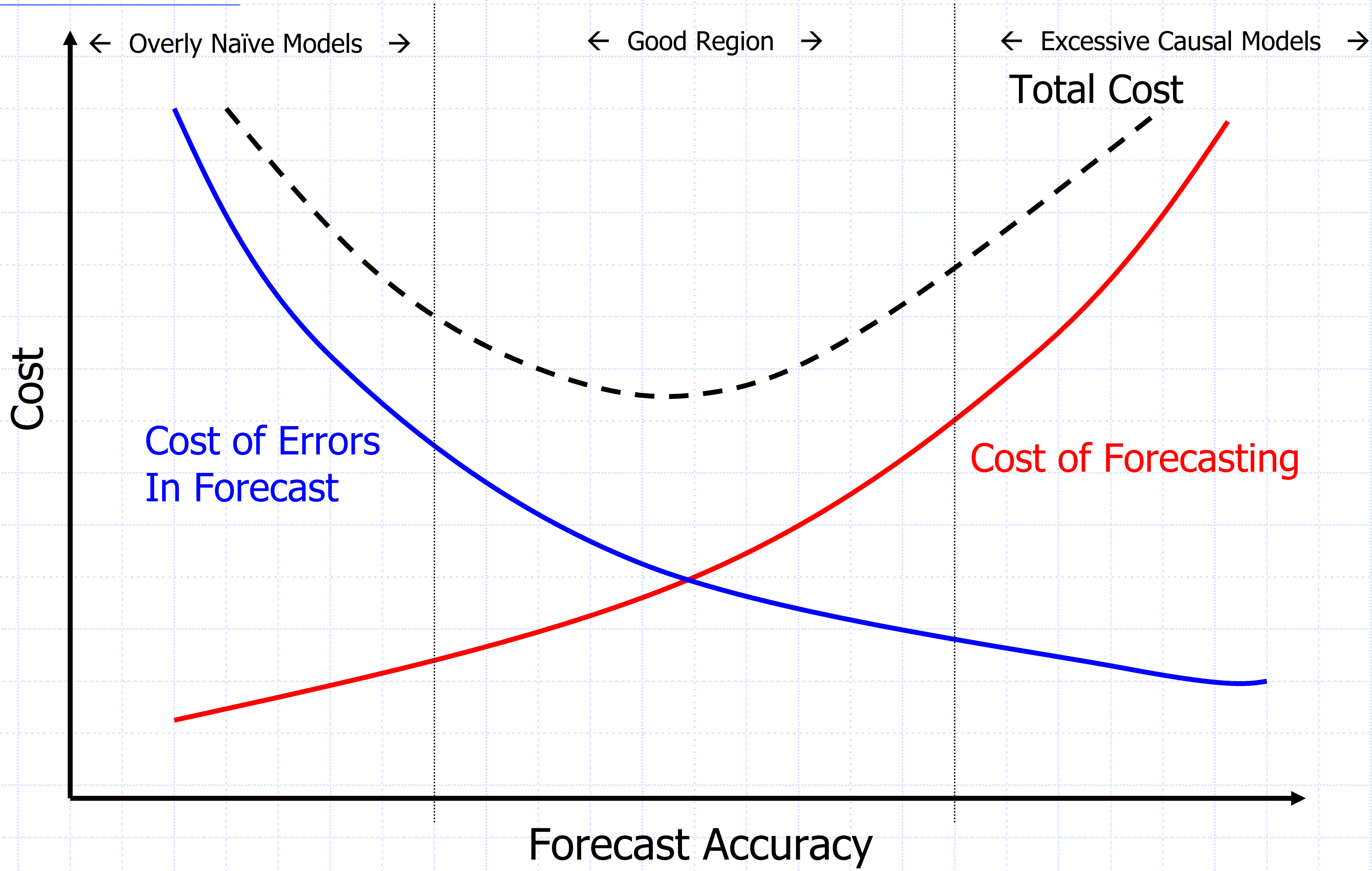
- Judgmental
  - Sales force surveys
  - Delphi techniques
  - Jury of experts
- Experimental
  - Customer surveys
  - Focus group sessions
  - Test Marketing
  - Simulation

Objective

- Time Series
  - "Black Box" Approach
  - Uses past to predict the future
- Causal / Relational
  - Econometric Models
  - Leading Indicators
  - Input-Output Models

Often times, you will need to use a combination of approaches

Cost of Forecasting vs Inaccuracy



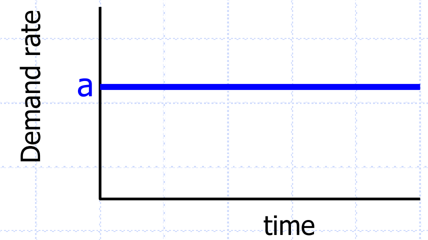
Time Series

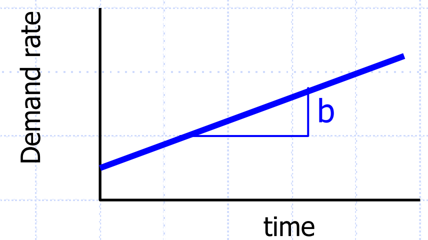
- The typical problem:
  - Generate the large number of short-term, SKU level, locally disaggregated demand forecasts required for production, logistics, and sales to operate successfully.
- Predominant use is for:
  - Forecasting product demand of . . .
  - Mature products over a . . .
  - Short time horizon (weeks, months, quarters, year) . . .
  - Using models to assist in the forecast where . . .
  - Demand of items is independent
- Special situations are treated differently
  - New product introduction
  - Old product retirement
  - Short life-cycle products
  - Erratic and sparse demand

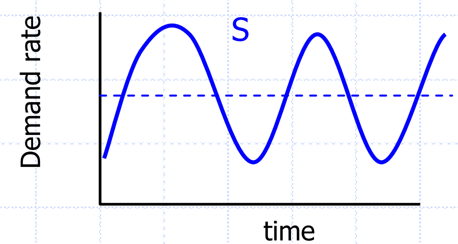
Time Series

Method of using past occurrences to model the future
Assumes some regular & recurring basis over time

- Basic Components
  - Level (a)
    - Value where demand hovers around
  - Trend (b)
    - Persistent movement in one direction
    - Typically linear but can be exponential, quadratic, etc.
  - Seasonal Variations (F)
    - Movement that is periodic to the calendar
    - Hourly, daily, weekly, monthly, quarterly, etc.
  - Cyclical Movements (C)
    - Periodic movement not tied to calendar
  - Random Fluctuations (e or ε)
    - Irregular and unpredictable variations, noise







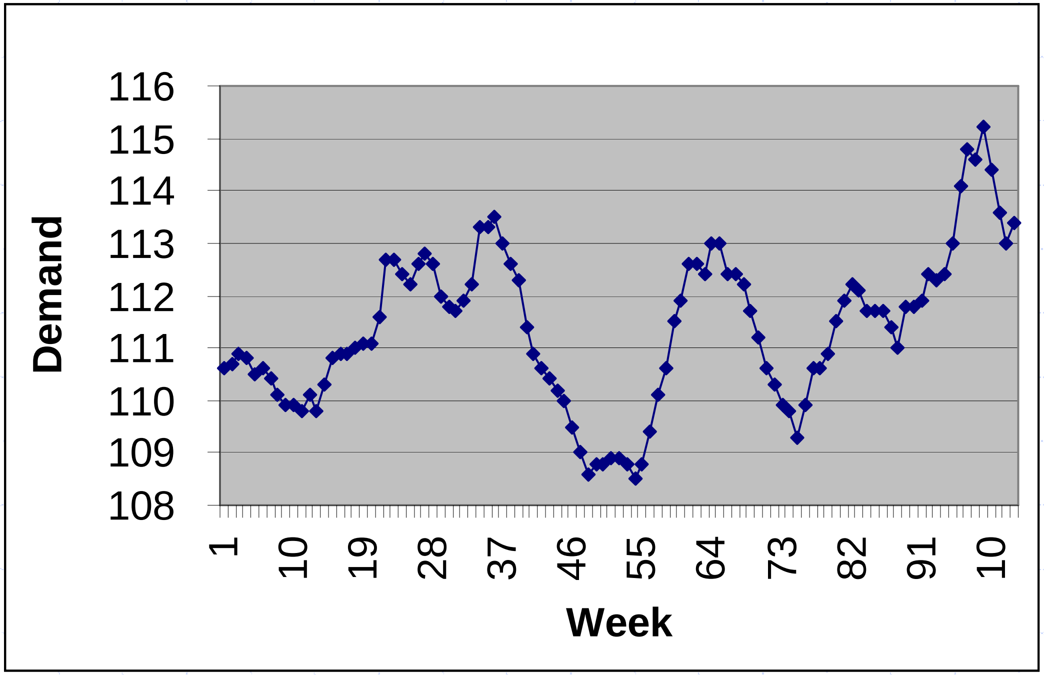
- Combine components to model demand in period t
  - **Multiplicative**: $x_t = (b)(F)(C)(e)$
  - **Additive**: $x_t = a + b(t) + F_t + C_t + e_t$
  - **Mixed**: Combination $x_t = a + b(F_t)t + e_t$

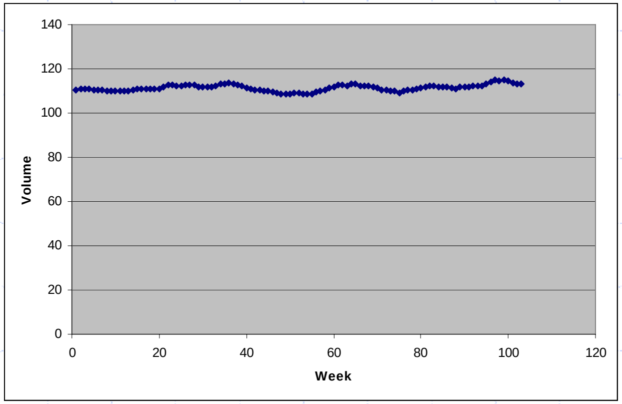
Time Series

- Simple Procedure
  1. Select an appropriate underlying model of the demand pattern over time
  2. Estimate and calibrate values for the model parameters
  3. Forecast future demand with the models and parameters selected
  4. Review model performance and adjust parameters and model accordingly

Time Series: Example

What is the forecast for period t+τ made at the end of period t, $\hat{x}_{t,t+\tau}$?





So, what can I say?

Time Series

- How important is the history? Two extreme assumptions . . . .

Cumulative Forecast

- All history matters equally
- Pure stationary demand

Underlying Model:

$$x_t = a + e_t$$

where:

$$e_t \sim \text{iid}\,(\mu=0\,,\ \sigma^2=V[e])$$

Forecasting Model:

$$\hat{x}_{t,t+1} = \frac{\sum_{i=1}^{t} x_i}{t}$$

Naïve Forecast

- Most recent dictates next
- Random Walk, Last is Next

Underlying Model:

$$x_t = x_{t-1} + e_t$$

where:

$$e_t \sim \text{iid}\,(\mu=0\,,\ \sigma^2=V[e])$$

Forecasting Model:

$$\hat{x}_{t,t+1} = x_t$$

Time Series

- Moving Average
  - Only include the last M observations
  - Compromise between cumulative and naïve
    - Cumulative model (M=n)
    - Naïve model (M=1)
  - Assumes that some step (S) occurred

Underlying Model:

$$x_t = a + e_t$$

where:

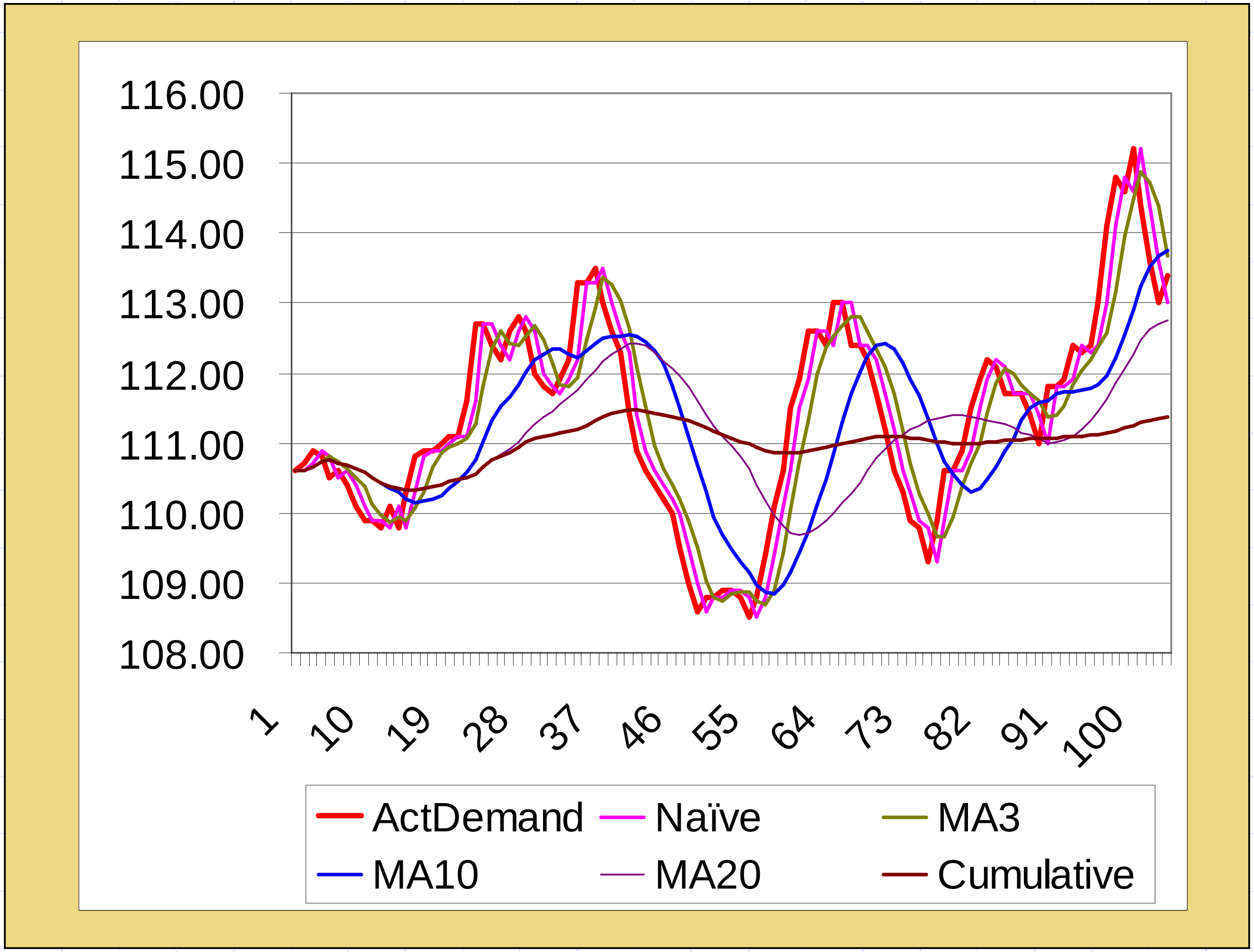
$$e_t \sim \text{iid}\,(\mu=0\,,\ \sigma^2=V[e])$$

Forecasting Model:

$$\hat{x}_{t,t+1} = \frac{\sum_{i=t+1-M}^{t} x_i}{M}$$

- So, some questions
  - How do we find M?
  - What trade-offs are involved?
  - How responsive are the three models?

Moving Average Forecasts



Time Series: Exponential Smoothing

- Why should past observations all be weighted the same?
- Value of observation degrades over time
- Introduce smoothing constant (α)

Underlying Model:

$$x_t = a + e_t$$

where:

$$e_t \sim \text{iid}\ (\mu=0\ ,\ \sigma^2=V[e])$$

Forecasting Model:

$$\hat{x}_{t,t+1} = \hat{x}_{t-1,t} + \alpha e_t \quad (0<\alpha<1)$$

or

$$\hat{x}_{t,t+1}=\alpha x_t + (1-\alpha)\hat{x}_{t-1,t}$$

Recall that

$$e_t = x_t - \hat{x}_{t-1,t}$$

Time Series: Exponential Smoothing

$$\hat{x}_{t,t+1} = \alpha x_t + (1-\alpha)\,\hat{x}_{t-1,t}$$

but recalling that $\hat{x}_{t-1,t} = \alpha x_{t-1} + (1-\alpha)\,\hat{x}_{t-2,t-1}$

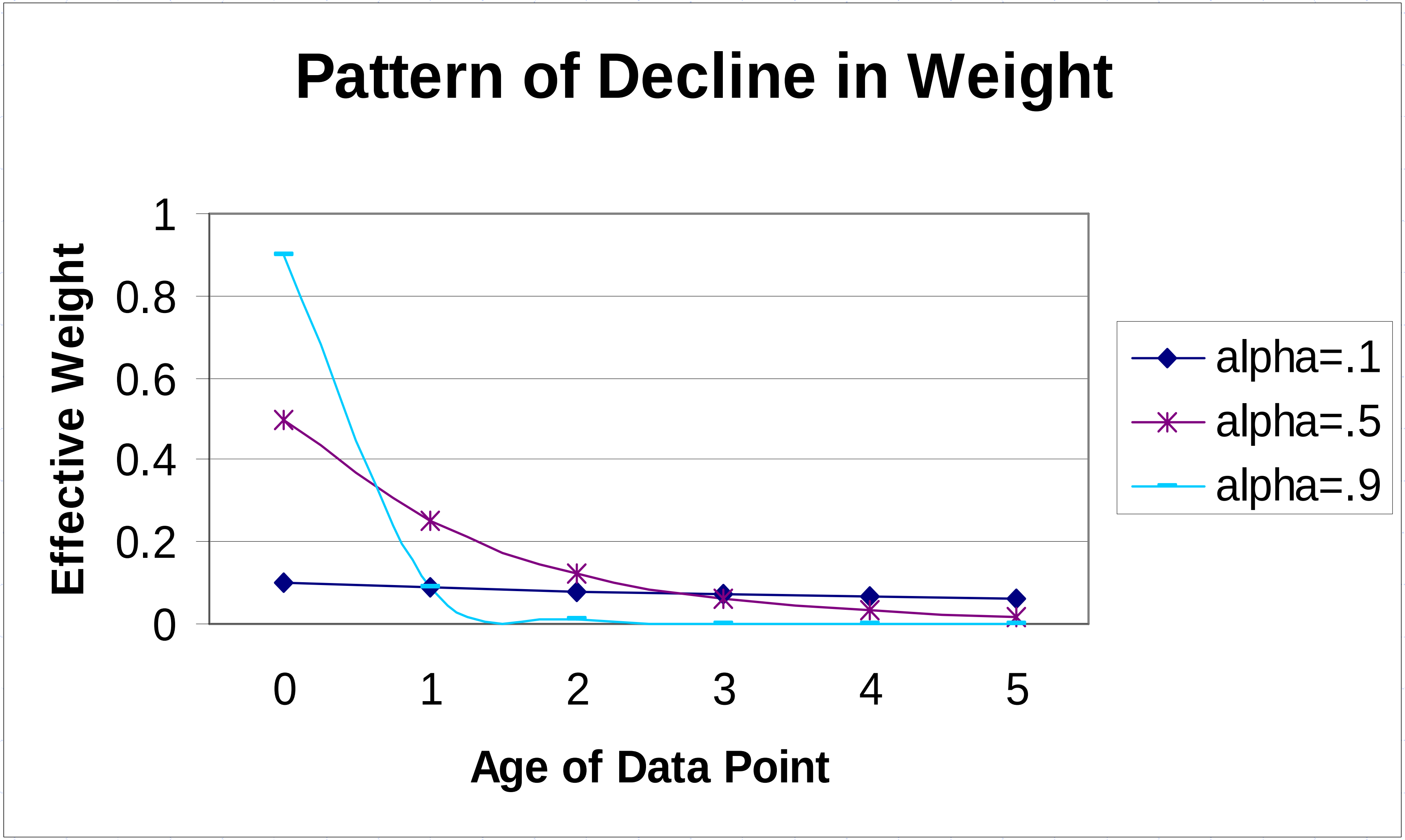
$$\hat{x}_{t,t+1} = \alpha x_t + (1-\alpha)(\alpha x_{t-1} + (1-\alpha)\,\hat{x}_{t-2,t-1})$$

$$\hat{x}_{t,t+1} = \alpha x_t + \alpha(1-\alpha)x_{t-1} + (1-\alpha)^2\,\hat{x}_{t-2,t-1}$$

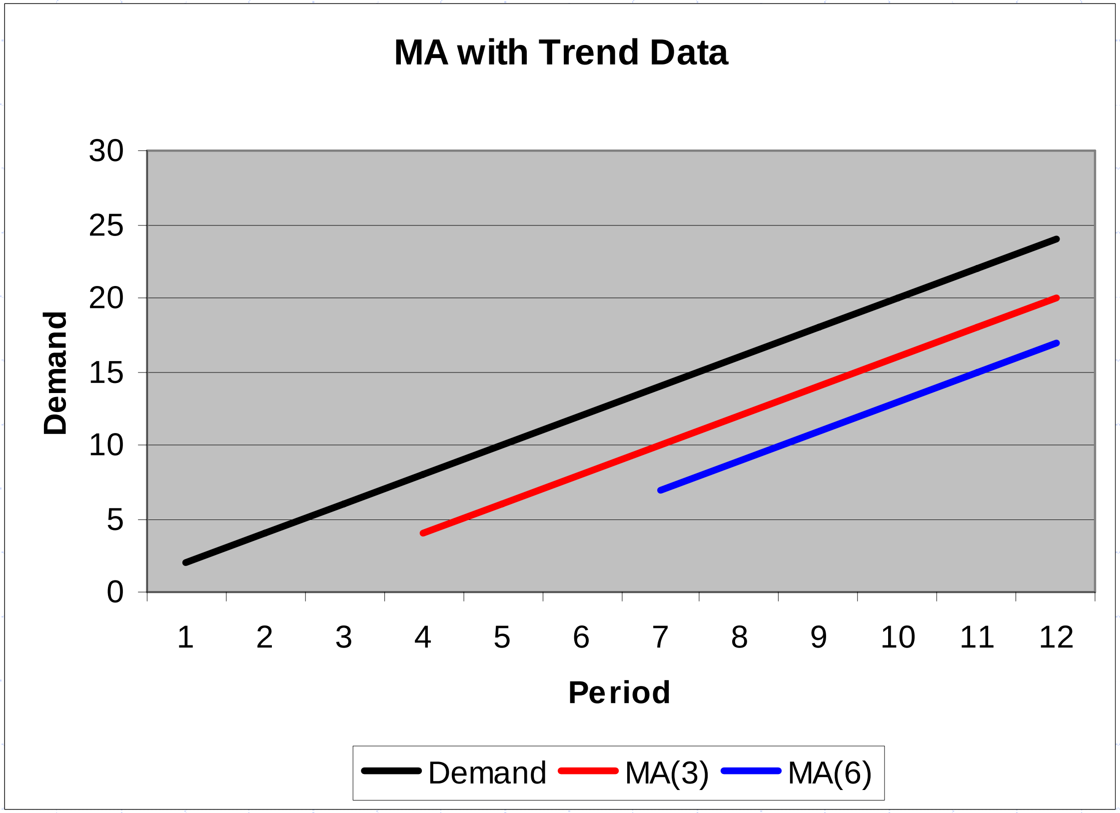
$$\hat{x}_{t,t+1} = \alpha x_t + \alpha(1-\alpha)x_{t-1} + \alpha(1-\alpha)^2 x_{t-2} + (1-\alpha)^3\,\hat{x}_{t-3,t-2}$$

$$\hat{x}_{t,t+1} = \alpha(1-\alpha)^0 x_t + \alpha(1-\alpha)^1 x_{t-1} + \alpha(1-\alpha)^2 x_{t-2} + \alpha(1-\alpha)^3 x_{t-3} \ldots$$

Time Series: Exponential Smoothing



Time Series: Non-Stationary Models



- Note that MA and standard Exp Smoothing will just lag a trend
- They only look at history to find the stationary level
- Need to capture the 'trend' or 'seasonality' factors

Time Series: Level & Trended Data

- Similar to exponential smoothing
- Holt's Method - smoothing constants for level (a) and trend (b) terms

Underlying Model:

$$x_t = a + bt + e_t$$

where: $e_t \sim \text{iid}\ (\mu=0\ ,\ \sigma^2=V[e])$

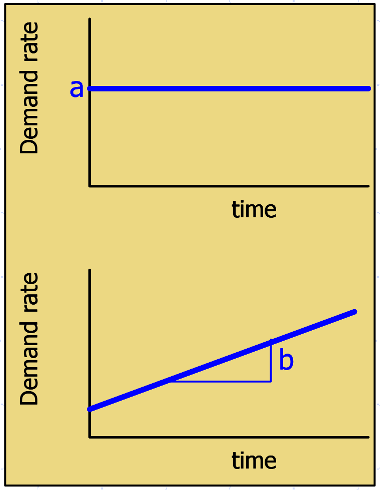
Forecasting Model:

$$\hat{x}_{t,t+\tau} = \hat{a}_t + \tau \hat{b}_t$$

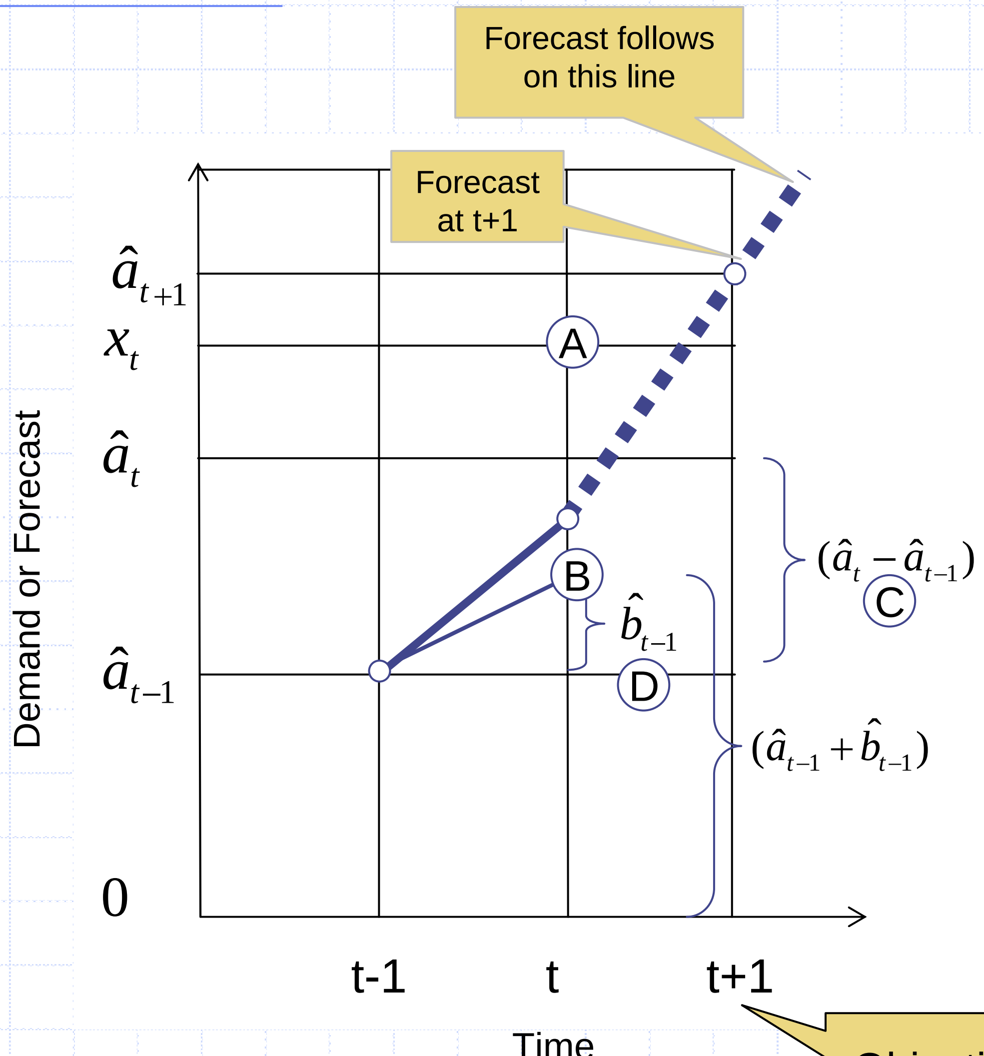
Where:

$$\hat{a}_t = \alpha x_t + (1-\alpha)(\hat{a}_{t-1} + \hat{b}_{t-1})$$

$$\hat{b}_t = \beta(\hat{a}_t - \hat{a}_{t-1}) + (1-\beta)\hat{b}_{t-1}$$



Time Series: Level & Trended Data



Source: Atul Agarwal MLOG'05

This is a linear weighted combination of level at (A) and (B)

$$\hat{a}_t = \underbrace{\alpha_{HW} x_t}_{A} + (1-\alpha_{HW})(\hat{a}_{t-1} + \underbrace{\hat{b}_{t-1}}_{B})$$

$$\hat{b}_t = \beta_{HW}\underbrace{(\hat{a}_t - \hat{a}_{t-1})}_{C} + (1-\beta_{HW})\underbrace{\hat{b}_{t-1}}_{D}$$

This is a linear weighted combination of slopes at (C) and (D)

Objective is to forecast t+1 & beyond

Time Series: Level & Seasonal Data

- Multiplicative model using exponential smoothing
- Introduces seasonal term, F, that covers P periods.

Underlying Model:

$$x_t = aF_t + e_t$$

where: $e_t \sim \text{iid}\ (\mu=0\ ,\ \sigma^2=V[e])$

Forecasting Model:

$$\hat{x}_{t,t+\tau} = \hat{a}_t\ \hat{F}_{t+\tau-P}$$

Where :

$$\hat{a}_t = \alpha(x_t/\hat{F}_{t-P}) + (1-\alpha)\hat{a}_{t-1}$$

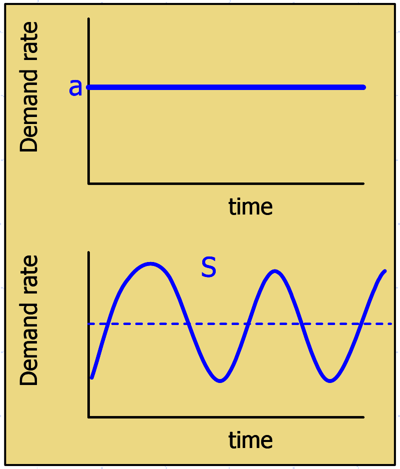
$$\hat{F}_t = \gamma(x_t/\hat{a}_t) + (1-\gamma)\hat{F}_{t-P}$$

An Example where P=4, t=12, τ=1:

$$\hat{x}_{12,13} = \hat{a}_{12}\hat{F}_9$$

$$\hat{a}_{12} = \alpha(x_{12}/\hat{F}_8) + (1-\alpha)\hat{a}_{12}$$

$$\hat{F}_{12} = \gamma(x_{12}/\hat{a}_{12}) + (1-\gamma)\hat{F}_8$$



Time Series: Level, Seasonal, & Trended Data

- Expand exponential model to include seasonality
- Winter's Method – Similar to Holt's Method with added term
- Seasonality is multiplicative

Underlying Model:

$$x_t = (a+bt)\,F_t + e_t$$

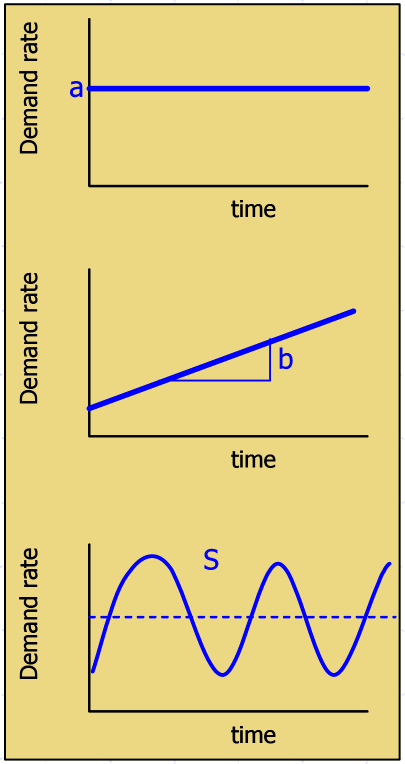
where: $e_t \sim \text{iid}\,(\mu=0\,,\,\sigma^2=V[e])$

Forecasting Model:

$$\hat{x}_{t,t+\tau} = (\hat{a}_t + \tau\hat{b}_t)\,\hat{F}_{t+\tau-P}$$

Where :

$$\hat{a}_t = \alpha(x_t/\hat{F}_{t-P}) + (1-\alpha)(\hat{a}_{t-1} + \hat{b}_{t-1})$$
$$\hat{b}_t = \beta(\hat{a}_t - \hat{a}_{t-1}) + (1-\beta)\,\hat{b}_{t-1}$$
$$\hat{F}_t = \gamma(x_t/\hat{a}_t) + (1-\gamma)\,\hat{F}_{t-P}$$



Comments on Time Series Models

- Most of the work is bookkeeping
  - Initialization procedures can be arbitrary
  - Adding seasonality greatly complicates calculations
- Most of the value comes from sharing with users
  - Provide insights into explaining abnormalities
  - Assist in initial formulations and models
- Picking appropriate smoothing factors
  - Level (α)
    - Stationary: ranges from 0.01 to 0.30 (0.1 reasonable)
    - Trend/Season: ranges from 0.02 to 0.51 (0.19 reasonable)
  - Trend (β)
    - Ranges from 0.005 to 0.176 (0.053 reasonable)
  - Seasonality (γ)
    - Ranges from 0.05 to 0.50 (0.10 reasonable)

Questions, Comments, Suggestions?

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