Inventory Management

Material Requirements Planning

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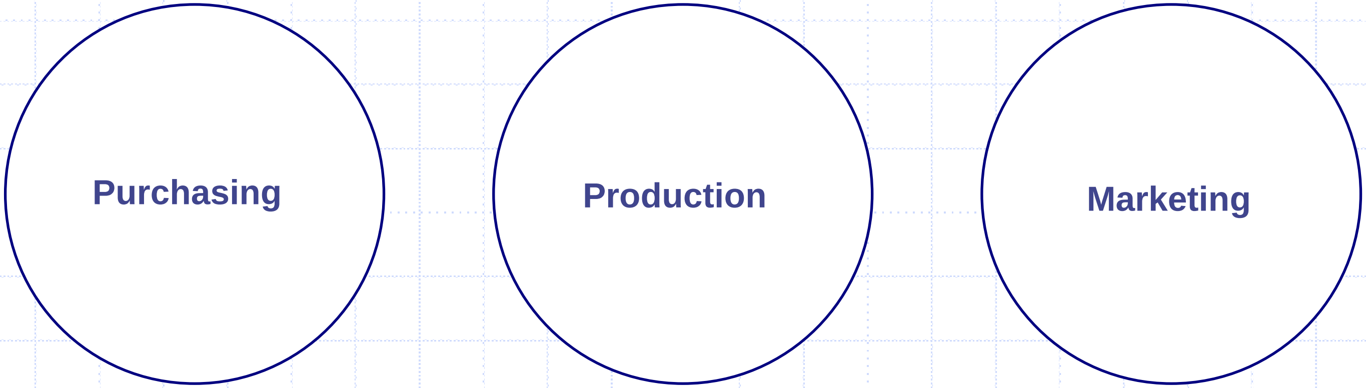
Assumptions: Basic MRP Model

- Demand
  - Constant vs **Variable**
  - **Known** vs Random
  - Continuous vs **Discrete**
- Lead time
  - Instantaneous
  - **Constant** or **Variable** (deterministic/stochastic)
- Dependence of items
  - Independent
  - Correlated
  - **Indentured**
- Review Time
  - **Continuous**
  - Periodic
- Number of Echelons
  - **One**
  - Multi (>1)
- Capacity / Resources
  - **Unlimited**
  - Limited (Constrained)
- Discounts
  - **None**
  - All Units or Incremental
- Excess Demand
  - **None**
  - All orders are backordered
  - Lost orders
  - Substitution
- Perishability
  - **None**
  - Uniform with time
- Planning Horizon
  - Single Period
  - **Finite Period**
  - Infinite
- Number of Items
  - One
  - **Many**
- Form of Product
  - **Single Stage**
  - Multi-Stage

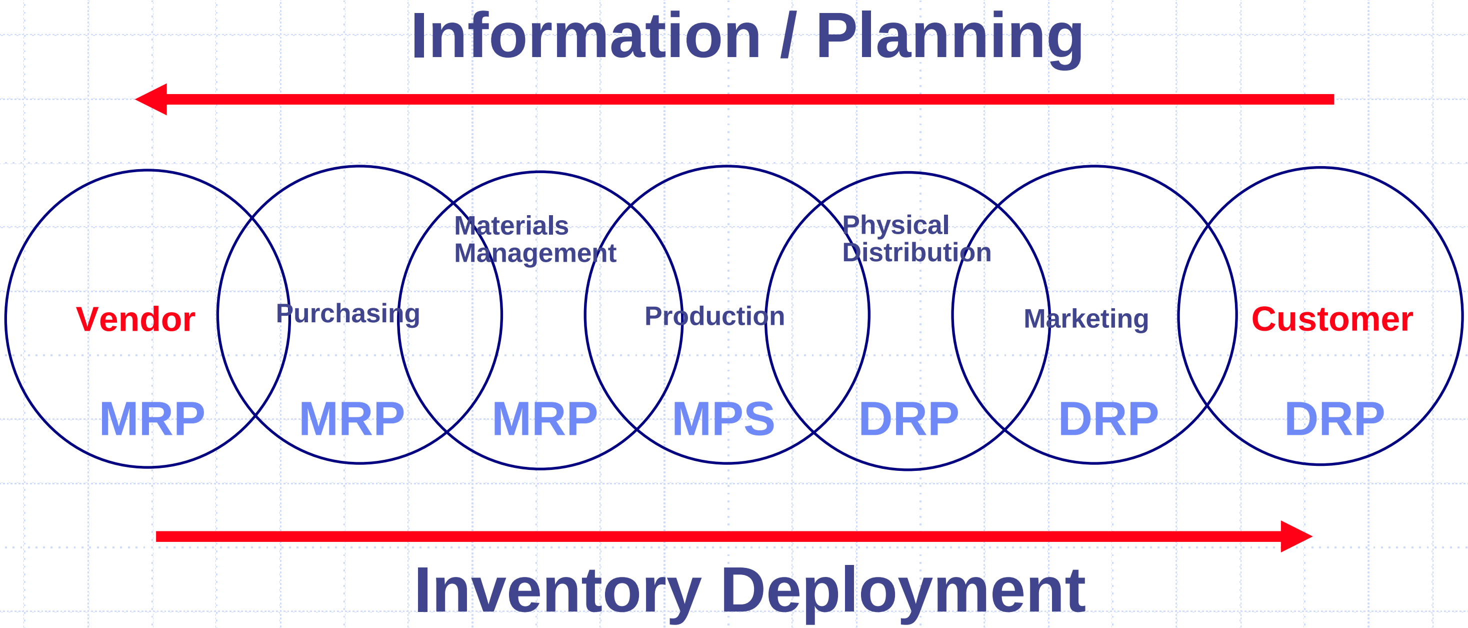
How many components are there?

Image of iPod Shuffle circuitry removed due to copyright restrictions.

Traditional Management



Supply Chain Integration



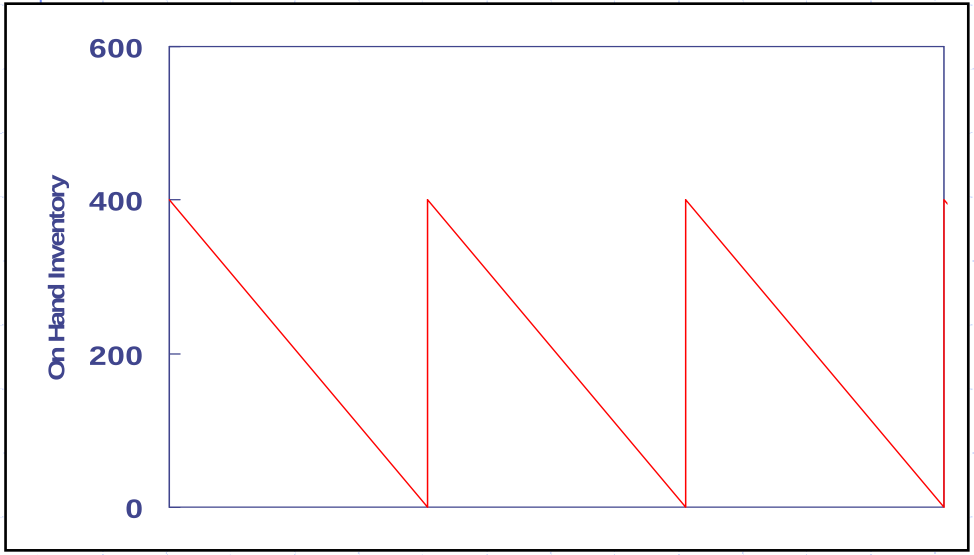
Material Requirements Planning
Master Production Scheduling
Distribution Requirements Planning

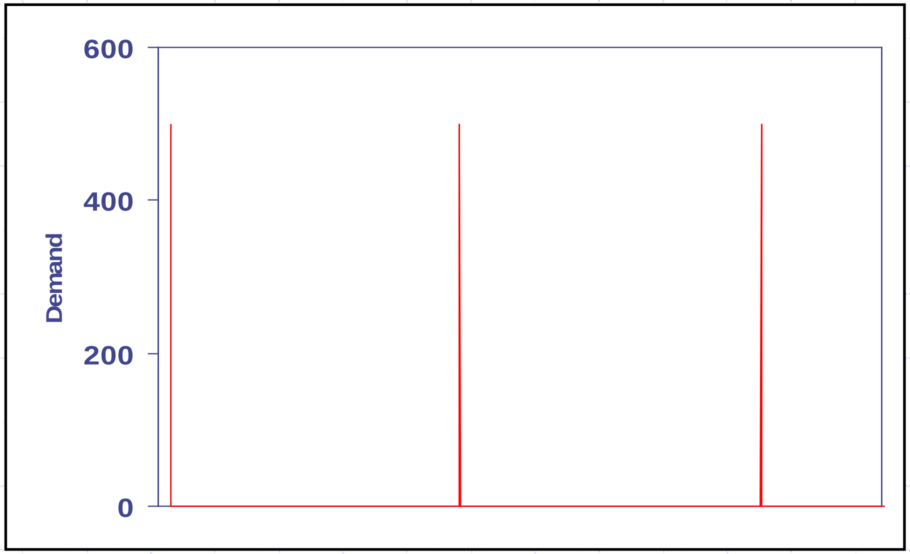
Inventory Management so far . . .

- Traditional techniques . . .
  - Forecast demand independently for each item based on usage history
  - Establish lot sizes independently for each item based on demand forecasts
  - Establish safety stocks independently for each item based on forecast errors
- Which make the following assumptions . . .
  - Demand is "Continuous‟
    [usage occurs in every period]
  - Demand is "Uniform"
    [average usage per period is stable over time]
  - Demand is "Random"
    [usage in any given period is not known in advance]

Cycle Stock with a Fixed Lot Size

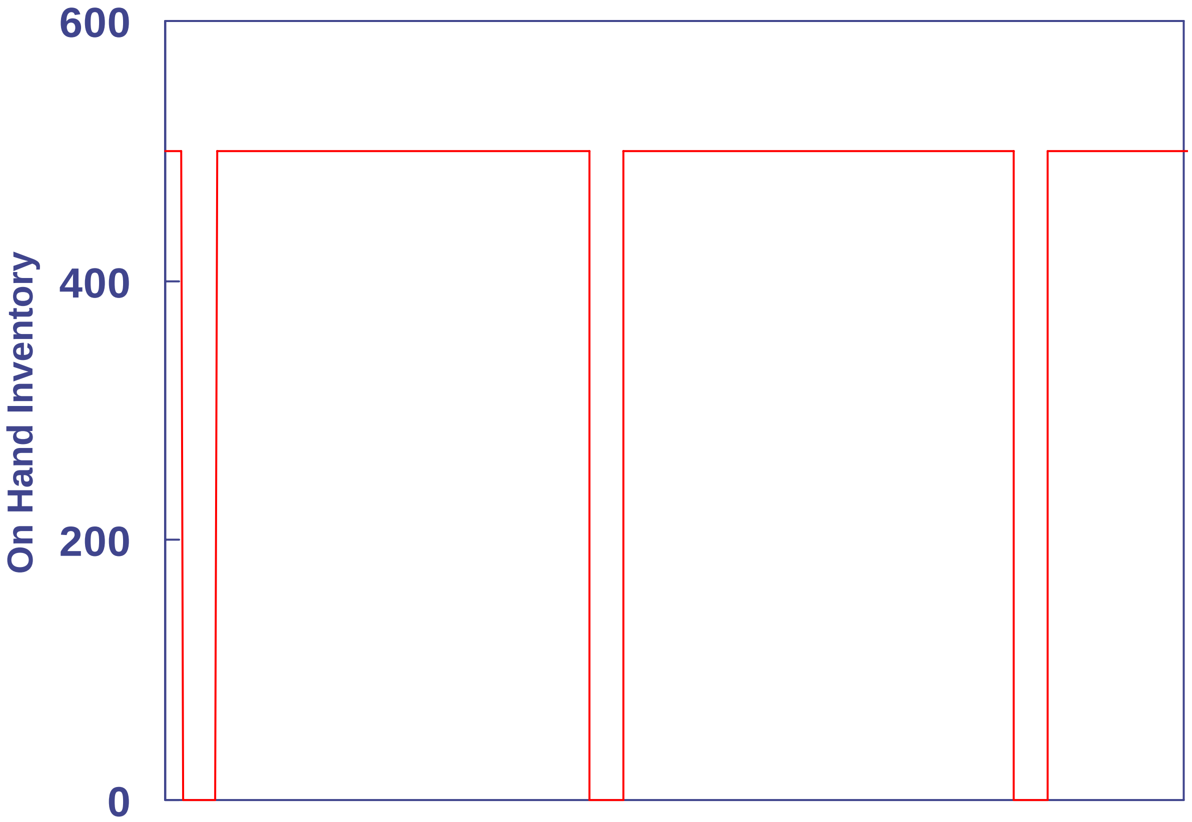
A= \$500, r=25%, v= \$50,
D = 2000 units/yr, **Q*=400 units**





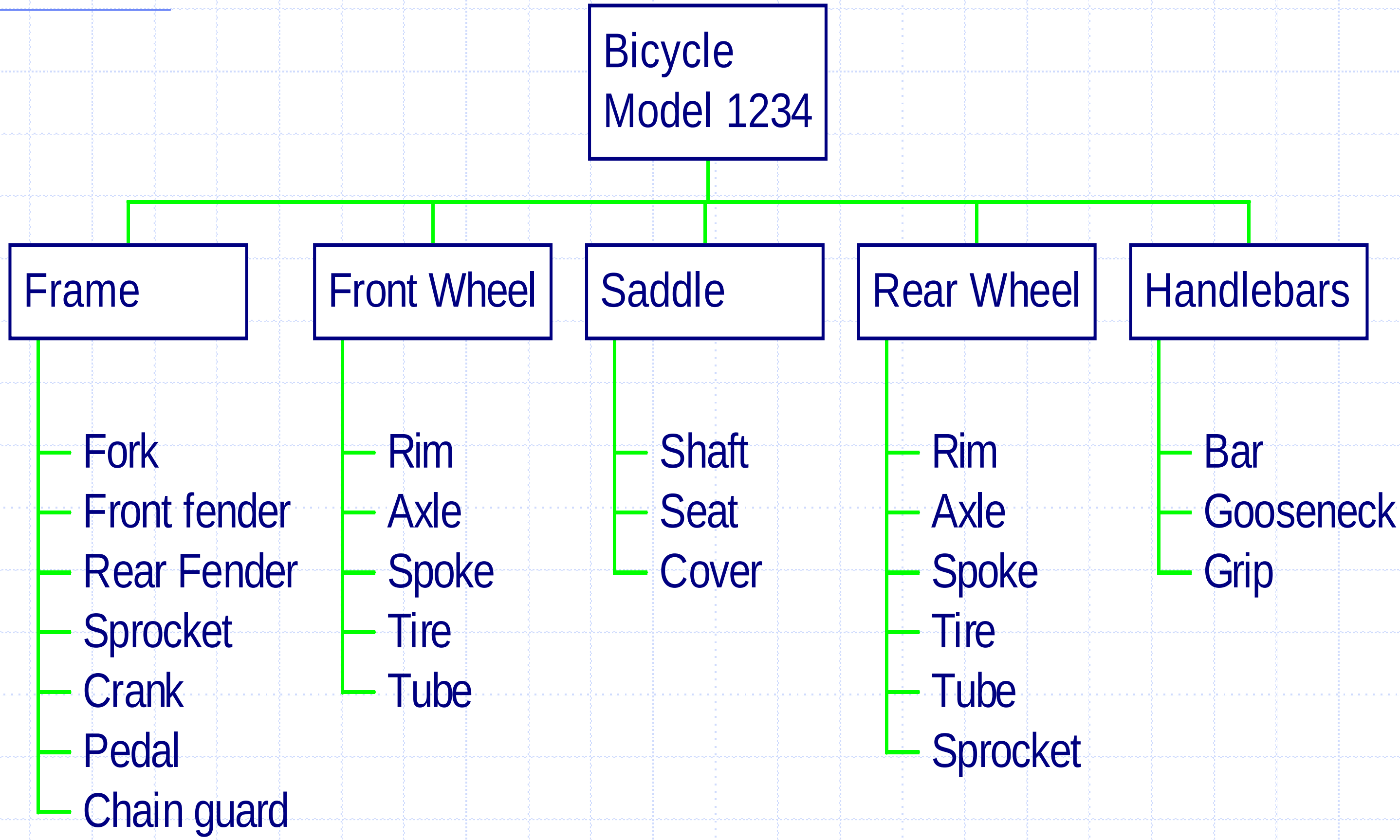
Problem: Intermittent Demand
4 production periods, 500 units/period,
Demand rate 2000/year

Fixed Lot Size with Intermittent Demand results in . . .



Can we do better?

Another Wrinkle . . . Product Indenture



Note that each item, sub-assembly, component etc. might feed into multiple end products

Combined Demand Impacts

- Suppose a widget is part of three items
  - Product A – 10 items per week – (3 Weeks OH)
  - Product B – 5 items per week – (2 Weeks OH)
  - Product C – 7 items per week - (4 Weeks OH)
- End demand looks like . . .

	1	2	3	4	5
A	10	10	10	10	10
B	5	5	5	5	5
C	7	7	7	7	7
Widget	22	22	22	22	22

Combined Demand Impacts

- But if ordered separately – what will widget demand look like?
  - Product A – 10 items per week – (3 Weeks OH)
  - Product B – 5 items per week – (2 Weeks OH)
  - Product C – 7 items per week - (4 Weeks OH)

	1	2	3	4	5
A	30	0	0	30	0
B	10	0	10	0	10
C	28	0	0	0	28
Widget	68	0	10	30	38

Important to synchronize

Push versus Pull Systems

- Simple Example
  - You make shovels that have 4 parts:
    - Metal Digger
    - Wooden Pole
    - 2 Screws
  - Production is 100 shovels per week:
    - Metal part is made in 400 item batches on first 2 days of the month
    - Handles are procured from Pole Co.
    - Assembly occurs during first week of each month
  - How should I manage my inventory for screws?
    - A=$0.25, v=$0.01, r=25%
    - D = 800*12=9600 units per year
    - L = 1 week
  - What are the values for . . .
    - Q* =
    - x_L =
    - RMSE(L) =

Push versus Pull Systems

- What is my policy if I follow a . . .
  - Standard EOQ policy?
    - Order ~1385 (~every other month)
    - What would the Inventory On Hand look like?
  - Standard (s,Q) policy?
    - So, since σ_L = 193, pick a CSL=95% k=1.64
    - s=185 + (1.64)193 = 502 units
    - Order 1385 units when inventory position ≤ 502
  - Standard (R,S) policy?
    - Select a monthly review policy (R=4 weeks)
    - x_{L+R}= 9600/(52/5) = 923 units
    - σ_{L+R} = 193(√5) = 432 units
    - S = 923 + (1.64)432 = 1631
    - Order up to 1631 units every 4 weeks
  - Other methods?

Material Requirements Planning

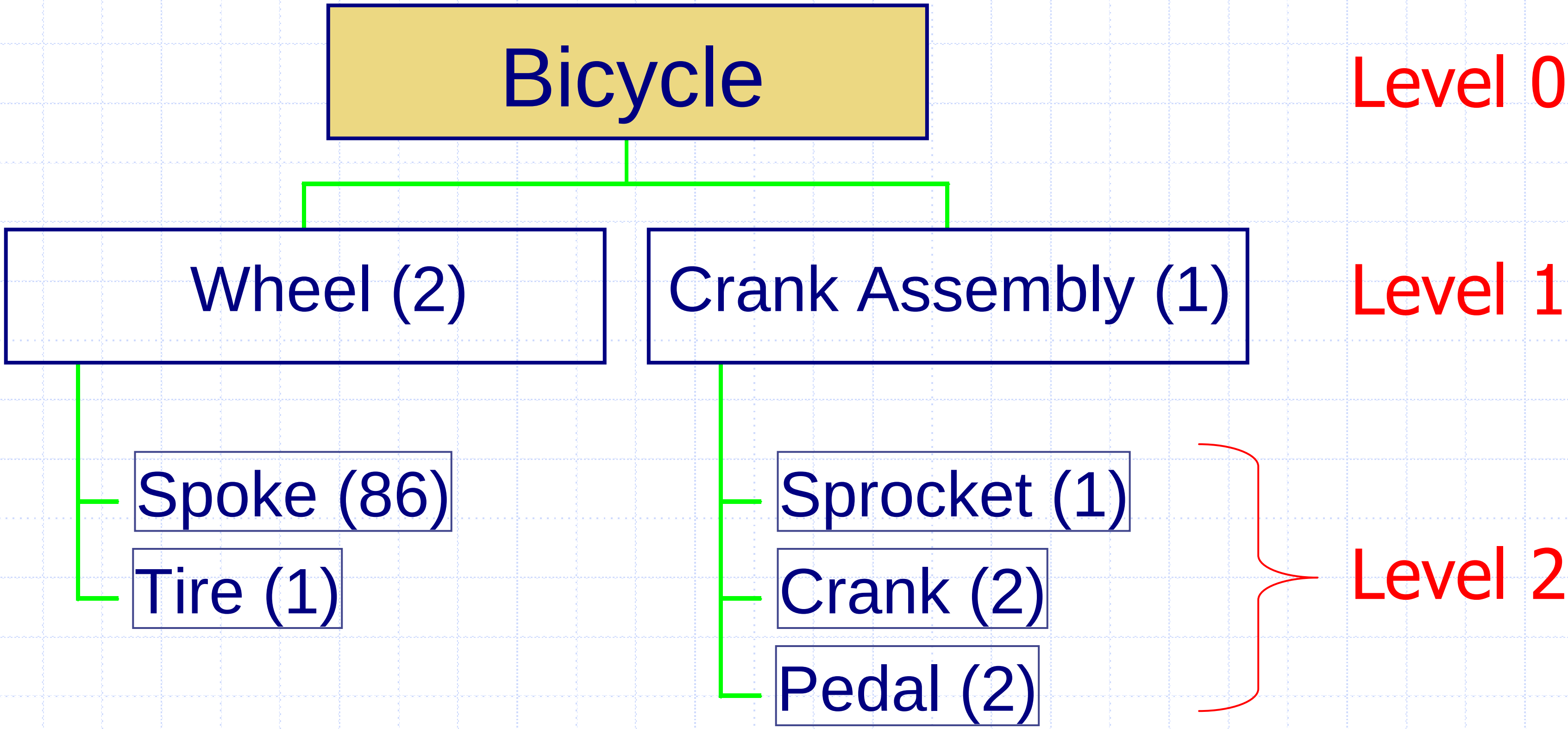
- Push vs Pull Systems
  - Push – MRP
    - "initiates production in anticipation of future demand"
  - Pull – JIT
    - "initiates production as a reaction to present demand"
- Major Premises
  - Inventory control in a production environment
  - Many products, many component parts
  - Complex product indenture structure
  - Production creates "lumpy" demand
- Major Concepts
  - Dependent demand versus independent demand
  - Requirements calculation versus demand forecasting
  - Schedule flow versus stockpile assets
  - Information replaces inventory

Material Requirements Planning

- Primary Questions
  - What are we going to make? => use forecast
  - What does it take to make it? => use res. req's & BOM
  - What do we have? => use inventory records
  - What do we need and when? => use mfg schedules
- Information Requirements
  - Master Production Schedule
  - Product Indenture Structure
  - Inventory Status
  - Ordering Data
- MRP Process
  - Requirements Explosion
  - Use of Bill of Materials (BOM)
  - Net from Gross Requirements
  - Requirements Time Phasing
  - Planned Order Release

Example: Bike Co.

BOM Explosion



Bill of Materials

Weekly buckets

Product	Sub-assembly	Component	Quantity	Lead Time
Bicycle			[1]	2
	Wheel		2	1
		Spoke	86	3
		Tire	1	2
	Crank Asm		1	1
		Sprocket	1	4
		Crank	2	3
		Pedal	2	3

MRP Approach:

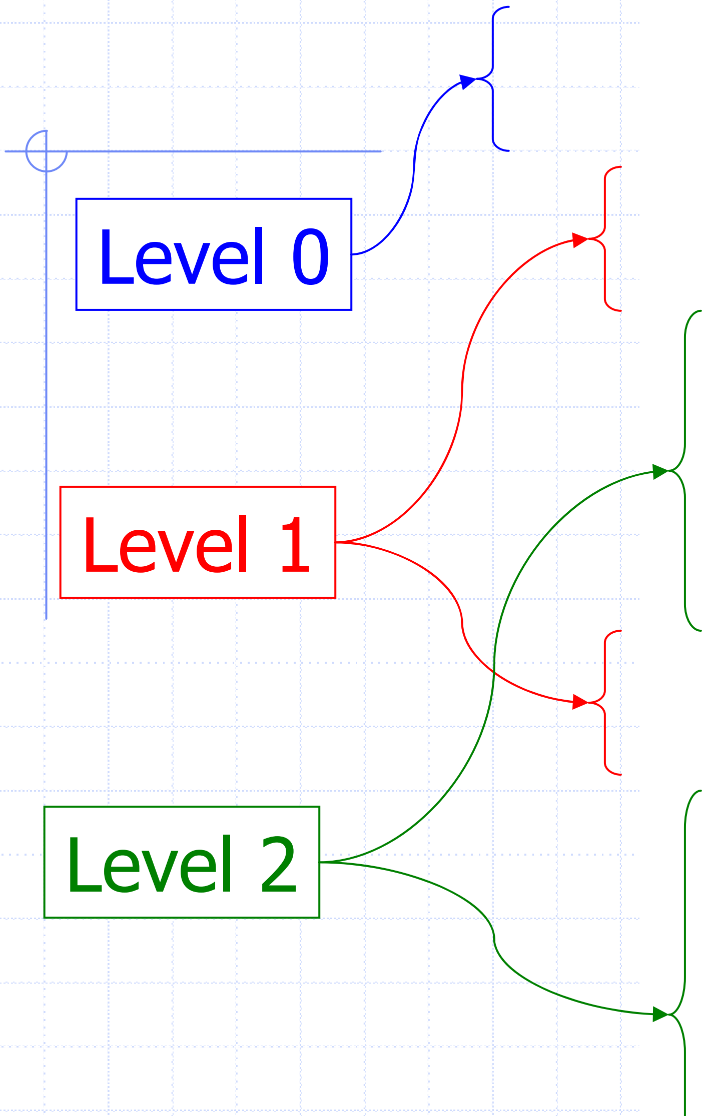
1. Start with Level i demand (i=0)
2. Find Gross Requirements (GR) and On Hand (OH) for Level i
3. Find Net Requirements (NR) for Level i+1 (NR=GR-OH)
4. Establish Planned Order Release (POR) for Level i using Level i lead times
5. Set GR for Level i+1 based on POR for Level i
6. Set i = i+1 and go to Step 2

The MRP Plan for the Bicycle

Objective:
Have materials ready for having 25 bikes in week 8

ITEM	PERIOD:	1	2	3	4	5	6	7	8
Bicycle	Rqmt								25
	On Hand								
	Due In								25
	POR						25		
Wheel	Rqmt						50		
	On Hand								
	Due In						50		
	POR					50			
Spoke	Rqmt					4300			
	On Hand								
	Due In					4300			
	POR		4300						

- Gross Requirement → Rqmt
- On Hand → On Hand
- Net Requirement → Due In
- Planned Order Release → POR



Item	Period:	1	2	3	4	5	6	7	8
Bicycle	GR								25
	OH								
	NR								25
	POR						25		
Wheel	GR						50		
	OH								
	NR						50		
	POR					50			
Spoke	GR					4300			
	OH								
	NR					4300			
	POR		4300						
Tire	GR					50			
	OH								
	NR					50			
	POR			50					
Crank Asm	GR						25		
	OH								
	NR						25		
	POR					25			
Sprocket	GR					25			
	OH								
	NR					25			
	POR	25							
Crank	GR					50			
	OH								
	NR					50			
	POR		50						
Pedal	GR					50			
	OH	20	20	20	20	20			
	NR					30			
	POR		30						

What is missing?

Ordering Plan

Two Issues

- How do we handle capacity constraints?
- How do we handle uncertainty?

Approach: Optimization (MILP)

Decision Variables:

Q_i = Quantity purchased in period i
Z_i = Buy variable = 1 if Q_i>0, =0 o.w.
B_i = Beginning inventory for period I
E_i = Ending inventory for period I

Data:

D_i = Demand per period, i = 1,,n
C_o = Ordering Cost
C_{hp} = Cost to Hold, $/unit/period
M = a very large number….

MILP Model

Objective Function:

- Minimize total relevant costs

Subject To:

- Beginning inventory for period 1 = 0
- Beginning and ending inventories must match
- Conservation of inventory within each period
- Nonnegativity for Q, B, E
- Binary for Z

Approach: Optimization (MILP)

$$Min\ TC = \sum_{i=1}^{n} C_O Z_i + \sum_{i=1}^{n} C_{HP} E_i$$

Objective Function

$s.t.$

$$B_1 = 0$$

$$B_i - E_{i-1} = 0 \qquad \forall i = 2,3,\ldots n$$

Beginning & Ending Inventory Constraints

$$E_i - B_i - Q_i = -D_i \qquad \forall i = 1,2,\ldots n$$

Conservation of Inventory Constraints

$$MZ_i - Q_i \geq 0 \qquad \forall i = 1,2,\ldots n$$

Ensures buys occur only if Q>0

$$B_i \geq 0 \qquad \forall i = 1,2,\ldots n$$

$$E_i \geq 0 \qquad \forall i = 1,2,\ldots n$$

$$Q_i \geq 0 \qquad \forall i = 1,2,\ldots n$$

$$Z_i = \{0,1\} \qquad \forall i = 1,2,\ldots n$$

Non-Negativity & Binary Constraints

MRP: Example

TOTAL COST			
$ 10,500.00			
END ITEM		Lead Time	2
$ 7,000.00		Setup	$ 1,000
		Holding	$ 5.00
		Capacity	1000

SIMPLE MRP SOLUTION

Notes:
- End Item requires 3 of the same Components
- There is a setup cost, holding cost, and (potential) capacity

Period	1	2	3	4	5	6	7	8	9	10	11	12
Gross Rqmt	0	0	0	0	0	30	50	200	30	70	180	40
Begin Inv	0	0	0	0	0	0	0	0	0	0	0	0
ORDER			0	0	0	30	50	200	30	70	180	40
Ending Inv	0	0	0	0	0	0	0	0	0	0	0	0
POR	0	0	0	30	50	200	30	70	180	40	0	0

COMPONENT		Lead Time	2
$ 3,500.00		QPA	3
		Setup	$ 500
		Holding	$ 1.00
		Capacity	2000

Lead time in Weeks

Quantity Per Assembly

Output from End Item Model becomes input to Component Model

Period	1	2	3	4	5	6	7	8	9	10	11	12
Gross Rqmt	0	0	0	90	150	600	90	210	540	120	0	0
Begin Inv	0	0	0	0	0	0	0	0	0	0	0	0
ORDER	0	0	0	90	150	600	90	210	540	120	0	0
Ending Inv	0	0	0	0	0	0	0	0	0	0	0	0
POR	0	90	150	600	90	210	540	120	0	0	0	0

MRP: Example

TOTAL COST											
$ 5,800.00					OPTIMIZE THE ITEM SCHEDULES						
END ITEM		Lead Time		2							
$ 4,300.00		Setup		$ 1,000							
		Holding		$ 5.00							
		Capacity		1000							

Notes:
- Solves the End Items and the Components models together
- The Order Quantities are the decision variables
- The Planned Order Release = f(Lead Time, Order)
- Capacity is not binding yet

Period	1	2	3	4	5	6	7	8	9	10	11	12
Gross Rqmt	0	0	0	0	0	30	50	200	30	70	180	40
Begin Inv	0	0	0	0	0	0	50	0	100	70	0	40
ORDER			0	0	0	80	0	300	0	0	220	0
Ending Inv	0	0	0	0	0	50	0	100	70	0	40	0
POR	0	0	0	80	0	300	0	0	220	0	0	0

COMPONENT		Lead Time		2
$ 1,500.00		QPA		3
		Setup		$ 500
		Holding		$ 1.00
		Capacity		2000

Note the bunching of orders!

Period	1	2	3	4	5	6	7	8	9	10	11	12
Gross Rqmt	0	0	0	240	0	900	0	0	660	0	0	0
Begin Inv	0	0	0	0	0	0	0	0	0	0	0	0
ORDER	0	0	0	240	0	900	0	0	660	0	0	0
Ending Inv	0	0	0	0	0	0	0	0	0	0	0	0
POR	0	240	0	900	0	0	660	0	0	0	0	0

MRP: Example

TOTAL COST												
$ 7,660.00						OPTIMIZE ITEM SCHEDULES						
END ITEM		Lead Time		2								
$ 4,300.00		Setup		$ 1,000		[Component capacity constraint binding]						
		Holding		$ 5.00								
		Capacity		1000								
Period	1	2	3	4	5	6	7	8	9	10	11	12
GR	0	0	0	0	0	30	50	200	30	70	180	40
BINV	0	0	0	0	0	0	50	0	100	70	0	40
ORDER			0	0	0	80	0	300	0	0	220	0
EINV	0	0	0	0	0	50	0	100	70	0	40	0
POR	0	0	0	80	0	300	0	0	220	0	0	0
COMPONENT		Lead Time		2								
$ 3,360.00		QPA		3								
		Setup		$ 500								
		Holding		$ 1.00								
		Capacity		400								
Period	1	2	3	4	5	6	7	8	9	10	11	12
GR	0	0	0	240	0	900	0	0	660	0	0	0
BINV	0	0	0	0	100	500	0	0	260	0	0	0
ORDER	0	0	0	340	400	400	0	260	400	0	0	0
EINV	0	0	0	100	500	0	0	260	0	0	0	0
POR	0	340	400	400	0	260	400	0	0	0	0	0

Introduce binding constraint

MRP: Example

TOTAL COST												
$ 8,700.00						OPTIMIZE ITEM SCHEDULES						
END ITEM		Lead Time		2								
$ 4,800.00		Setup		$ 1,000		[Component capacity constraint "more binding"]						
		Holding		$ 5.00								
		Capacity		1000		NOTE: Component constraint redefines the End Item schedule.						
Period	1	2	3	4	5	6	7	8	9	10	11	12
GR	0	0	0	0	0	30	50	200	30	70	180	40
BINV	0	0	0	0	0	0	50	0	0	70	0	40
ORDER			0	0	0	80	0	200	100	0	220	0
EINV	0	0	0	0	0	50	0	0	70	0	40	0
POR	0	0	0	80	0	200	100	0	220	0	0	0
COMPONENT		Lead Time		2								
$ 3,900.00		QPA		3								
		Setup		$ 500								
		Holding		$ 1.00								
		Capacity		300								
Period	1	2	3	4	5	6	7	8	9	10	11	12
GR	0	0	0	240	0	600	300	0	660	0	0	0
BINV	0	0	0	0	60	360	60	60	360	0	0	0
ORDER	0	0	0	300	300	300	300	300	300	0	0	0
EINV	0	0	0	60	360	60	60	360	0	0	0	0
POR	0	300	300	300	300	300	300	0	0	0	0	0

Tighten binding constraint

MRP: Example

TOTAL COST

$ 9,100.00

NON-OPTIMAL SEQUENTIAL SOLUTION

END ITEM		
$ 4,300.00	Lead Time	2
	Setup	$ 1,000
	Holding	$ 5.00
	Capacity	1000

Notes:

- Solves the End Items and the Components models separately
- What is the impact? insight?
- Who wins? Loses?

Period	1	2	3	4	5	6	7	8	9	10	11	12
GR	0	0	0	0	0	30	50	200	30	70	180	40
BINV	0	0	0	0	0	0	50	0	100	70	0	40
ORDER			0	0	0	80	0	300	0	0	220	0
EINV	0	0	0	0	0	50	0	100	70	0	40	0
POR	0	0	0	80	0	300	0	0	220	0	0	0

COMPONENT		
$ 4,800.00	Lead Time	2
	QPA	3
	Setup	$ 500
	Holding	$ 1.00
	Capacity	300

Keep tight constraint

Period	1	2	3	4	5	6	7	8	9	10	11	12
GR	0	0	0	240	0	900	0	0	660	0	0	0
BINV	0	0	0	300	360	660	60	60	360	0	0	0
ORDER	0	0	300	300	300	300	0	300	300	0	0	0
EINV	0	0	300	360	660	60	60	360	0	0	0	0
POR	300	300	300	300	0	300	300	0	0	0	0	0

Handling Uncertainty

- Safety Stock
  - Add to existing stock levels
  - Where would this be applied?

- Safety Times
  - Pad the planned lead times
  - Where would this be applied?

Optimal Lead Time Padding

Let:

t	= Delivery Time, a random variable
t'	= Forecasted Delivery Time
σ	= Standard Deviation of the Forecast Error
T_p	= Padded Lead time = $t' + k\sigma$
Q	= Lot Size in units
v	= Unit Cost
r	= Holding Cost per unit per time period
C_d	= Shortage Cost per time period

$$TC[T_p] = \sum_{t=0}^{T_p} rvQ\left(T_p - t\right)P[t] + \sum_{t=T_p+1}^{\infty} C_d(t - T_p)P[t]$$

$$CSL^* = \frac{C_d}{C_d + rvQ}$$

Optimal Lead Time Padding

$$TC[T_p] = \left(\sum_{t=0}^{T_p} rvQT_p P[t] \right) - \left(\sum_{t=0}^{T_p} rvQtP[t] \right) + \left(\sum_{t=T_p+1}^{\infty} C_d t P[t] \right) - \left(\sum_{t=T_p+1}^{\infty} C_d T_p P[t] \right)$$

$$\frac{dTC[T_p]}{dT_p} = rvQ \left(\sum_{t=0}^{T_p} P[t] \right) - (0) + (0) - \left(C_d \sum_{t=T_p+1}^{\infty} T_p P[t] \right) = 0$$

$$rvQ(Prob[NoStockout]) - (C_d(Prob[Stockout])) = 0$$

$$rvQ(CSL^*) = C_d(1 - CSL^*)$$

$$CSL^* = \frac{C_d}{C_d + rvQ}$$

Optimal Lead Time Padding

Example:

v	= \$5.00/unit	Q	= 1000 units
r	= 36% annual	t'	= 10 days
rv	= .005 dollars/unit/day	σ	= 3 days
C_d	= \$500 per day	($t \sim$ normal)	

$CSL^* =$

$k^* =$

$T_p^* =$

Benefits of MRP

- Lower Inventory Levels
  - Able to better manage components
  - Increased visibility
- Fewer Stock outs
  - Relationships are defined and explicit
  - Allows for coordination with MPS
- Less Expediting
  - Due to increased visibility
- Fewer Production Disruptions
  - Input needs are explicitly modeled
  - Plans are integrated

Shortcomings of MRP

- MRP is a scheduling, not a stockage, algorithm
  - Replaces the forecasting mechanism
  - Considers indentured structures
- MRP does not address how to determine lot size
  - Does not explicitly consider costs
  - Wide use of Lot for Lot in practice
- MRP systems do not inherently deal with uncertainty
  - User must enter these values – by item by production level
  - Typical use of "safety time" rather than "safety stock"
- MRP assumes constant, known leadtimes
  - By component and part and production level
  - But lead time is often a function of order size and other activity
- MRP does not provide incentives for improvement
  - Requires tremendous amount of data and effort to set up
  - Initial values are typically inflated to avoid start up issues
  - Little incentive to correct a system "that works"

MRP: Evolution of Concepts

- Simple MRP
  - Focus on "order launching‟
  - Used within production – not believed outside
- Closed Loop MRP
  - Focus on production scheduling
  - Interacts with the MPS to create feasible plans
- MRP II [Manufacturing Resource Planning]
  - Focus on integrated financial planning
  - Treats the MPS as a decision variable
  - Capacity is considered (Capacity Resource Planning)
- Enterprise Resource Planning Systems
  - Common, centralized data for all areas
  - Implementation is costly and effort intensive
  - Forces business rules on companies

Questions?
Comments?
Suggestions?

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