Single Period Inventory Models

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Outline

- Single period inventory decisions
- Calculating the optimal order size
  - Numerically
    - Using spreadsheet
    - Using simulation
  - Analytically
- The profit function
  - For specific distributions
- Level of Service
- Extensions:
  - Fixed costs
  - Risks
  - Initial inventory
  - Elastic demand

Single Period Ordering

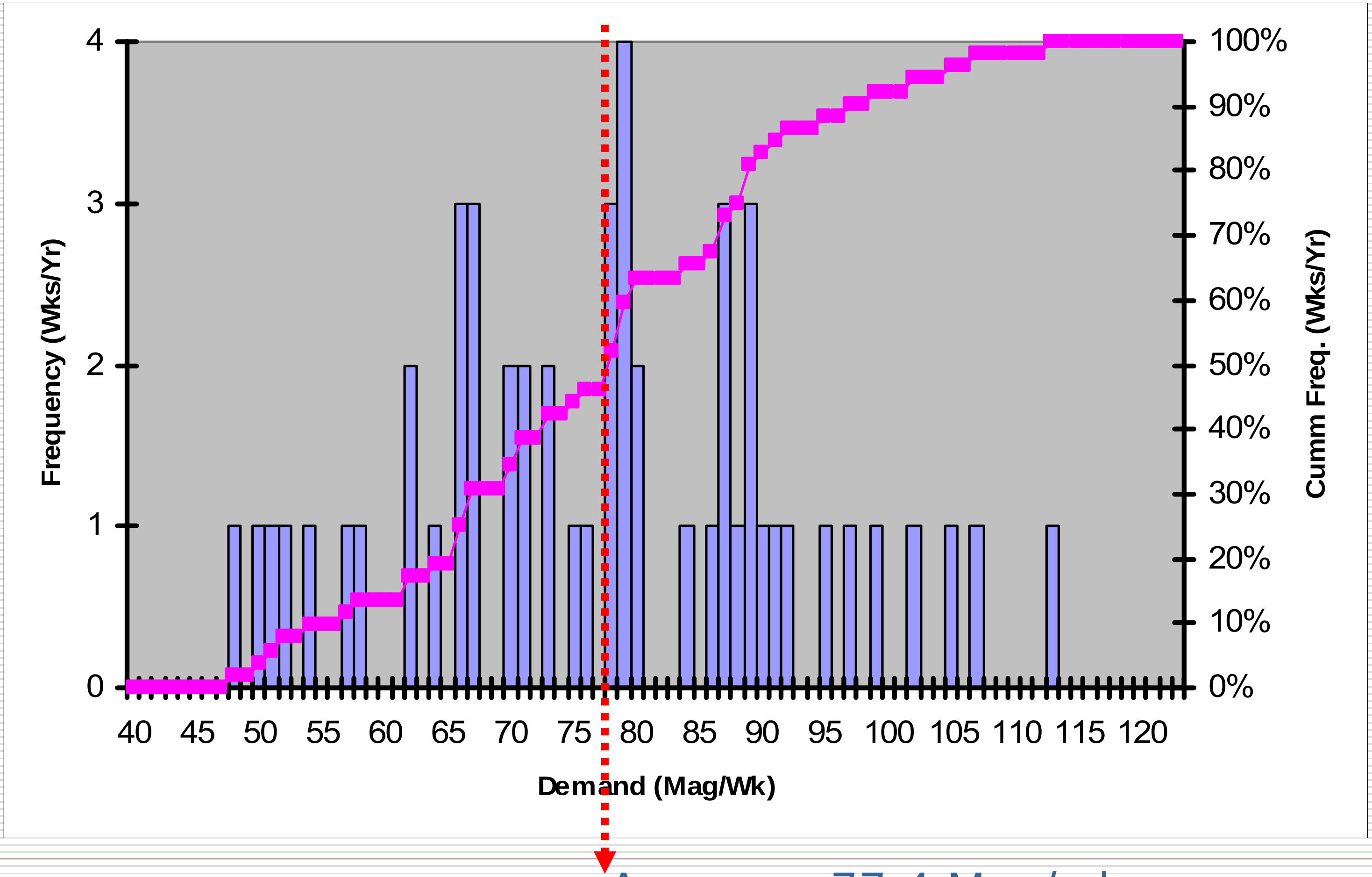
Selling Magazines

- Weekly demand:

90	48	87	78	58	71	102	87	66	79	97	75	89
57	86	95	67	89	70	113	52	84	62	91	71	66
99	73	92	66	67	89	87	64	70	54	67	88	62
79	79	105	76	73	78	50	107	80	78	51	79	80

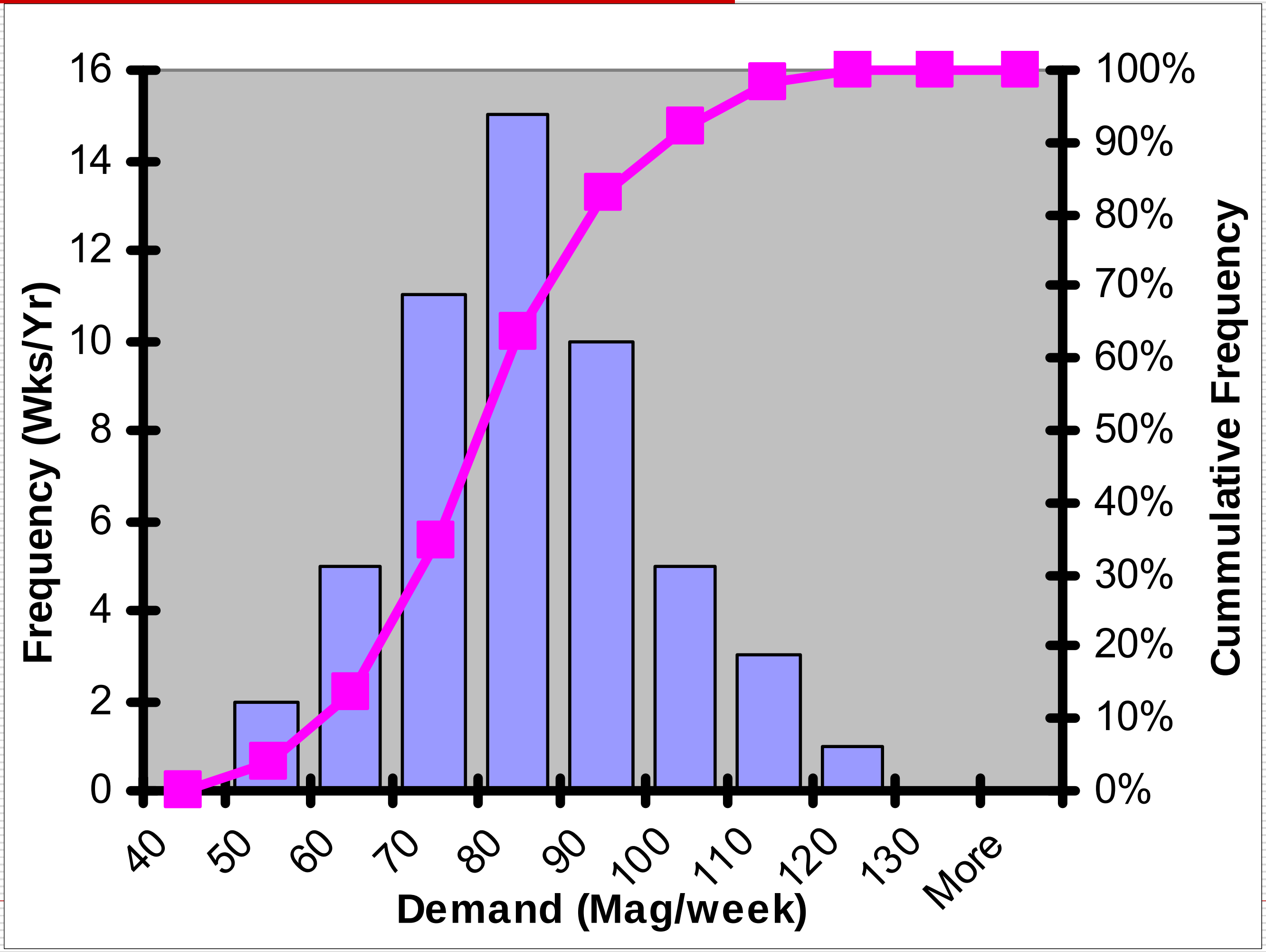
  - Total: 4023 magazines
  - Average: 77.4 Mag/week
  - Min: 51; max: 113 Mag/week

Detailed Histogram



Average=77.4 Mag/wk

Histogram



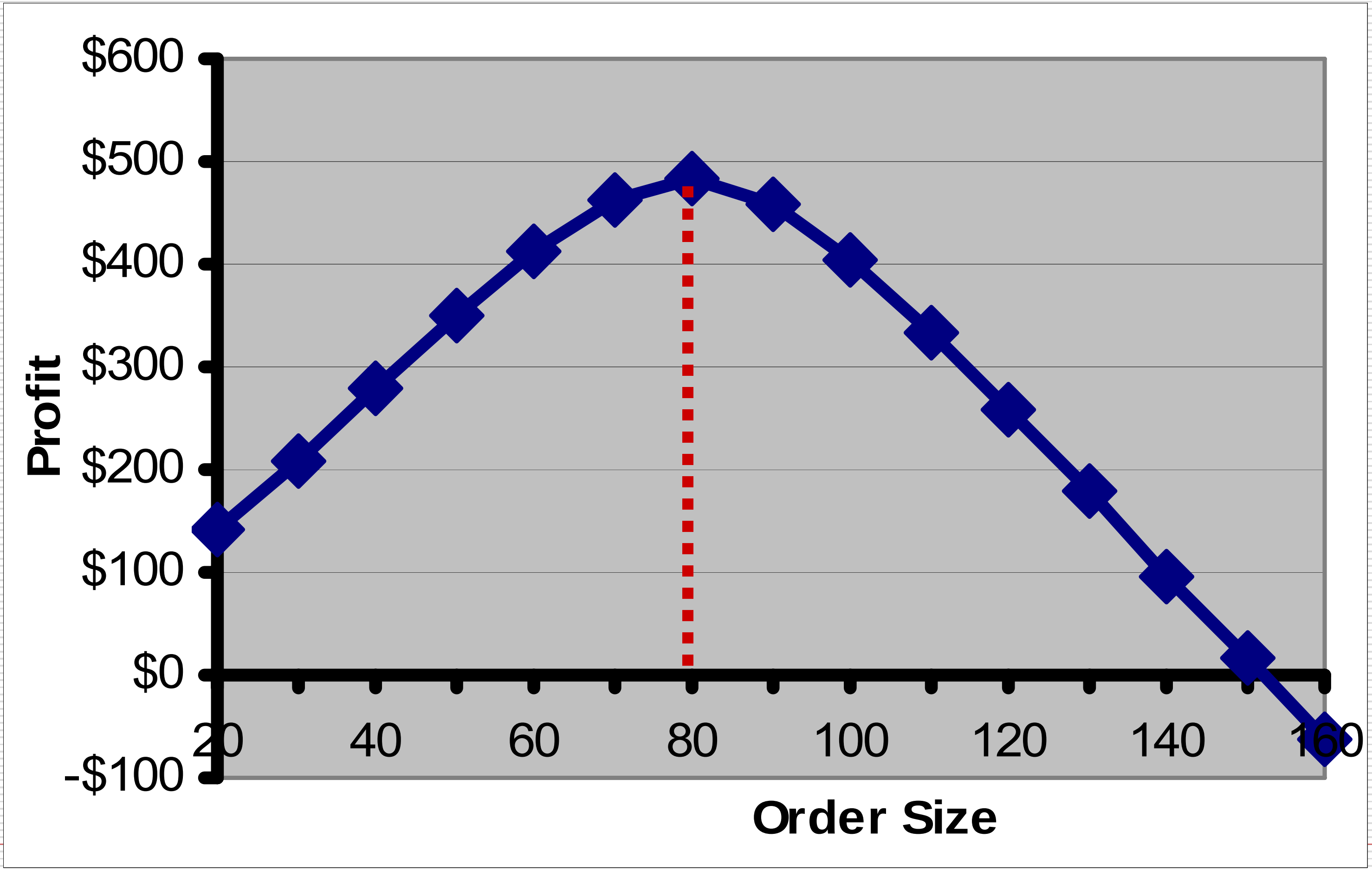
16
14
12
10
8
6
4
2
0
Frequency (Wks/Yr)
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Cummulative Frequency
40
50
60
70
80
90
100
110
120
130
More
Demand (Mag/week)

The Ordering Decision (Spreadsheet)

- Assume: each magazine sells for: $15
- Cost of each magazine: $8

	Order:	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160
d/wk	**Prob.**															
40	0.00	$140	$210	$280	$200	$120	$40	-$40	-$120	-$200	-$280	-$360	-$440	-$520	-$600	-$680
50	0.04	$140	$210	$280	$350	$270	$190	$110	$30	-$50	-$130	-$210	-$290	-$370	-$450	-$530
60	0.10	$140	$210	$280	$350	$420	$340	$260	$180	$100	$20	-$60	-$140	-$220	-$300	-$380
70	0.21	$140	$210	$280	$350	$420	$490	$410	$330	$250	$170	$90	$10	-$70	-$150	-$230
80	0.29	$140	$210	$280	$350	$420	$490	$560	$480	$400	$320	$240	$160	$80	$0	-$80
90	0.19	$140	$210	$280	$350	$420	$490	$560	$630	$550	$470	$390	$310	$230	$150	$70
100	0.10	$140	$210	$280	$350	$420	$490	$560	$630	$700	$620	$540	$460	$380	$300	$220
110	0.06	$140	$210	$280	$350	$420	$490	$560	$630	$700	$770	$690	$610	$530	$450	$370
120	0.02	$140	$210	$280	$350	$420	$490	$560	$630	$700	$770	$840	$760	$680	$600	$520
130	0.00	$140	$210	$280	$350	$420	$490	$560	$630	$700	$770	$840	$910	$830	$750	$670
Exp. Profit:		**$140**	**$210**	**$280**	**$350**	**$414**	**$464**	**$482**	**$457**	**$403**	**$334**	**$257**	**$177**	**$97**	**$17**	**-$63**

Expected Profits



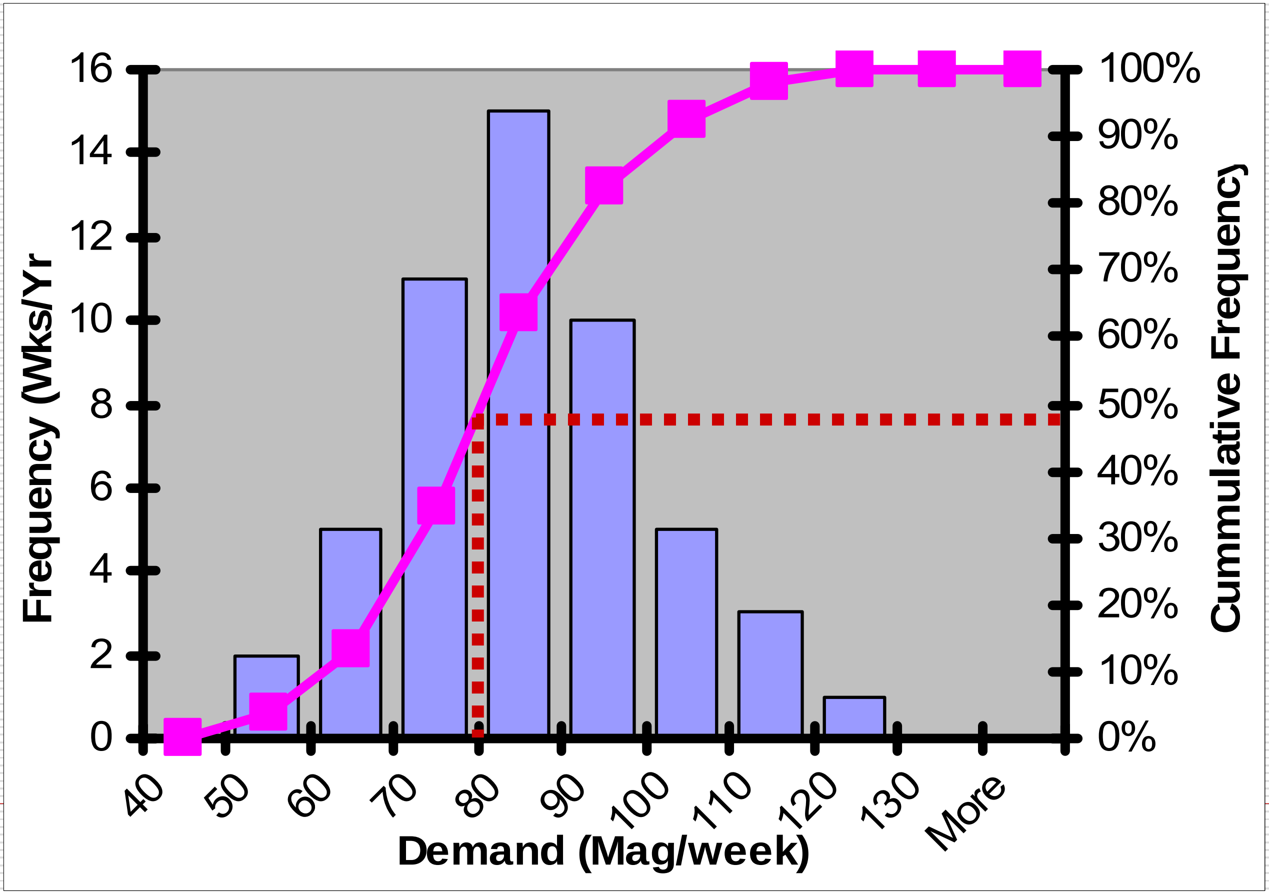
Optimal Order (Analytical)

- The optimal order is Q*
- At Q* - what is the probability of selling one more magazine ?
- The expected profit from ordering the (Q*+1)st magazine is:
- The optimum is where the total expected profit from ordering one more magazine is zero:
  -

$$\Pr(\text{Demand} \leq Q^*) = \frac{\text{REV-COST}}{\mathit{REV}}$$

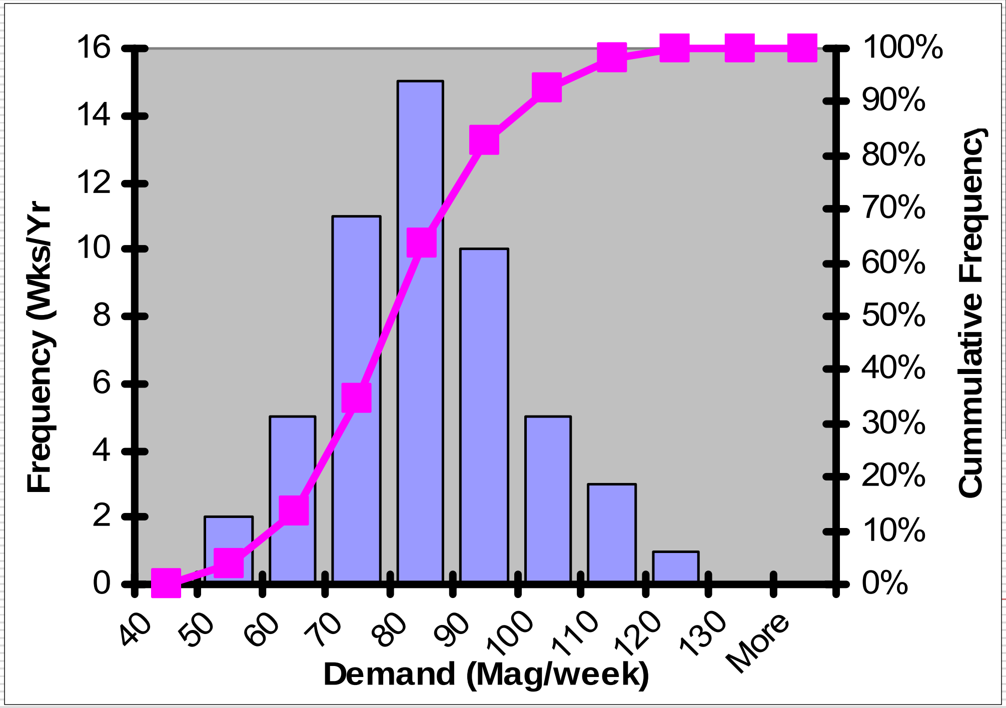
Optimal Order

The "critical ratio": $\Pr(\text{Demand} \leq Q^*) = \frac{\text{REV-COST}}{REV} = \frac{15-8}{15} = 0.47$



Salvage Value

Salvage value = \$4/Mag. Critcal Ratio $= \frac{REV - COST}{REV - SLV} = \frac{15-8}{15-4} = 0.64$



The Profit Function

- Revenue from sold items
- Revenue or costs associated with unsold items. These may include revenue from salvage or cost associated with disposal.
- Costs associated with not meeting customers’ demand. The lost sales cost can include lost of good will and actual penalties for low service.
- The cost of buying the merchandise in the first place.

The Profit Function

$$E[Sales] = Q \bullet \int_{X=Q}^{\infty} f(x)dx + \int_{x=0}^{Q} x \bullet f(x)dx$$

$$E[Unsold] = \int_{x=0}^{Q} (Q - x) \bullet f(x)dx = Q - E[Sales]$$

$$E[Lost\ Sales] = \int_{X=Q}^{\infty} (x - Q) \bullet f(x)dx = \mu - E[Sales]$$

$$E[Profit] = R \bullet E[Sales] + S \bullet E[Unsold] - L \bullet E[Lost\ \ Sales] - C \bullet Q$$

The Profit Function – Simple Case

$$E[Profit] = R \bullet E[Sales] - C \bullet Q$$

Optimal Order:

$$\frac{d}{dQ}E[Profit] = \left(1 - F(Q)\right) \bullet R - C = 0$$

$$\frac{d}{dQ}E[Sales] = 1 - F(Q)$$

$$F[Q^*] = \frac{R - C}{R}$$ and: $$Q^* = F^{-1}\left[\frac{R - C}{R}\right]$$

Level of Service

- Cycle Service – The probability that there will be a stock-out during a cycle
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- Fill Rate - The probability that a specific customer will encounter a stock-out
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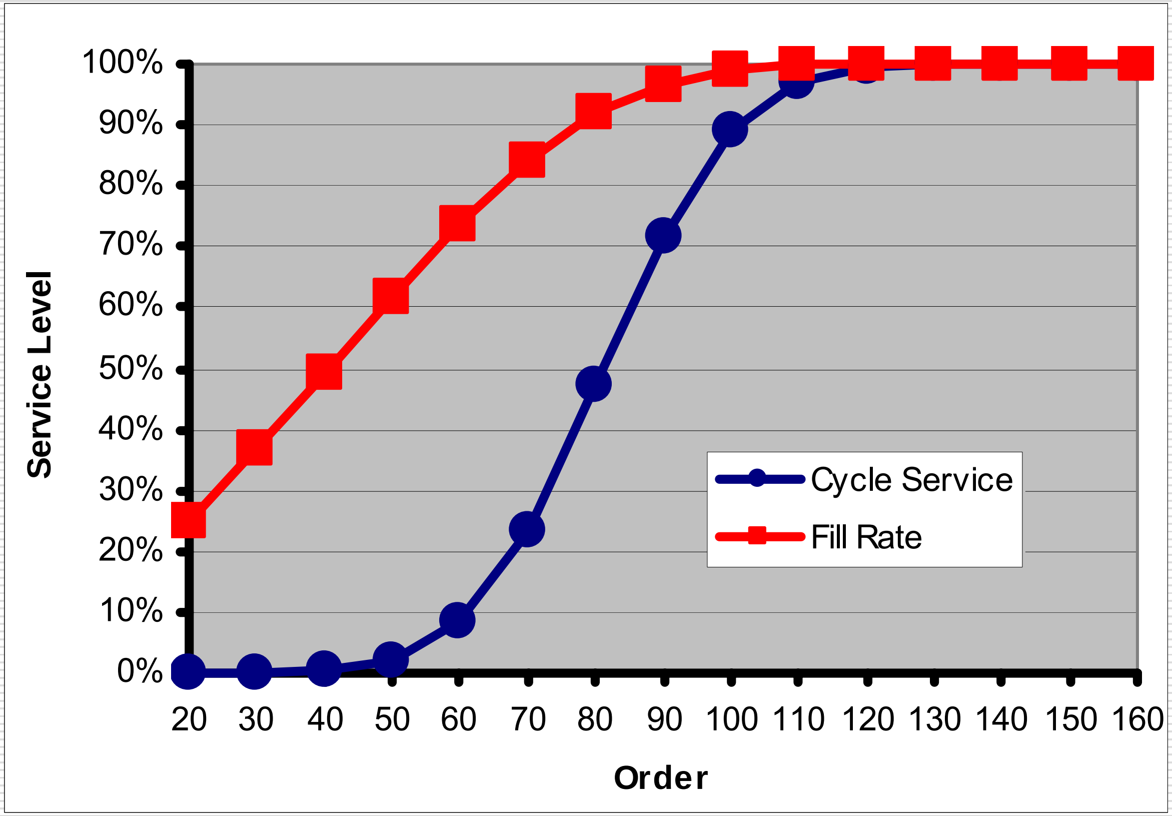
Level of Service

REV=$15
COST=$8



Normal Distribution of Demand

$$X \sim N(\mu, \sigma)$$

$$E[sales] = Q - \sigma \bullet \left(z \bullet \Phi(z) + \phi(z) \right)$$

$$z = \frac{Q - \mu}{\sigma}$$

$$E[Profit] = (R - C) \bullet Q - R \bullet \sigma \cdot \left[z \bullet \Phi(z) + \phi(z) \right]$$

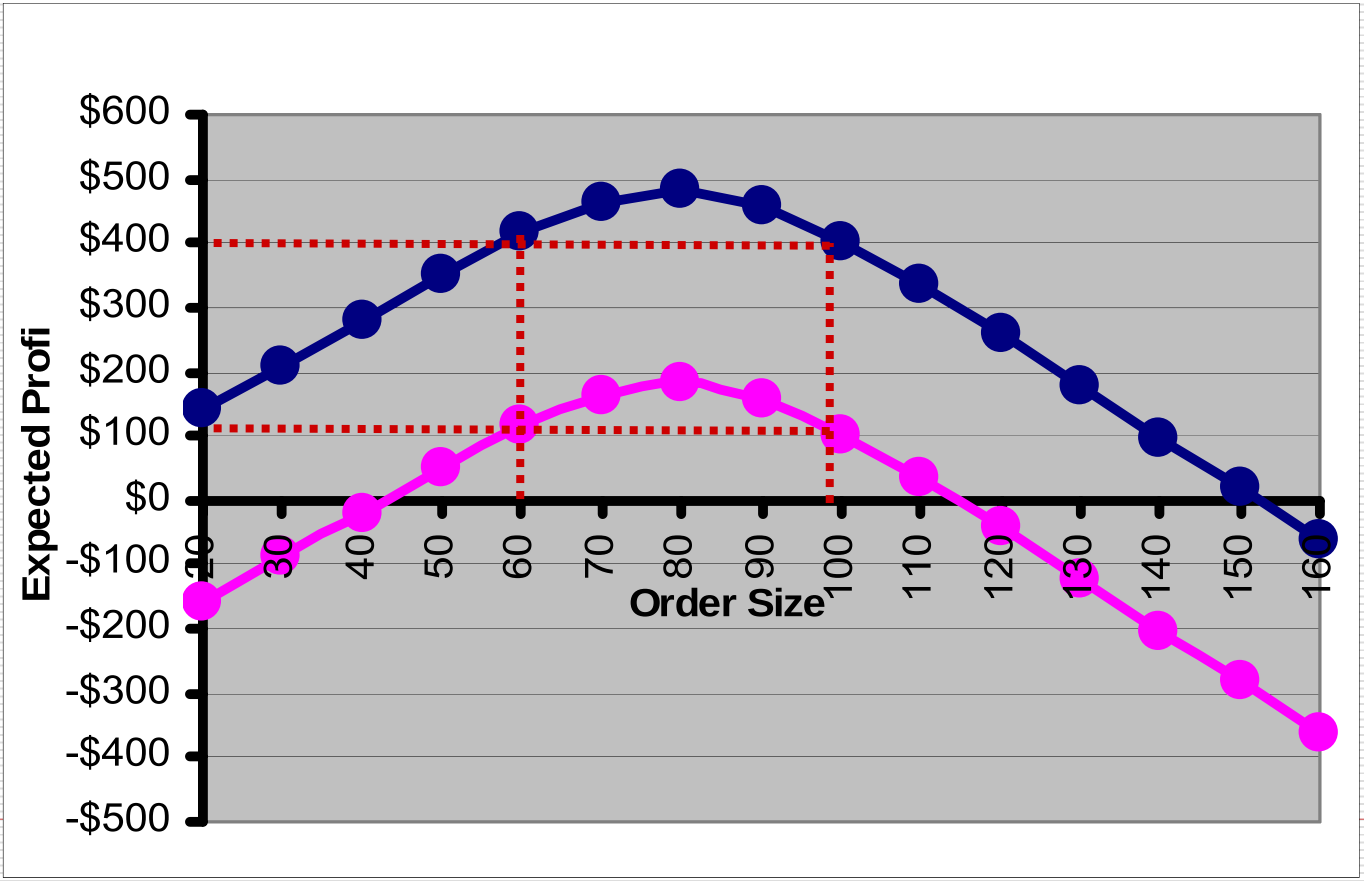
$$Q^* = NORMINV\left(\frac{R - C}{R} \right) =$$

$$= NORMINV\left(\frac{15 - 8}{15} \right) = 76 \text{ Mags}$$

REV=$15
COST=$8

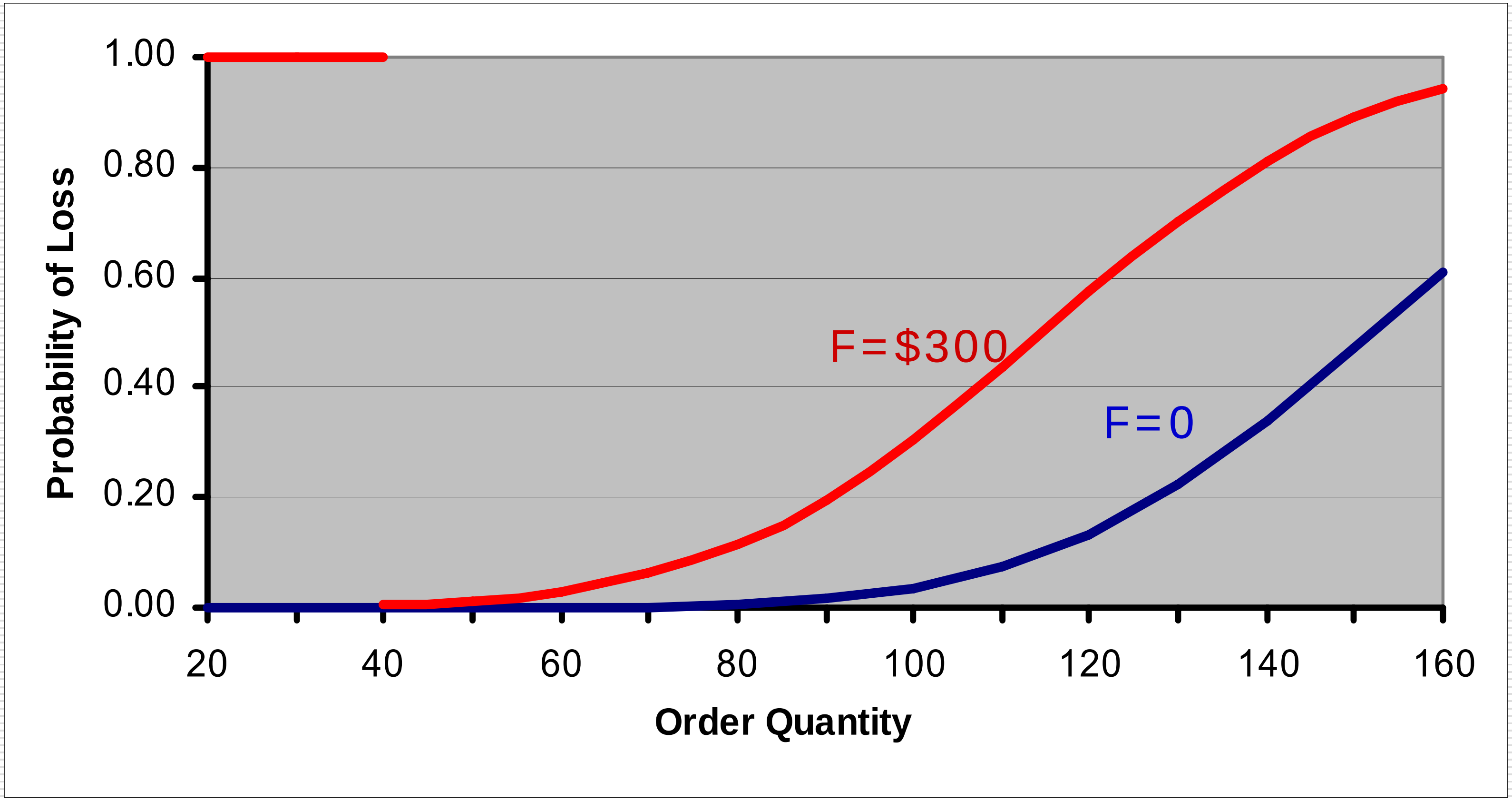
Incorporating Fixed Costs

With fixed costs of $300/order:



REV=$15
COST=$8

Risk of Loss



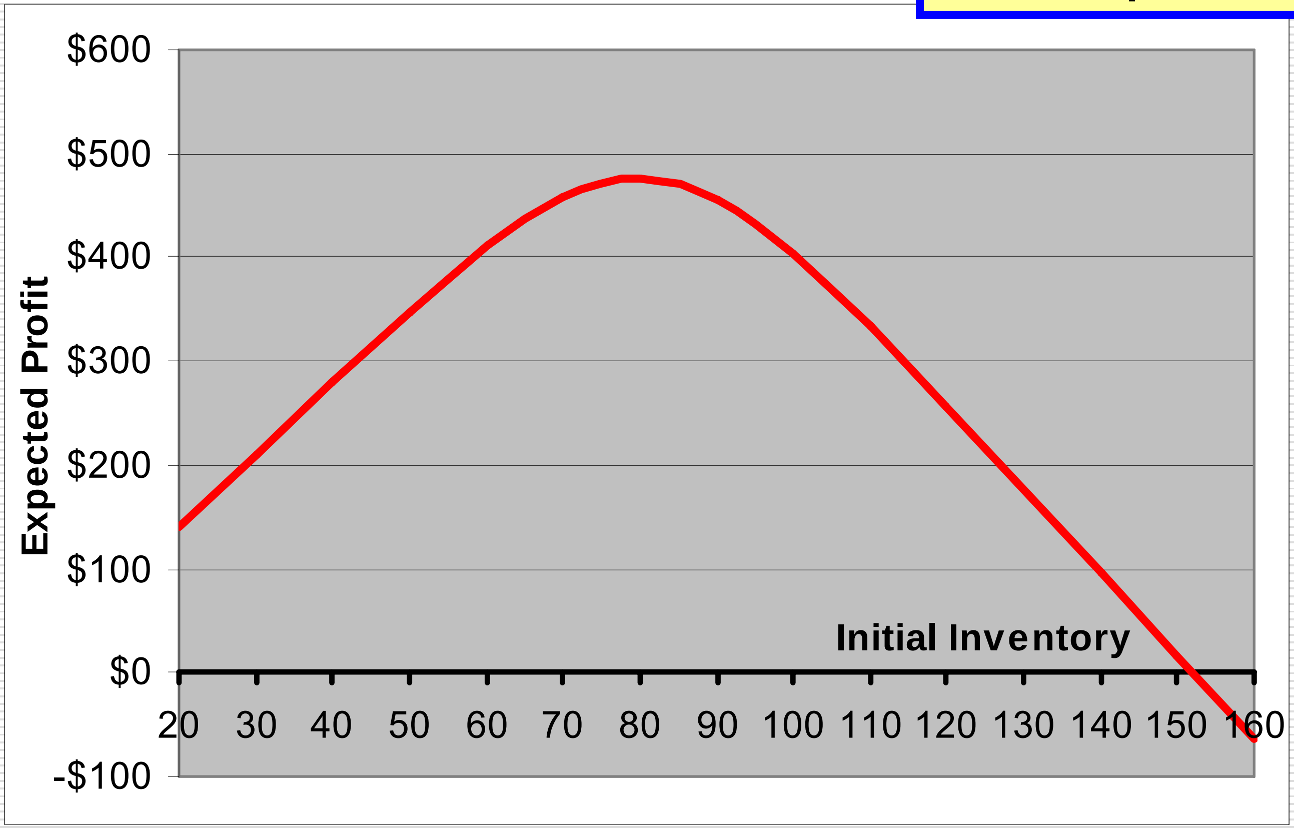
Ordering with Initial Inventory

- Given initial Inventory: Q_0, how to order?
- Cost of initial inventory
- With fixed costs, order only if the expected profits from ordering are more than the ordering costs

Ordering with Fixed Costs and Initial Inventory

REV=$15
COST=$8

Example: F = $150

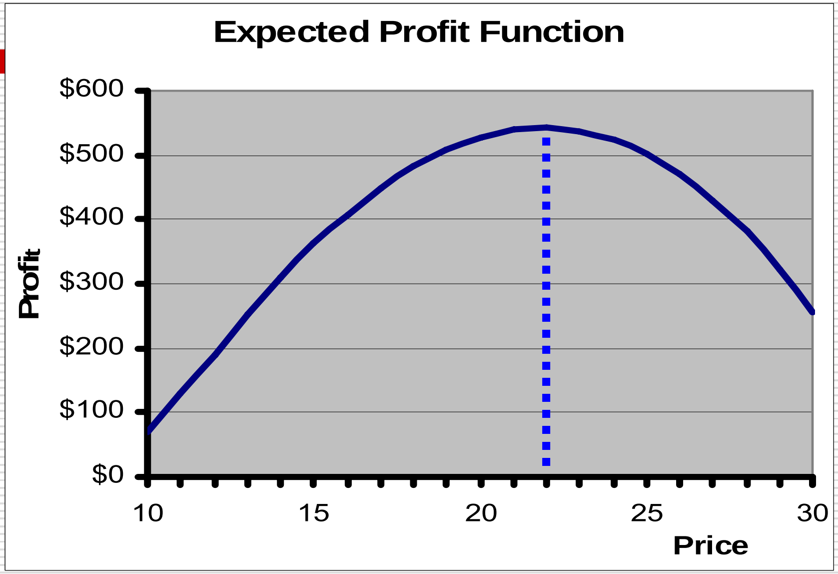


- If initial inventory is LE 46, order up to 80
- If initial inventory is GE 47, order nothing

Elastic Demand

- $\mu = D(P)$; $\sigma = f(\mu)$
- Procedure:

1. Set P
2. Calculate μ
3. Calculate σ
4. $Q^* = F^{-1}\left(\frac{P-C}{P}\right)$
5. Calculate optimal expected profits as a function of P.



Rev = $15
Cost = $8
μ(p) = 165-5*p
σ = μ/2

P* = $22
Q* = 65 Mag
μ(p) = 56 Mag
σ = 28
Exp. Profit = $543

Elastic Demand:
Numerical Optimization

Screenshots removed due to copyright restrictions.

Any Questions?

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