Inventory Management
Special Cases
Probabilistic Demand

Chris Caplice
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Special Inventory Cases

- Class A items – worth spending more time on
- Class C items – worth spending less time on
- Fashion or Perishable items – worth handling differently
- Indentured items – worth handling differently

What form of inventory policy?

- No hard and fast rules, but some rules of thumb

When & how to spend more time to manage A’ inventory

Type of Item,	**Continuous Review**	**Periodic Review**
A Items	(s, S)	(R, s, S)
B Items	(s,Q)	(R, S)
C Items		Manual ~ (R,S)

When & how to spend less time to manage or reduce ‘C’ inventory

Comparison of Approaches

	A Items	B Items	C Items
Type of records	Extensive, Transactional	Moderate	None – use a rule
Level of Management Reporting	Frequent (Monthly or more)	Infrequently - Aggregated	Only as Aggregate
Interaction with Demand	Manual input Ascertain predictability Manipulate (pricing etc.)	Modified Forecast (promotions etc.)	Simple Forecast at best
Interaction with Supply	Actively Manage	Manage by Exception	None
Initial Deployment	Minimize exposure (high v)	Steady State	Steady State
Frequency of Policy Review	Very Frequent (monthly or more)	Moderate – Annually or Event Based	Very Infrequent
Importance of Parameter Precision	Very High – accuracy worthwhile	Moderate – rounding & approximation is ok	Very Low
Shortage Strategy	Actively manage (confront)	Set service levels & manage by exception	Set & forget service levels
Demand Distribution	Consider alternatives to Normal as situation fits	Normal	N/A

Managing Class A Inventory

- When does it make sense to spend more time?
  - Tradeoff between complexity and 'other' costs
  - Is the savings worth the extra effort?
- Adding precision
  - Finding 'optimal' parameters
  - Using more complex policies

Dictates whether item is Class A or not

$$TC = vD + A\left(\frac{D}{Q}\right) + vr\left(\frac{Q}{2} + k\sigma_L\right) + B_1\left(\frac{D}{Q}\right)P[SO]$$

$$TC = vD + A\left(\frac{D}{Q}\right) + vr\left(\frac{Q}{2} + k\sigma_L\right) + B_2 v\left(\frac{D}{Q}\right)\sigma_L G_u(k)$$

Managing Class A Inventory

- Two Types of Class A items:
  - Fast moving but cheap (big D little v → Q>1)
  - Slow moving but expensive (big v little D → Q=1)
- Impacts the probability distribution used
  - Fast Movers - Normal Distribution
    - Good enough for B items
    - OK for A items if $x_L \geq 10$ or $x_{L+R} \geq 10$
  - Slow Movers – Poisson Distribution (& others)
    - More complicated to handle
    - Ok for A items if $x_L < 10$ or $x_{L+R} < 10$

Fast Moving A Items

- Finding Better (s,Q) Parameters
  - Solve for k* and Q* simultaneously (why?)
  - Assume ~Normal & B_1 (Cost per Stockout Occasion)

$$TRC = A\left(\frac{D}{Q}\right) + vr\left(\frac{Q}{2} + k\sigma_L\right) + B_1\left(\frac{D}{Q}\right)p_{x\geq}(k)$$

$$\frac{\partial TRC}{\partial Q} = 0 \qquad \frac{\partial TRC}{\partial k} = 0$$

$$\frac{\partial TRC}{\partial Q} = -A\left(\frac{D}{Q^2}\right) + \frac{vr}{2} - B_1\left(\frac{D}{Q^2}\right)p_{k\geq}(k) = 0$$

$$\frac{\partial TRC}{\partial k} = 0 + vr\sigma_L - B_1\left(\frac{D}{Q}\right)f_x(k) = 0$$

Note that:

$$\frac{\partial p_{k\geq}(k)}{\partial k} = -f_x(k)$$

$$f_x(k) = \frac{e^{\left(\frac{-x^2}{2}\right)}}{\sqrt{2\pi}}$$

Fast Moving A Items

- Finding Better (s,Q) Parameters
  - End up with two equations
  - How do we solve for (s*, Q*)?
  - Will the new optimal Q* be > or < than the EOQ?
  - Will the optimal k* be > or < than the old k?
  - What is the impact on safety stock? Cycle stock?

$$Q^* = EOQ\sqrt{1+\frac{B_1\, p_{x\geq}(k)}{A}}$$

$$k^* = \sqrt{2\ln\left(\frac{DB_1}{\sqrt{2\pi}Qvr\sigma_L}\right)}$$

Fast Moving A Items

- Establish an (s,S) policy
  - If IP<s then order up to S items (=S-IP)
  - More complicated due to ‘undershoots’
  - See SPP Section 8.5
- Establish an (R,s,S) policy
  - Every R time units, if IP<s then order up to S items (=S-IP)
  - Even more complicated – but can be programmed
  - See SPP Section 8.6

Slow Moving A Items

- Normal distribution may not make sense – why?
- Poisson distribution
  - Probability of x events occurring w/in a time period
  - Mean = Variance = λ

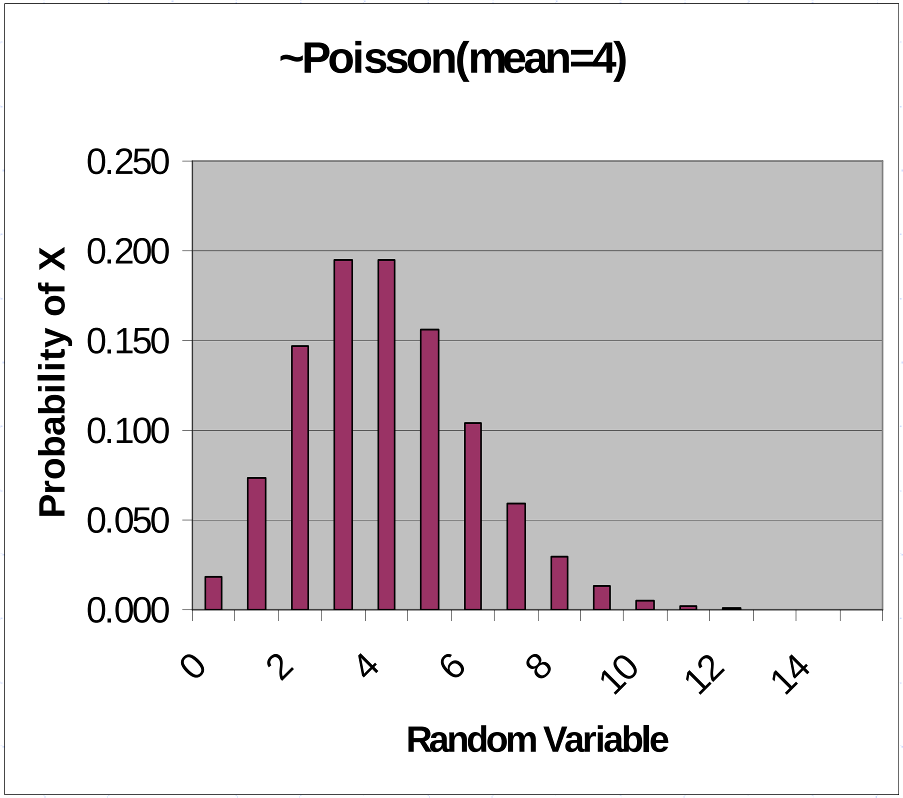
$$p_k(x_0) = \frac{e^{-\lambda}\lambda^{x_0}}{x_0!} \quad for\ x_0 = 0,1,2,\ldots$$

$$p_{k\le}(x_0) = \sum_{k=0}^{x_0} \frac{e^{-\lambda}\lambda^{k}}{k!}$$

In Excel:

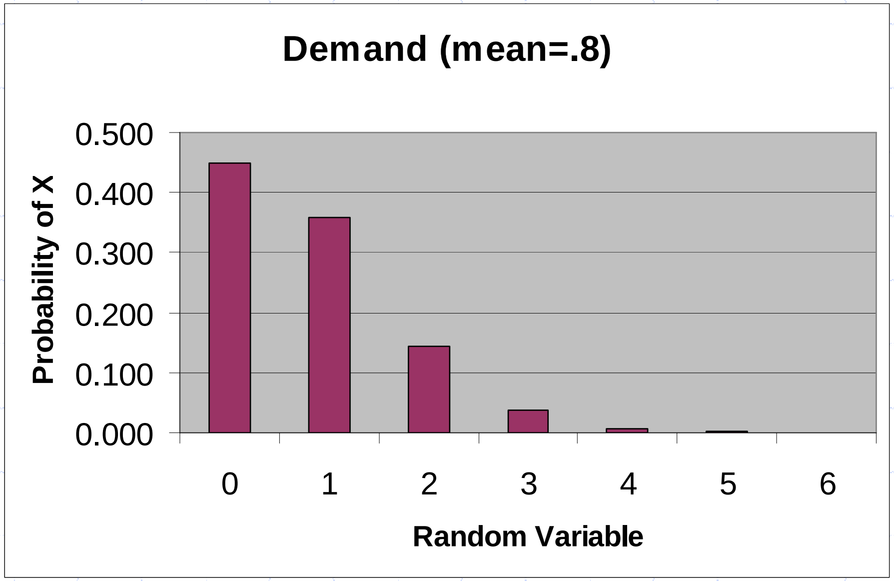
$p_k(x_0)$ =POISSON(x_0,λ,0)

$p_{k\le}(x_0)$ =POISSON(x_0,λ,1)



Example

- Suppose demand ~P(λ=0.8) items per week. We want to set an (R,S) policy for an IFR=.90 where R=1 wk
- We know that
  - IFR = 1-(E[US]/E[Demand in Period])=1-(E[US]/λ)
  - E[US] = (1-IFR)λ = (1-.90)(.8) = 0.08 units
- How do I find an S so that EUS≤0.08?



x	P[x]	F[x]	L[x]
0	44.9%	44.9%	
1	35.9%	80.9%	
2	14.4%	95.3%	
3	3.8%	99.1%	
4	0.8%	99.9%	
5	0.1%	100.0%	
6	0.0%	0.0%	

Loss Function for Discrete Function

- We find the loss function, $L(X_i)$, for each value of X given the cumulative probability $F(X_i)$.
- Start with first value
  - $L(X_1) = \text{mean} - X_1$
  - $L(X_2) = L(X_1) - (X_2 - X_1)(1-F(X_1))$
  - $L(X_3) = L(X_2) - (X_3 - X_2)(1-F(X_2))$
  - . . . .
  - $L(X_i) = L(X_{i-1}) - (X_i - X_{i-1})(1-F(X_{i-1}))$

x	P[x]	F[x]	L[x]
0	**44.9%**	**44.9%**	**0.80**
1	**35.9%**	**80.9%**	**0.25**
2	**14.4%**	**95.3%**	**0.06**
3	**3.8%**	**99.1%**	**0.01**
4	**0.8%**	**99.9%**	**0.009**
5	**0.1%**	**100.0%**	**0.0088**
6	**0.0%**	**0.0%**	**0.00878**

- So, set S=2 since L(2)=0.06
  - Policy is order up to 2 units every week
- More methods in SPP Section 8.3

Managing "C" Inventory

- Establish simple reorder rules
  - Periodic rather than continuous
  - Set for all C items collectively (if possible)
  - Look to reduce the number of order cycles
- Identify & Dispose of Dead Inventory
  - Which items to dispose?
    - Look at DOS (days of supply) for each item = IOH/D
    - Consider getting rid of items that have DOS > x years
  - How much to get rid of?
    - Decision rule: IOH – EOQ – D(v-salvage)/(vr)
  - What do you do with it?
  - When can you not never get rid of C or D or FF items?

Managing “C” Inventory

- To Stock or Not to Stock?
  - Buy-to-order versus buy-to-stock decision
  - Factors
    - System cost for stocking an item
    - Variable cost differential for buy-to-order vs buy-to-stock
    - Cost of temporary backorder
  - Decision Rule in SPP Section 9.5
    - Essentially trade off between cost to order and frequency of demand

Questions?
Comments?
Suggestions?