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6.976
High Speed Communication Circuits and Systems
Lecture 12
Noise in Voltage Controlled Oscillators

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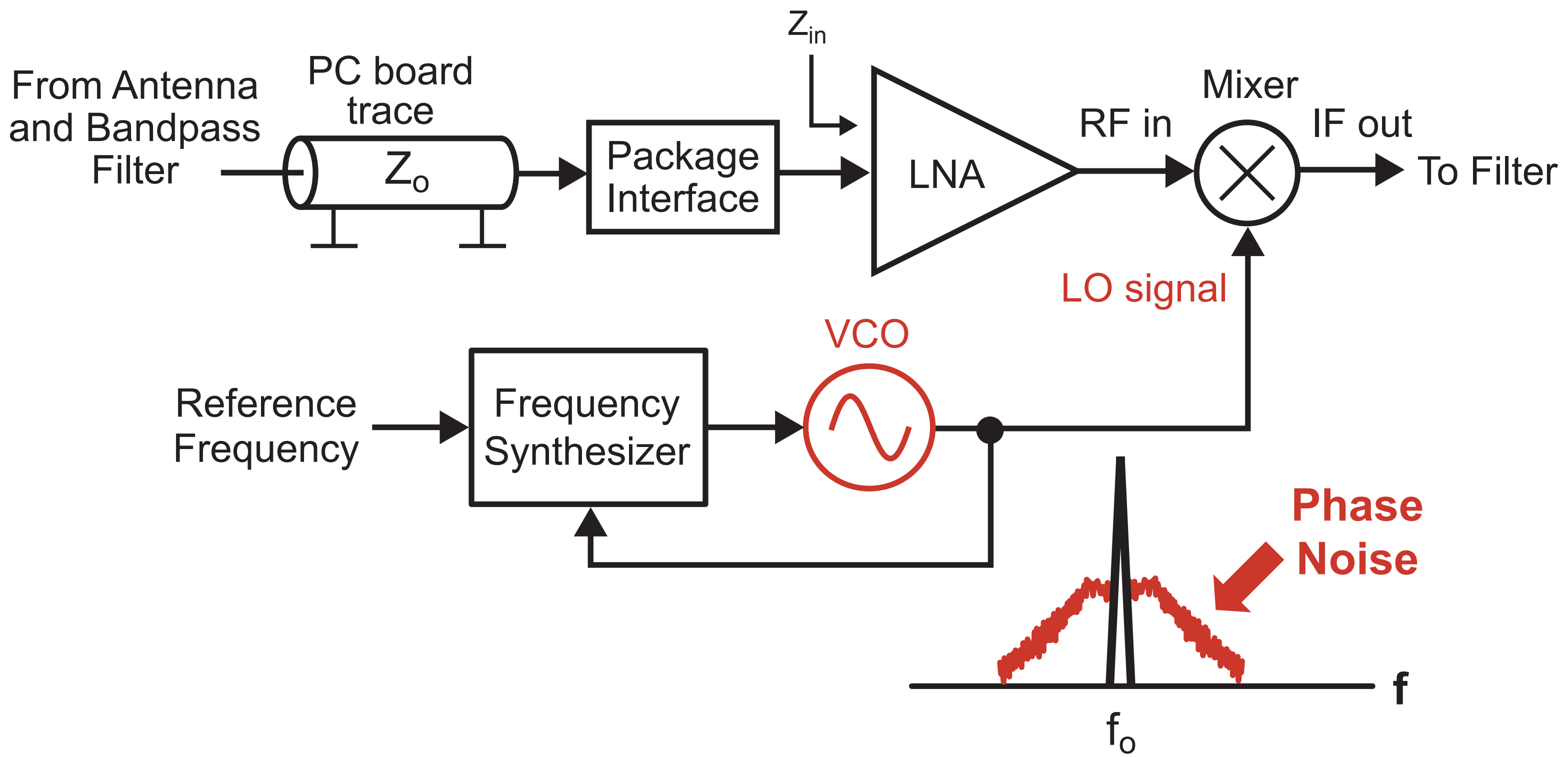
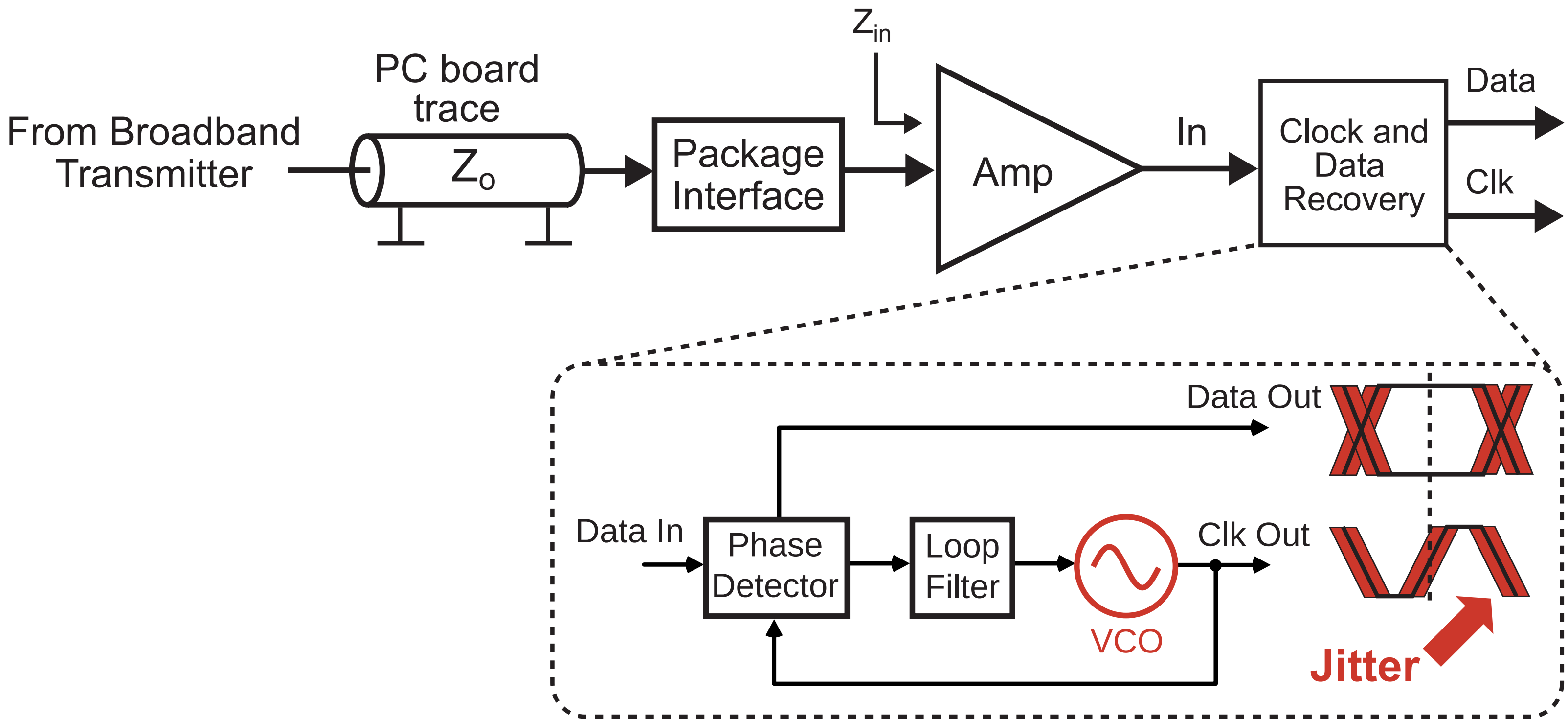
VCO Noise in Wireless Systems



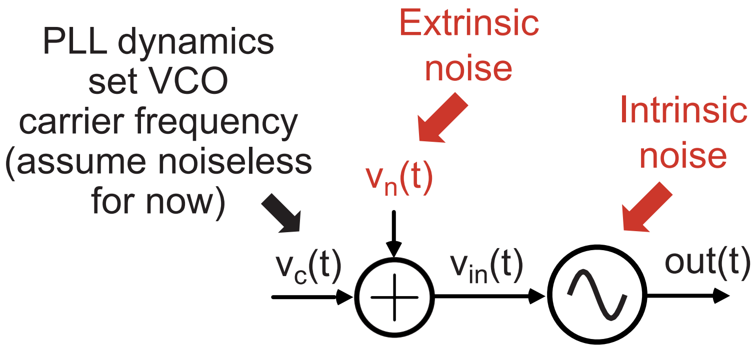
- VCO noise has a negative impact on system performance
  - Receiver – lower sensitivity, poorer blocking performance
  - Transmitter – increased spectral emissions (output spectrum must meet a mask requirement)
- Noise is characterized in frequency domain

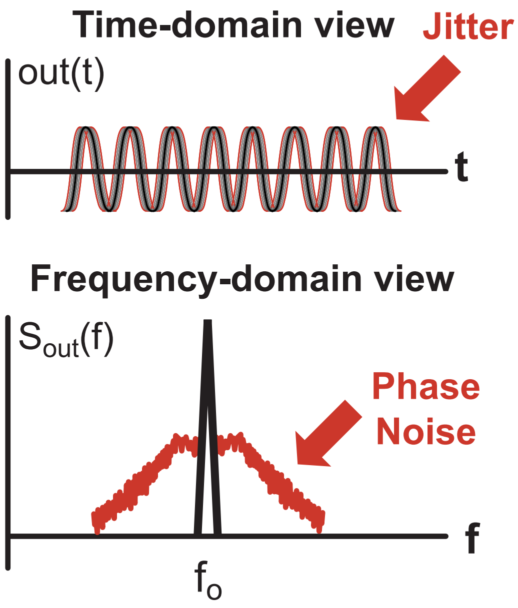
VCO Noise in High Speed Data Links



- VCO noise also has a negative impact on data links
  - Receiver – increases bit error rate (BER)
  - Transmitter – increases jitter on data stream (transmitter must have jitter below a specified level)
- Noise is characterized in the time domain

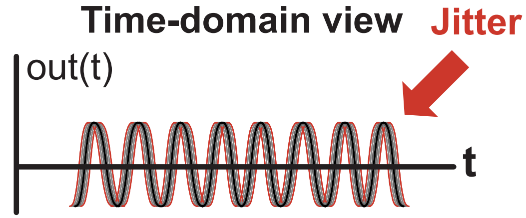
Noise Sources Impacting VCO

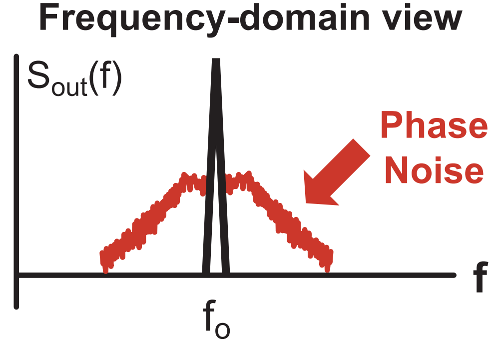




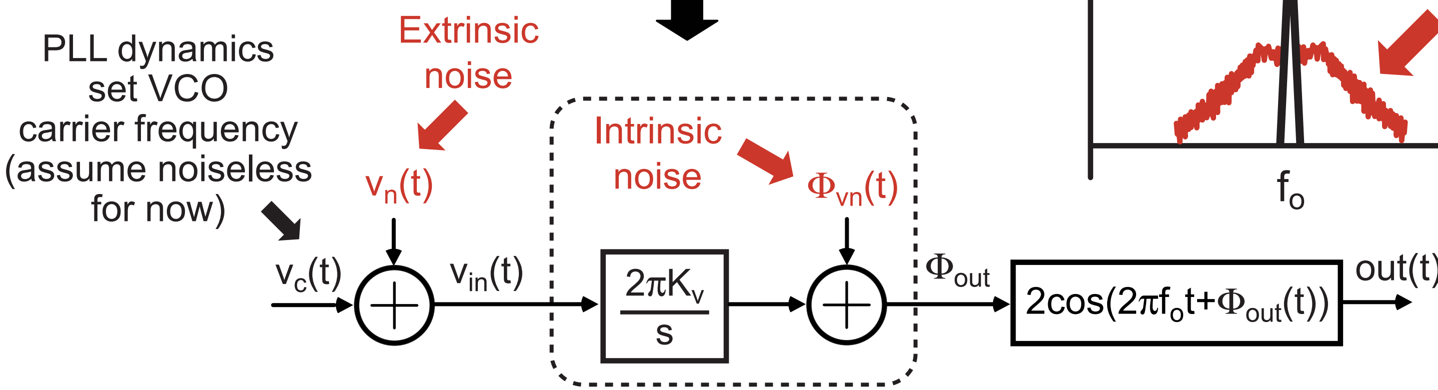
- **Extrinsic noise**
  - **Noise from other circuits (including PLL)**
- **Intrinsic noise**
  - **Noise due to the VCO circuitry**

VCO Model for Noise Analysis





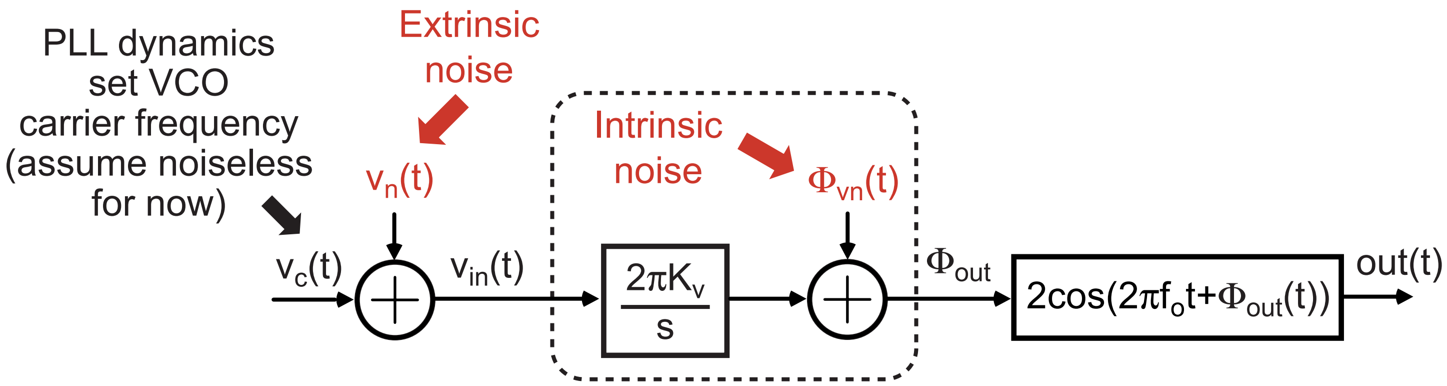
Note: K_v units are Hz/V



- **We will focus on phase noise (and its associated jitter)**
  - **Model as phase signal in output sine waveform**

$$out(t) = 2\cos(2\pi f_o t + \underline{\Phi_{out}(t)})$$

Simplified Relationship Between Φ_{out} and Output



$$out(t) = 2\cos(2\pi f_o t + \Phi_{out}(t))$$

- **Using a familiar trigonometric identity**

$$out(t) = 2\cos(2\pi f_o t)\cos(\Phi_{out}(t)) - 2\sin(2\pi f_o t)\sin(\Phi_{out}(t))$$

- **Given that the phase noise is small**

$$\cos(\Phi_{out}(t)) \approx 1, \quad \sin(\Phi_{out}(t)) \approx \Phi_{out}(t)$$

$$\Rightarrow \quad out(t) = 2\cos(2\pi f_o t) - 2\sin(2\pi f_o t)\Phi_{out}(t)$$

Calculation of Output Spectral Density

$$out(t) = 2\cos(2\pi f_o t) - 2\sin(2\pi f_o t)\Phi_{out}(t)$$

- **Calculate autocorrelation**

$$R\{out(t)\} = R\{2\cos(2\pi f_o t)\} + R\{2\sin(2\pi f_o t)\} \cdot R\{\Phi_{out}(t)\}$$

- **Take Fourier transform to get spectrum**

$$S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}$$

  - **Note that * symbol corresponds to convolution**
- **In general, phase spectral density can be placed into one of two categories**
  - **Phase noise – $\Phi_{out}(t)$ is non-periodic**
  - **Spurious noise - $\Phi_{out}(t)$ is periodic**

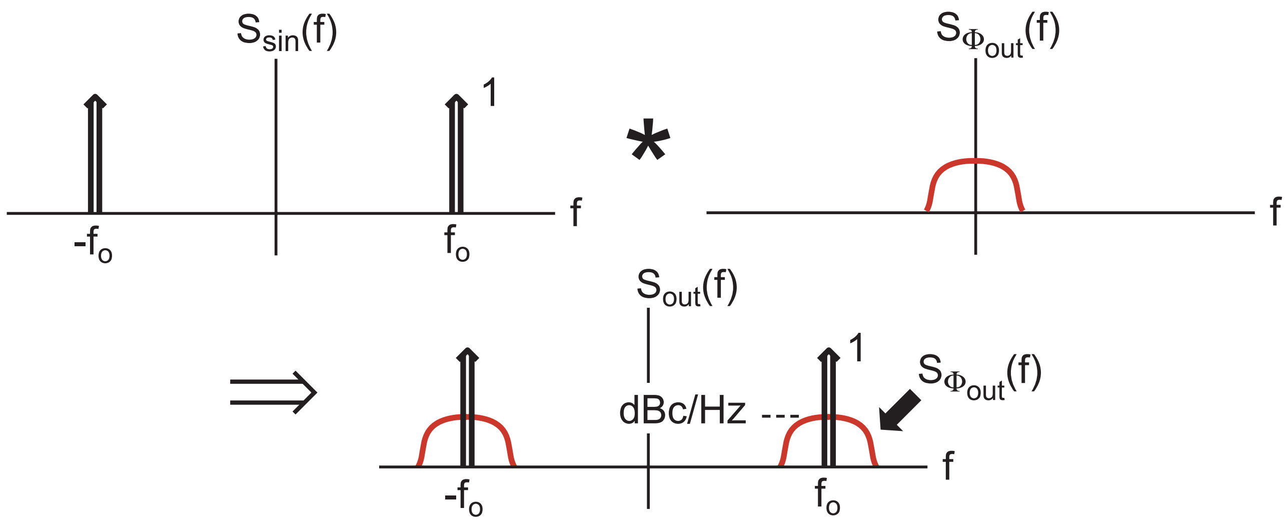
Output Spectrum with Phase Noise

- Suppose input noise to VCO ($v_n(t)$) is bandlimited, non-periodic noise with spectrum $S_{vn}(f)$
  - In practice, derive phase spectrum as

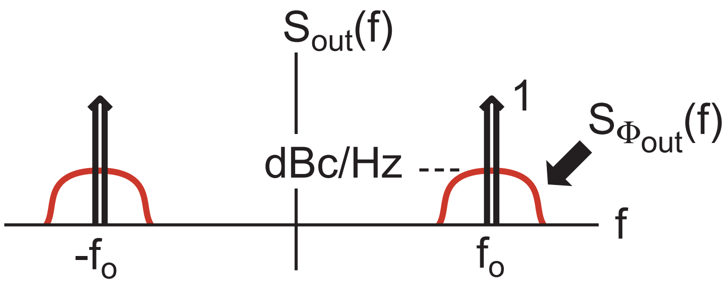
$$S_{\Phi_{out}}(f) = \left(\frac{K_v}{f}\right)^2 S_{v_n}(f)$$

- Resulting output spectrum

$$S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}$$



Measurement of Phase Noise in dBc/Hz



- **Definition of L(f)**

$$L(f) = 10\log\left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}}\right)$$

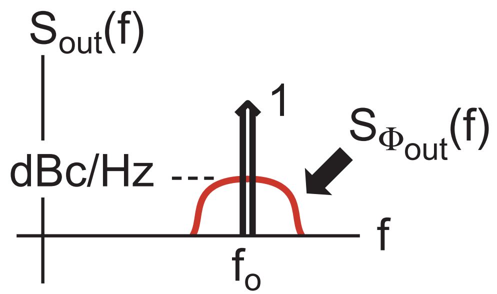
  - **Units are dBc/Hz**

- **For this case**

$$L(f) = 10\log\left(\frac{2S_{\Phi_{out}}(f)}{2}\right) = 10\log(S_{\Phi_{out}}(f))$$

  - **Valid when $\Phi_{out}(t)$ is small in deviation (i.e., when carrier is not modulated, as currently assumed)**

Single-Sided Version



- **Definition of L(f) remains the same**

$$L(f) = 10\log\left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}}\right)$$

  - **Units are dBc/Hz**

- **For this case**

$$L(f) = 10\log\left(\frac{S_{\Phi_{out}}(f)}{1}\right) = 10\log(S_{\Phi_{out}}(f))$$

  - **So, we can work with either one-sided or two-sided spectral densities since L(f) is set by *ratio* of noise density to carrier power**

Output Spectrum with Spurious Noise

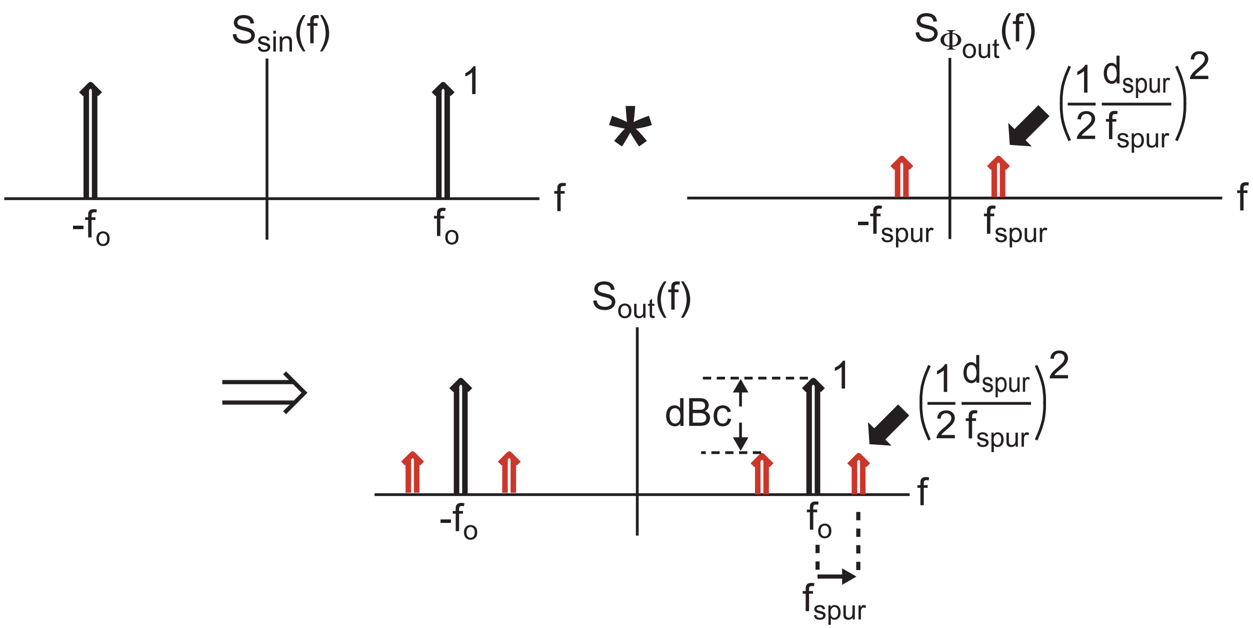
- **Suppose input noise to VCO is**

$$v_n(t) = \frac{d_{spur}}{K_v}\cos(2\pi f_{spur} t)$$

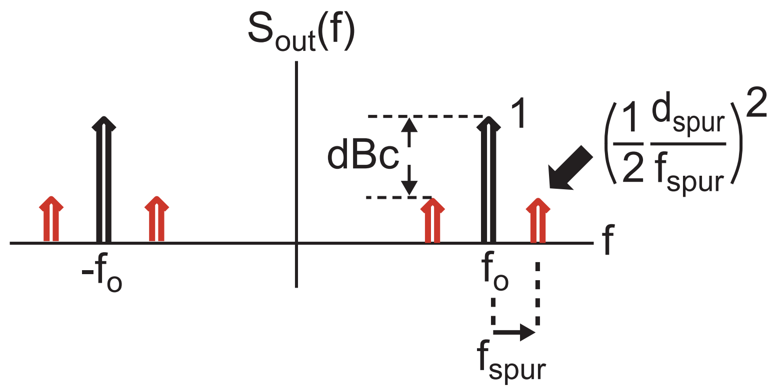
$$\Rightarrow \quad \Phi_{out}(t) = 2\pi K_v \int v_n(t)dt = \frac{d_{spur}}{f_{spur}}\sin(2\pi f_{spur} t)$$

- **Resulting output spectrum**

$$S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}$$



Measurement of Spurious Noise in dBc



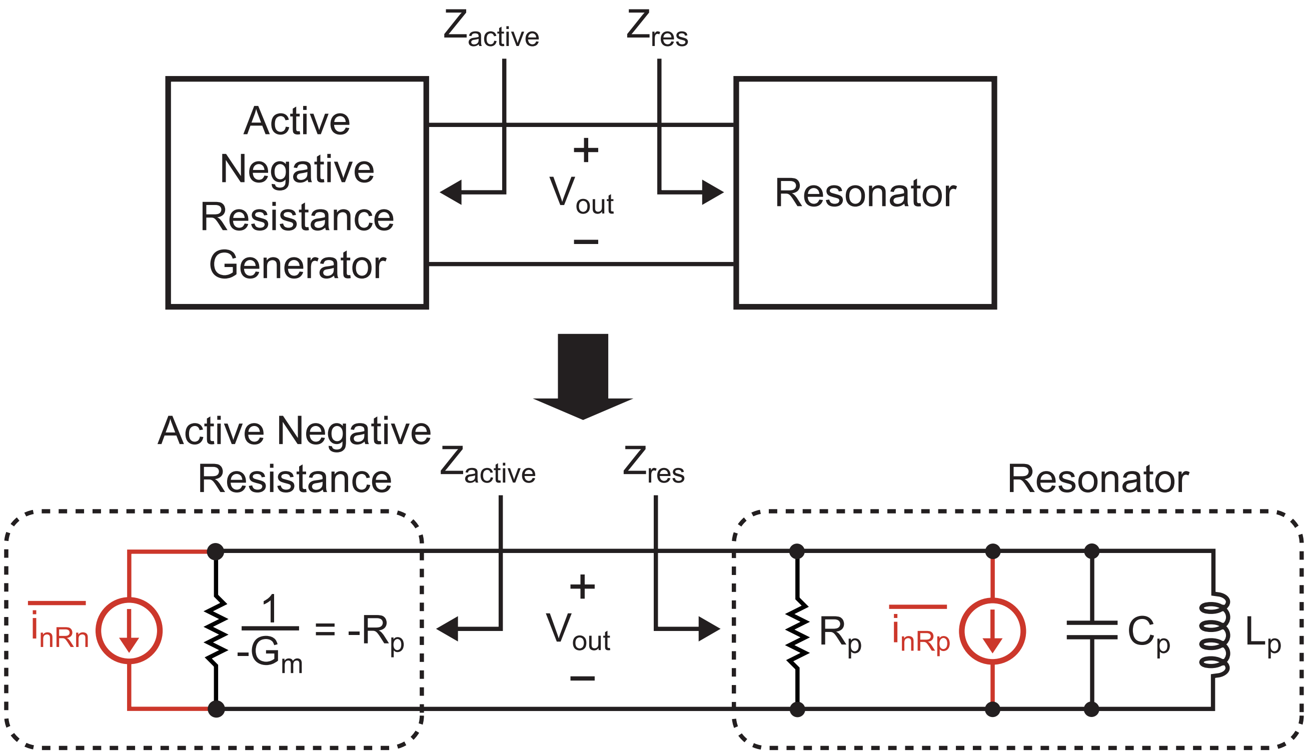
- **Definition of dBc**

$$10 \log \left(\frac{\text{Power of tone}}{\text{Power of carrier}} \right)$$

  - **We are assuming double sided spectra, so integrate over positive and negative frequencies to get power**
    - Either single or double-sided spectra can be used in practice
- **For this case**

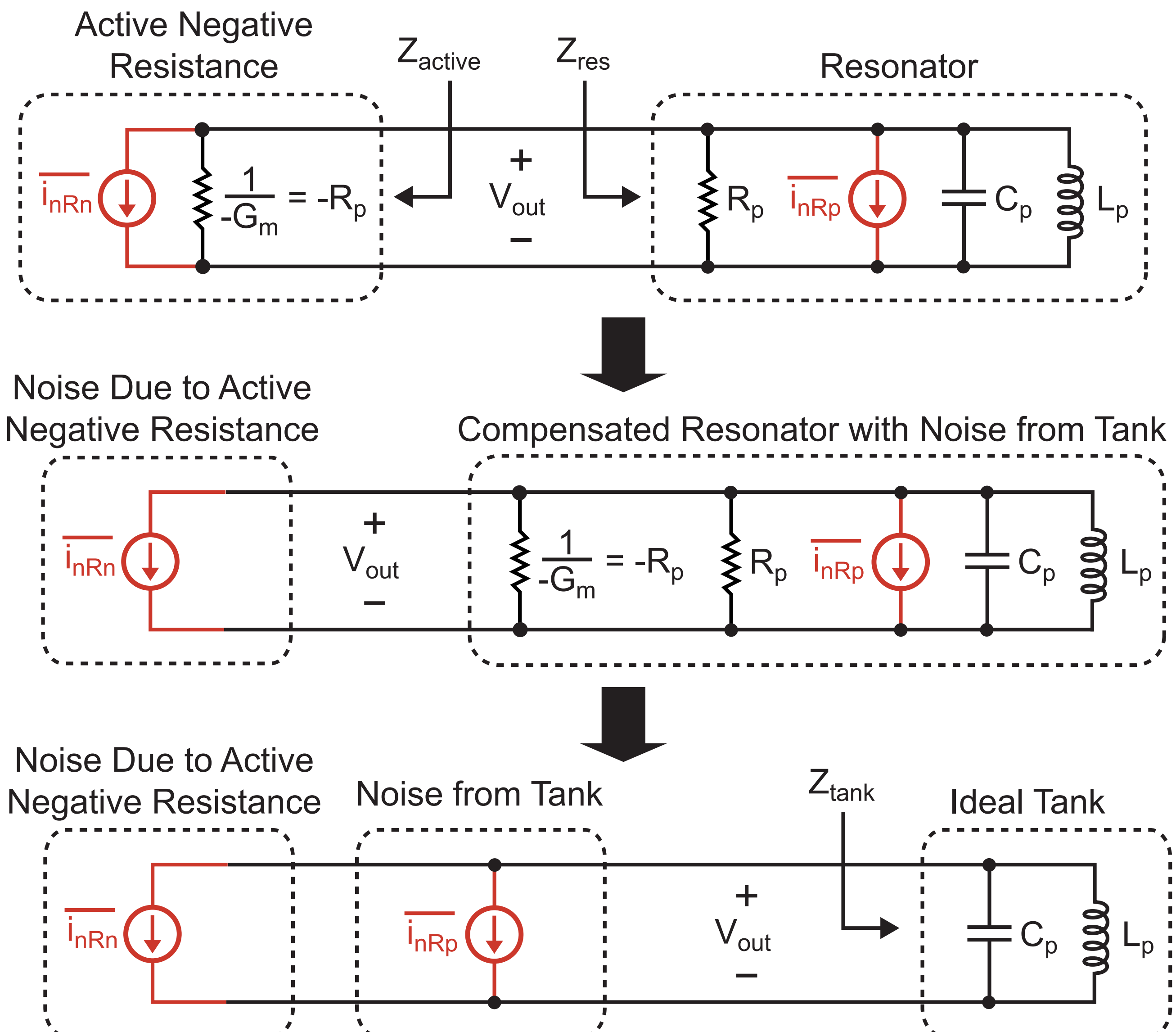
$$10 \log \left(\frac{2\left(\frac{d_{spur}}{2 f_{spur}}\right)^2}{2} \right) = 20 \log \left(\frac{d_{spur}}{2 f_{spur}} \right) \ dBc$$

Calculation of Intrinsic Phase Noise in Oscillators

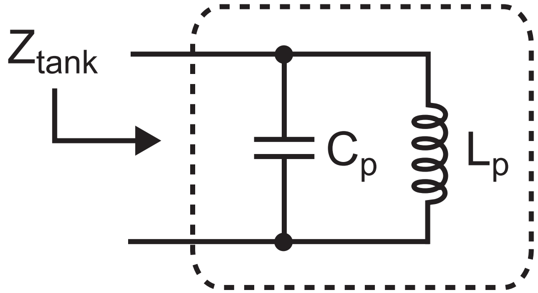


- **Noise sources in oscillators are put in two categories**
  - **Noise due to tank loss**
  - **Noise due to active negative resistance**
- **We want to determine how these noise sources influence the phase noise of the oscillator**

Equivalent Model for Noise Calculations



Calculate Impedance Across Ideal LC Tank Circuit



$$Z_{tank}(w) = \frac{1}{jwC_p} || jwL_p = \frac{jwL_p}{1 - w^2 L_p C_p}$$

- **Calculate input impedance about resonance**

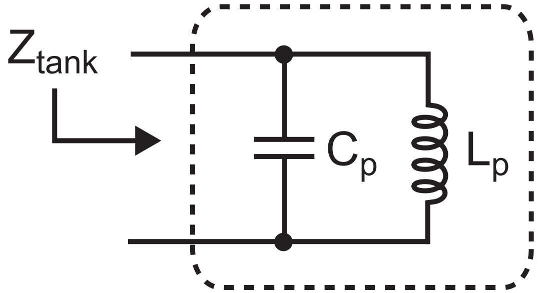
$$\text{Consider } w = w_o + \Delta w, \text{ where } w_o = \frac{1}{\sqrt{L_p C_p}}$$

$$Z_{tank}(\Delta w) = \frac{j(w_o + \Delta w)L_p}{1 - (w_o + \Delta w)^2 L_p C_p}$$

$$= \frac{j(w_o + \Delta w)L_p}{\underbrace{1 - w_o^2 L_p C_p}_{= 0} - 2\Delta w(w_o L_p C_p) - \underbrace{\Delta w^2 L_p C_p}_{\text{negligible}}} \approx \frac{j(w_o + \Delta w)L_p}{-2\Delta w(w_o L_p C_p)}$$

$$\Rightarrow Z_{tank}(\Delta w) \approx \frac{jw_o L_p}{-2\Delta w(w_o L_p C_p)} = \boxed{-\frac{j}{2}\frac{1}{w_o C_p}\left(\frac{w_o}{\Delta w}\right)}$$

A Convenient Parameterization of LC Tank Impedance



$$Z_{tank}(\Delta w) \approx -\frac{j}{2}\frac{1}{w_o C_p}\left(\frac{w_o}{\Delta w}\right)$$

- Actual tank has loss that is modeled with R_p
  - Define Q according to actual tank

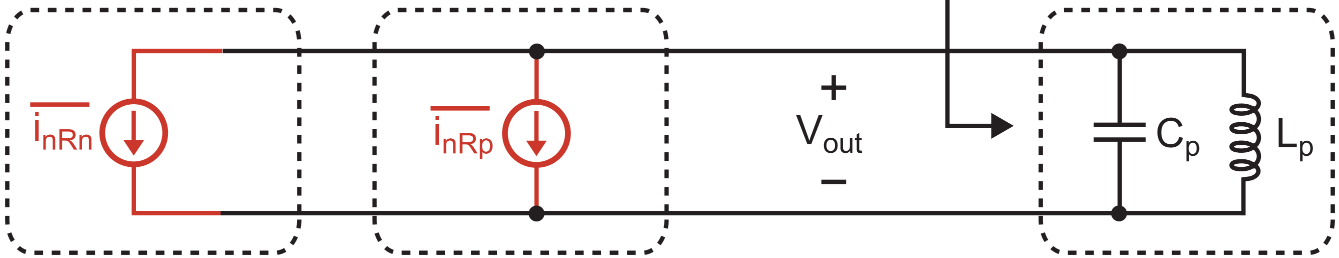
$$Q = R_p w_o C_p \quad \Rightarrow \quad \frac{1}{w_o C_p} = \frac{R_p}{Q}$$

- Parameterize ideal tank impedance in terms of Q of actual tank

$$Z_{tank}(\Delta w) \approx -\frac{j}{2}\frac{R_p}{Q}\left(\frac{w_o}{\Delta w}\right)$$

$$\Rightarrow \quad |Z_{tank}(\Delta f)|^2 \approx \boxed{\left(\frac{R_p}{2Q}\frac{f_o}{\Delta f}\right)^2}$$

Overall Noise Output Spectral Density



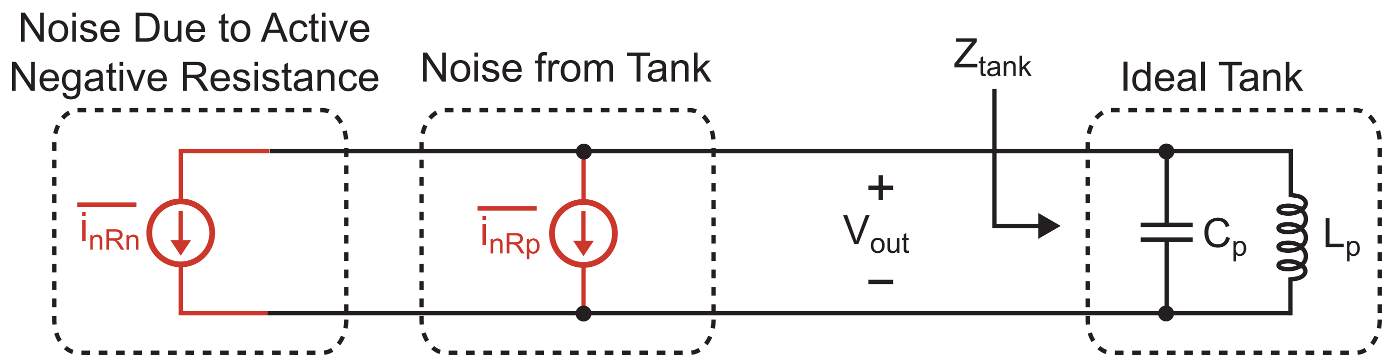
- **Assume noise from active negative resistance element and tank are uncorrelated**

$$\frac{\overline{v_{out}^2}}{\Delta f} = \left(\frac{\overline{i_{nRp}^2}}{\Delta f} + \frac{\overline{i_{nRn}^2}}{\Delta f}\right) |Z_{tank}(\Delta f)|^2$$

$$= \frac{\overline{i_{nRp}^2}}{\Delta f} \left(1 + \frac{\overline{i_{nRn}^2}}{\Delta f} \Big/ \frac{\overline{i_{nRp}^2}}{\Delta f}\right) |Z_{tank}(\Delta f)|^2$$

  - **Note that the above expression represents total noise that impacts both amplitude and phase of oscillator output**

Parameterize Noise Output Spectral Density



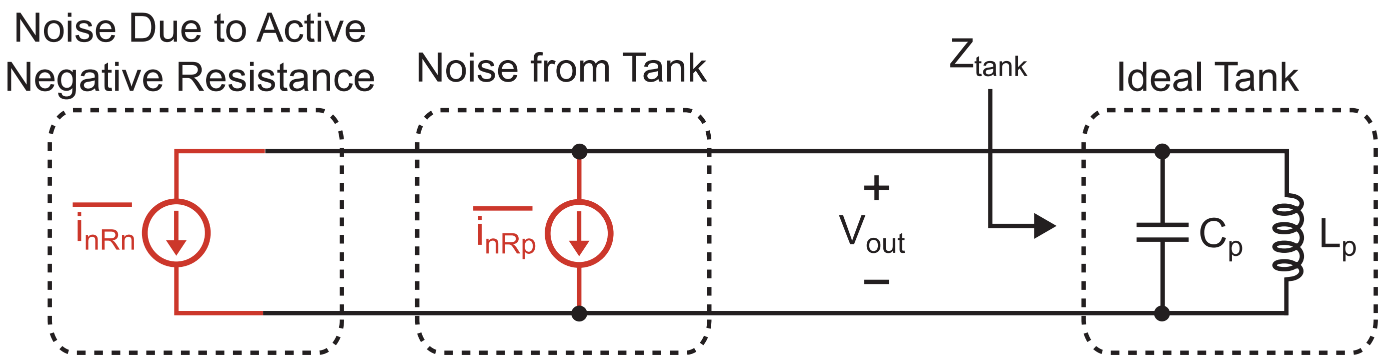
- **From previous slide**

$$\frac{\overline{v_{out}^2}}{\Delta f} = \frac{\overline{i_{nRp}^2}}{\Delta f}\underbrace{\left(1 + \frac{\overline{i_{nRn}^2}}{\Delta f} \Big/ \frac{\overline{i_{nRp}^2}}{\Delta f}\right)}_{F(\Delta f)} |Z_{tank}(\Delta f)|^2$$

- **F(Δf) is defined as**

$$F(\Delta f) = \frac{\text{total noise in tank at frequency } \Delta f}{\text{noise in tank due to tank loss at frequency } \Delta f}$$

Fill in Expressions



- **Noise from tank is due to resistor R_p**

$$\frac{\overline{i^2_{nRp}}}{\Delta f} = 4kT\frac{1}{R_p} \quad \text{(single-sided spectrum)}$$

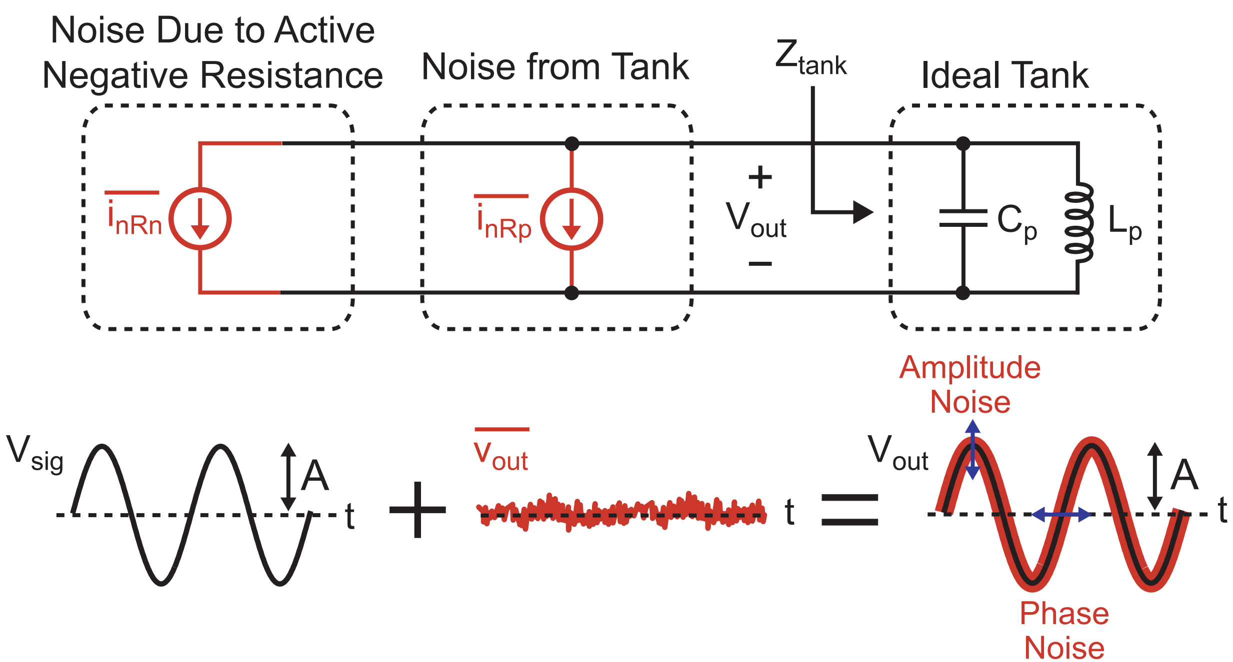
- **$Z_{tank}(\Delta f)$ found previously**

$$|Z_{tank}(\Delta f)|^2 \approx \left(\frac{R_p}{2Q}\frac{f_o}{\Delta f}\right)^2$$

- **Output noise spectral density expression (single-sided)**

$$\frac{\overline{v^2_{out}}}{\Delta f} = 4kT\frac{1}{R_p}F(\Delta f)\left(\frac{R_p}{2Q}\frac{f_o}{\Delta f}\right)^2 = 4kTF(\Delta f)R_p\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2$$

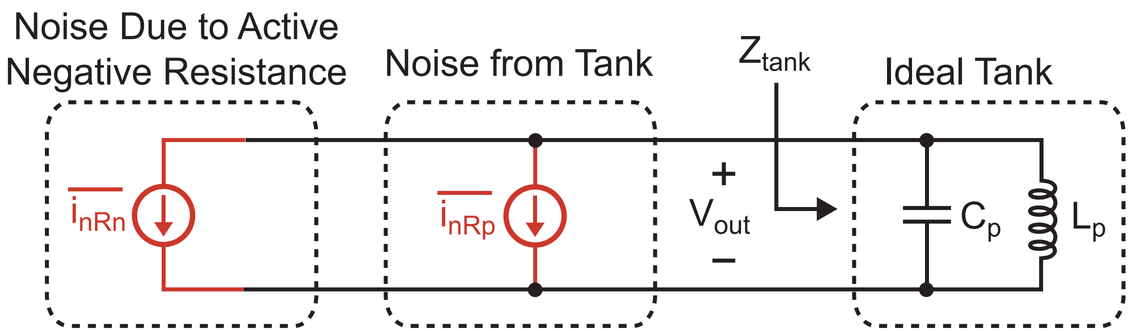
Separation into Amplitude and Phase Noise



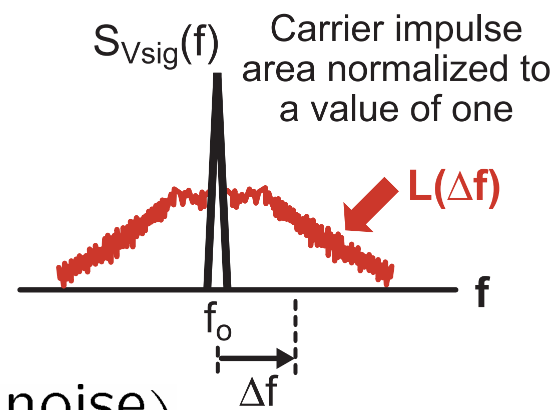
- **Equipartition theorem (see Tom Lee, p 534) states that noise impact splits evenly between amplitude and phase for V_{sig} being a sine wave**
  - **Amplitude variations suppressed by feedback in oscillator**

$$\Rightarrow \quad \frac{\overline{v_{out}^2}}{\Delta f}\Bigg|_{\text{phase}} = \boxed{2kTF(\Delta f)R_p\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2} \quad \text{(single-sided)}$$

Output Phase Noise Spectrum (Leeson's Formula)



Output Spectrum



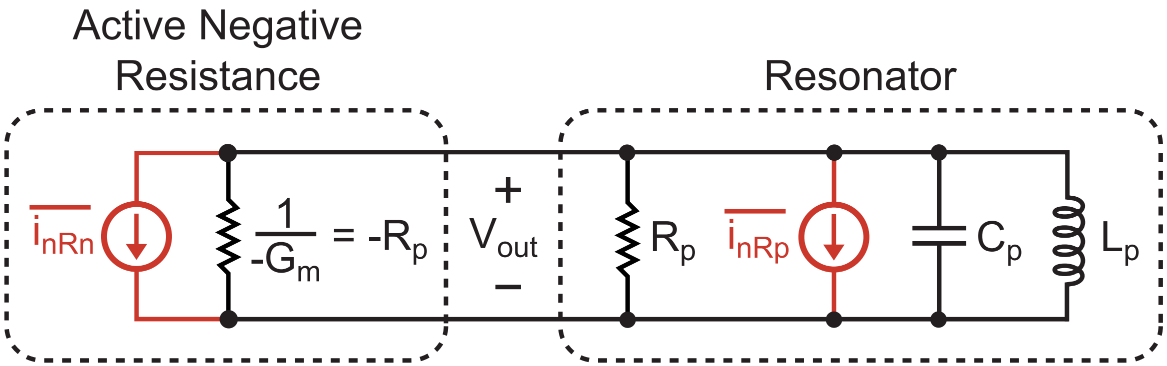
$$L(\Delta f) = 10\log\left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}}\right)$$

- **All power calculations are referenced to the tank loss resistance, R_p**

$$P_{sig} = \frac{V^2_{sig,rms}}{R_p} = \frac{(A/\sqrt{2})^2}{R_p}, \quad S_{noise}(\Delta f) = \frac{1}{R_p}\frac{\overline{v^2_{out}}}{\Delta f}$$

$$L(\Delta f) = 10\log\left(\frac{S_{noise}(\Delta f)}{P_{sig}}\right) = 10\log\left(\frac{2kTF(\Delta f)}{P_{sig}}\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2\right)$$

Example: Active Noise Same as Tank Noise

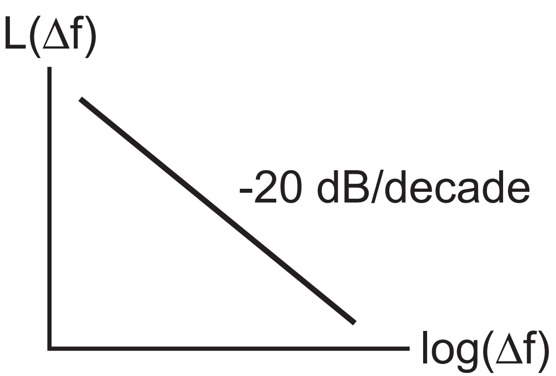


- Noise factor for oscillator in this case is

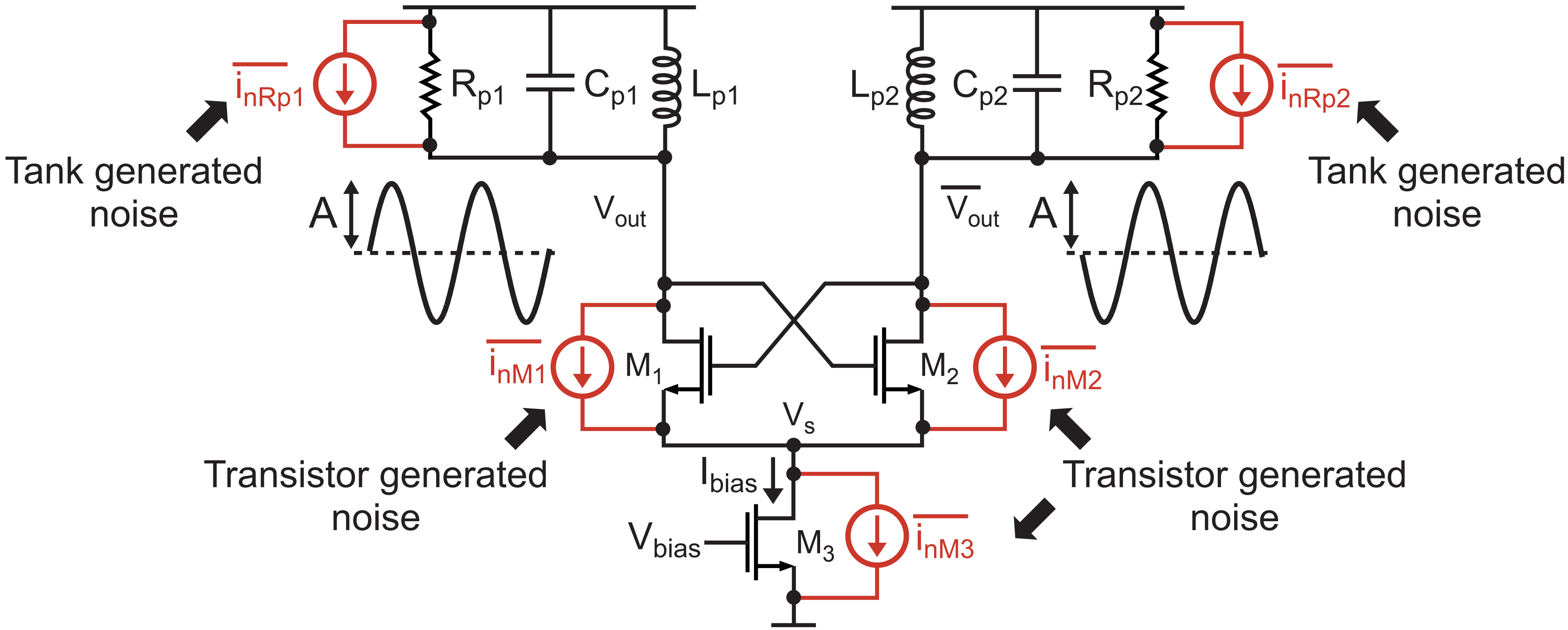
$$F(\Delta f) = 1 + \frac{\overline{i^2_{nRn}}}{\Delta f} \Big/ \frac{\overline{i^2_{nRp}}}{\Delta f} = 2$$

- Resulting phase noise

$$L(\Delta f) = 10\log\left(\frac{4kT}{P_{sig}}\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2\right)$$

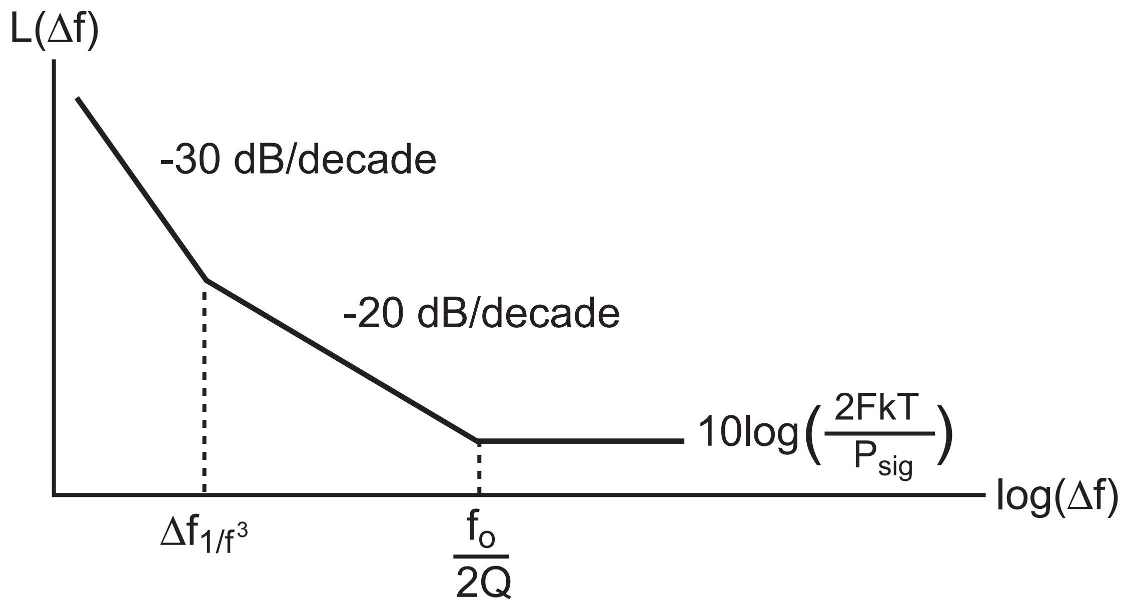


The Actual Situation is Much More Complicated



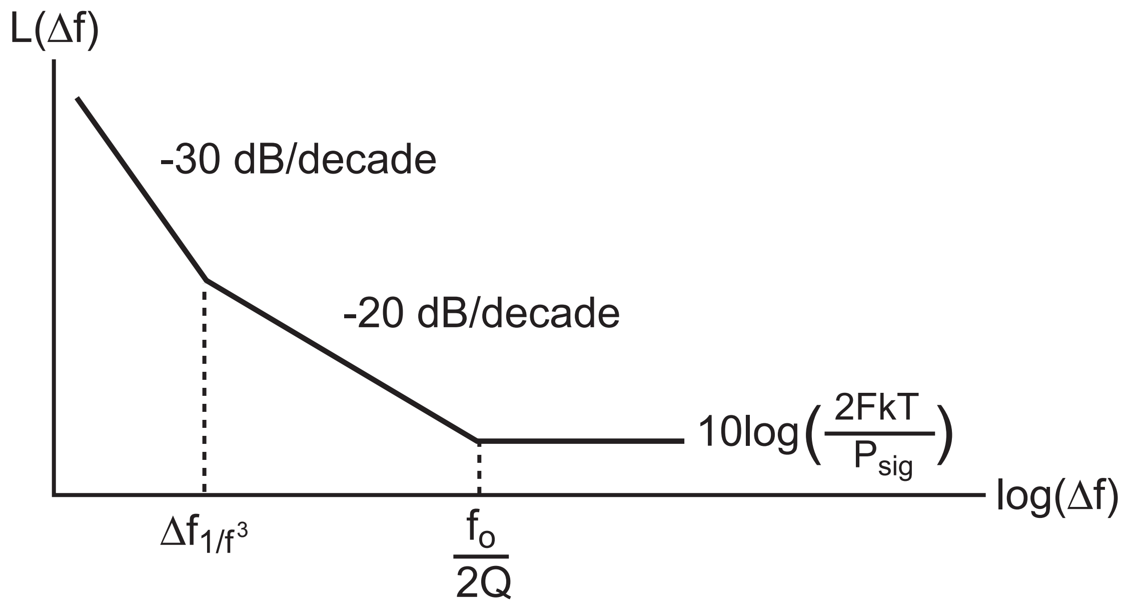
- **Impact of tank generated noise easy to assess**
- **Impact of transistor generated noise is complicated**
  - **Noise from M_1 and M_2 is modulated on and off**
  - **Noise from M_3 is modulated before influencing V_{out}**
  - **Transistors have 1/f noise**
- **Also, transistors can degrade Q of tank**

Phase Noise of A Practical Oscillator



- **Phase noise drops at -20 dB/decade over a wide frequency range, but deviates from this at:**
  - **Low frequencies – slope increases (often -30 dB/decade)**
  - **High frequencies – slope flattens out (oscillator tank does not filter all noise sources)**
- **Frequency breakpoints and magnitude scaling are not readily predicted by the analysis approach taken so far**

Phase Noise of A Practical Oscillator

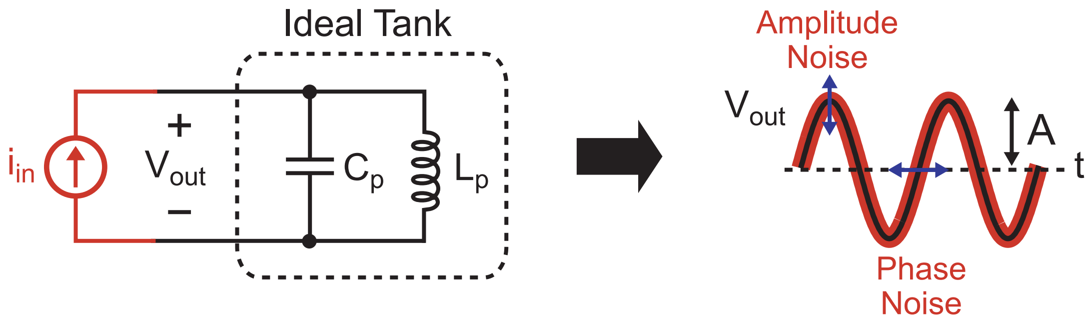


- **Leeson proposed an ad hoc modification of the phase noise expression to capture the above noise profile**

$$L(\Delta f) = 10\log\left(\frac{2FkT}{P_{sig}}\left(1+\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2\right)\left(1+\frac{\Delta f_{1/f^3}}{|\Delta f|}\right)\right)$$

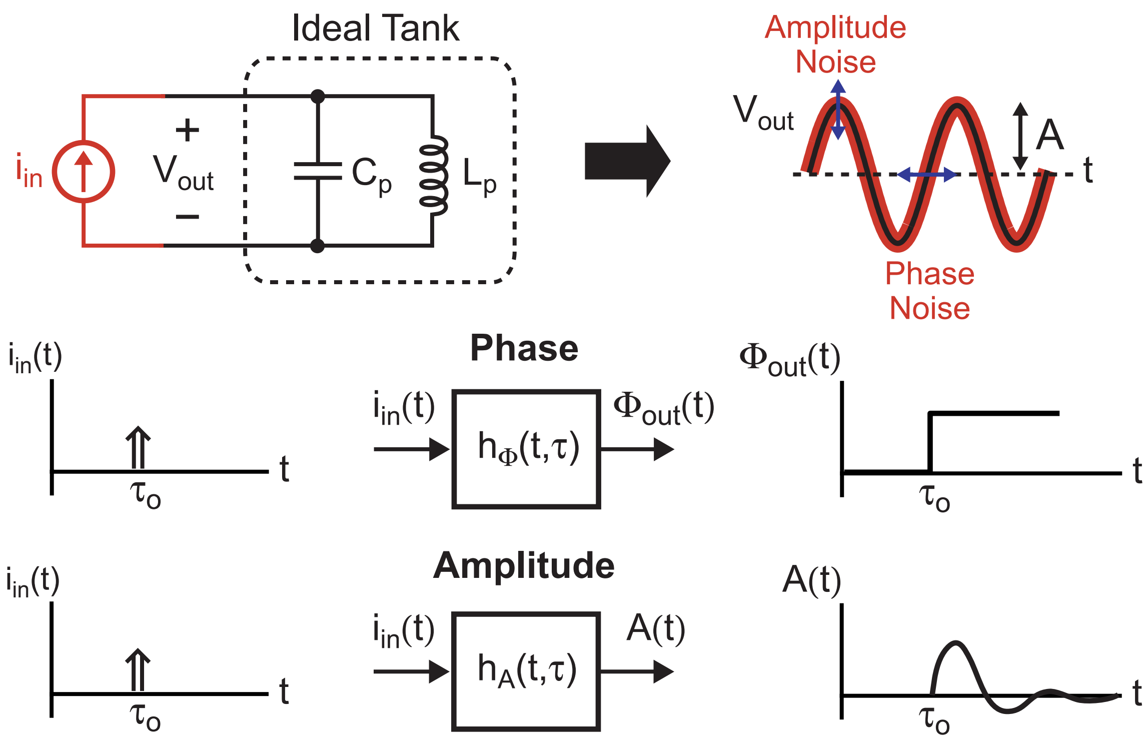
  - **Note: he assumed that F(Δf) was constant over frequency**

A More Sophisticated Analysis Method



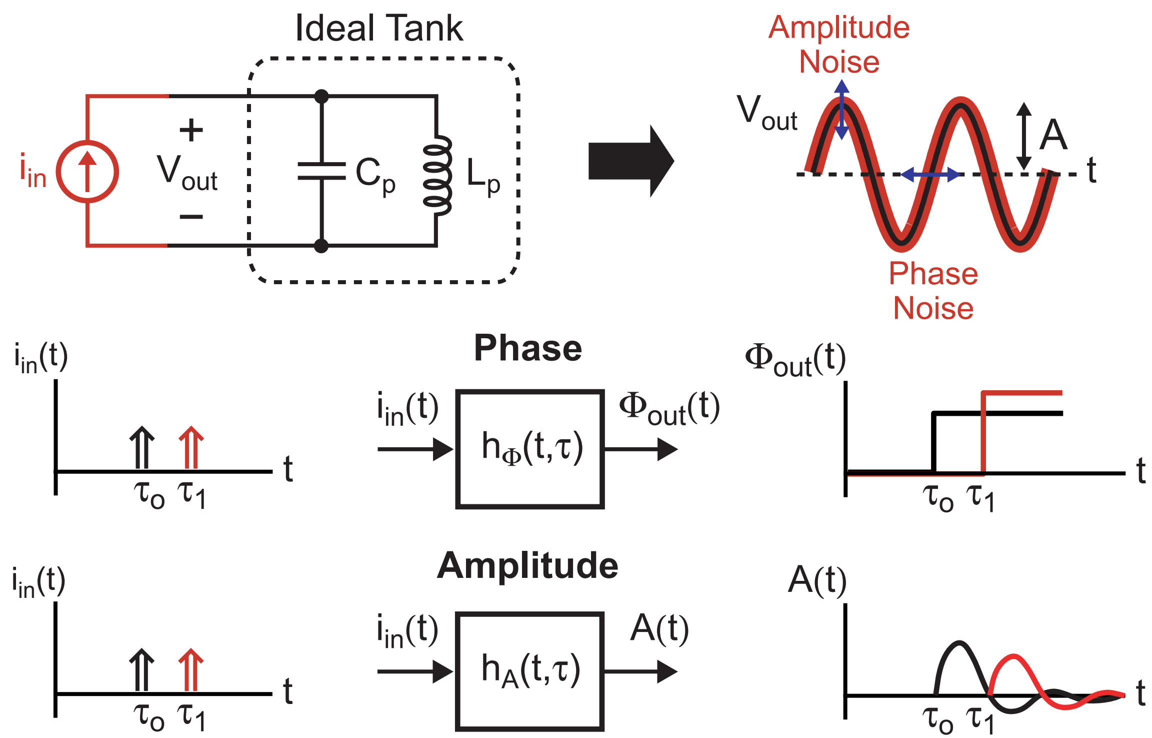
- **Our concern is what happens when noise current produces a voltage across the tank**
  - **Such voltage deviations give rise to both amplitude and phase noise**
  - **Amplitude noise is suppressed through feedback (or by amplitude limiting in following buffer stages)**
    - Our main concern is phase noise
- **We argued that impact of noise divides equally between amplitude and phase for sine wave outputs**
  - **What happens when we have a non-sine wave output?**

Modeling of Phase and Amplitude Perturbations



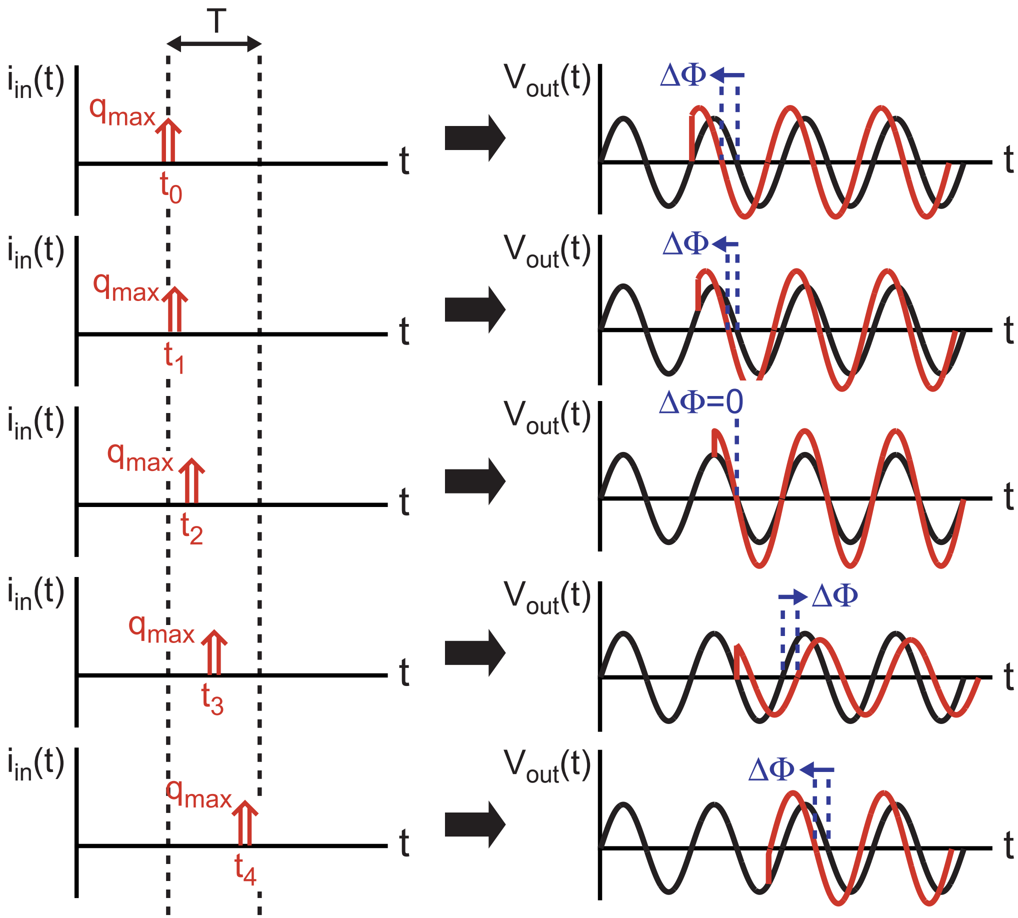
- **Characterize impact of current noise on amplitude and phase through their associated impulse responses**
  - **Phase deviations are accumulated**
  - **Amplitude deviations are suppressed**

Impact of Noise Current is Time-Varying



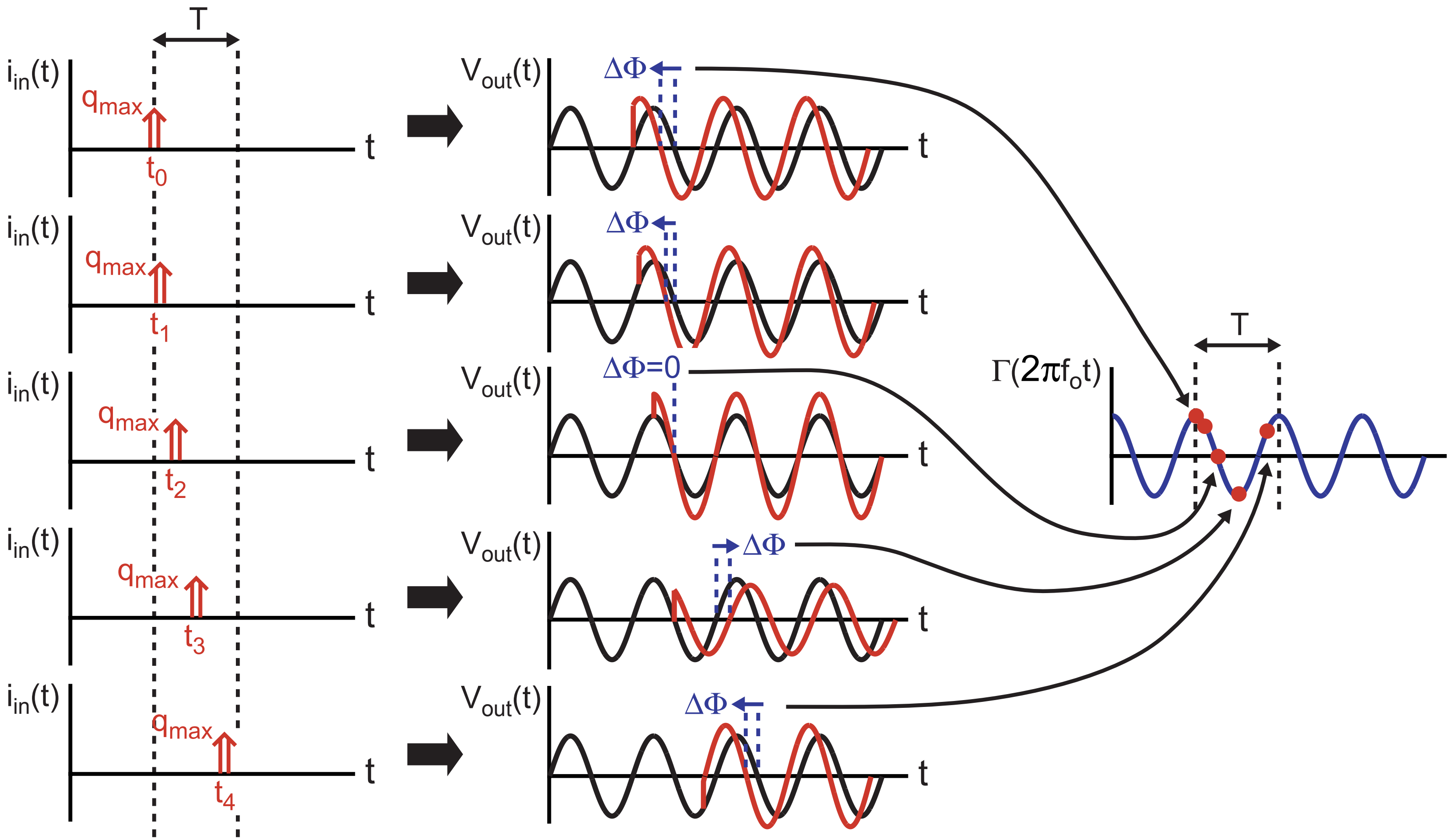
- **If we vary the time at which the current impulse is injected, its impact on phase and amplitude changes**
  - **Need a time-varying model**

Illustration of Time-Varying Impact of Noise on Phase



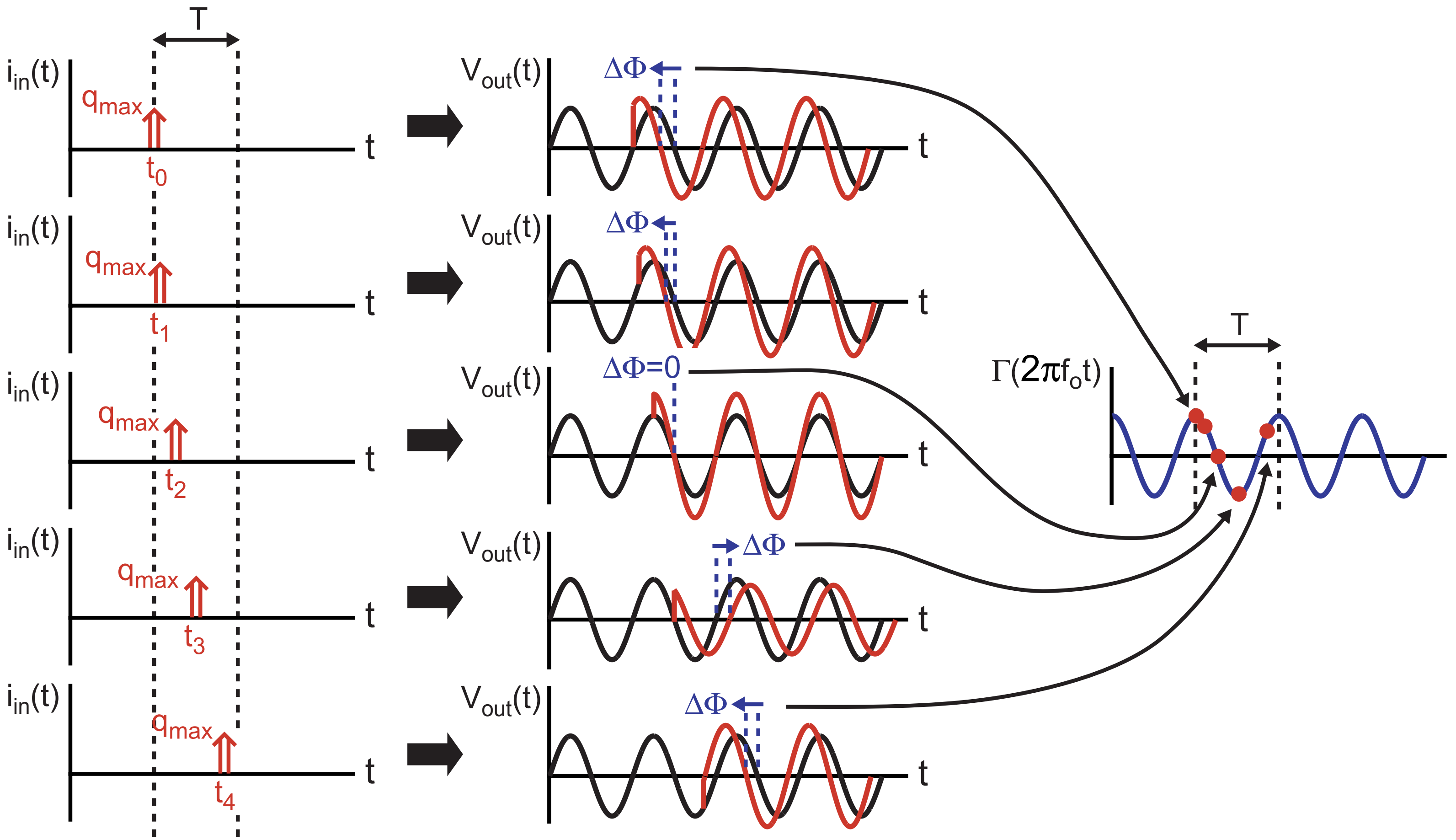
- **High impact on phase when impulse occurs close to the zero crossing of the VCO output**
- **Low impact on phase when impulse occurs at peak of output**

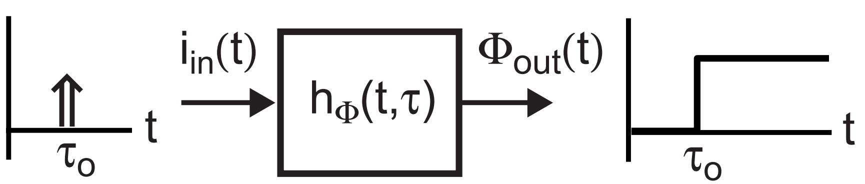
Define Impulse Sensitivity Function (ISF) – $\Gamma(2\pi f_o t)$



- **ISF constructed by calculating phase deviations as impulse position is varied**
  - **Observe that it is periodic with same period as VCO output**

Parameterize Phase Impulse Response in Terms of ISF





$$h_\Phi(t,\tau) = \frac{\Gamma(2\pi f_o \tau)}{q_{max}} u(t-\tau)$$

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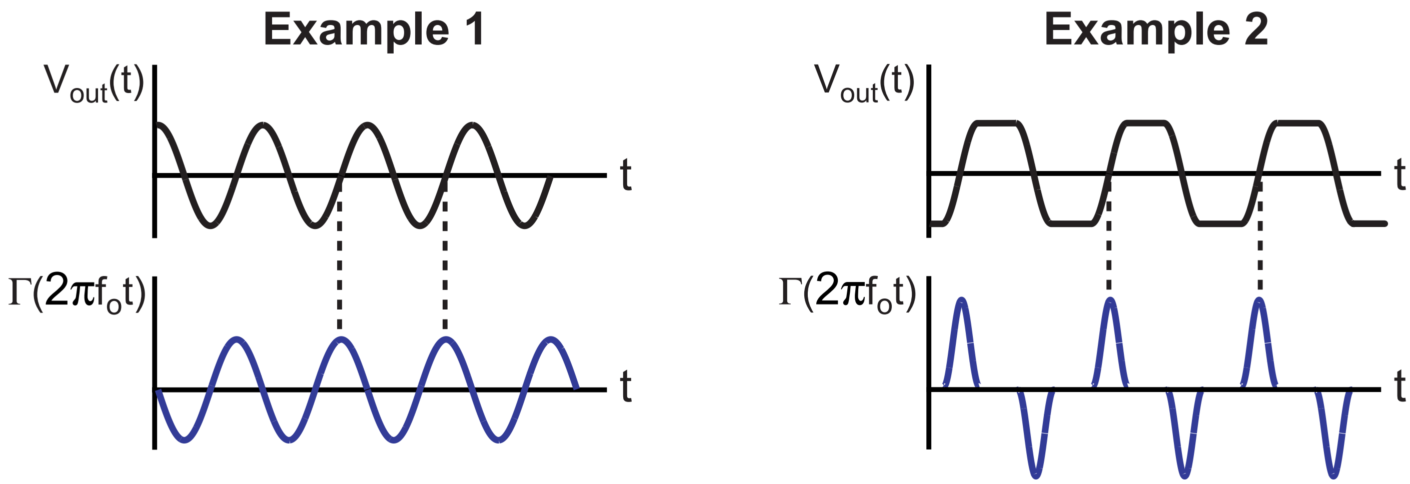
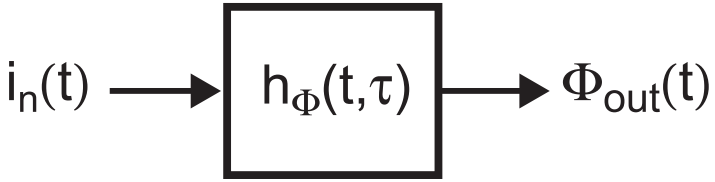
Examples of ISF for Different VCO Output Waveforms



- **ISF (i.e., Γ) is approximately proportional to derivative of VCO output waveform**
  - **Its magnitude indicates where VCO waveform is most sensitive to noise current into tank with respect to creating phase noise**
- **ISF is periodic**
- **In practice, derive it from simulation of the VCO**

Phase Noise Analysis Using LTV Framework



- Computation of phase deviation for an arbitrary noise current input

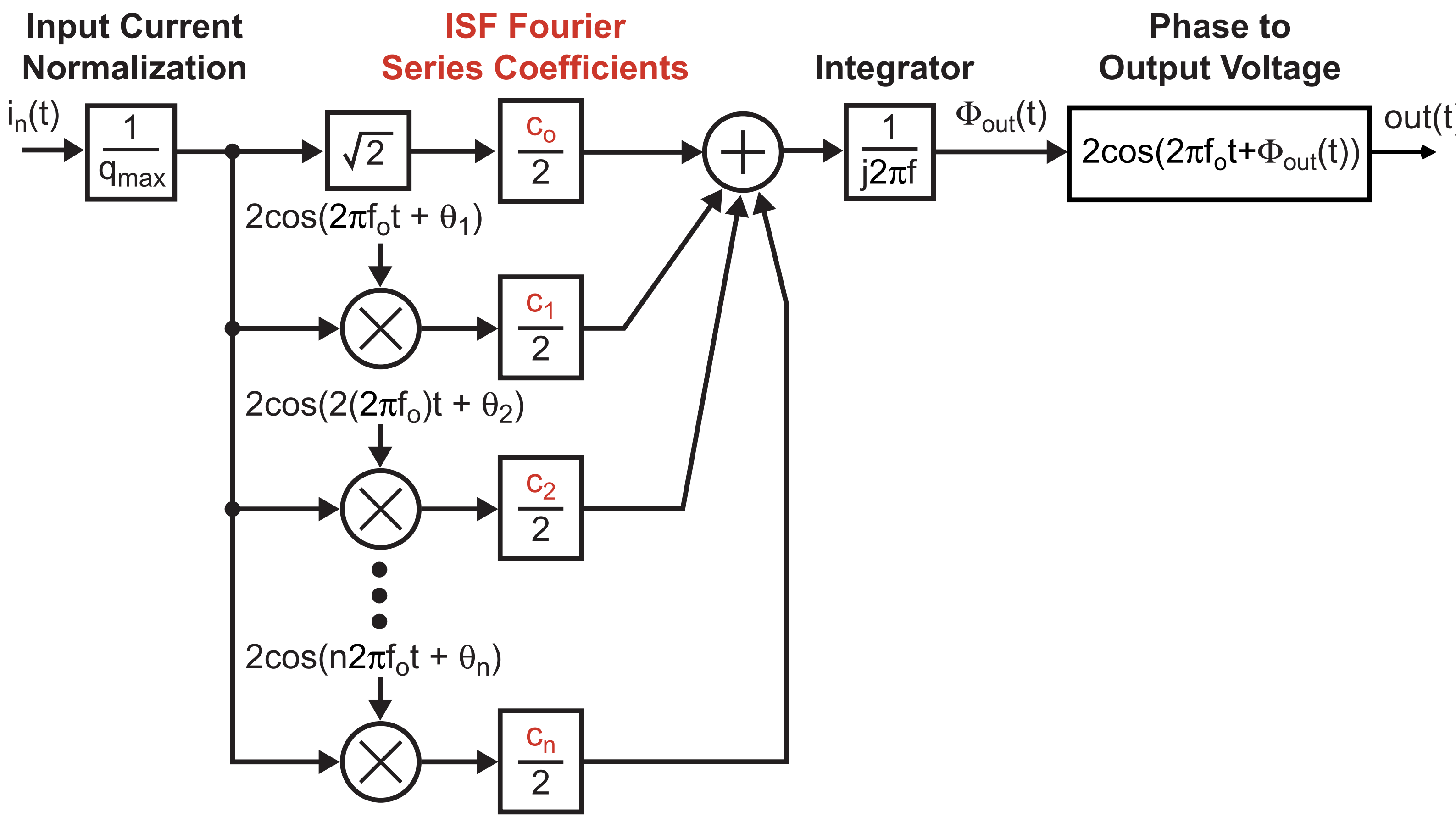
$$\Phi_{out}(t) = \int_{-\infty}^{\infty} h_\Phi(t,\tau) i_n(\tau) d\tau = \frac{1}{q_{max}} \int_{-\infty}^{t} \Gamma(2\pi f_o \tau) i_n(\tau) d\tau$$

- Analysis simplified if we describe ISF in terms of its Fourier series (note: c_o here is different than book)

$$\Gamma(2\pi f_o \tau) = \frac{c_o}{\sqrt{2}} + \sum_{n=1}^{\infty} c_n \cos(n 2\pi f_o \tau + \theta_n)$$

$$\Rightarrow \boxed{\Phi_{out}(t) = \int_{-\infty}^{t} \left(\frac{c_o}{\sqrt{2}} + \sum_{n=1}^{\infty} c_n \cos(n 2\pi f_o \tau + \theta_n) \right) \frac{i_n(\tau)}{q_{max}} d\tau}$$

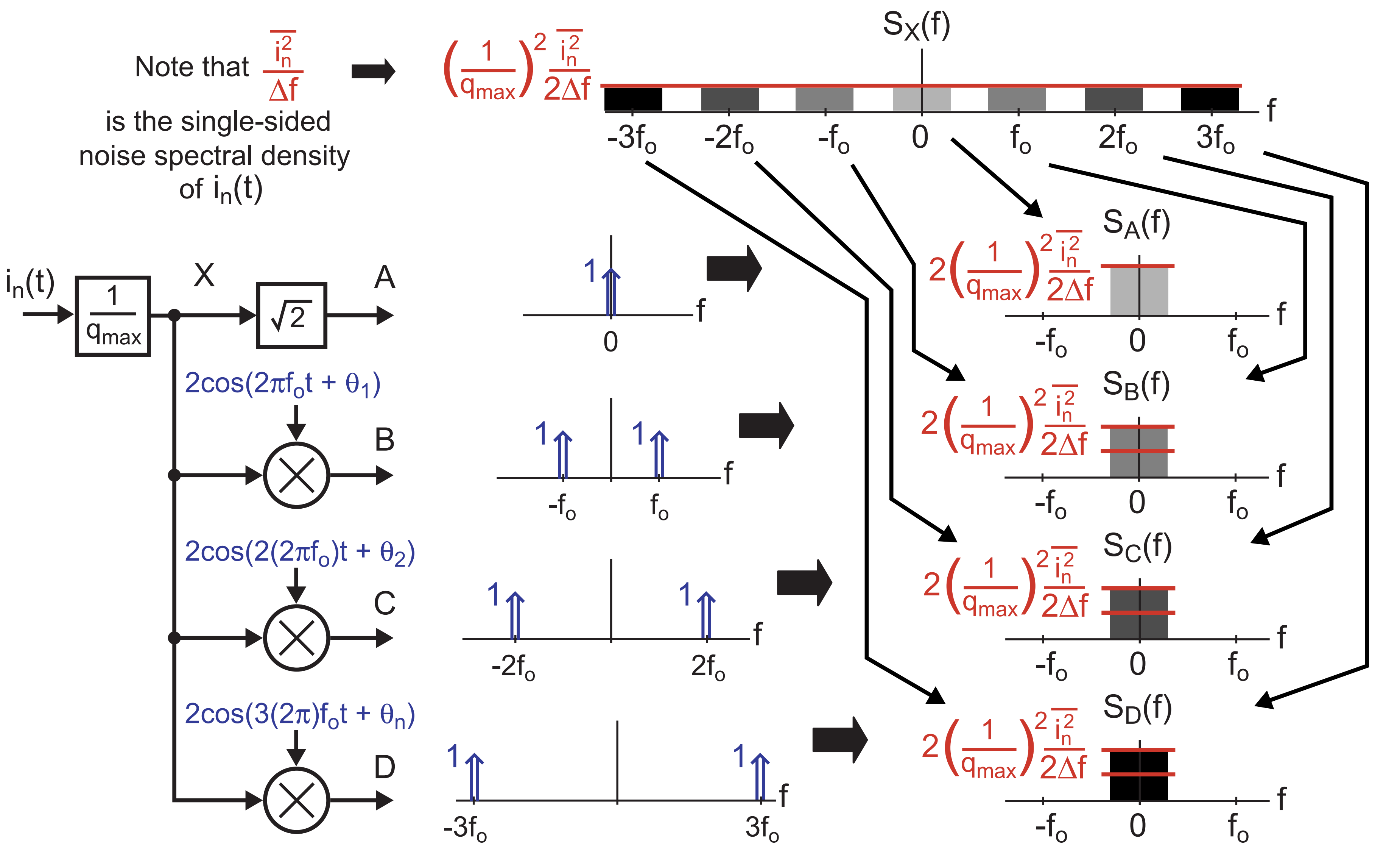
Block Diagram of LTV Phase Noise Expression



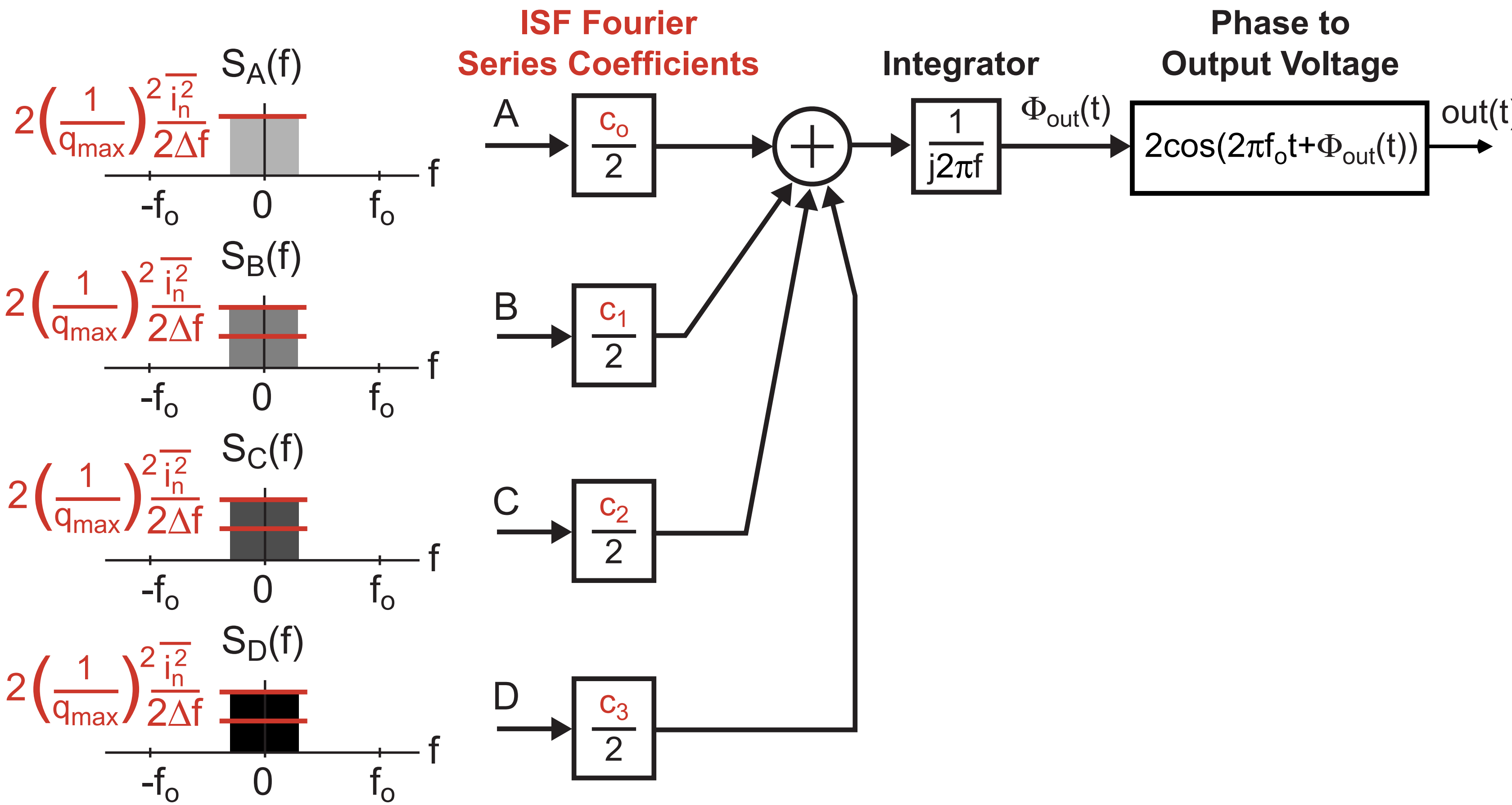
- **Noise from current source is mixed down from different frequency bands and scaled according to ISF coefficients**

Phase Noise Calculation for White Noise Input (Part 1)

Note that $\frac{\overline{i_n^2}}{\Delta f}$ is the single-sided noise spectral density of $i_n(t)$



Phase Noise Calculation for White Noise Input (Part 2)



$$S_{\Phi_{out}}(f) = \left|\frac{1}{j2\pi f}\right|^2 \left(\left(\frac{c_o}{2}\right)^2 S_A(f) + \left(\frac{c_1}{2}\right)^2 S_B(f) + \cdots\right)$$

Spectral Density of Phase Signal

- **From the previous slide**

$$S_{\Phi_{out}}(f) = \left(\frac{1}{2\pi f}\right)^2 \left(\left(\frac{c_o}{2}\right)^2 S_A(f) + \left(\frac{c_1}{2}\right)^2 S_B(f) + \cdots\right)$$

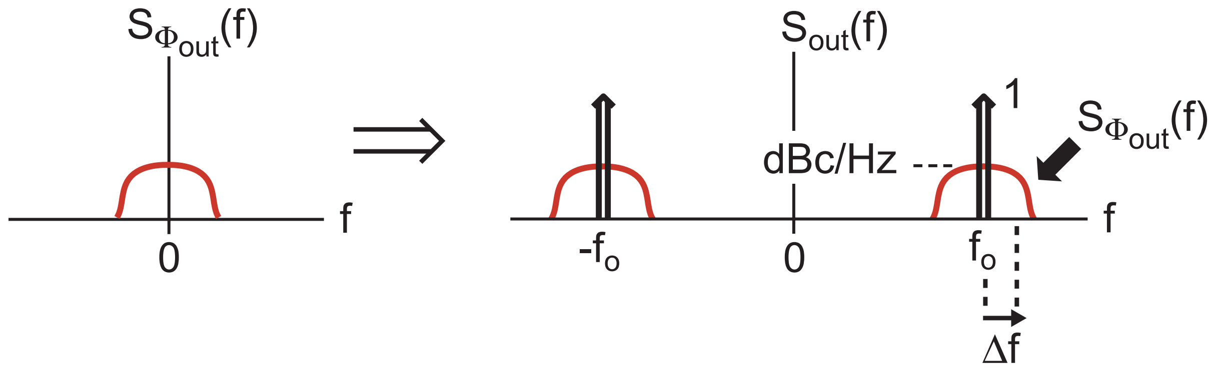
- **Substitute in for $S_A(f)$, $S_B(f)$, etc.**

$$S_{\Phi_{out}}(f) = \left(\frac{1}{2\pi f}\right)^2 \left(\left(\frac{c_o}{2}\right)^2 + \left(\frac{c_1}{2}\right)^2 + \cdots\right) 2 \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{2\Delta f}$$

- **Resulting expression**

$$S_{\Phi_{out}}(f) = \left(\frac{1}{2\pi f}\right)^2 \left(\sum_{n=0}^{\infty} c_n^2\right) \frac{1}{4} \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}$$

Output Phase Noise



- **We now know**

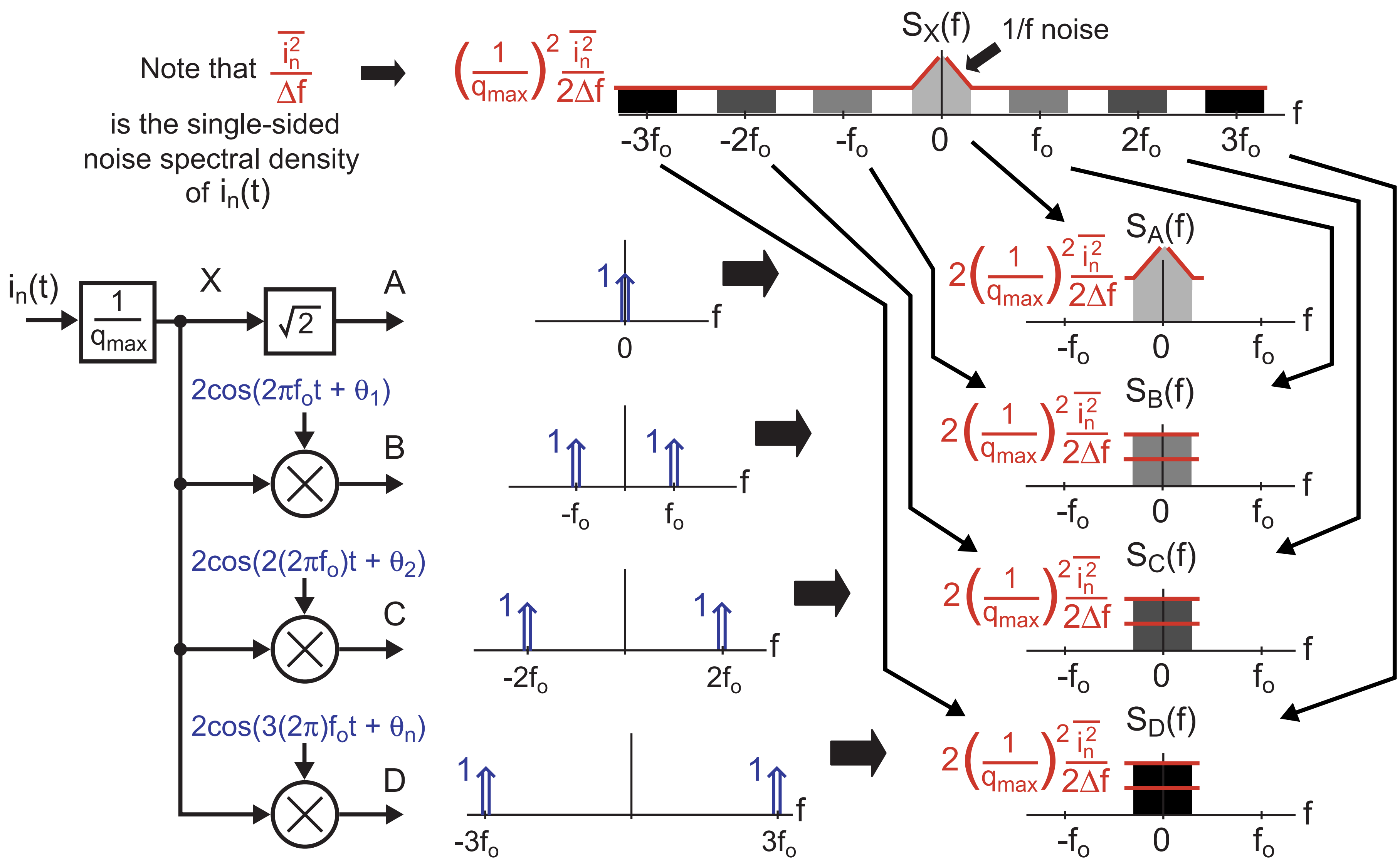
$$S_{\Phi_{out}}(f) = \left|\frac{1}{2\pi f}\right|^2 \left(\sum_{n=0}^{\infty} c_n^2\right) \frac{1}{4} \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}$$

$$L(\Delta f) = 10\log(S_{\Phi_{out}}(\Delta f))$$

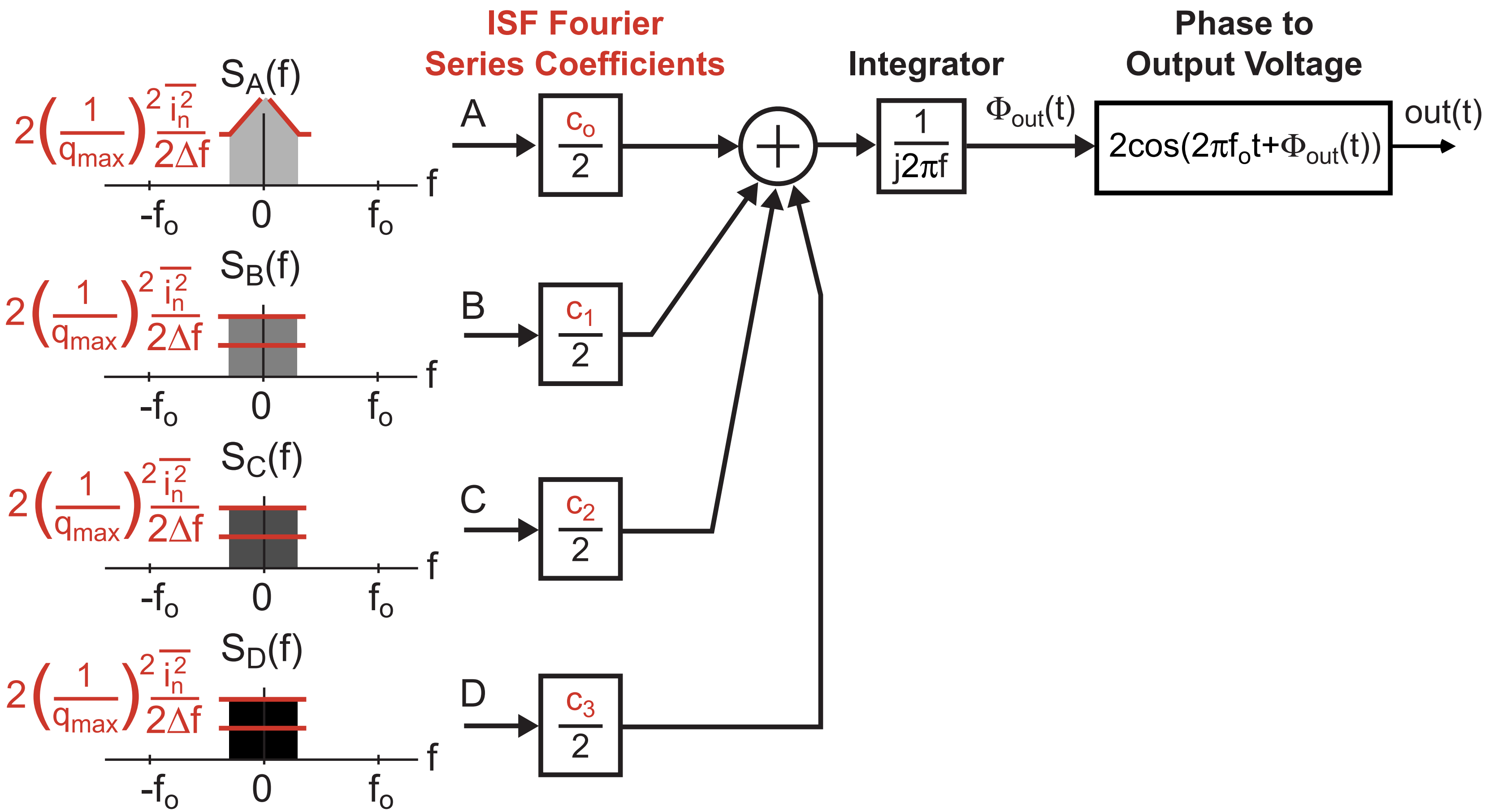
- **Resulting phase noise**

$$L(\Delta f) = 10\log\left(\left(\frac{1}{2\pi \Delta f}\right)^2 \left(\sum_{n=0}^{\infty} c_n^2\right) \frac{1}{4} \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\right)$$

The Impact of 1/f Noise in Input Current (Part 1)



The Impact of 1/f Noise in Input Current (Part 2)



$$S_{\Phi_{out}}(f)\Big|_{1/f^3} = \left|\frac{1}{j2\pi f}\right|^2 \left(\frac{c_o}{2}\right)^2 S_A(f)$$

Calculation of Output Phase Noise in $1/f^3$ region

- From the previous slide

$$S_{\Phi_{out}}(f)\Big|_{1/f^3} = \left(\frac{1}{2\pi f}\right)^2 \left(\frac{c_o}{2}\right)^2 S_A(f)$$

- Assume that input current has 1/f noise with corner frequency $f_{1/f}$

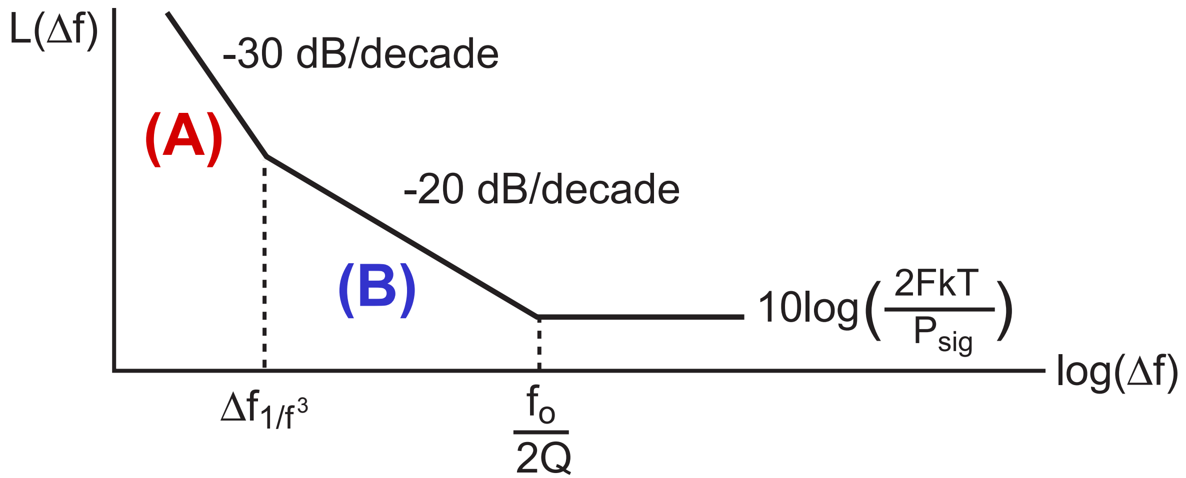
$$S_A(f) = \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\left(\frac{f_{1/f}}{\Delta f}\right)$$

- Corresponding output phase noise

$$L(\Delta f)\Big|_{1/f^3} = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2\left(\frac{c_o}{2}\right)^2 S_A(f)\right)$$

$$= 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 (c_o^2)\,\frac{1}{4}\left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\left(\frac{f_{1/f}}{\Delta f}\right)\right)$$

Calculation of 1/f³ Corner Frequency



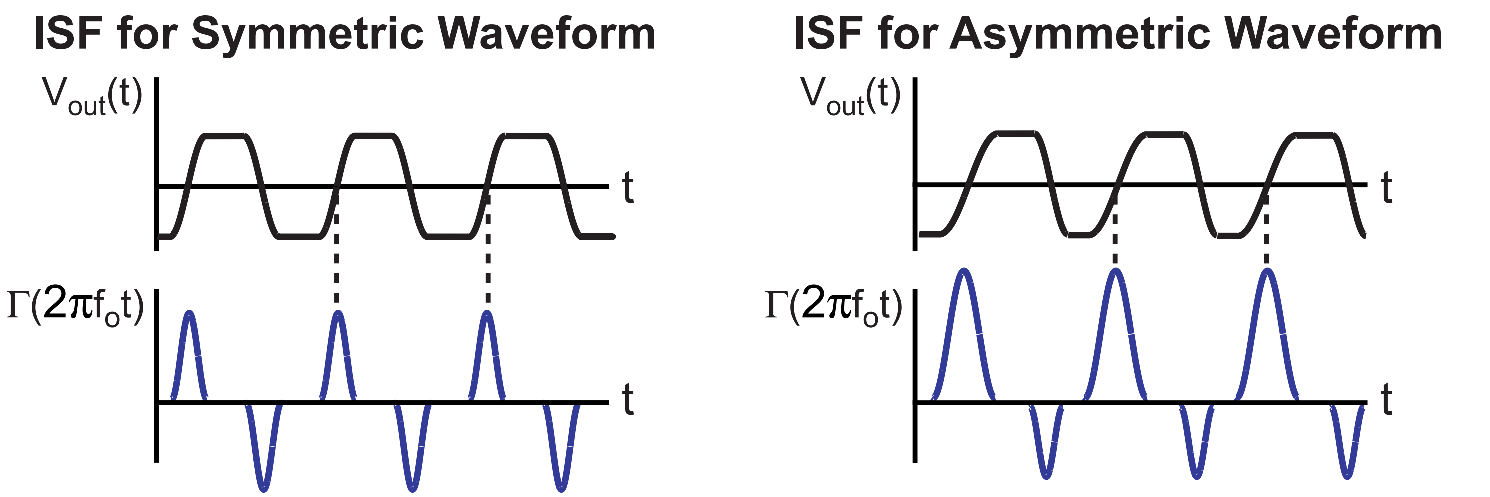
(A) $$L(\Delta f)\Big|_{1/f^3} = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 (c_o^2)\,\frac{1}{4}\left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\left(\frac{f_{1/f}}{\Delta f}\right)\right)$$

(B) $$L(\Delta f) = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 \left(\sum_{n=0}^{\infty} c_n^2\right)\frac{1}{4}\left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\right)$$

(A) = (B) at:

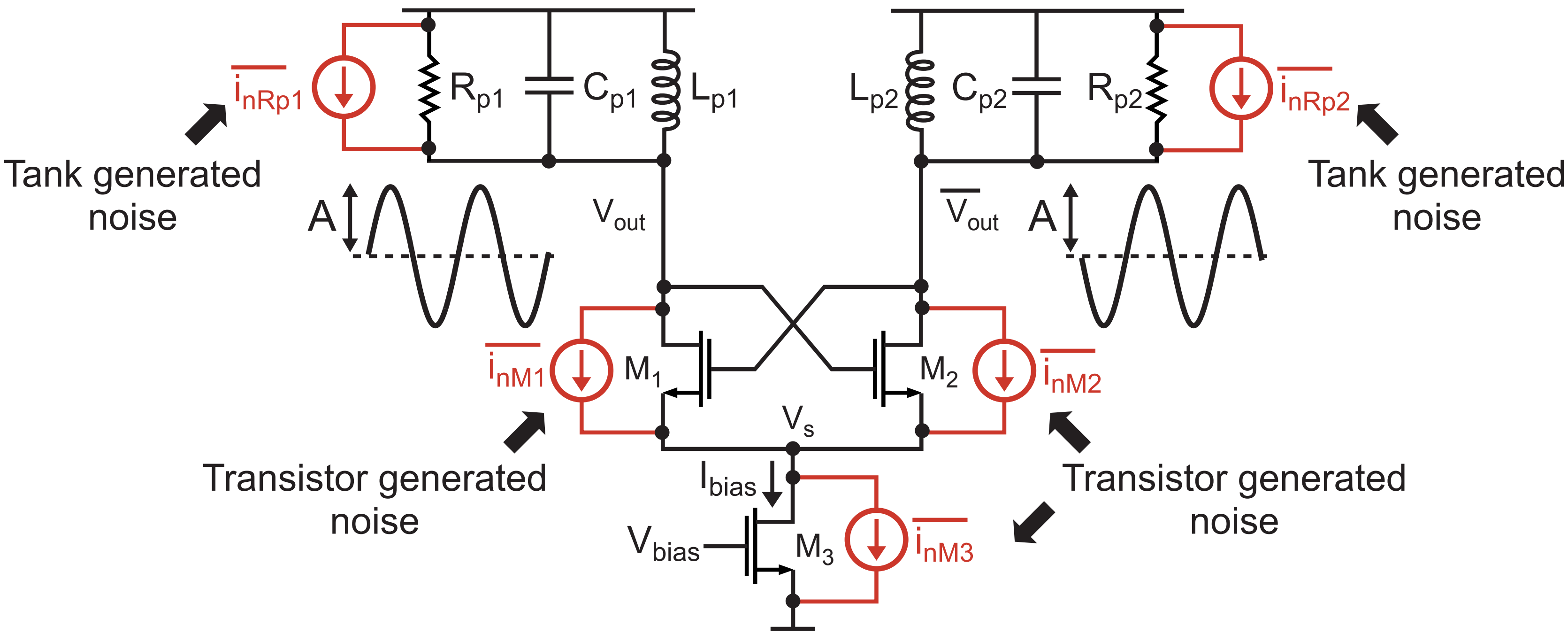
$$\Rightarrow \quad \Delta f_{1/f^3} = \left(c_o^2 \Big/ \sum_{n=0}^{\infty} c_n^2\right) f_{1/f}$$

Impact of Oscillator Waveform on $1/f^3$ Phase Noise



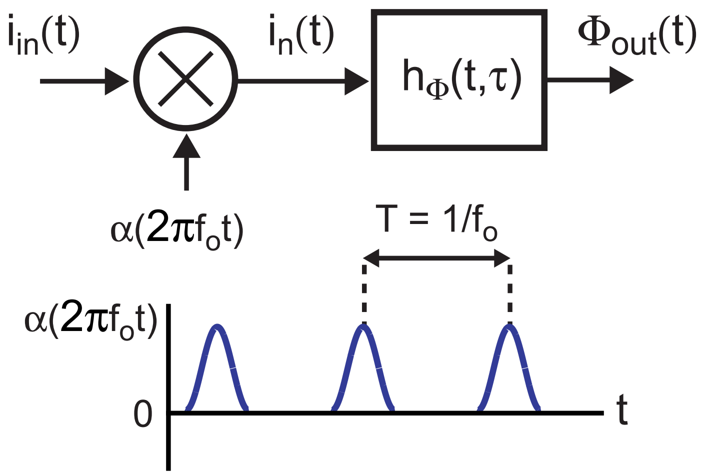
- Key Fourier series coefficient of ISF for $1/f^3$ noise is c_o
  - If DC value of ISF is zero, c_o is also zero
- For symmetric oscillator output waveform
  - DC value of ISF is zero – no upconversion of flicker noise! (i.e. output phase noise does not have $1/f^3$ region)
- For asymmetric oscillator output waveform
  - DC value of ISF is nonzero – flicker noise has impact

Issue – We Have Ignored Modulation of Current Noise



- **In practice, transistor generated noise is modulated by the varying bias conditions of its associated transistor**
  - **As transistor goes from saturation to triode to cutoff, its associated noise changes dramatically**
- **Can we include this issue in the LTV framework?**

Inclusion of Current Noise Modulation



- **Recall**

$$\Phi_{out}(t) = \int_{-\infty}^{\infty} h_\Phi(t,\tau) i_n(\tau) d\tau = \frac{1}{q_{max}} \int_{-\infty}^{t} \Gamma(2\pi f_o \tau) i_n(\tau) d\tau$$

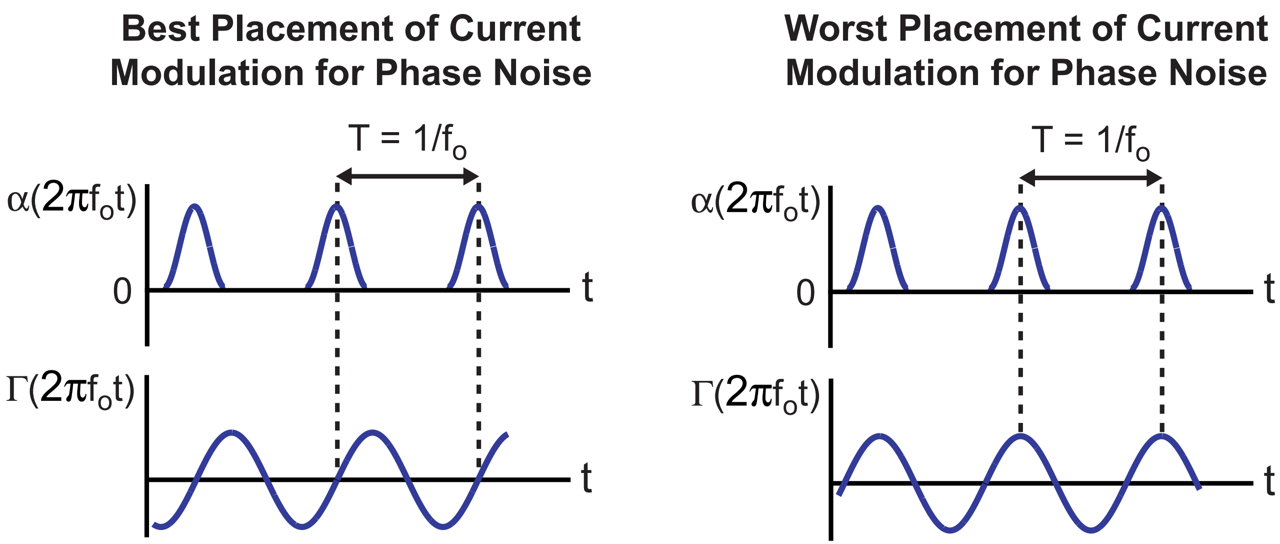
- **By inspection of figure**

$$\Rightarrow \quad \Phi_{out}(t) = \frac{1}{q_{max}} \int_{-\infty}^{t} \underline{\Gamma(2\pi f_o \tau)\alpha(2\pi f_o \tau)} i_{in}(\tau) d\tau$$

- **We therefore apply previous framework with ISF as**

$$\boxed{\Gamma_{eff}(2\pi f_o \tau) = \Gamma(2\pi f_o \tau)\alpha(2\pi f_o \tau)}$$

Placement of Current Modulation for Best Phase Noise

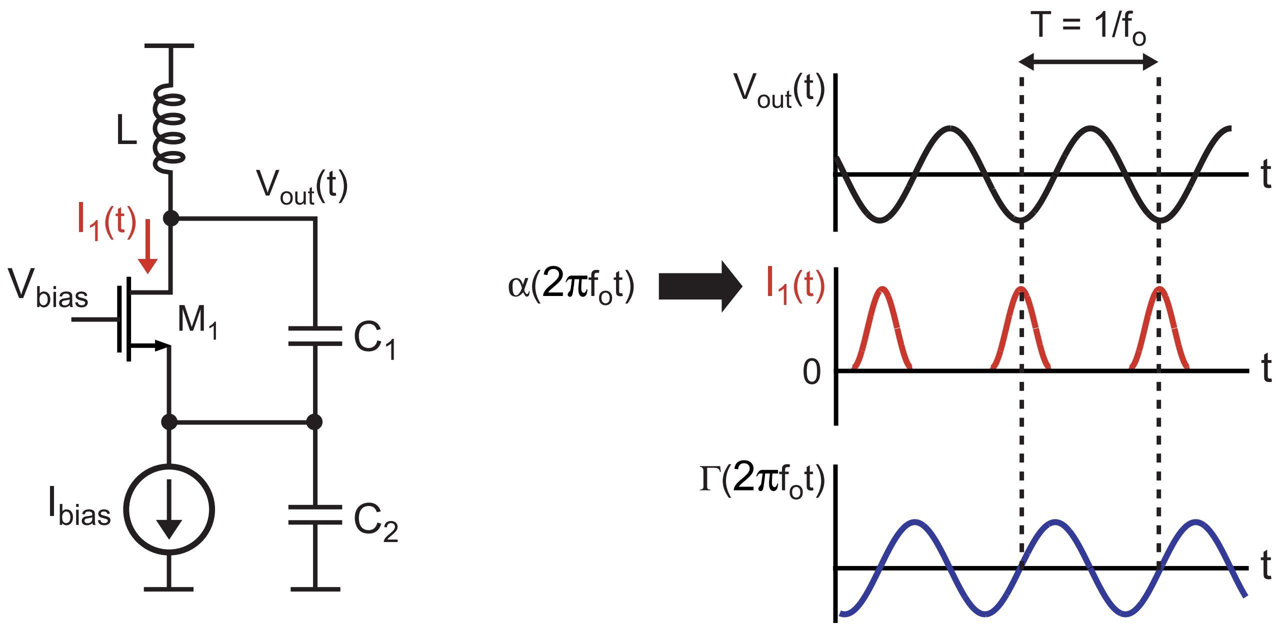


- **Phase noise expression (ignoring 1/f noise)**

$$L(\Delta f) = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 \left(\sum_{n=0}^{\infty} c_n^2\right) \frac{1}{4} \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\right)$$

- **Minimum phase noise achieved by minimizing sum of square of Fourier series coefficients (i.e. rms value of Γ_{eff})**

Colpitts Oscillator Provides Optimal Placement of α

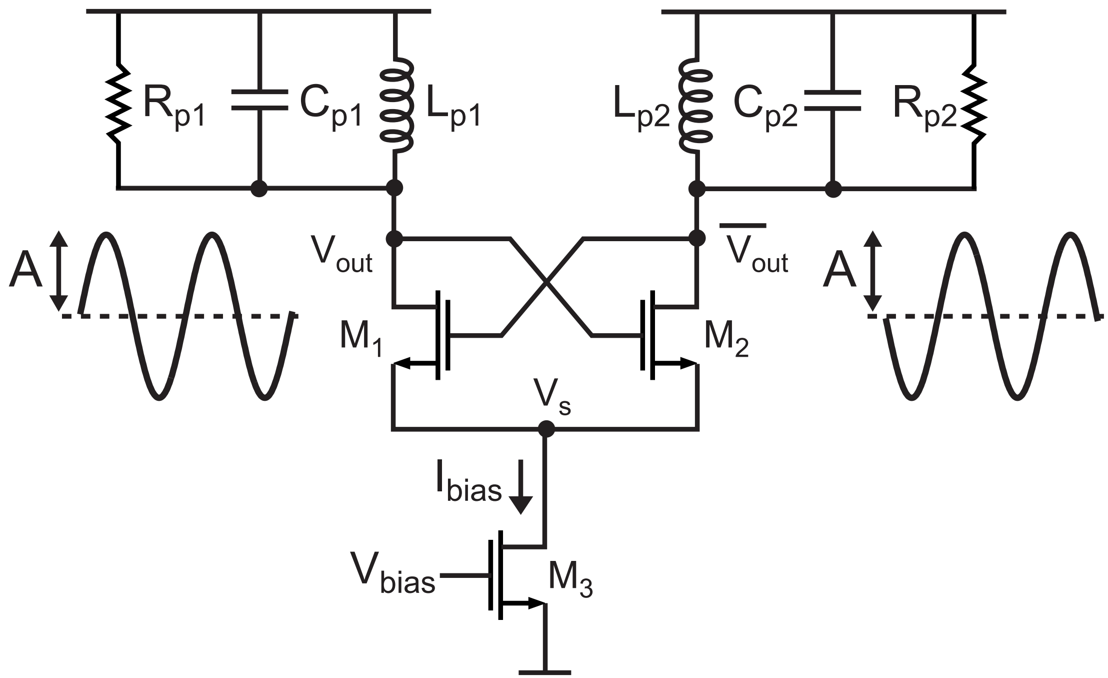


- Current is injected into tank at bottom portion of VCO swing
  - Current noise accompanying current has minimal impact on VCO output phase

Summary of LTV Phase Noise Analysis Method

- **Step 1: calculate the impulse sensitivity function of each oscillator noise source using a simulator**
- **Step 2: calculate the noise current modulation waveform for each oscillator noise source using a simulator**
- **Step 3: combine above results to obtain $\Gamma_{eff}(2\pi f_o t)$ for each oscillator noise source**
- **Step 4: calculate Fourier series coefficients for each $\Gamma_{eff}(2\pi f_o t)$**
- **Step 5: calculate spectral density of each oscillator noise source (before modulation)**
- **Step 6: calculate overall output phase noise using the results from Step 4 and 5 and the phase noise expressions derived in this lecture (or the book)**

Alternate Approach for Negative Resistance Oscillator

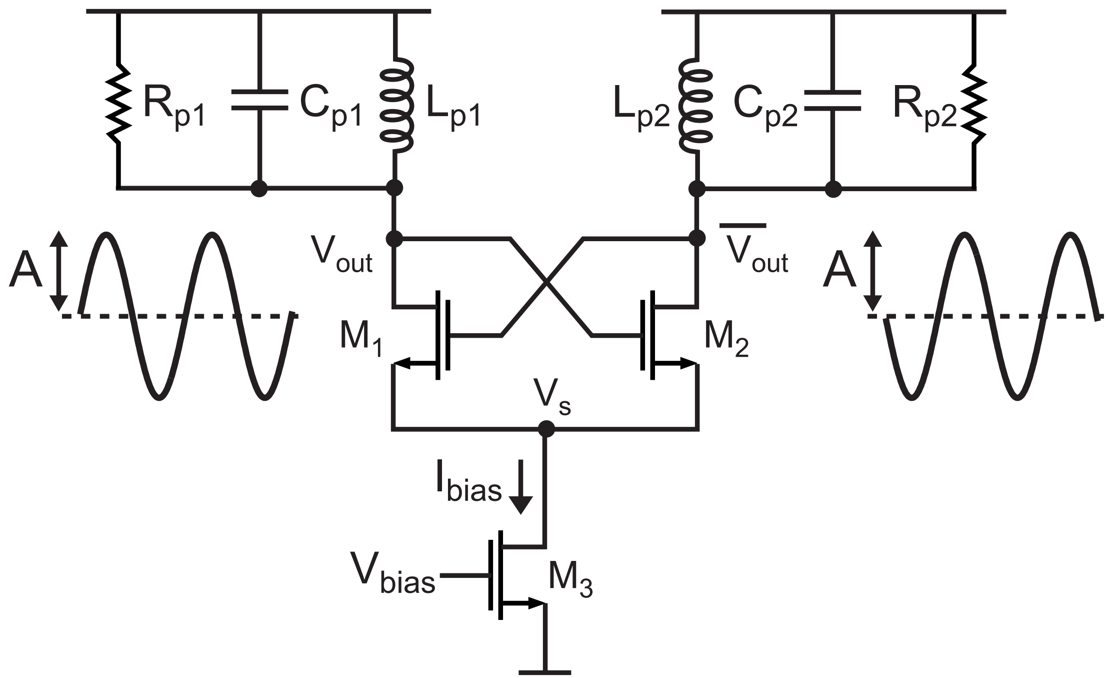


- **Recall Leeson's formula**

$$L(\Delta f) = 10\log\left(\frac{2kTF(\Delta f)}{P_{sig}}\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2\right)$$

  - **Key question: how do you determine F(Δf)?**

F(Δf) Has Been Determined for This Topology



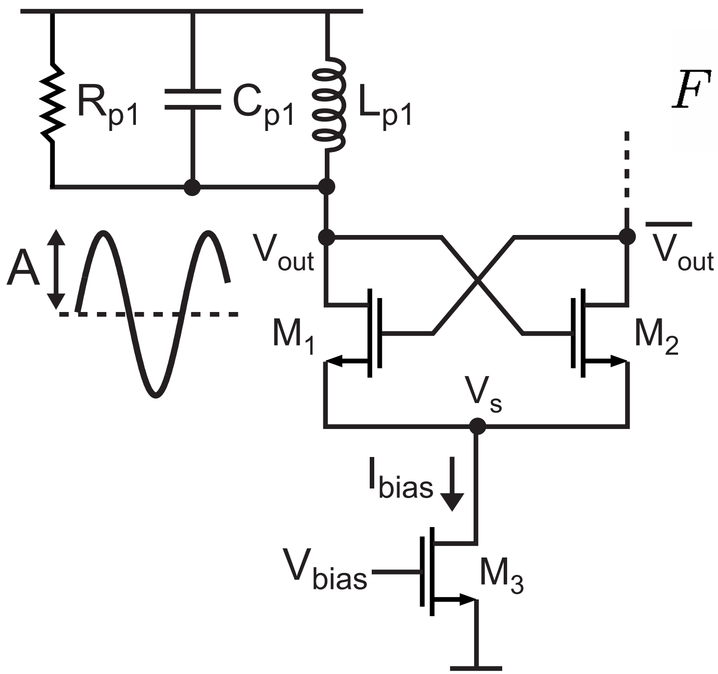
- Rael et. al. have come up with a closed form expression for F(Δf) for the above topology
- In the region where phase noise falls at -20 dB/dec:

$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A} + \gamma \frac{4}{9} g_{do,M3} R_p \quad (R_p = R_{p1} = R_{p2})$$

References to Rael Work

- Phase noise analysis
  - J.J. Rael and A.A. Abidi, “Physical Processes of Phase Noise in Differential LC Oscillators”, Custom Integrated Circuits Conference, 2000, pp 569-572
- Implementation
  - Emad Hegazi et. al., “A Filtering Technique to Lower LC Oscillator Phase Noise”, JSSC, Dec 2001, pp 1921-1930

Designing for Minimum Phase Noise



$$F(\Delta f) = \underbrace{1}_{(A)} + \underbrace{\frac{2\gamma I_{bias} R_p}{\pi A}}_{(B)} + \underbrace{\gamma \frac{4}{9} g_{do,M3} R_p}_{(C)}$$

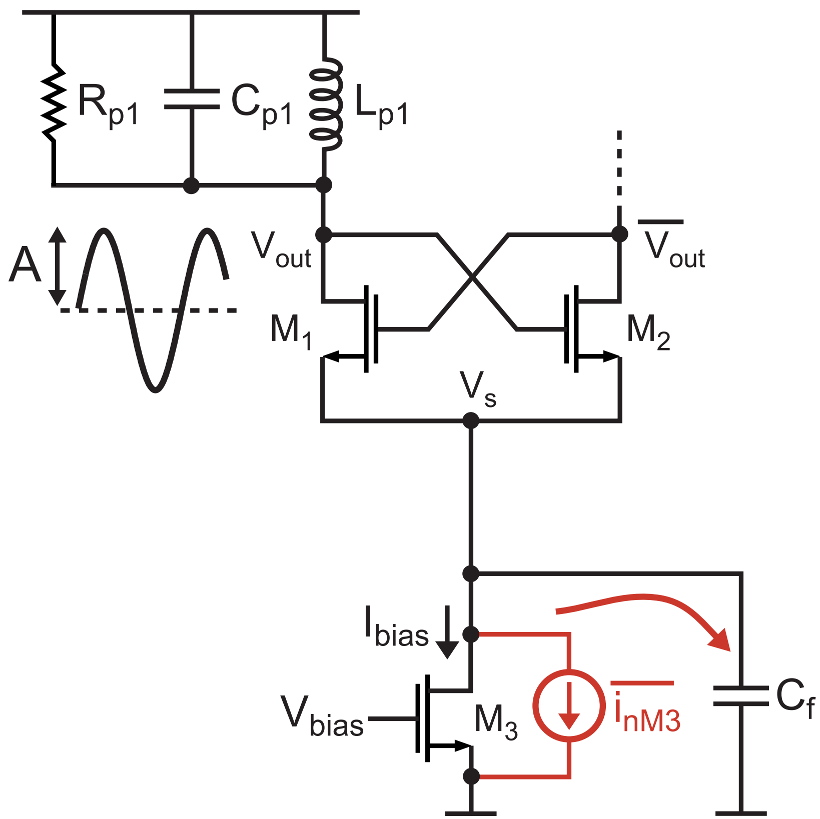
(A) Noise from tank resistance

(B) Noise from M_1 and M_2

(C) Noise from M_3

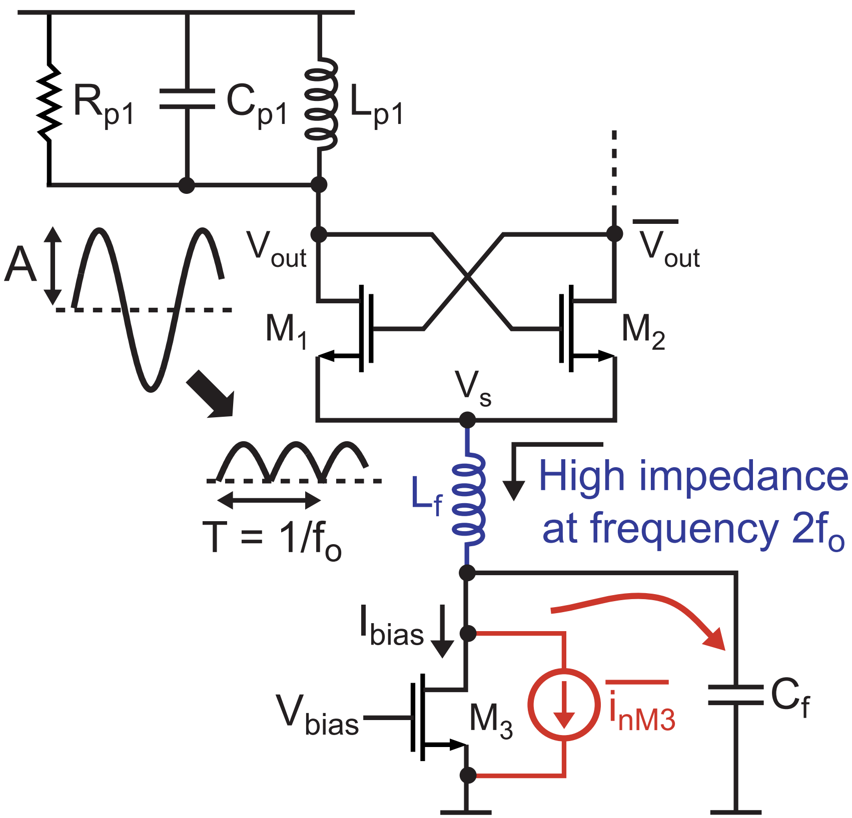
- **To achieve minimum phase noise, we'd like to minimize F(Δf)**
- **The above formulation provides insight of how to do this**
  - **Key observation: (C) is often quite significant**

Elimination of Component (C) in F(Δf)



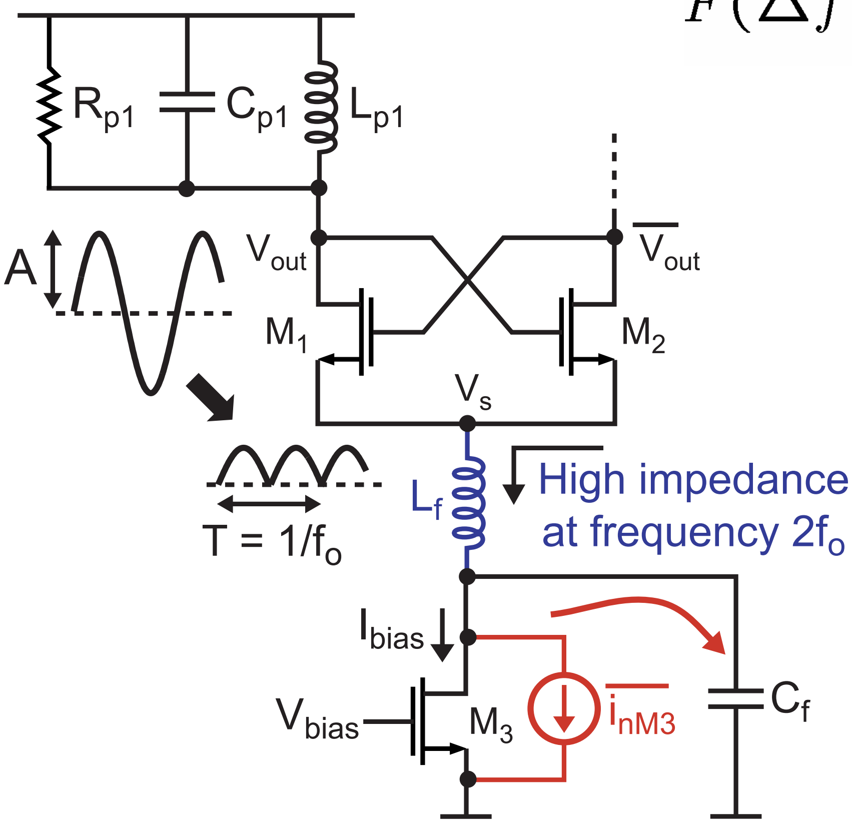
- **Capacitor C_f shunts noise from M_3 away from tank**
  - **Component (C) is eliminated!**
- **Issue – impedance at node V_s is very low**
  - **Causes M_1 and M_2 to present a low impedance to tank during portions of the VCO cycle**
    - Q of tank is degraded

Use Inductor to Increase Impedance at Node V_s



- **Voltage at node V_s is a rectified version of oscillator output**
  - **Fundamental component is at twice the oscillation frequency**
- **Place inductor between V_s and current source**
  - **Choose value to resonate with C_f and parasitic source capacitance at frequency $2f_o$**
- **Impedance of tank not degraded by M_1 and M_2**
  - **Q preserved!**

Designing for Minimum Phase Noise – Next Part



$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A} + \gamma \frac{4}{9} g_{do,M3} R_p$$

(A) (B) (C)

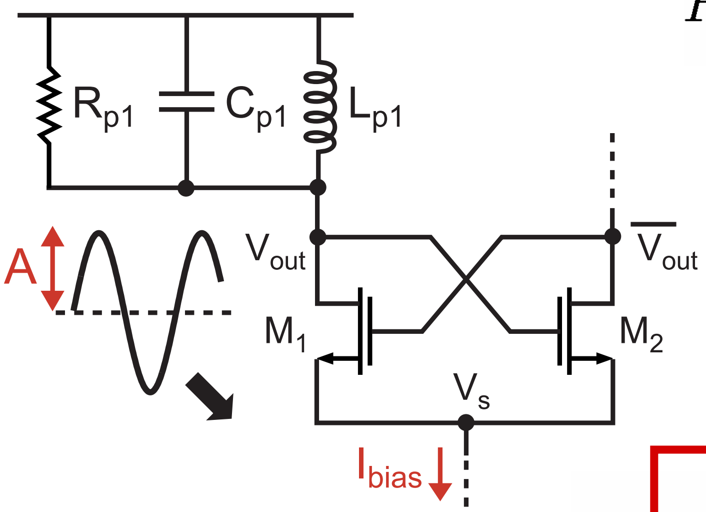
(A) Noise from tank resistance

(B) Noise from M_1 and M_2

(C) ~~Noise from M_3~~

- **Let's now focus on component (B)**
  - **Depends on bias current and oscillation amplitude**

Minimization of Component (B) in F(Δf)



$$F(\Delta f) = 1 + \underbrace{\frac{2\gamma I_{bias} R_p}{\pi A}}_{(B)}$$

- **Recall from Lecture 11**

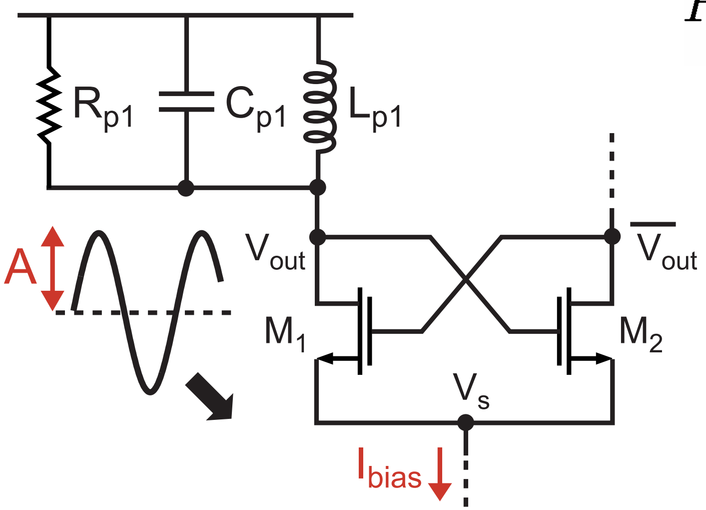
$$A = \frac{2}{\pi} I_{bias} R_p$$

$$\Rightarrow \boxed{F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi(2/\pi) I_{bias} R_p} = 1 + \gamma}$$

- **So, it would seem that I_{bias} has no effect!**
  - **Not true – want to maximize A (i.e. P_{sig}) to get best phase noise, as seen by:**

$$L(\Delta f) = 10 \log\left(\frac{2kTF(\Delta f)}{P_{sig}}\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2\right)$$

Current-Limited Versus Voltage-Limited Regimes



$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A}$$

(B)

- **Oscillation amplitude, A, cannot be increased above supply imposed limits**
- **If I_{bias} is increased above the point that A saturates, then (B) increases**

- **Current-limited regime: amplitude given by** $A = \frac{2}{\pi} I_{bias} R_p$
- **Voltage-limited regime: amplitude saturated**

Best phase noise achieved at boundary between these regimes!

Final Comments

- **Hajimiri method useful as a numerical procedure to determine phase noise**
  - **Provides insights into 1/f noise upconversion and impact of noise current modulation**
- **Rael method useful for CMOS negative-resistance topology**
  - **Closed form solution of phase noise!**
  - **Provides a great deal of design insight**
- **Another numerical method**
  - **Spectre RF from Cadence now does a reasonable job of estimating phase noise for many oscillators**
    - Useful for verifying design ideas and calculations