Intro to Practical Digital Communications

Lecture 2

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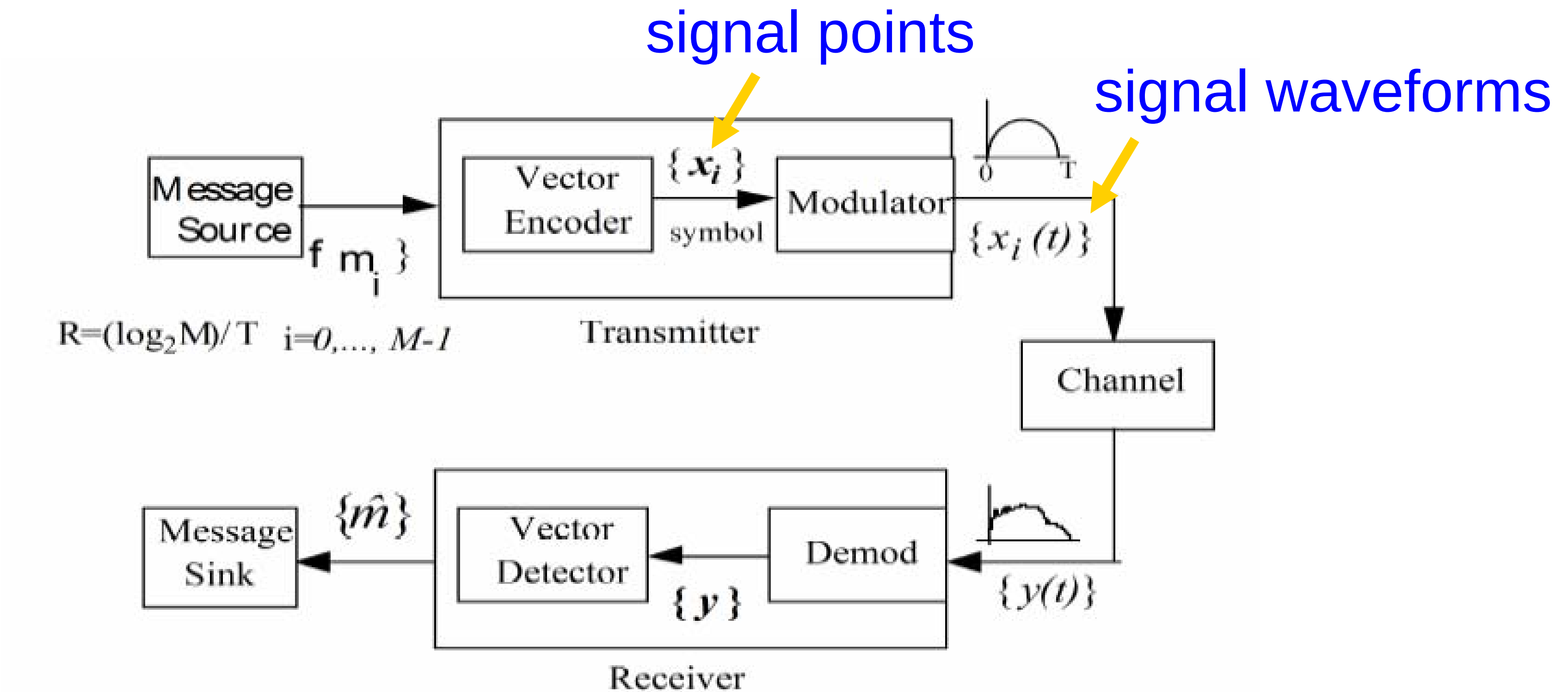
Massachusetts Institute of Technology

Discrete data transmission

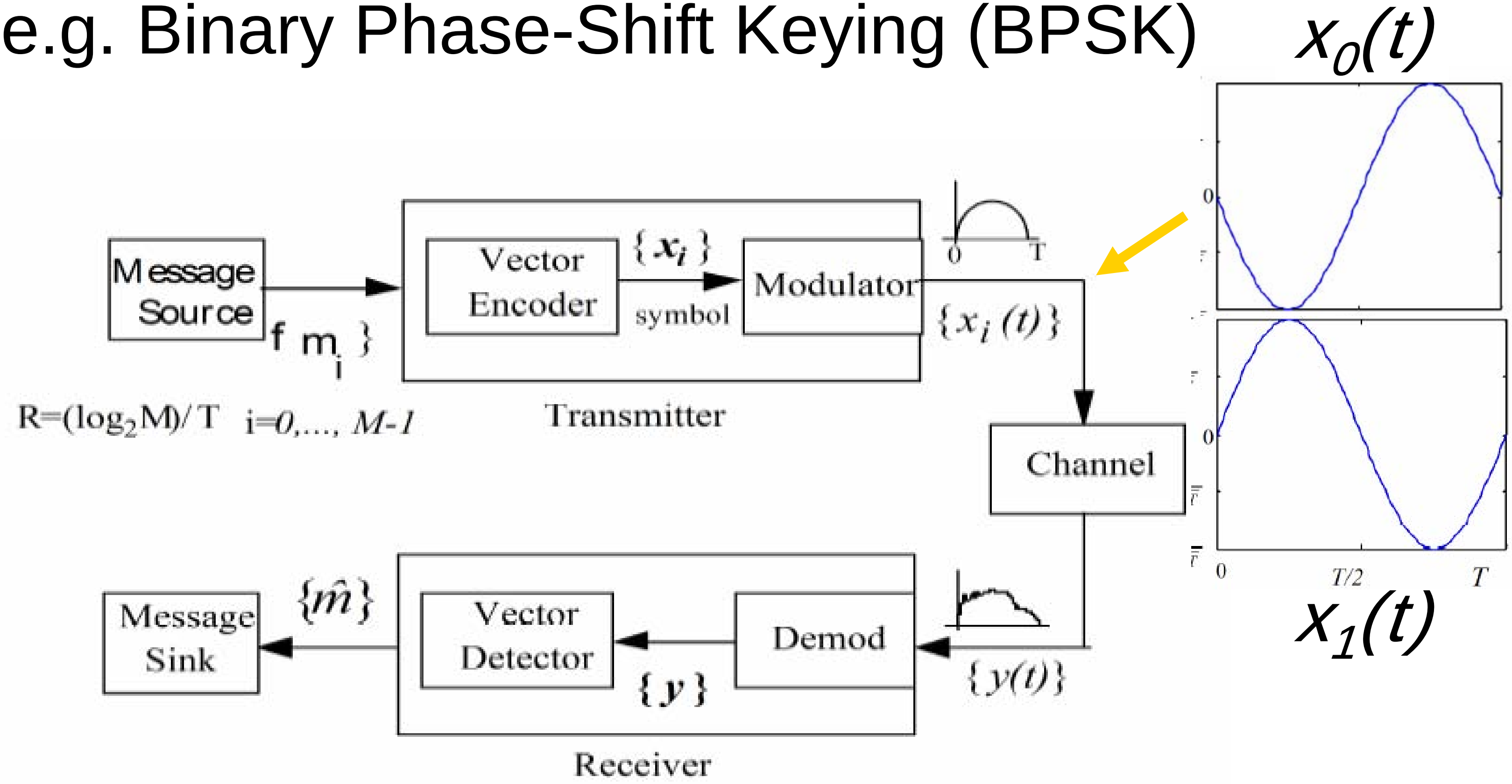
- Messages are encoded into signal points



- Signal points are mapped to signal waveforms
  - Modulation

Modulation and de-modulation

- e.g. Binary Phase-Shift Keying (BPSK)



Vector signal representation

- Maps continuous signals to discrete vectors
  - Significantly simplifies system analysis

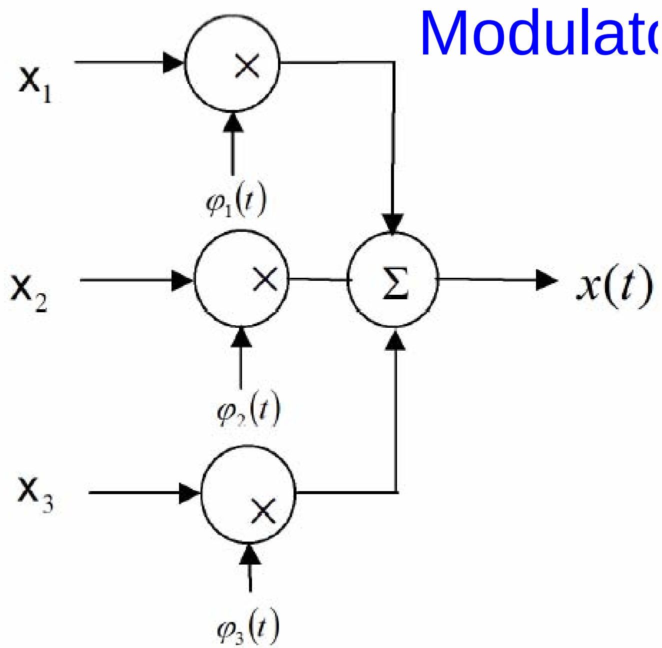
signal waveforms

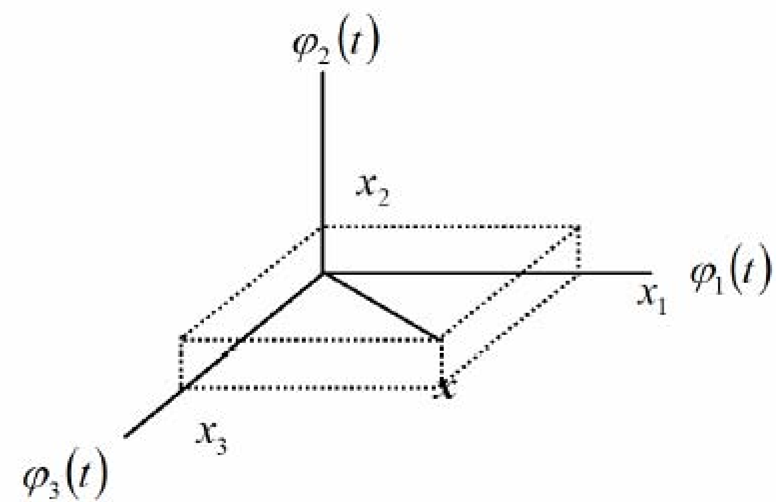
signal points

basis f

$$x(t) \triangleq \sum_{n=1}^{N} x_n \varphi_n(t)$$

Modulat

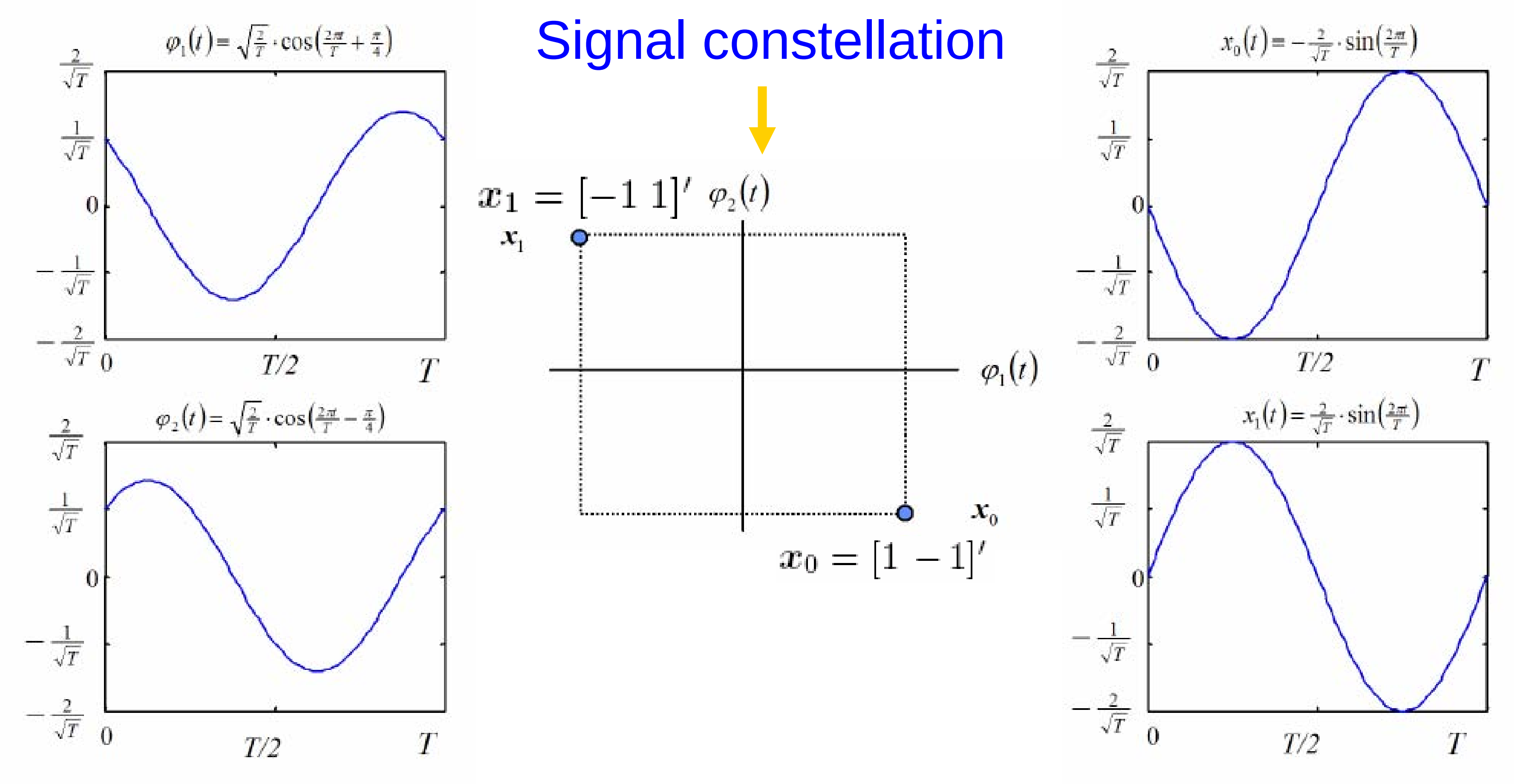




$$P_x \triangleq \frac{\mathcal{E}_x}{T}$$

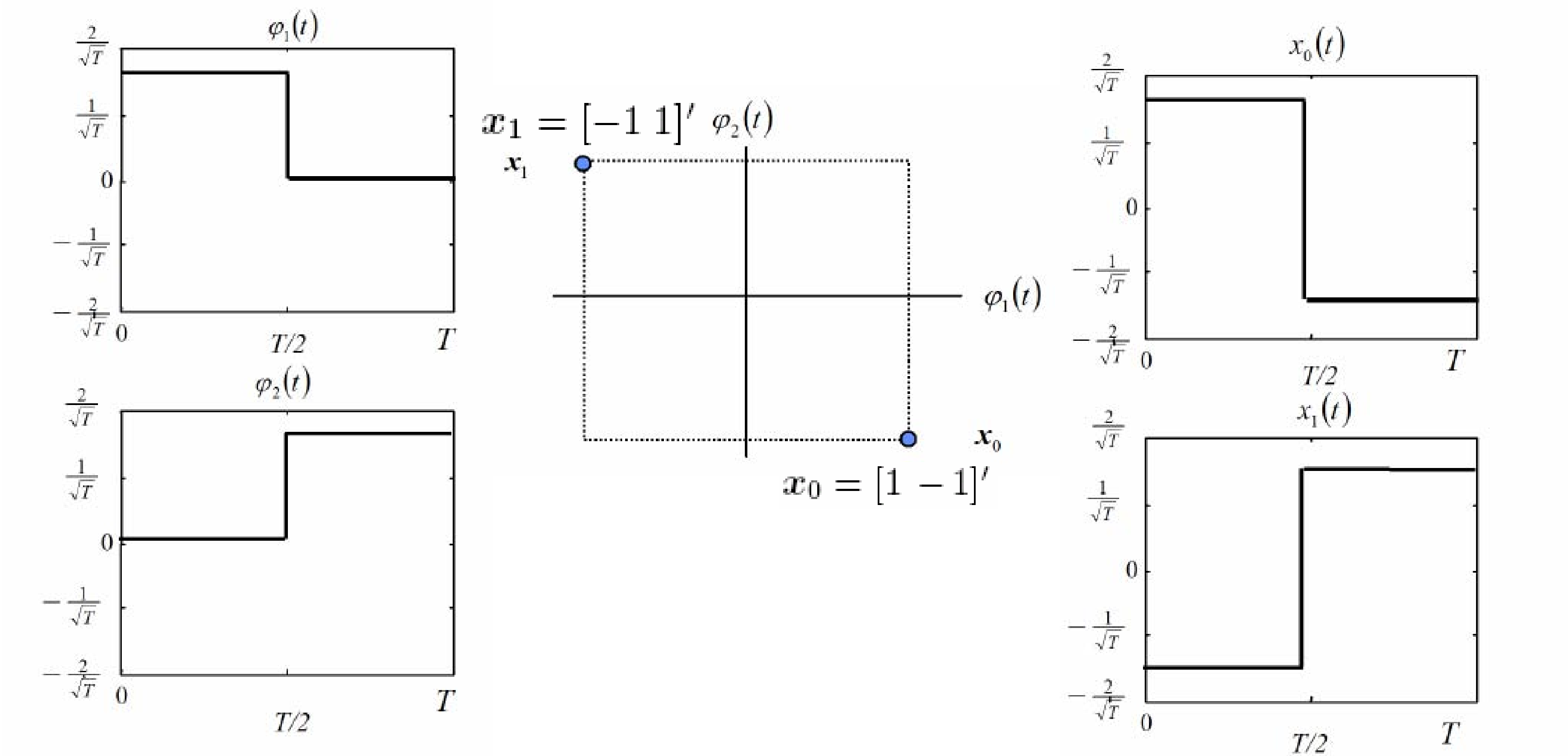
$$\mathcal{E}_x \triangleq E\left[||x||^2\right] = \sum_{i=0}^{M-1} ||x_i||^2 p_x(i)$$

BPSK example



- What is the information rate (R) of this modulation?

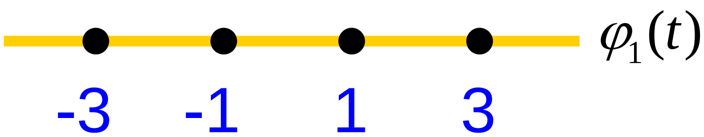
Manchester modulation example (Ethernet)



- Different waveforms can have same vector representations

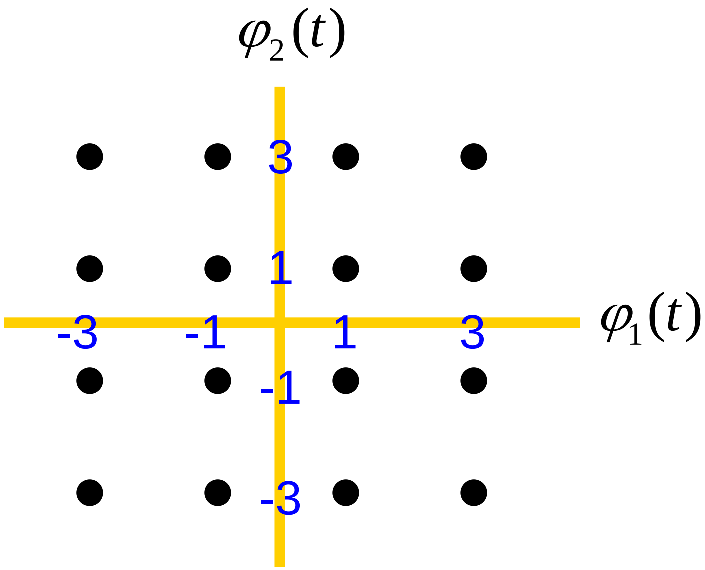
More constellations

Pulse Amplitude Modulation (PAM)



e.g. PAM4

Quadrature Amplitude Modulation (QAM)



e.g. 16-QAM

PAM and QAM have pulses as basis functions

How do we choose basis functions?

- Need to be orthonormal – (b/c of demodulation)

$$\int_{-\infty}^{\infty} \varphi_m(t)\varphi_n(t)dt = \delta_{mn} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$$

- Inner products
  - Continuous

$$\langle u(t), v(t)\rangle \triangleq \int_{-\infty}^{\infty} u(t)v(t)dt$$

  - Discrete

$$\langle \boldsymbol{u}, \boldsymbol{v}\rangle \triangleq \boldsymbol{u}^* \boldsymbol{v} = \sum_{n=1}^{N} u_n v_n$$

- Invariant to choice of basis functions

$$u(t) = \textstyle\sum_{n=1}^{N} u_n \varphi_n(t) \ \ and \ \ v(t) = \textstyle\sum_{n=1}^{N} v_n \varphi_n(t)$$

$$\begin{aligned}
\langle u(t), v(t)\rangle &= \int_{-\infty}^{\infty} u(t)v(t)dt = \int_{-\infty}^{\infty} \sum_{n=1}^{N}\sum_{m=1}^{N} u_n v_m \varphi_n(t)\varphi_m(t)dt \\
&= \sum_{n=1}^{N}\sum_{m=1}^{N} u_n v_m \int_{-\infty}^{\infty} \varphi_n(t)\varphi_m(t)dt = \sum_{m=1}^{N}\sum_{n=1}^{N} u_n v_m \delta_{nm} = \sum_{n=1}^{N} u_n v_n \\
&= \langle \boldsymbol{u}, \boldsymbol{v}\rangle \ \ \text{QED.}
\end{aligned}$$

- Average energy of the constellation
  - Invariant to the choice of basis functions

Constellation energy

- Implications of the inner product invariance to basis functions

$$\langle u(t), v(t)\rangle = \langle \boldsymbol{u}, \boldsymbol{v}\rangle$$

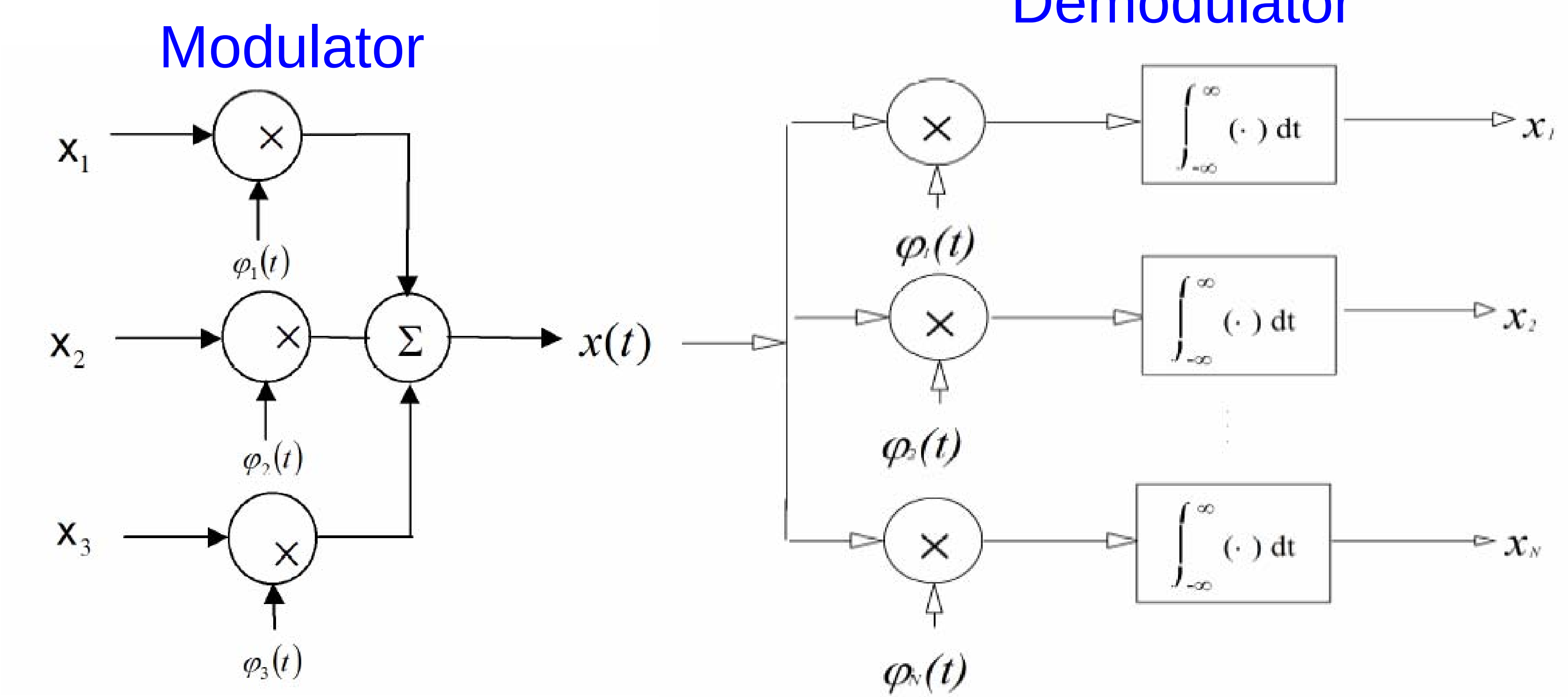
- If energy is a signal, it is the same regardless of the mod waveform used
  - As long as basis functions are orthogonal
  - Parseval's identity

$$\mathcal{E}_{\boldsymbol{x}} = E\left[||\boldsymbol{x}||^2\right] = E\left[\int_{-\infty}^{\infty} x^2(t)dt\right]$$

$$\begin{aligned} E\left[\langle u(t), v(t)\rangle\right] &= E\left[\langle \boldsymbol{x}, \boldsymbol{x}\rangle\right] \\ &= E\left[\sum_{n=1}^{N} x_n x_n\right] \\ &= E\left[\|\boldsymbol{x}\|^2\right] \\ &= \mathcal{E}_{\boldsymbol{x}} \ \text{QED}. \end{aligned}$$

Correlative demodulator

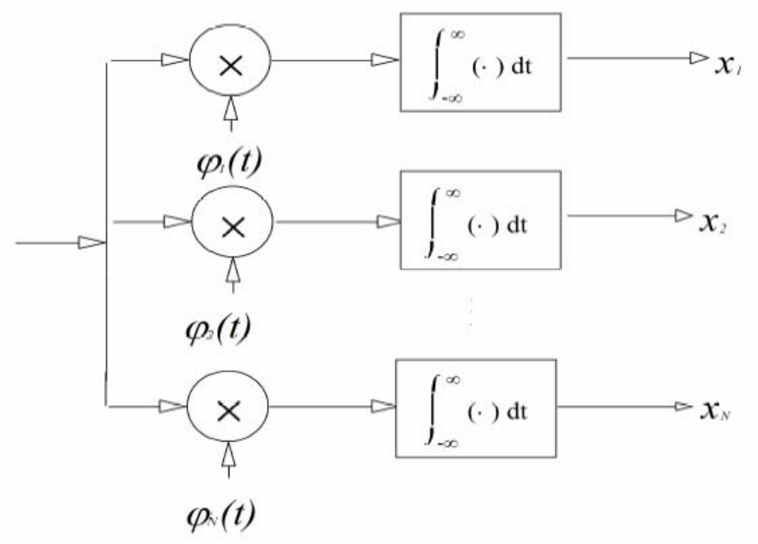
$$\int_{-\infty}^{\infty} \varphi_m(t)\varphi_n(t)dt = \delta_{mn} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$$



- Straightforward demodulator implementation
  - Use the fact that basis functions are orthogonal
  - Collect the signal energy
  - Hard to build in practice

Practical implementation

Correlative demodulator



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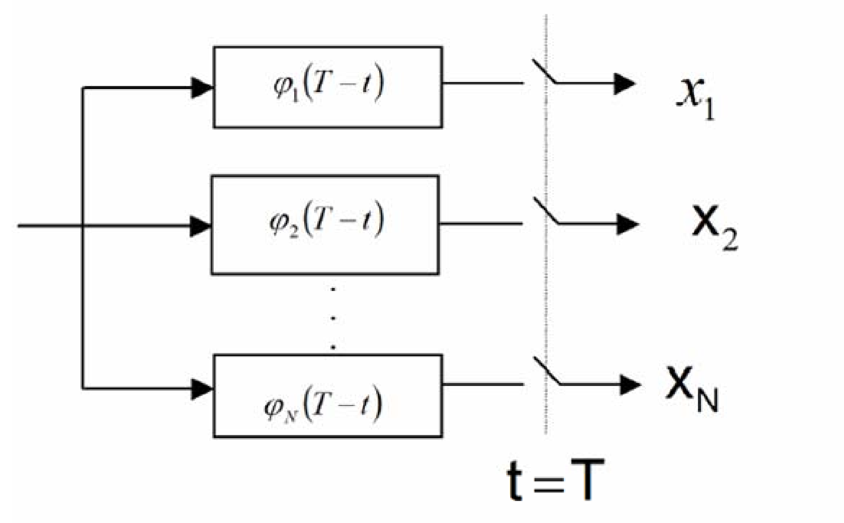
Matched-filter demodulator



- Note $x_n = \int_0^T x(t)\varphi_n(t)dt$ equivalent to $x(t) * \varphi_n(T-t)|_{t=T}$
- Can implement with an “integrate-and-dump”

Summary

- In this course you'll be able to learn
  - Practical digital communication techniques
    - Hands-on, little math
  - Hardware implementations
    - Algorithmic transformations
    - Micro-architectures
    - ASIC flow and behavioral modeling
- In other words, everything you'll need to start building cutting-edge digital communication systems
- Started intro to digital communications
  - Modulation – signal constellation, basis functions
  - Demodulation – basis function invariance, matched-filter
  - Next – basics of detection, signalling on band-limited channels

Sources

- VppSim/CppSim is a tool developed by prof. Michael Perrott
- Digital communications material is adapted from prof. John Cioffi's Stanford Course readers
  - http://www.stanford.edu/class/ee379a,b,c/