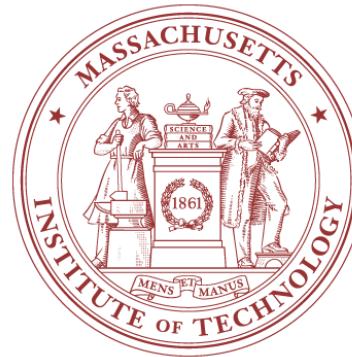


# Viterbi Algorithm - Implementation

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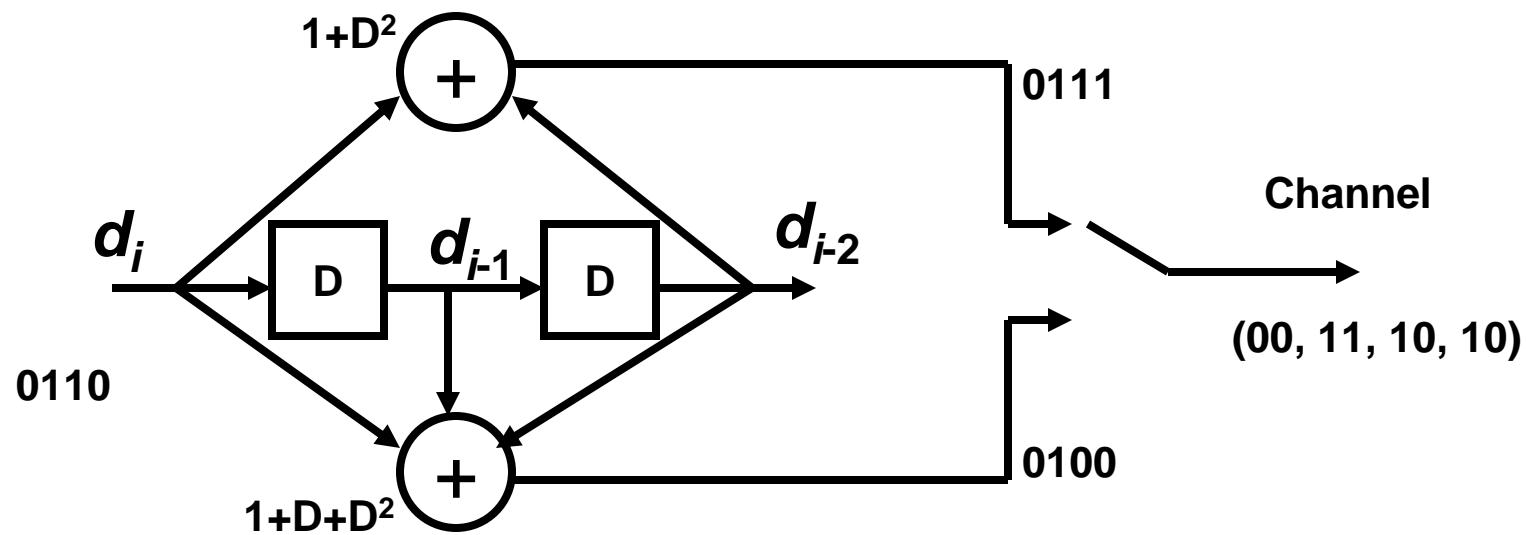
Lecture 14  
Vladimir Stojanović



6.973 Communication System Design – Spring 2006  
Massachusetts Institute of Technology

# Convolutional Codes

- Adding redundancy



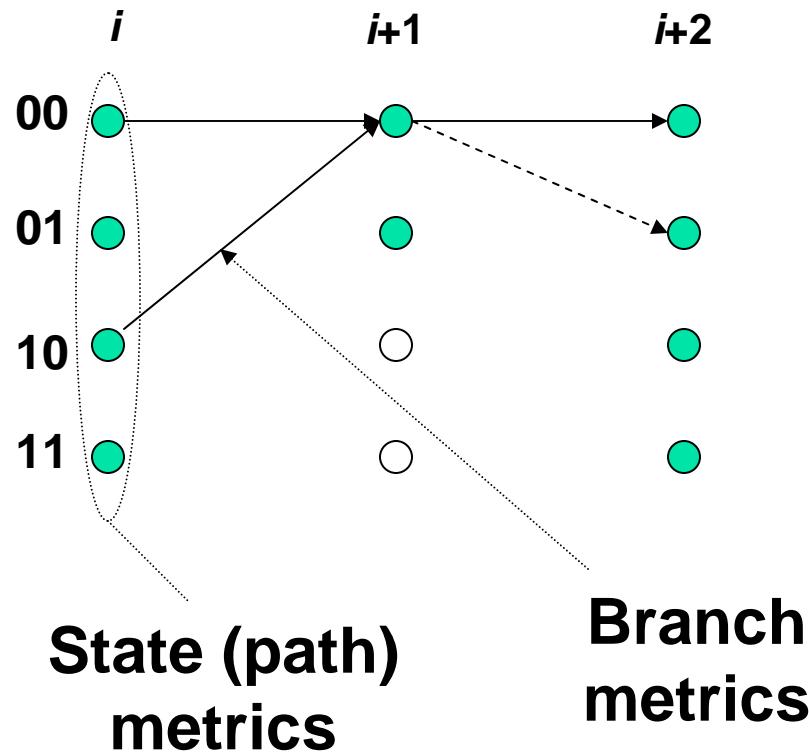
- Generators:

$$G_1 = 101$$

$$G_2 = 111$$



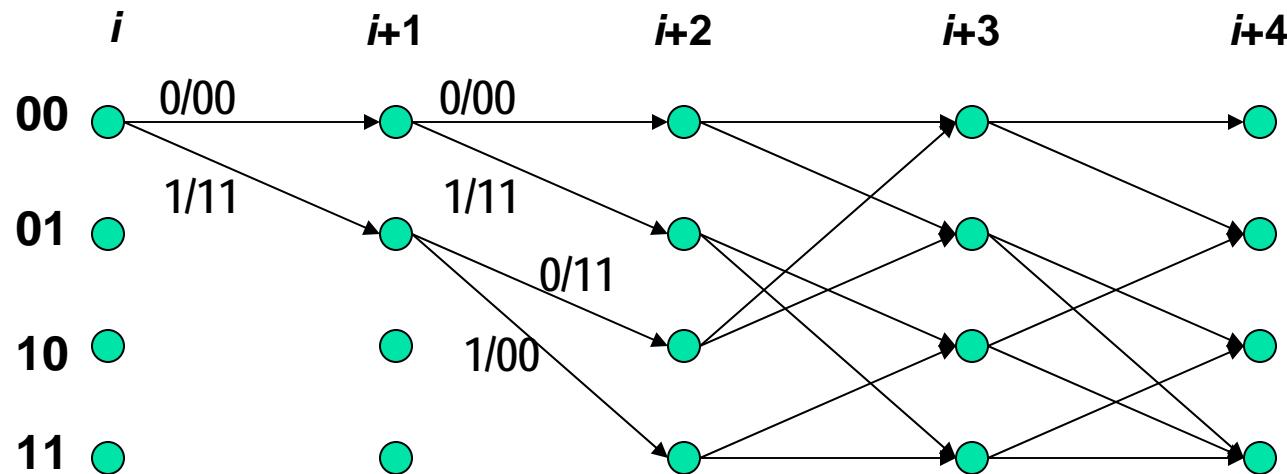
# Decisions at Each Step



- In Gaussian channel:  
 $bm = (y_k - s_k)^2$
  - In BSC:  
 $bm = d_H(y_k, s_k) = |y_k - s_k|$
- $d_H$  is Hamming distance
- If received  $y_k = 10$   
 $d_H(10, 00) = 1$   
 $d_H(10, 01) = 2$

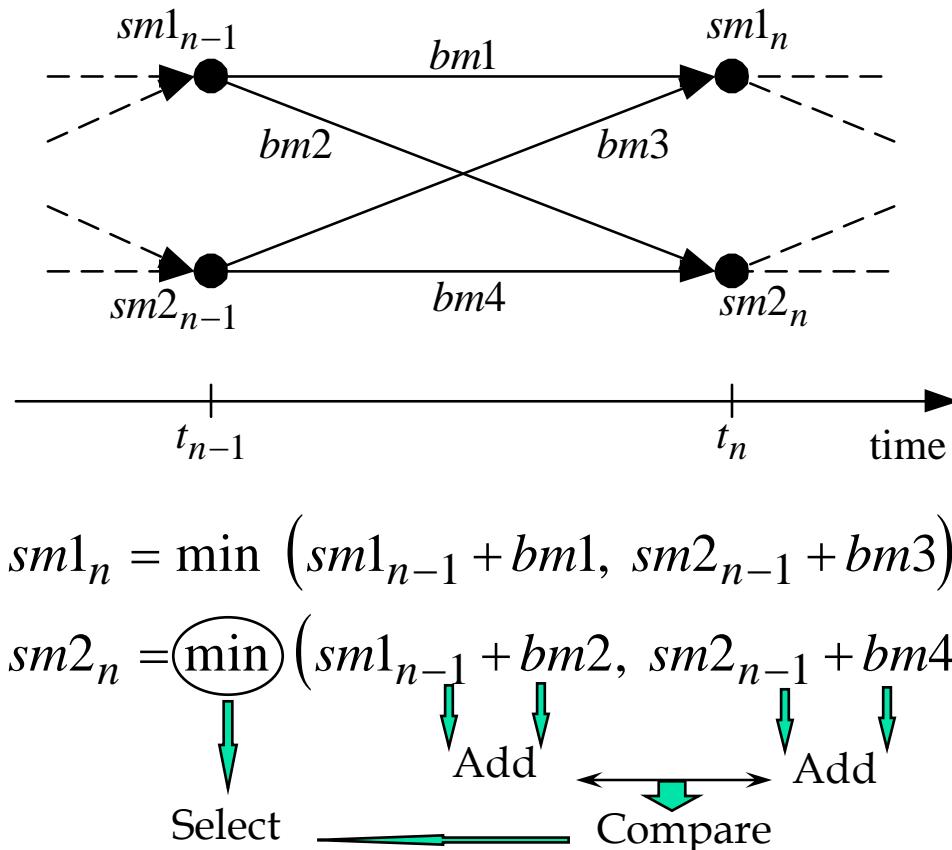
# Trellis Diagram

- Time-indexed state diagram



# The Viterbi Algorithm

- Illustrated by 2-state trellis



# Viterbi Decoder

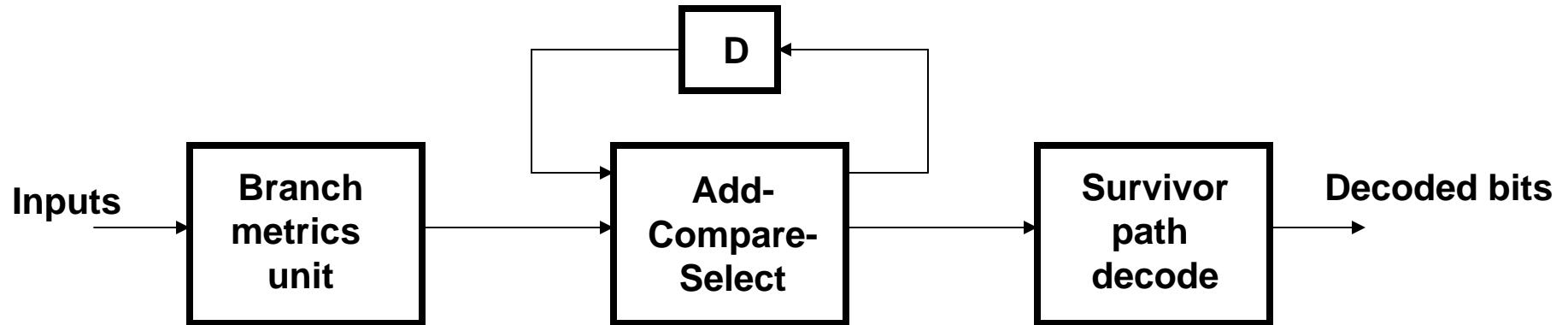
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- Implements Viterbi algorithm
- Three main components
  - Branch metric calculation
  - Path metric accumulation (add-compare-select recursion)
  - Survivor path decode



# Viterbi Decoder

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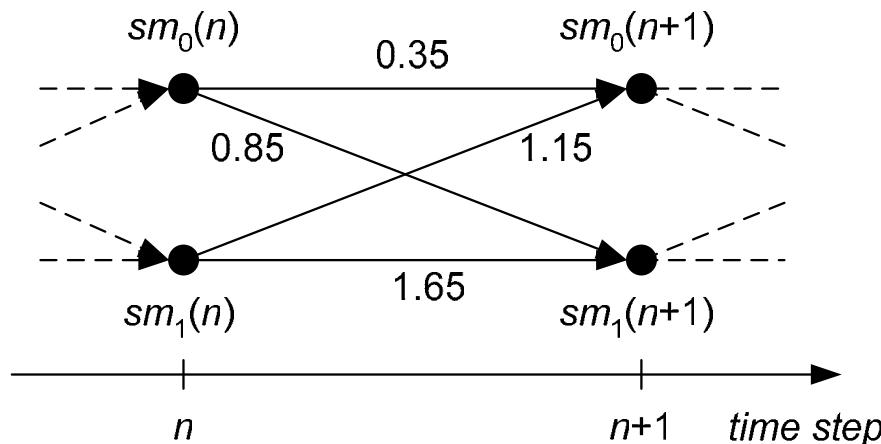


- ❑ Branch metrics unit
  - Calculates the distances between the received signal and the ideal signals
- ❑ Add-Compare-Select unit
  - Accumulates path metrics
- ❑ Survivor path decode
  - Keeps track of the path through the trellis



# Calculating Branch Metrics

- ❑ Assume  $G_1$  and  $G_2$  both output a 0
- ❑ Soft decoder inputs are 0.1 and 0.25 instead
- ❑ Branch metrics:
  - $bm_{00} = |0 - 0.1| + |0 - 0.25| = 0.35$
  - $bm_{01} = |0 - 0.1| + |1 - 0.25| = 0.85$
  - $bm_{10} = |1 - 0.1| + |0 - 0.25| = 1.15$
  - $bm_{11} = |1 - 0.1| + |1 - 0.25| = 1.65$



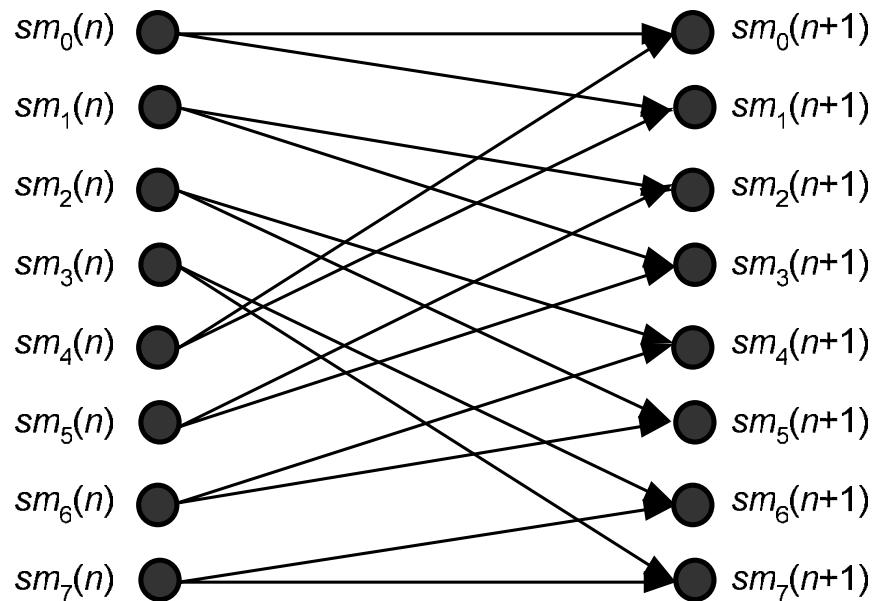
# Calculating Branch Metrics

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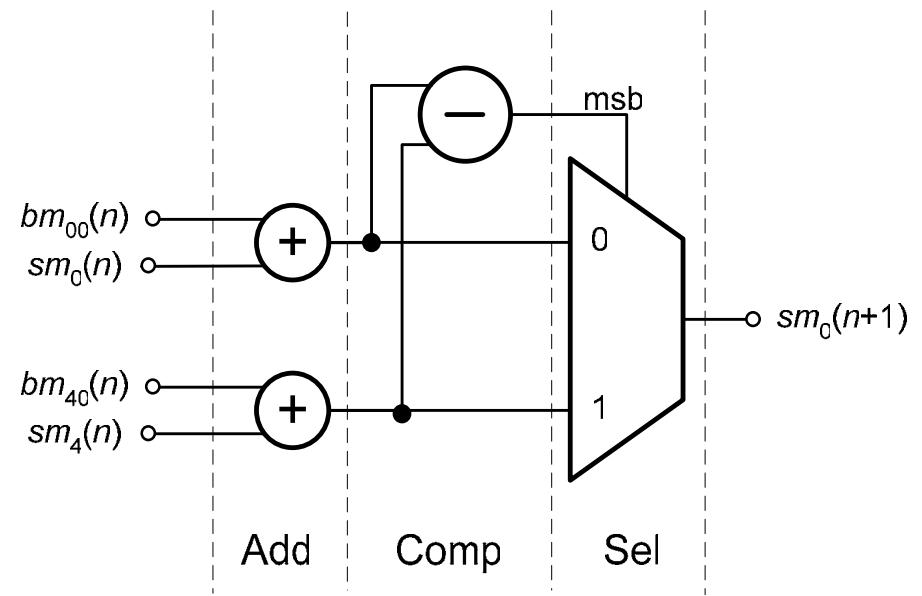
- Euclidean distances (AWGN channel)
- $bm_i = (y_k - s_k)^2$
- $s_k$  are usually integers  
$$bm_i = y_k^2 - 2C_i y_k + C_i^2$$
- Since  $y_k$  terms are common to all branch metrics they drop out in ACS comparison, and can be eliminated.
- $C_i^2$  are precomputed and  $2C_i y_k$  are shifts and adds.



# Add-Compare-Select Recursion

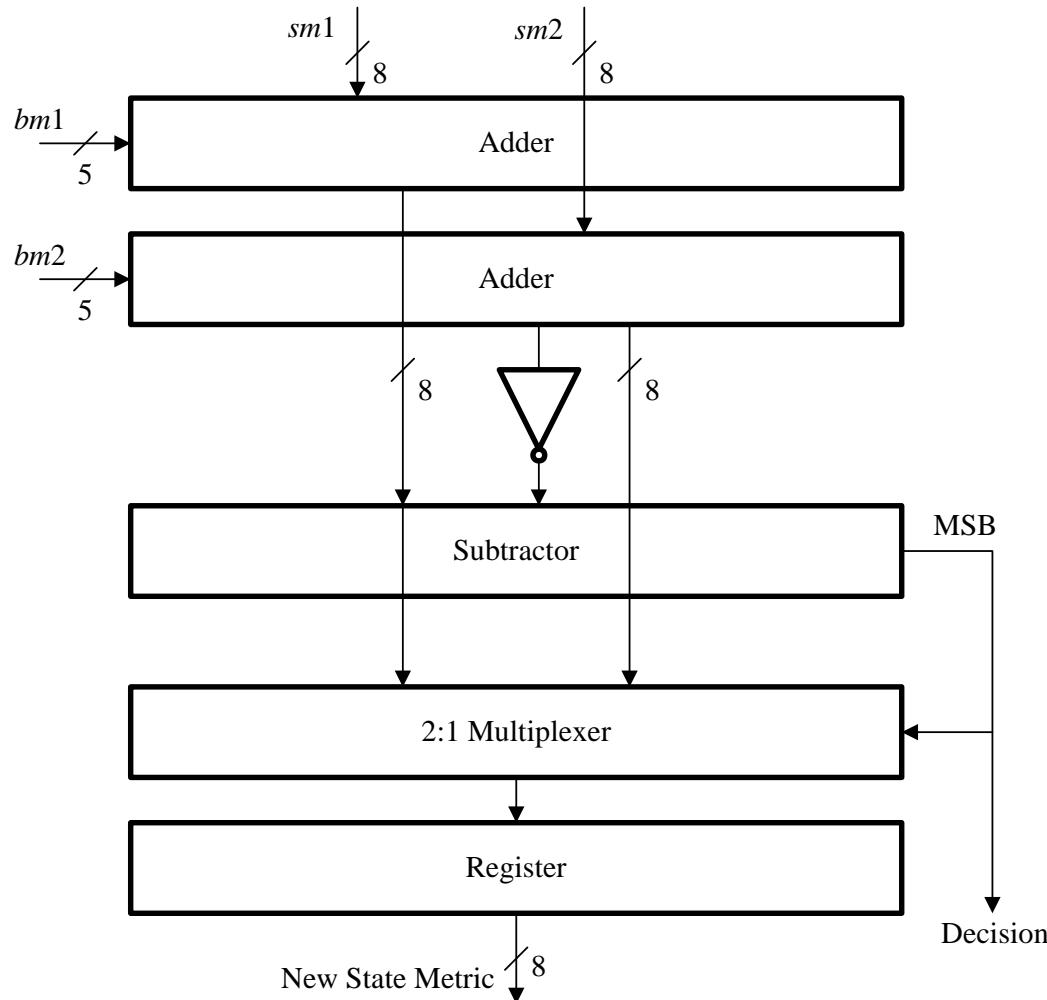


Eight state trellis

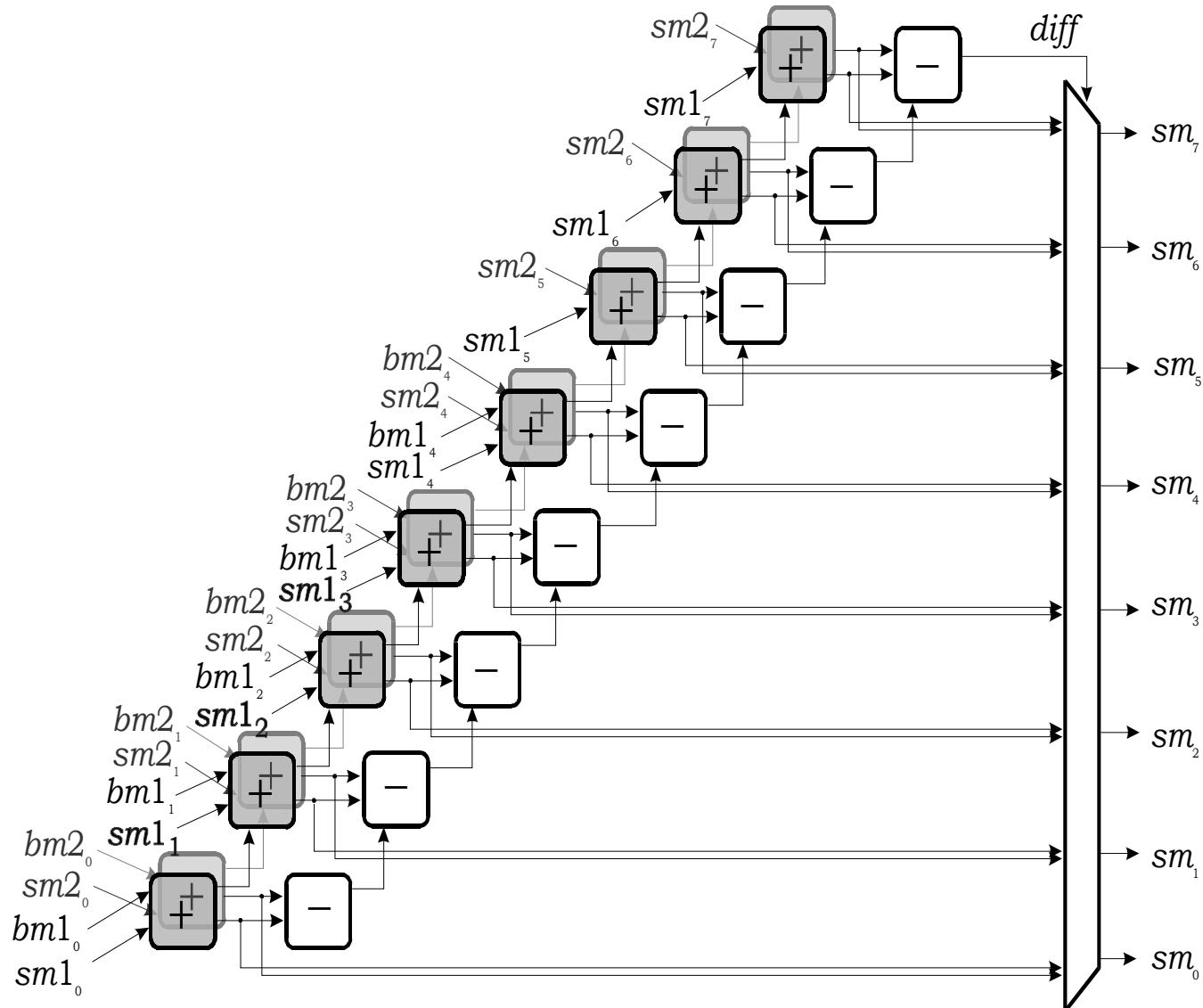


Conventional  
add-compare-select unit

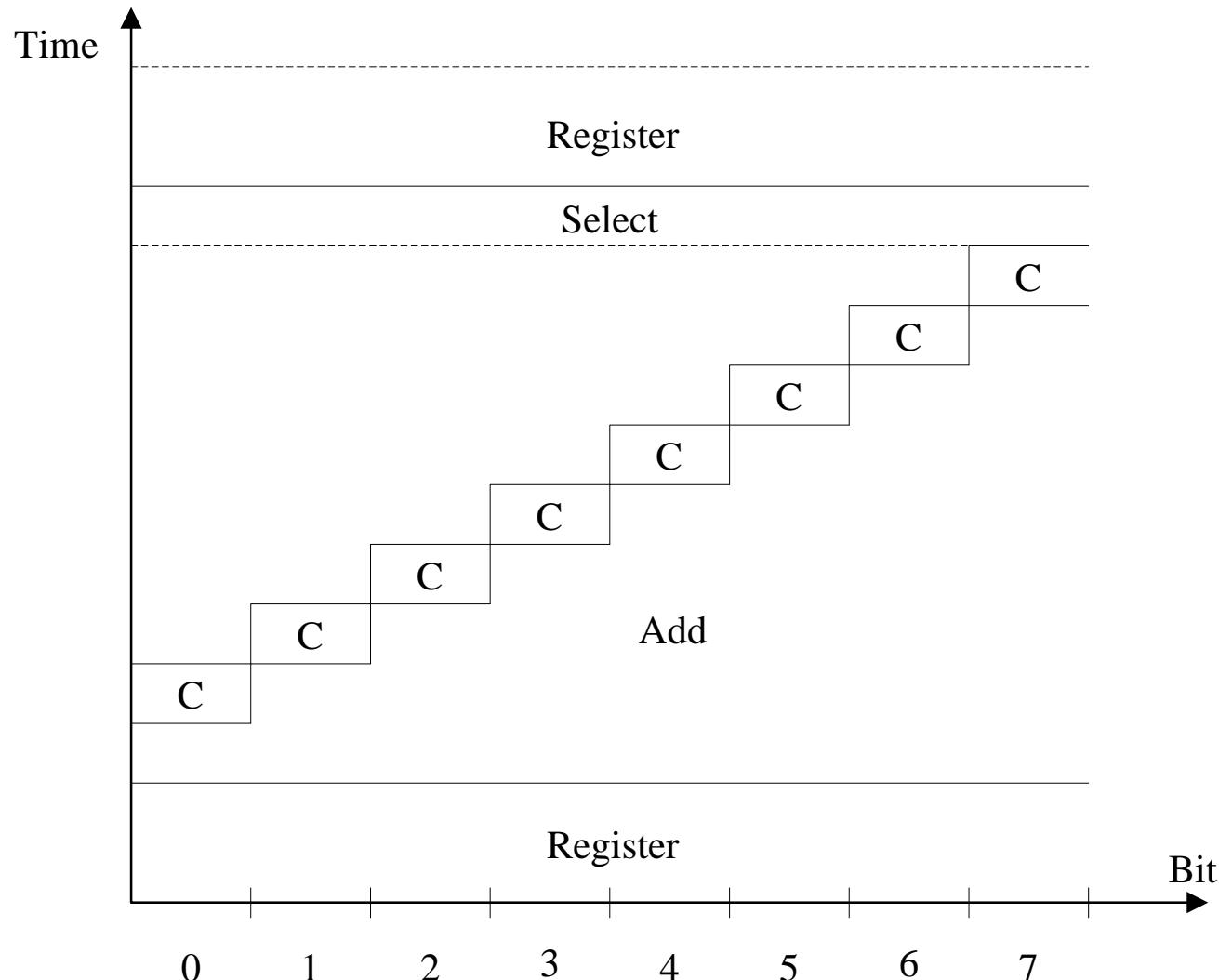
# Add-Compare-Select Recursion



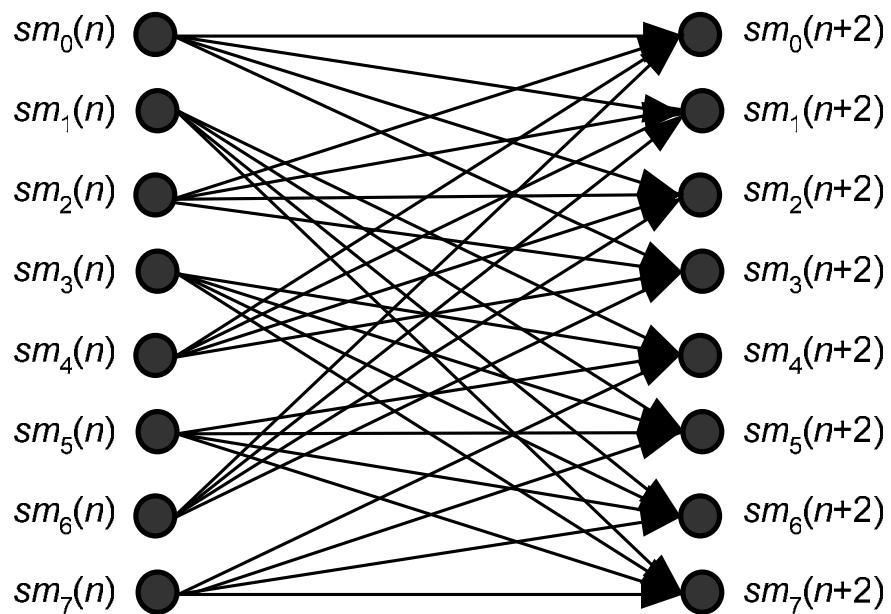
# Add-Compare-Select Recursion



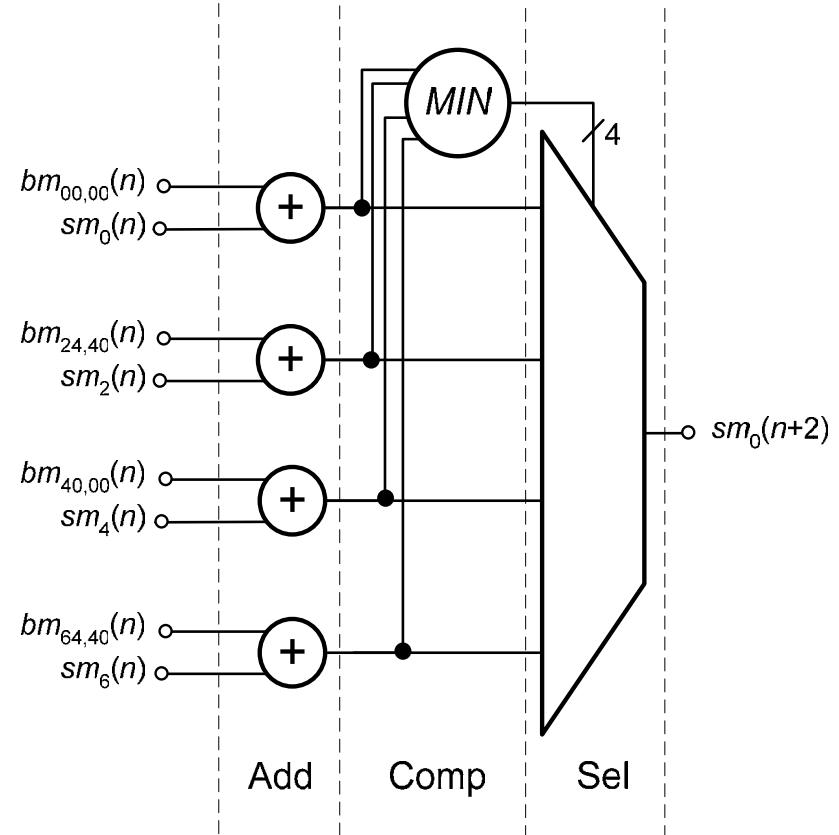
# Add-Compare-Select Recursion



# Add-Compare-Select Recursion



One step lookahead  
applied to an eight-state trellis

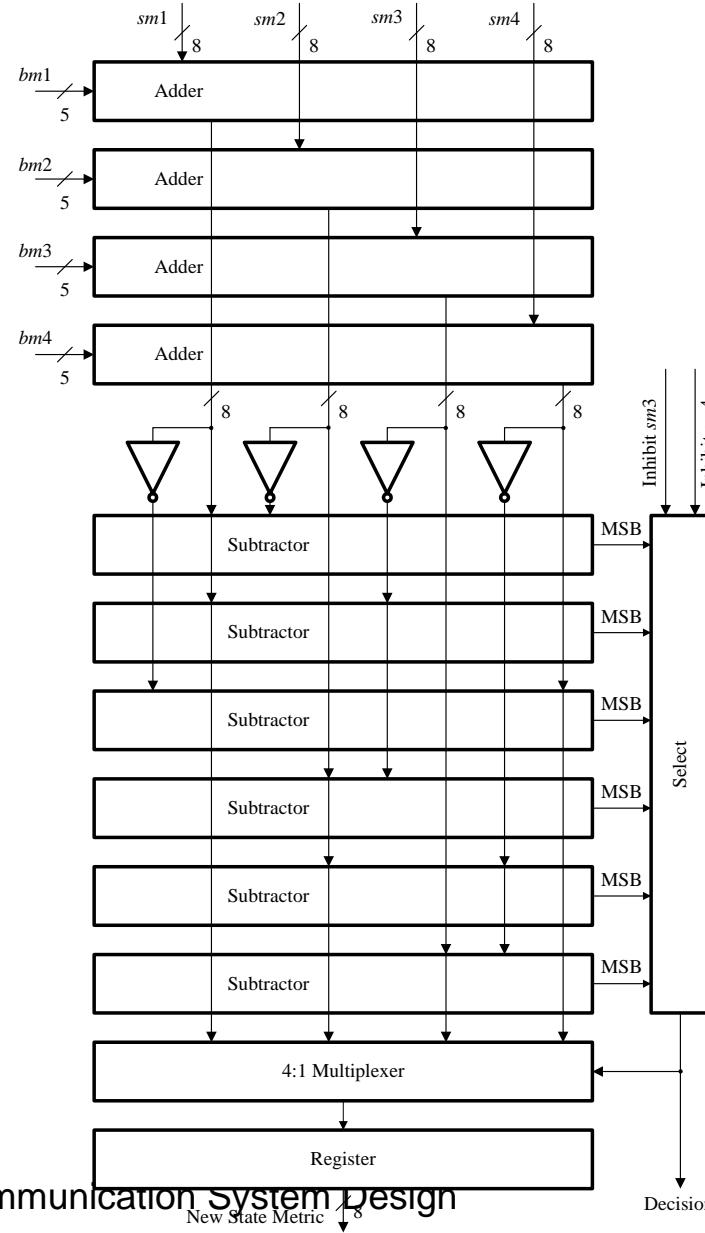


Radix-4  
add-compare-select unit

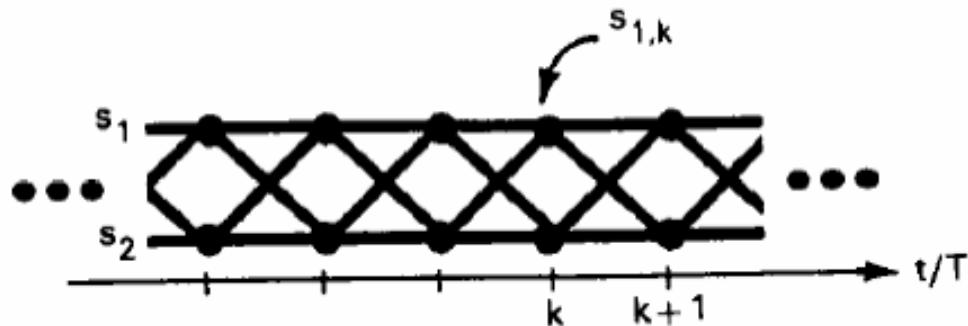


# Add-Compare-Select Recursion

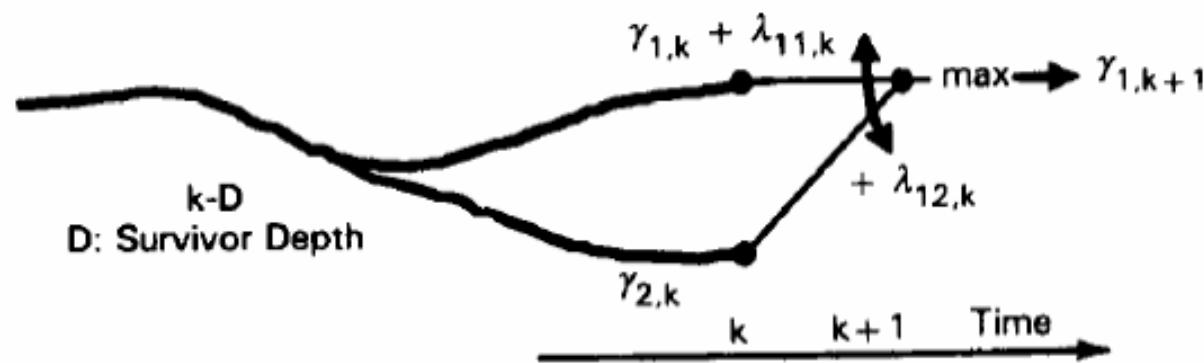
**4-way ACS:**  
**2 additions +**  
**6 comparisons**



# 2-state example



a. A trellis with  $N=2$  states



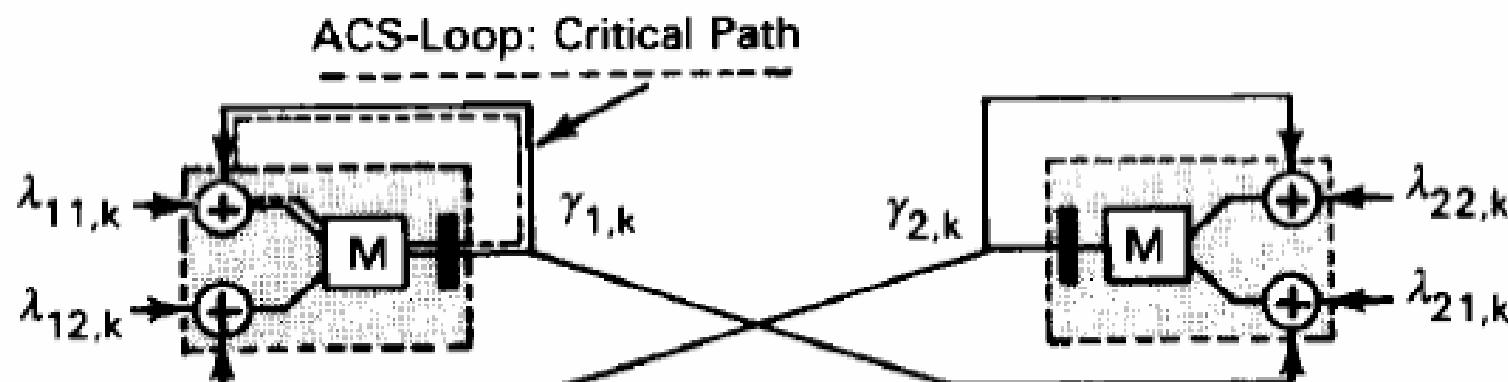
b. Decoding the optimum path to node  $s_{1,k+2}$  at time  $k+1$  and paths showing that they merge when traced back D steps.

Figure from Fettweis, G., and H. Meyr. "High-speed Parallel Viterbi Decoding: Algorithm and VLSI-architecture." *IEEE Communications Magazine* 29 (1991): 46-55. Copyright 1991 IEEE. Used with permission.





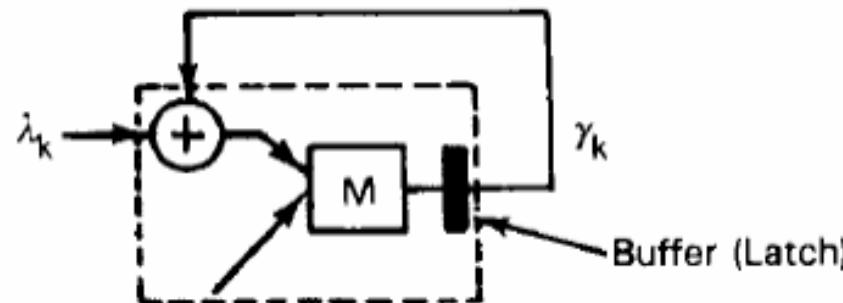
a. Example of a state diagram with  $N=2$  states



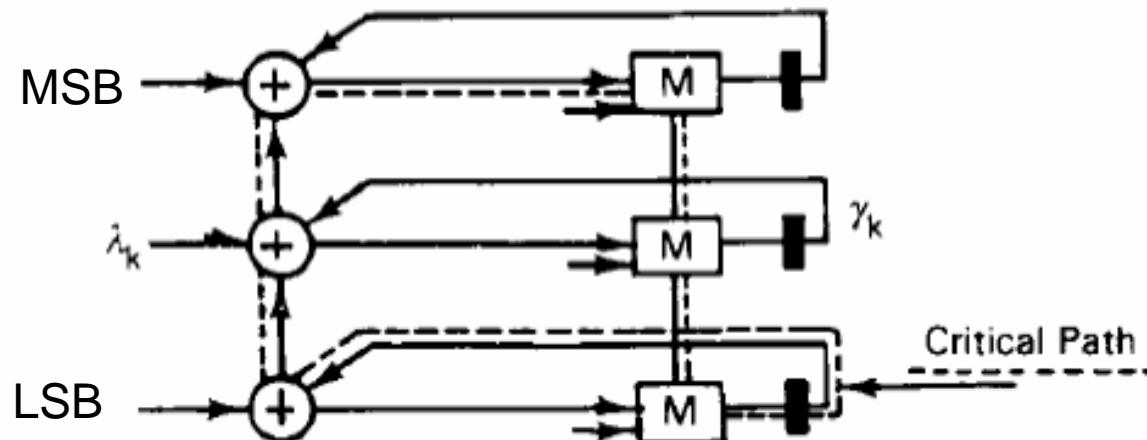
b. ACSU for the state diagram given

Figure from Fettweis, G., and H. Meyr. "High-speed Parallel Viterbi Decoding: Algorithm and VLSI-architecture." *IEEE Communications Magazine* 29 (1991): 46-55. Copyright 1991 IEEE. Used with permission.

# Bit-level view



a. The ACS-loop which comprises an addition and a maximum selection (M).

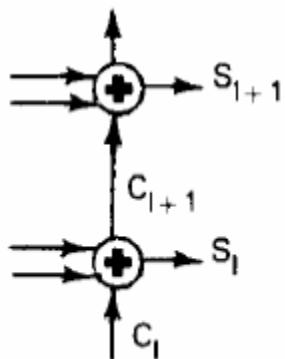


b. A 3-bit implementation of the ACS-loop.

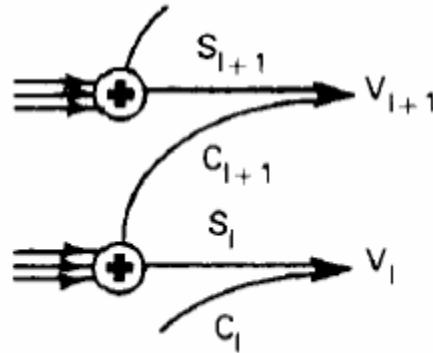
Figure from Fettweis, G., and H. Meyr. "High-speed Parallel Viterbi Decoding: Algorithm and VLSI-architecture." *IEEE Communications Magazine* 29 (1991): 46-55. Copyright 1991 IEEE. Used with permission.



# Turn into forward path and pipeline

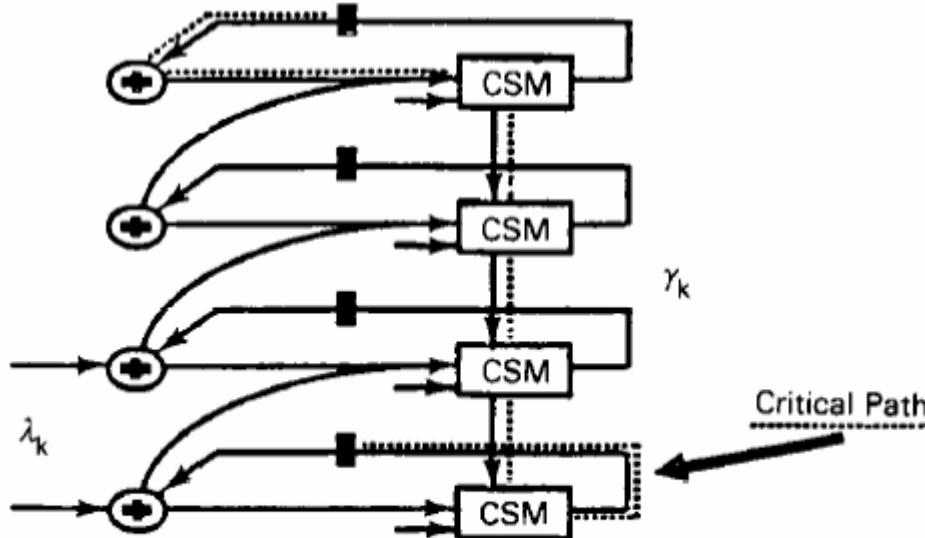


a. Carry-ripple addition.



b. Carry-save addition.

$$S = \sum_i s_i 2^i$$

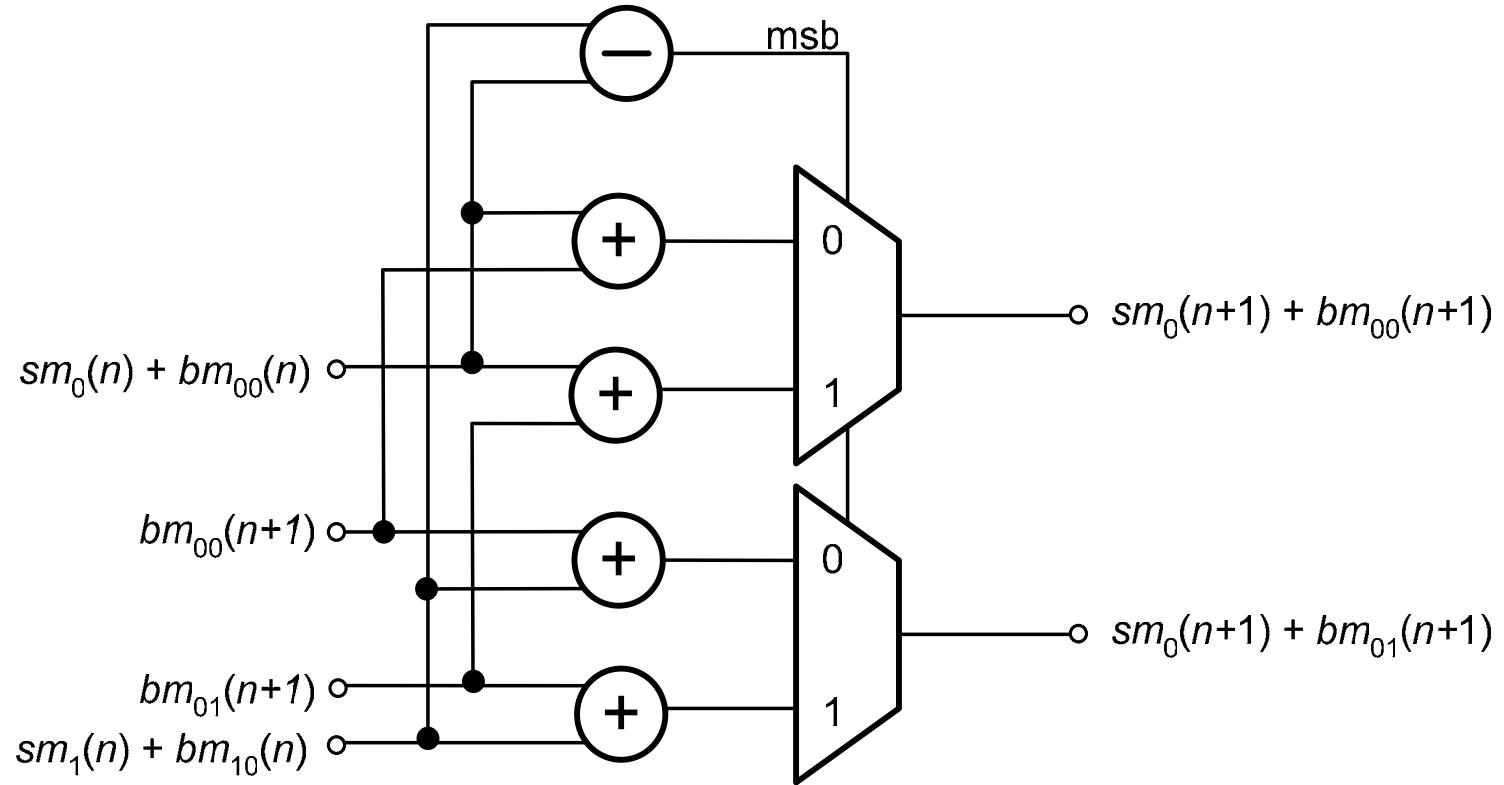


4-bit ACS-loop with CS-arithmetic.

Figures from Fettweis, G., and H. Meyr. "High-speed Parallel Viterbi Decoding: Algorithm and VLSI-architecture." *IEEE Communications Magazine* 29 (1991): 46-55. Copyright 1991 IEEE. Used with permission.



# Add-Compare-Select Recursion



Parallel compare-select-add unit



# References

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- ❑ Slides from Borivoje Nikolic and Bob Brodersen
- ❑ G. Fettweis and H. Meyr "High-speed parallel Viterbi decoding: algorithm and VLSI-architecture," *Communications Magazine, IEEE* vol. 29, no. 5, pp. 46-55, 1991.

