Shape Influence in Geodesic Active Contours

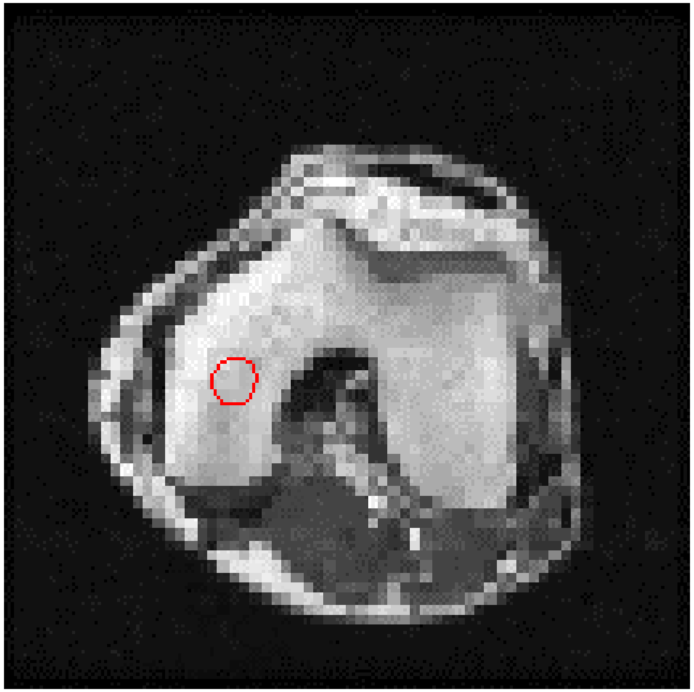
Leventon, Grimson, Faugeras

Presenter: Polina Golland

Images courtesy of Michael Leventon.

Segmentation

- Active Contours, ‘Snakes’, Level Sets



Geodesic Active Contours

- Snake methodology defines an energy function E(C) over a curve C as

$$E(\mathcal{C}) = \beta \int |\mathcal{C}'(q)|^2 dq - \lambda \int |\nabla I(\mathcal{C}(q))| dq$$

- Caselles, *et al*. reduced the minimization problem to the expression.

$$\min_{\mathcal{C}(q)} \int g(|\nabla I(\mathcal{C}(q))|)\ |\mathcal{C}'(q)|\ dq$$

where g is a function of the image gradient of the form $\frac{1}{1+|\nabla I|^2}$.

- The following curve evolution equation can be derived using Euler-Lagrange.

$$\frac{\partial \mathcal{C}(t)}{\partial t} = g\kappa\mathcal{N} - (\nabla g \cdot \mathcal{N})\mathcal{N}$$

where κ is the curvature and N is the normal.

- By defining an embedding function u of the curve C, the update equation for the higher dimensional surface is given by (Osher, Sethian '88):

$$\frac{\partial u}{\partial t} = g\ \kappa\ |\nabla u| + \nabla u \cdot \nabla g$$

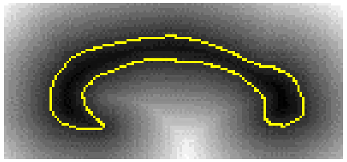
Shape Prior for Segmentation

- Train on a set of shapes
  - Mean shape
  - PCA-based model of variation

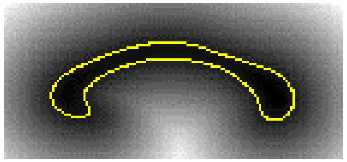
- Bias the segmentation towards likely shapes

Training Data

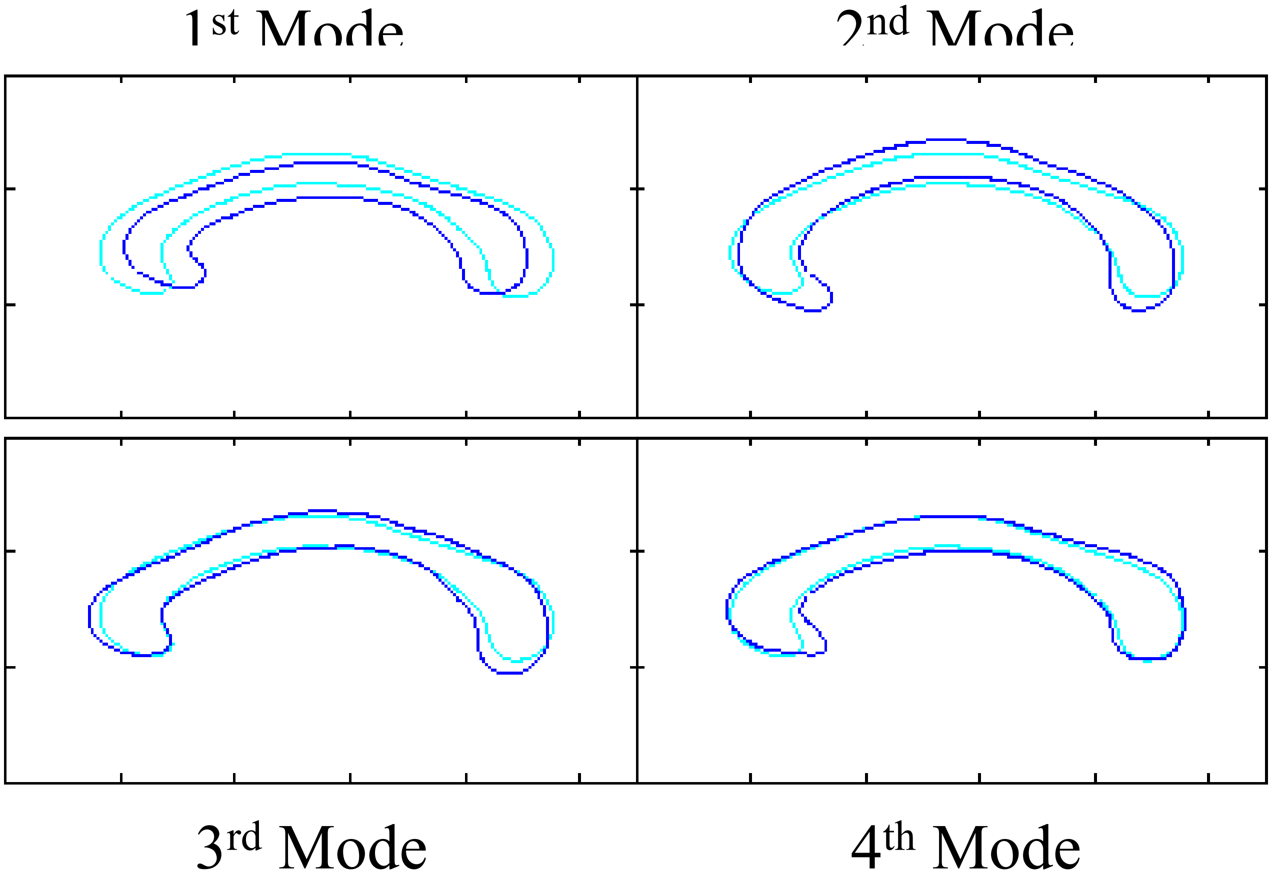
- The training set, T, consists of a set of surfaces: $T = \{u_1, u_2, \ldots, u_n\}$

$T = \{$  $, \ldots\}$

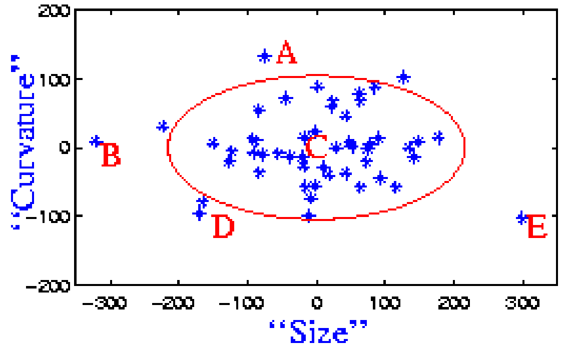
- The mean shape

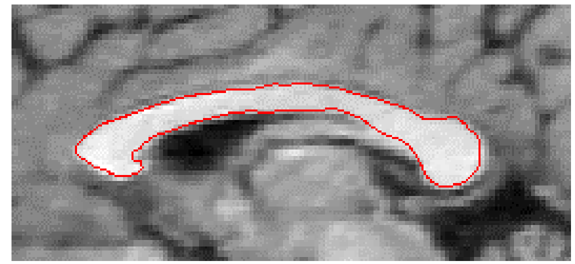
$\mu =$ 

Principal Modes of Variation
(using PCA)

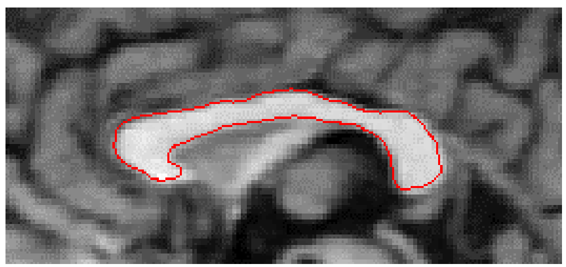


Shape Distribution

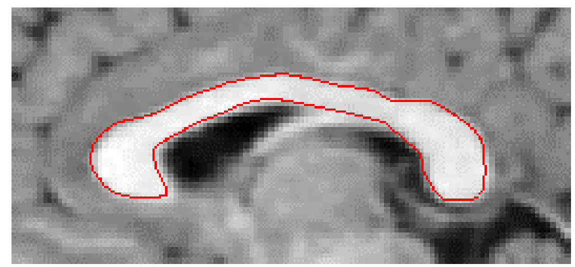




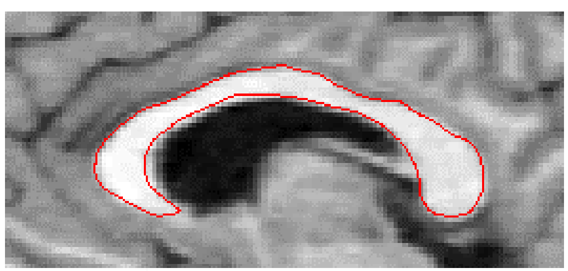
A



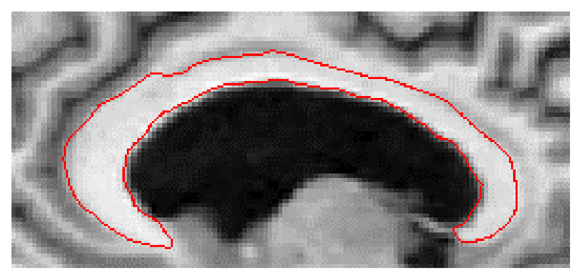
B



C



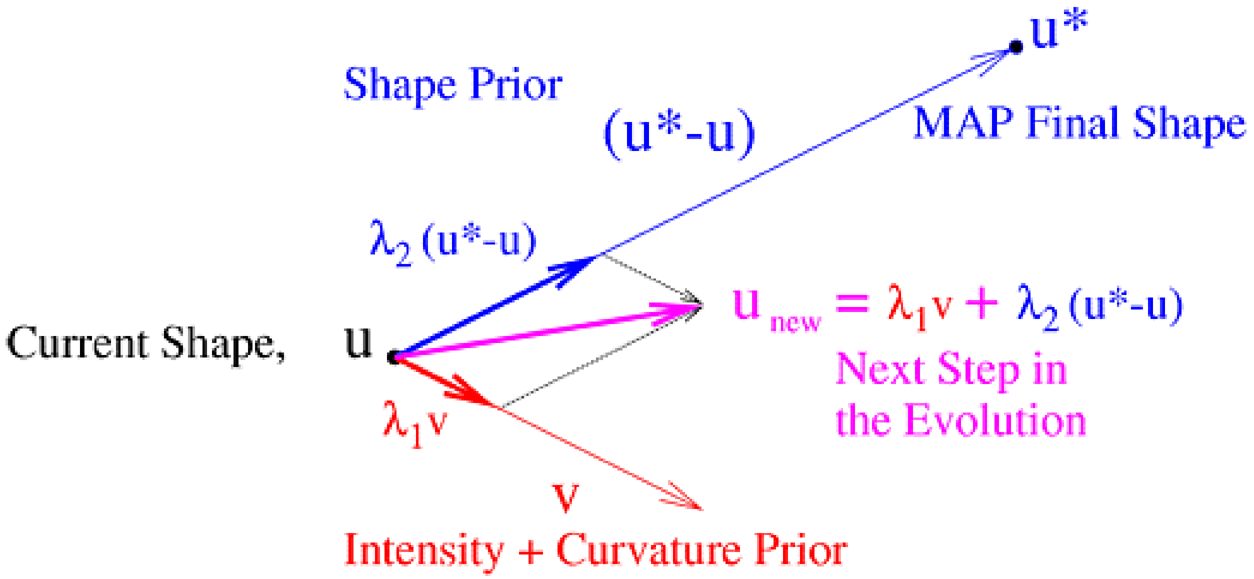
D



E

Modified Evolution Equation

$$u(t+1) = u(t) + \lambda_1 \left(g\,(c+\kappa)\,|\nabla u(t)| + \nabla u(t) \cdot \nabla g \right) + \lambda_2 \left(u^*(t) - u(t) \right)$$



Shape+Pose Estimation

- Given the current contour

$$\langle \alpha_{\mathrm{MAP}}, p_{\mathrm{MAP}} \rangle = \underset{\alpha, p}{\operatorname{argmax}} \, P(\alpha, p \mid u, \nabla I)$$

- Probability model

$$\begin{aligned} P(\alpha, p \mid u, \nabla I) &= \frac{P(u, \nabla I \mid \alpha, p) P(\alpha, p)}{P(u, \nabla I)} \\ &= \frac{P(u \mid \alpha, p) P(\nabla I \mid \alpha, p, u) P(\alpha) P(p)}{P(u, \nabla I)} \end{aligned}$$

Shape+Pose Estimation (cont'd)

$$P(u \mid \alpha, p) P(\nabla I \mid \alpha, p, u) P(\alpha) P(p)$$

- Inside term

$$P(u \mid \alpha, p) = \exp\left(-V_{outside}\right)$$

- Gradient Term

$$P(\nabla I \mid u^*, u) = \exp\left(-\left| h(u^*) - |\nabla I| \right|^2\right)$$

- Shape prior: Gaussian (PCA model)
- Pose prior: uniform over the image

Modified Evolution Equation

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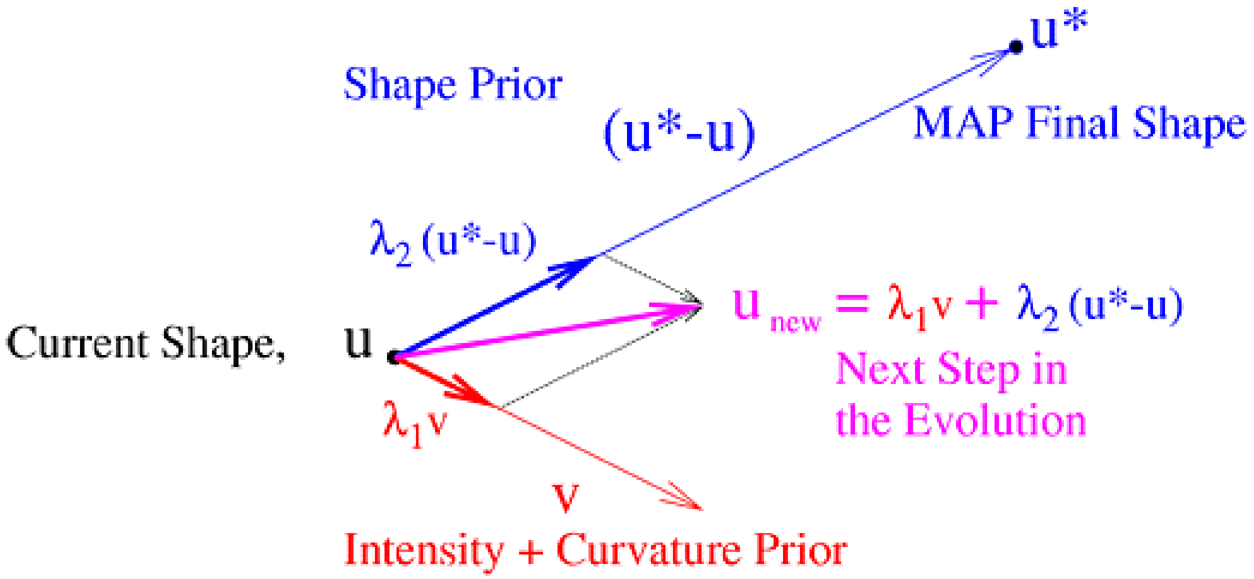
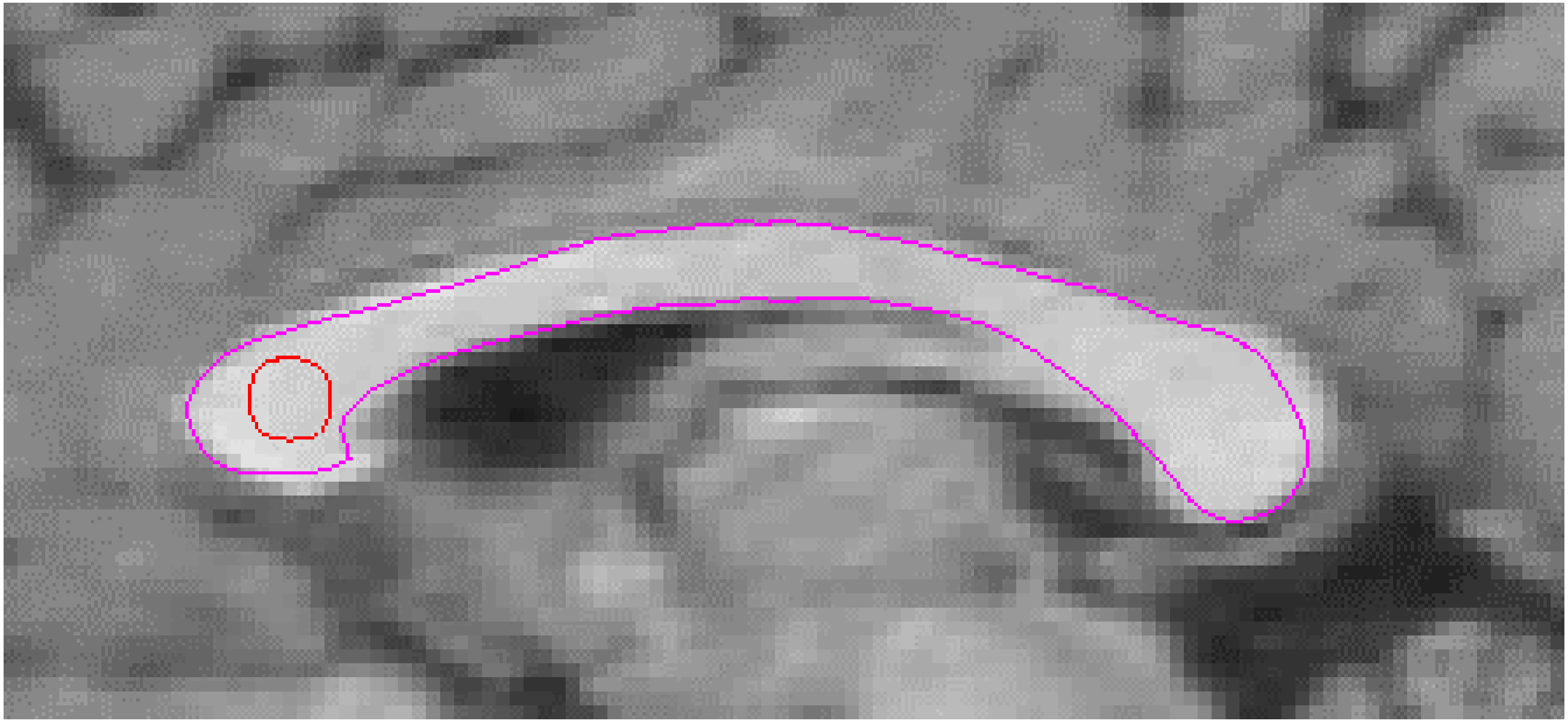


Corpus Callosum Segmentation



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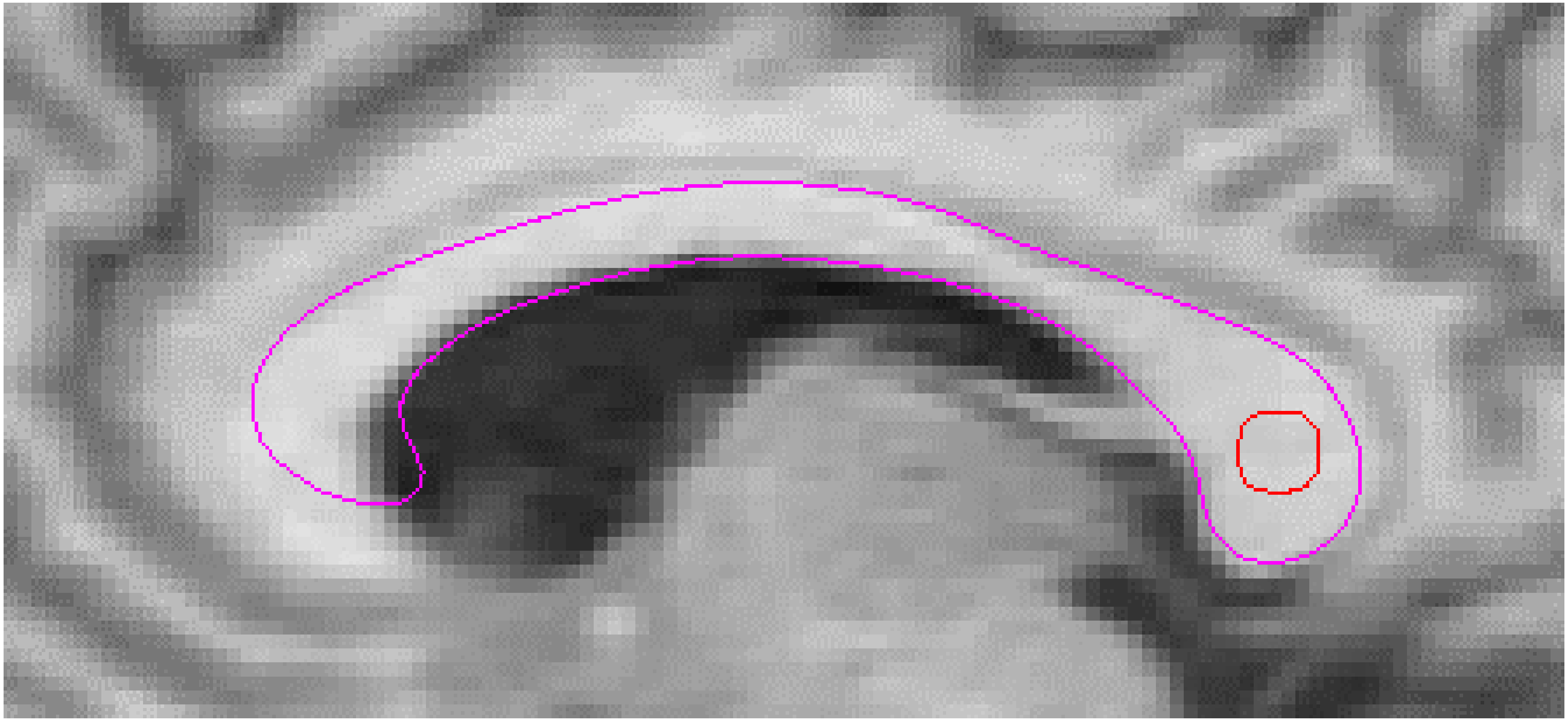
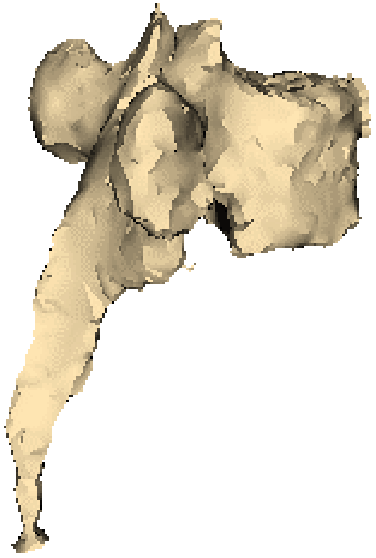


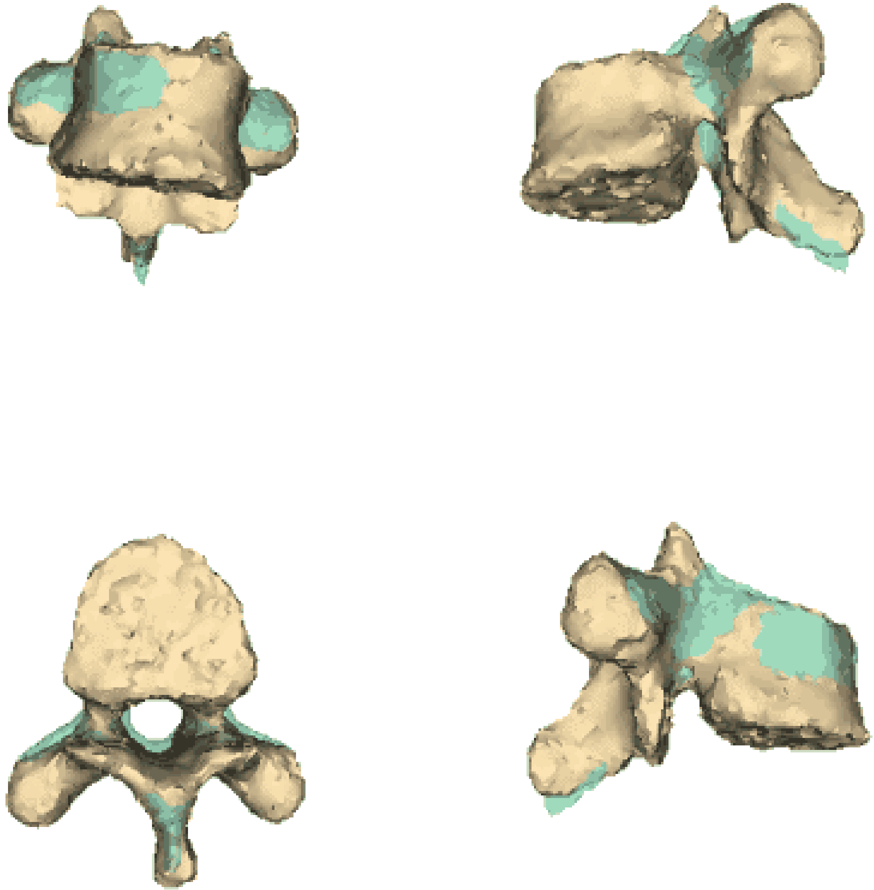
Spine Modes

- 3D Models of seven thoracic vertebrae (T3-T9)

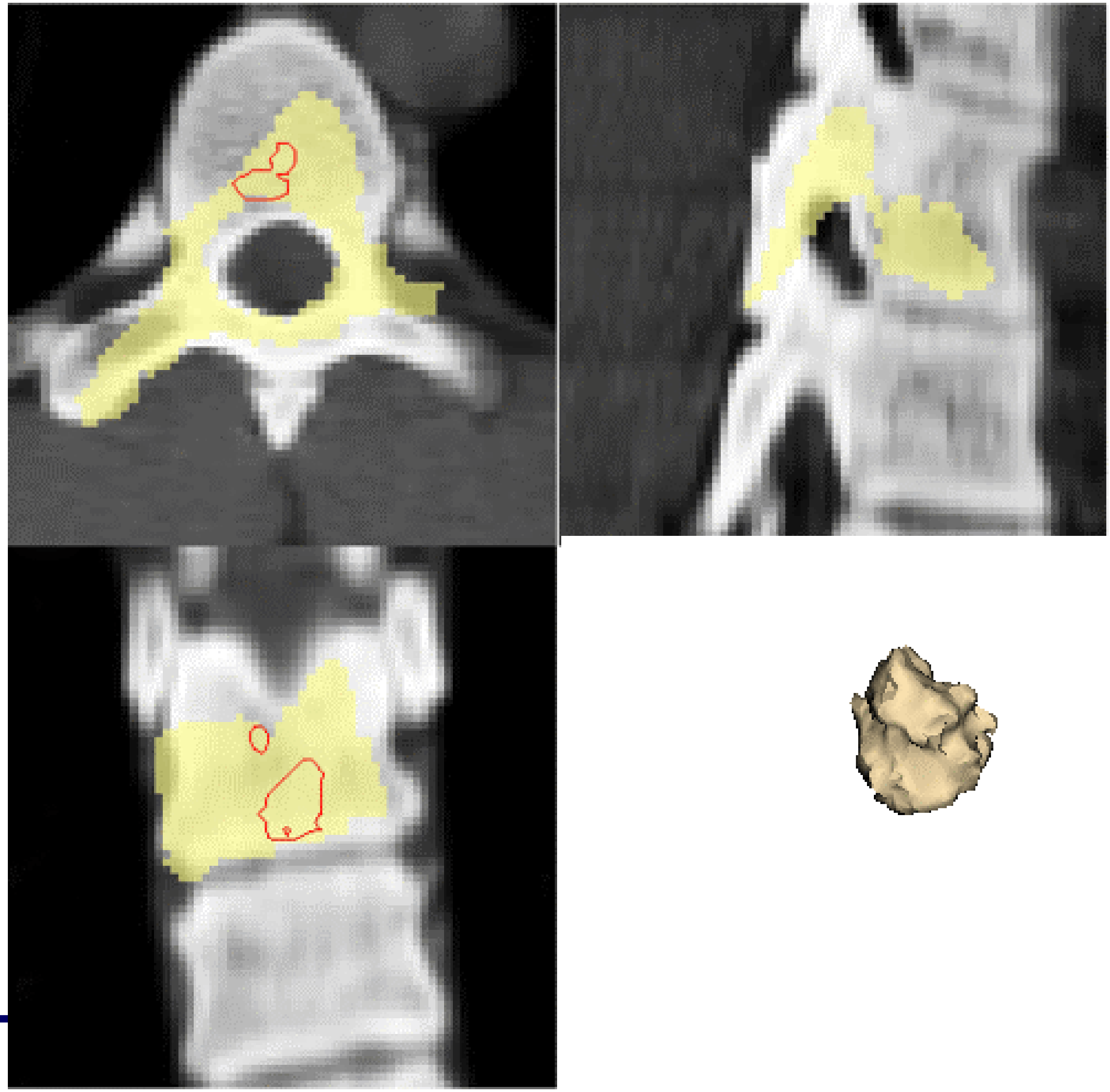


Spine Mean Shape

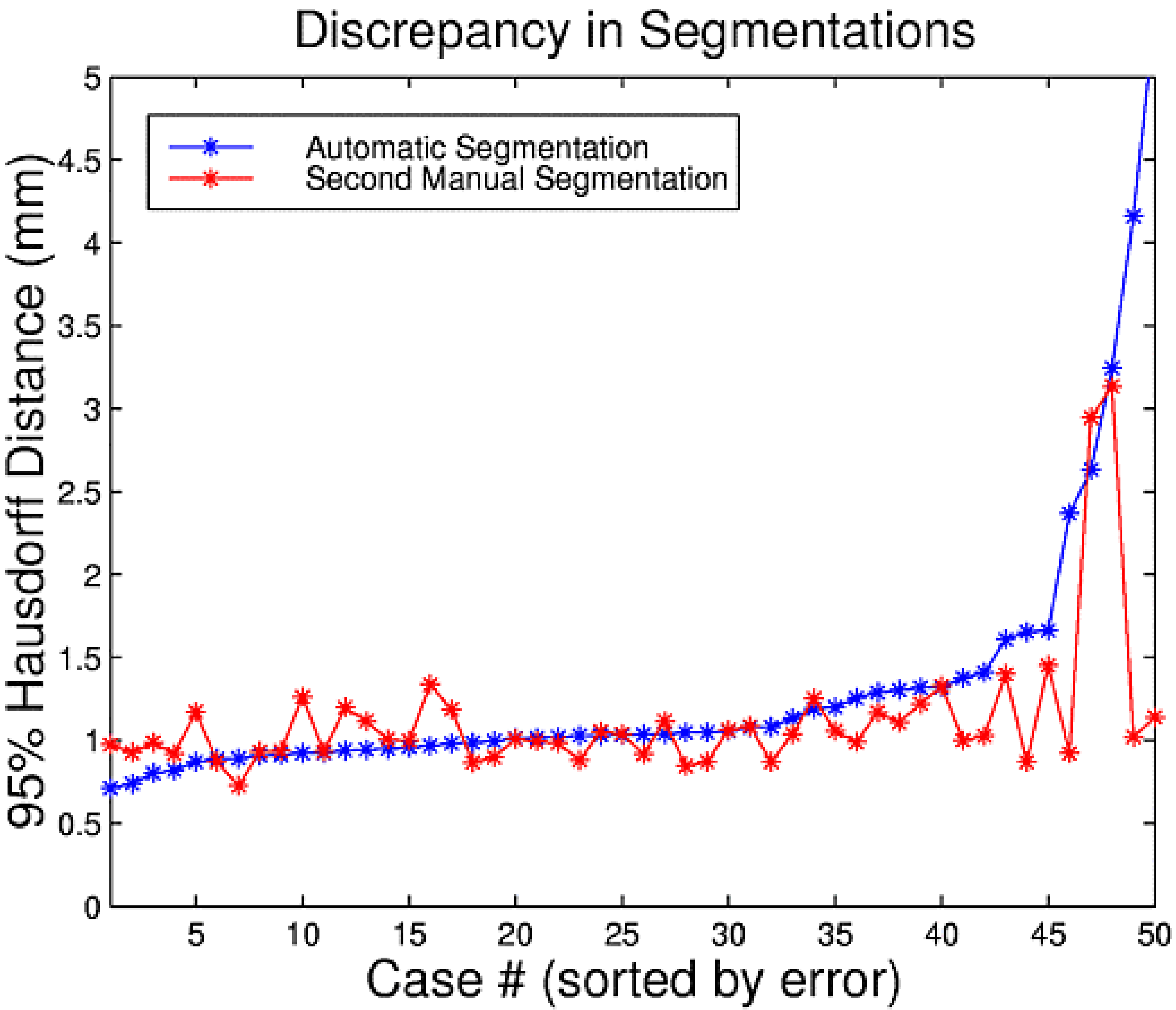
Spine 1st Mode of Variation



Segmentation of the Vertebrae



Comparison to human expert



Summary

- Introduced shape prior into curve evolution
  - Previous work – Fourier decomposition and ASH
  - This one is the first for the level set formulation
- PCA on training examples
- 2D and 3D
- Several follow-up methods that perform optimization differently.