6.852: Distributed Algorithms
Fall, 2009

Class 19

Today's plan

- Techniques for implementing concurrent objects:
  - Coarse-grained mutual exclusion
  - Fine-grained locking (mutex and read/write)
  - Optimistic locking
  - Lock-free/nonblocking algorithms
  - "Lazy" synchronization
- We illustrate on list-based sets, but the techniques apply to other data structures
- Reading:
  - Herlihy, Shavit, Chapter 9
- Next:
  - Transactional memory
  - HS, Chapter 18
  - Guerraoui, Kapalka

Shared-memory model

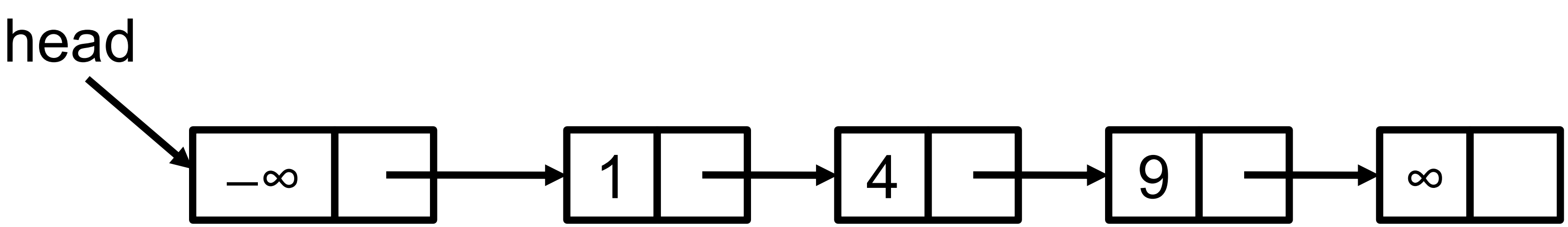
- Shared variables
- At most one memory access per atomic step.
- Read/write access
- Synchronization primitives:
  - Compare-and-swap (CAS)
  - Load-linked/store-conditional (LL/SC)
  - Assume lock and unlock methods for every object.
- Most (not all) of our algorithms use locking.
- Memory management:
  - Allocate objects dynamically, assume unlimited supply.
  - In practice, would garbage collect and reuse, but we won't worry about this.
- Assume no failures (mostly).

Correctness guarantees

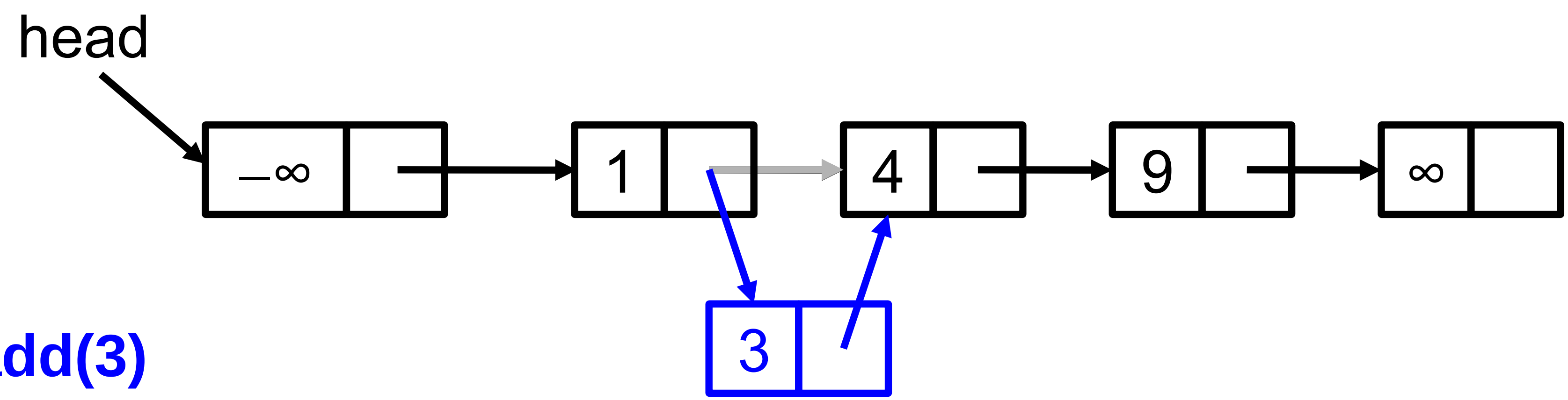
- Linearizability (atomicity) of object operations.
- Liveness properties:
  - Different guarantees for different algorithms.
  - Progress:
    - Some operations keep completing.
  - Lockout-freedom (AKA starvation-freedom):
    - Every operation completes.
  - "Nonblocking" conditions:
    - Wait-freedom: Even if other processes stop, a particular operation by a process that keeps taking steps eventually finishes.
    - Lock-freedom: Even if some processes stop, if some keep taking steps, then some operation finishes.
    - Can think of the stopped processes as failing, or as going slowly.
    - Captures the idea that slow processes don't block others.
    - Rules out locking strategies.
- Performance
  - Worst-case (time bounds) vs. average case (throughput).
  - No good formal models

List-based sets

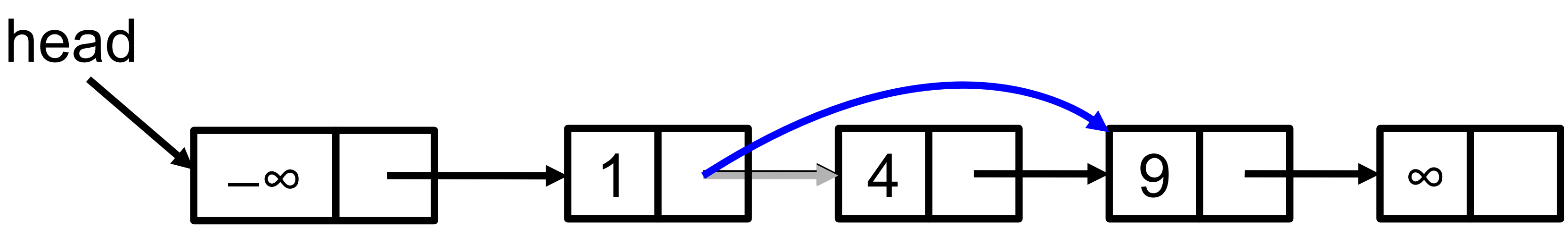
- Data type: Set S of integers (no duplicates)
  - S.add(x): Boolean: S := S ∪ {x}; return true iff x not already in S
  - S.remove(x): Boolean: S := S \ {x}; return true iff x in S initially
  - S.contains(x): Boolean: return true iff x in S (no change to S)
- Simple ordered linked-list-based implementation
  - Illustrates techniques useful for pointer-based data structures.
  - Unless set is small, this is a poor data structure for this specific data type--better to use arrays, hash tables, etc.



Sequential list-based set



add(3)



remove(4)

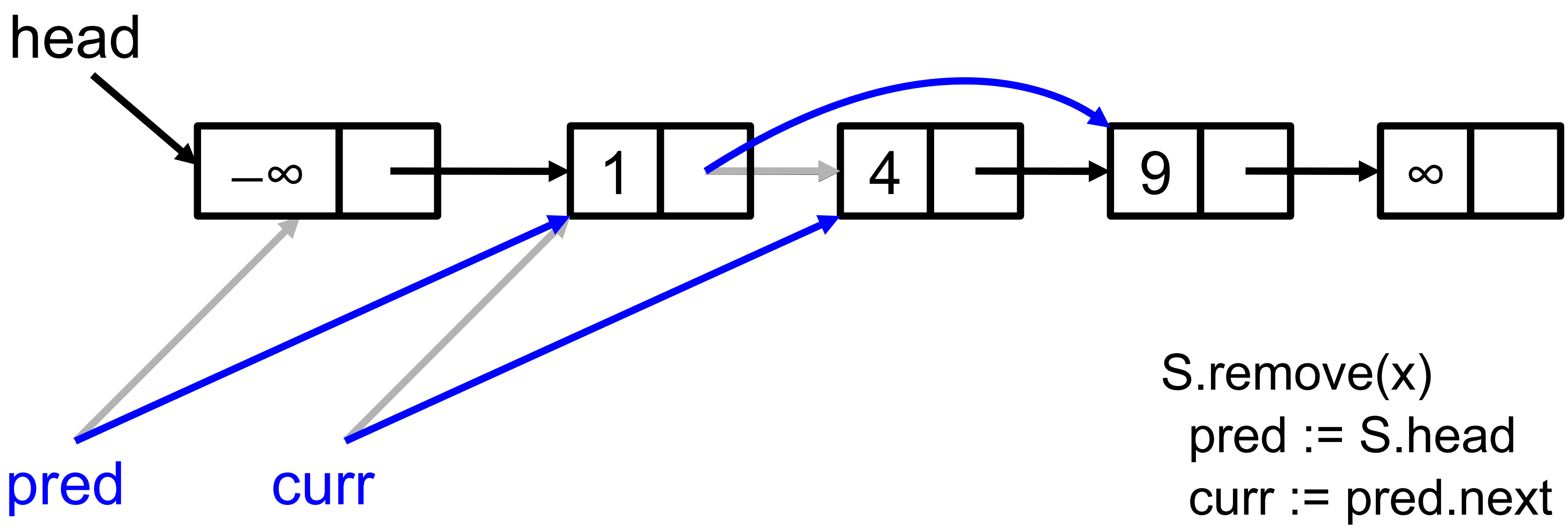
Sequential list-based set

```
S.add(x)
  pred := S.head
  curr := pred.next
  while (curr.key < x)
    pred := curr
    curr := pred.next
  if curr.key = x then
    return false
  else
    node := new Node(x)
    node.next := curr
    pred.next := node
    return true
```

```
S.remove(x)
  pred := S.head
  curr := pred.next
  while (curr.key < x)
    pred := curr
    curr := pred.next
  if curr.key = x then
    pred.next := curr.next
    return true
  else
    return false
```

```
S.contains(x)
  curr := S.head
  while (curr.key < x)
    curr := curr.next
  if curr.key = x then
    return true
  else
    return false
```

Sequential list-based set



remove(4)

```
S.remove(x)
  pred := S.head
  curr := pred.next
  while (curr.key < x)
    pred := curr
    curr := pred.next
  if curr.key = x then
    pred.next := curr.next
    return true
  else
    return false
```

Correctness

- Assume algorithm queues up operations, runs them sequentially.
- Atomicity (linearizability):
  - Show the algorithm implements a canonical atomic set object.
  - Use forward simulation relation: Set consists of those elements that are reachable from the head of the list via list pointers.
  - When do "perform" steps occur?
    - add(x): If successful, then when pred.next := node, else any time during the operation.
    - remove(x): If successful, then when pred.next := curr.next, else any time during the operation.
    - contains(x): Any time during the operation.
  - Proof uses invariants saying that the list is ordered and contains no duplicates.
- Liveness: Lockout-free, but blocking (not wait-free or lock-free)

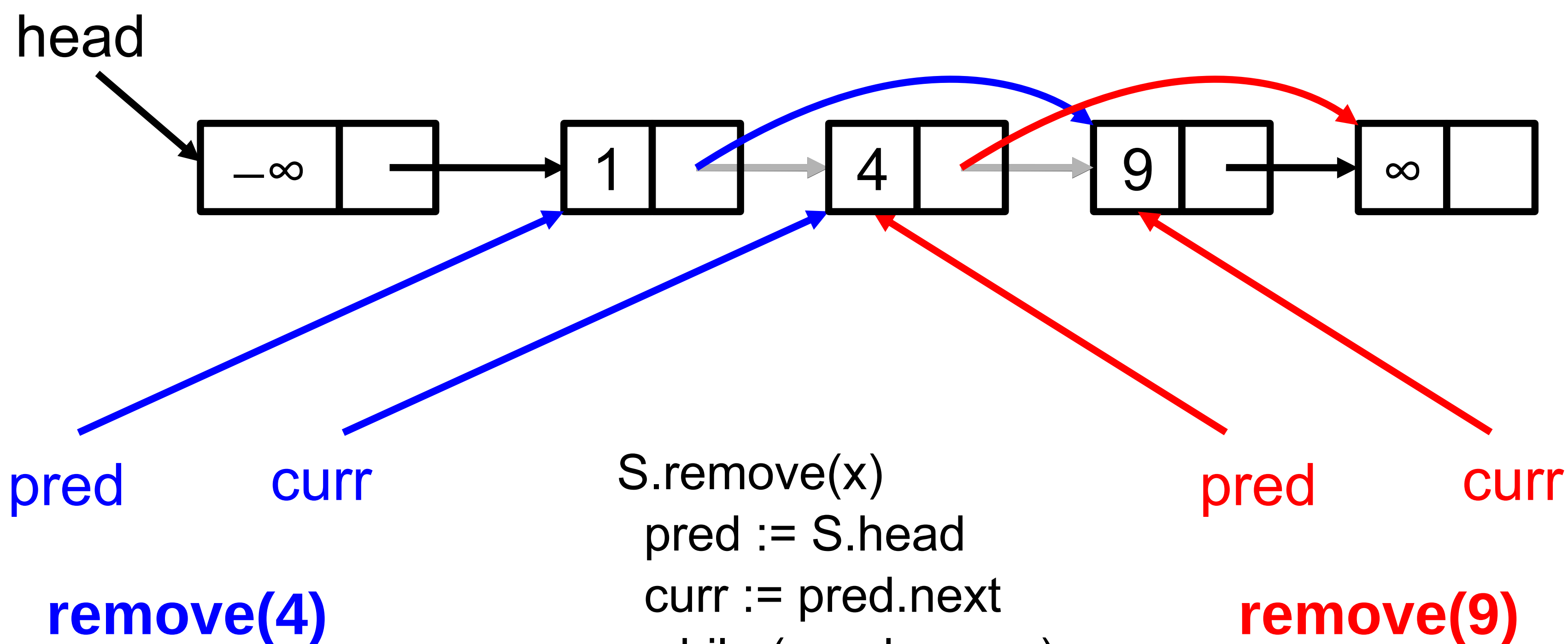
Invariants

- Keys strictly increase down the list.
  - List is ordered.
  - No duplicates.
- Keys of first and last nodes (i.e., the "sentinels") are $-\infty$ and ∞ respectively.
- pred.key < x
- pred.key < curr.key
- pred.next ≠ null
- …

Allowing concurrent access

- Can this algorithm tolerate concurrent execution of the operations by different processes?
- What can go wrong?
- How can we fix it?

Concurrent operations (bad)




```
S.remove(x)
  pred := S.head
  curr := pred.next
  while (curr.key < x)
    pred := curr
    curr := pred.next
  if curr.key = x then
    pred.next := curr.next
    return true
  else
    return false
```

Techniques for managing concurrent operations

- Coarse-grained mutual exclusion
- Fine-grained locking
- Optimistic locking
- Lock-free/nonblocking algorithms
- “Lazy” synchronization

Coarse-grained mutual exclusion

- Each process acquires a global lock, for the entire time it is executing significant steps of an operation implementation.

Coarse-grained locking

Why can we unlock early here?

```
S.add(x)
  S.lock()
  pred := S.head
  curr := pred.next
  while (curr.key < x)
    pred := curr
    curr := pred.next
  if curr.key = x then
    S.unlock()
    return false
  else
    node := new Node(x)
    node.next := curr
    pred.next := node
    S.unlock()
   return true
```

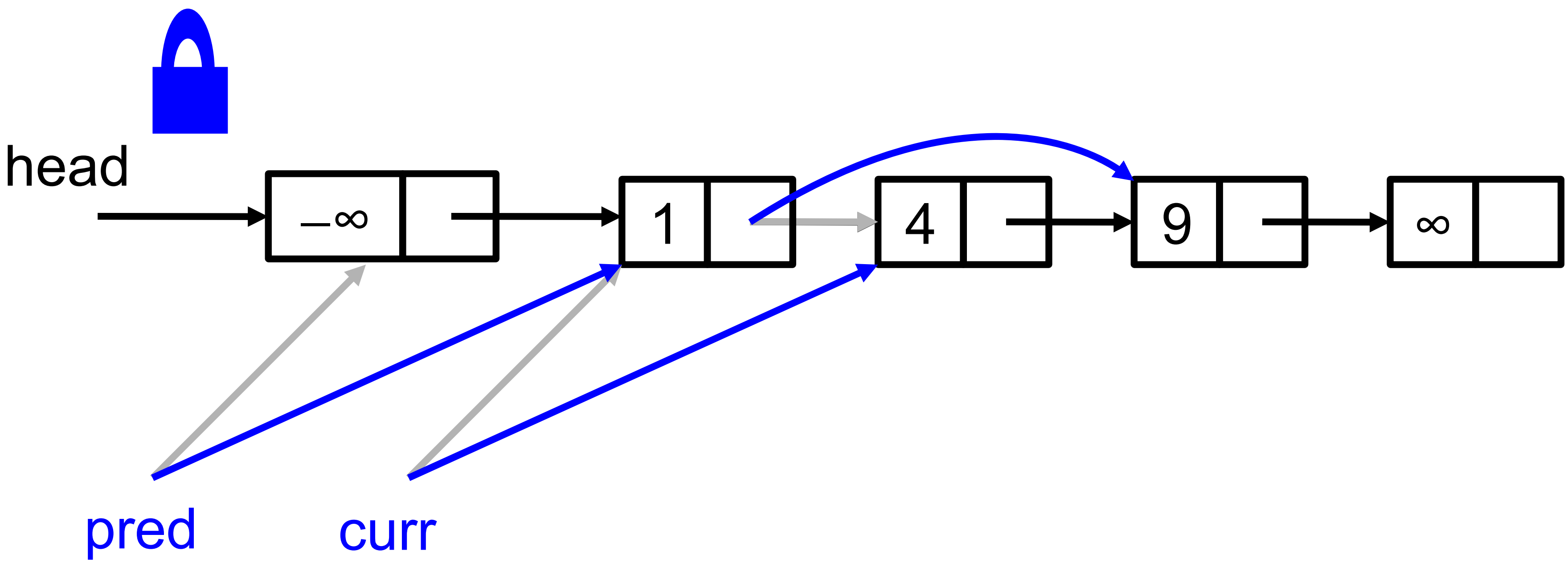
```
S.lock()
pred := S.head
curr := pred.next
while (curr.key < x)
  pred := curr
  curr := pred.next
if curr.key = x then
  pred.next := curr.next
  S.unlock()
  return true
else
  S.unlock()
  return false
```

```
S.contains(x)
  S.lock()
  curr := S.head
  while (curr.key < x)
    curr := curr.next
  S.unlock()
  if curr.key = x then
    return true
  else
    return false
```

Correctness

- Similar to sequential implementation.
- Atomicity:
  - Show the algorithm implements a canonical atomic set object.
  - Use forward simulation: S = elements that are reachable in the list
  - When do "perform" steps occur?
    - add(x): If successful, then when pred.next := node, else any time the lock is held.
    - remove(x): If successful, then when pred.next := curr.next, else any time the lock is held.
    - contains(x): Any time the lock is held.
  - Invariant: If an operation holds the lock, then any node it visits is reachable in the list.
- Liveness:
  - Guarantees progress, assuming that the lock does.
  - May or may not be lockout-free, depending on whether the lock is.
  - Blocking (not wait-free or lock-free):
    - Everything comes to a halt if someone stops while holding the lock.

Coarse-grained locking



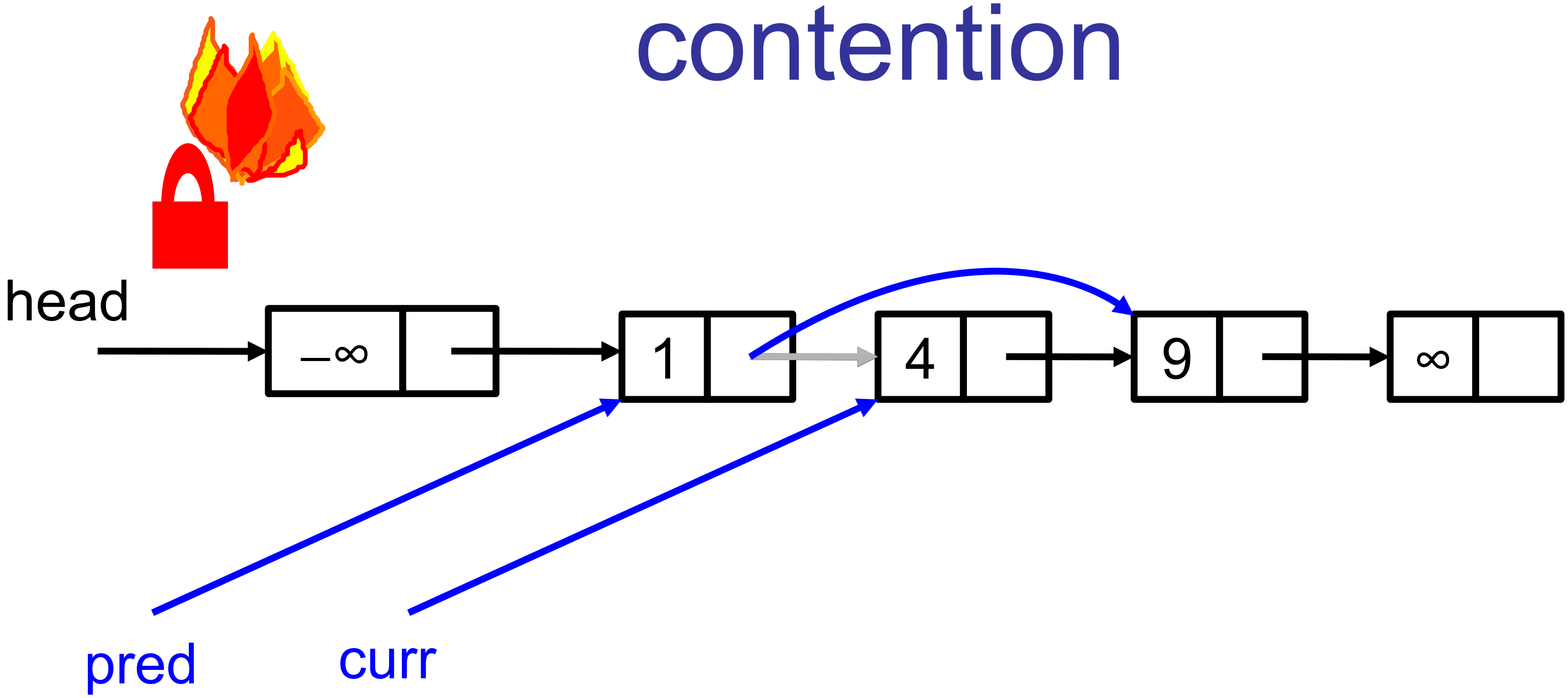
remove(4)

Coarse-grained locking

For many applications, this is the best solution!
(Don't underrate simplicity.)

- Easy
  - to write,
  - to prove correct.
- Guarantees progress
- If we use queue locks, it's lockout-free.
- But:
  - Blocking (not wait-free, not lock-free)
  - Poor performance when contention is high
    - Essentially no concurrent access.
    - But often good enough for low contention.

Coarse-grained locking with high contention



remove(4) **remove(9)** **add(6)** **contains(4)** **add(3)**

Improving coarse-grained locking

- Reader/writer locks
  - Multiple readers can hold the lock simultaneously, but writer cannot share with anyone else (reader or writer).
- Using reader/writer lock for coarse-grained locking, in the list-based set implementation:
  - Contains takes only a read lock
    - Can be a big win if contains is the most common operation.
  - What about add or remove that returns false?
    - Let add/remove start with a read lock, then “upgrade” to a write lock if needed.
    - If it can’t upgrade, abandon/restart the operation.

Fine-grained locking

- Associate locks with smaller pieces of data, not entire data structure.
- Process acquires/releases locks as it executes steps of an operation.
- Operations that work on disjoint pieces of data proceed concurrently.

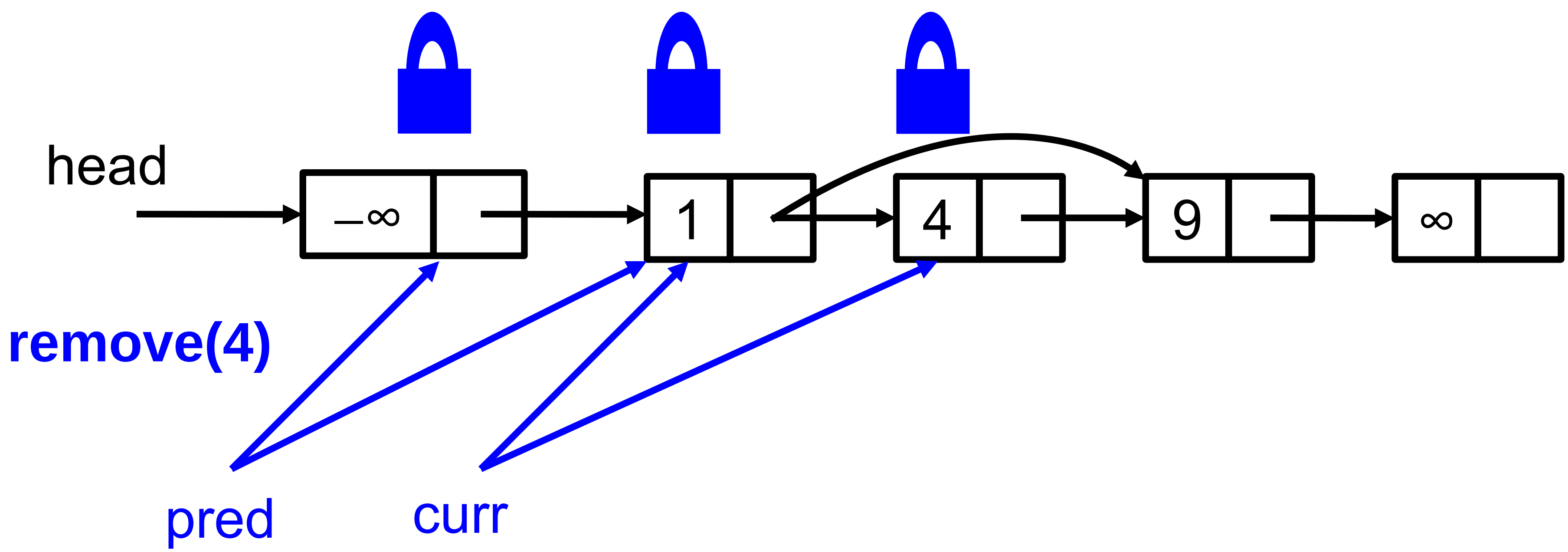
Two-phase locking

- Finish acquiring all locks before releasing any.
  - Typically, release all locks at end of the op: "strict 2-phase locking".
- Easy to prove atomicity:
  - Serialize each operation at any point when it holds all its locks.
  - For strict 2-phase locking, usually the end of the operation.
  - Algorithm behaves like sequential algorithm, with operations performed in order of serialization points.
- But acquiring all the locks at once can be costly (delays).
- Must avoid deadlock, e.g., by acquiring locks in predetermined order.

- Naïve 2-phase locking for list-based set implementation:
  - Lock each node as visited, using a mutex lock.
  - Avoids deadlock by acquiring all locks in list order.
  - Doesn't help performance.
  - Using reader/writer locks might help performance, but introduces new deadlock possibilities.

Hand-over-hand locking

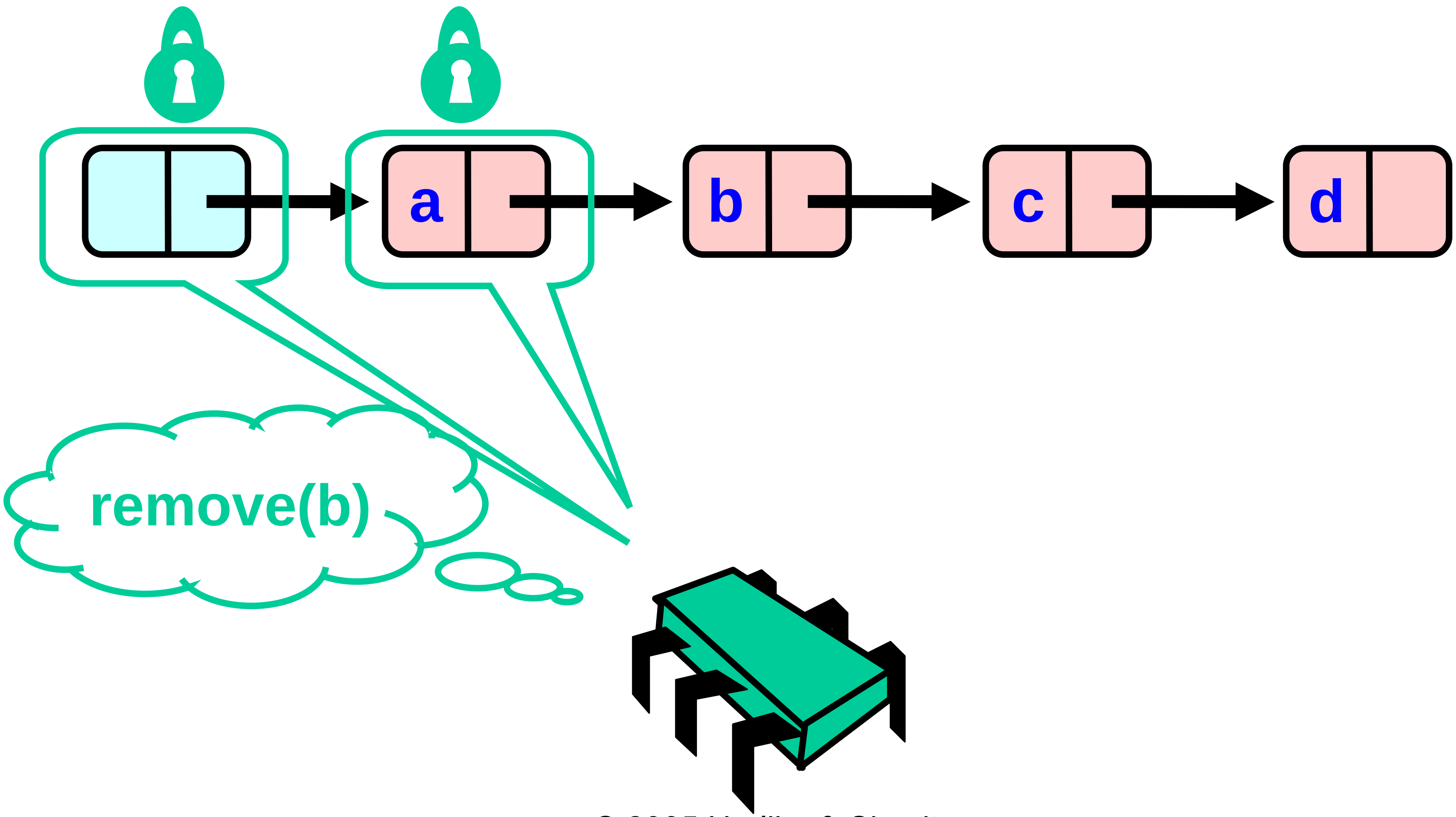
- Fine-grained locking, but not “two-phase”
  - Atomicity doesn't follow from general rule; trickier to prove.
- Each process holds at most two locks at a time.
  - Acquires lock for successor before releasing lock for predecessor.
- Keeps operations “pipelined”.



Hand-over-hand locking

- Must we lock a node we are trying to remove?
- Can’t we just lock its predecessor, while resetting the predecessor’s next pointer?
- No. Counterexample (from Herlihy and Shavit’s slides):

Removing a Node



From *The Art of Multiprocessor Programming*, Maurice Herlihy and Nir Shavit.

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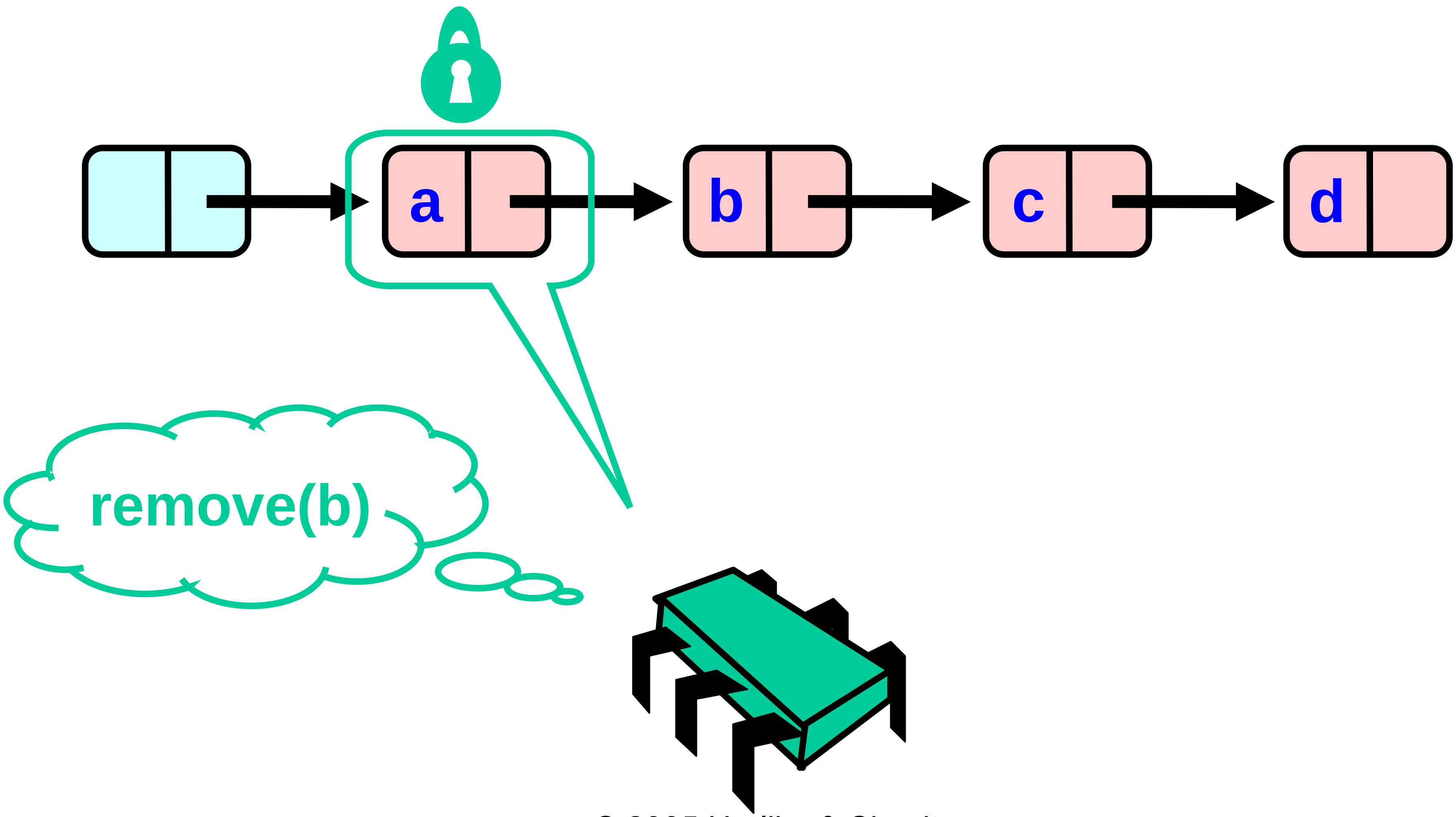

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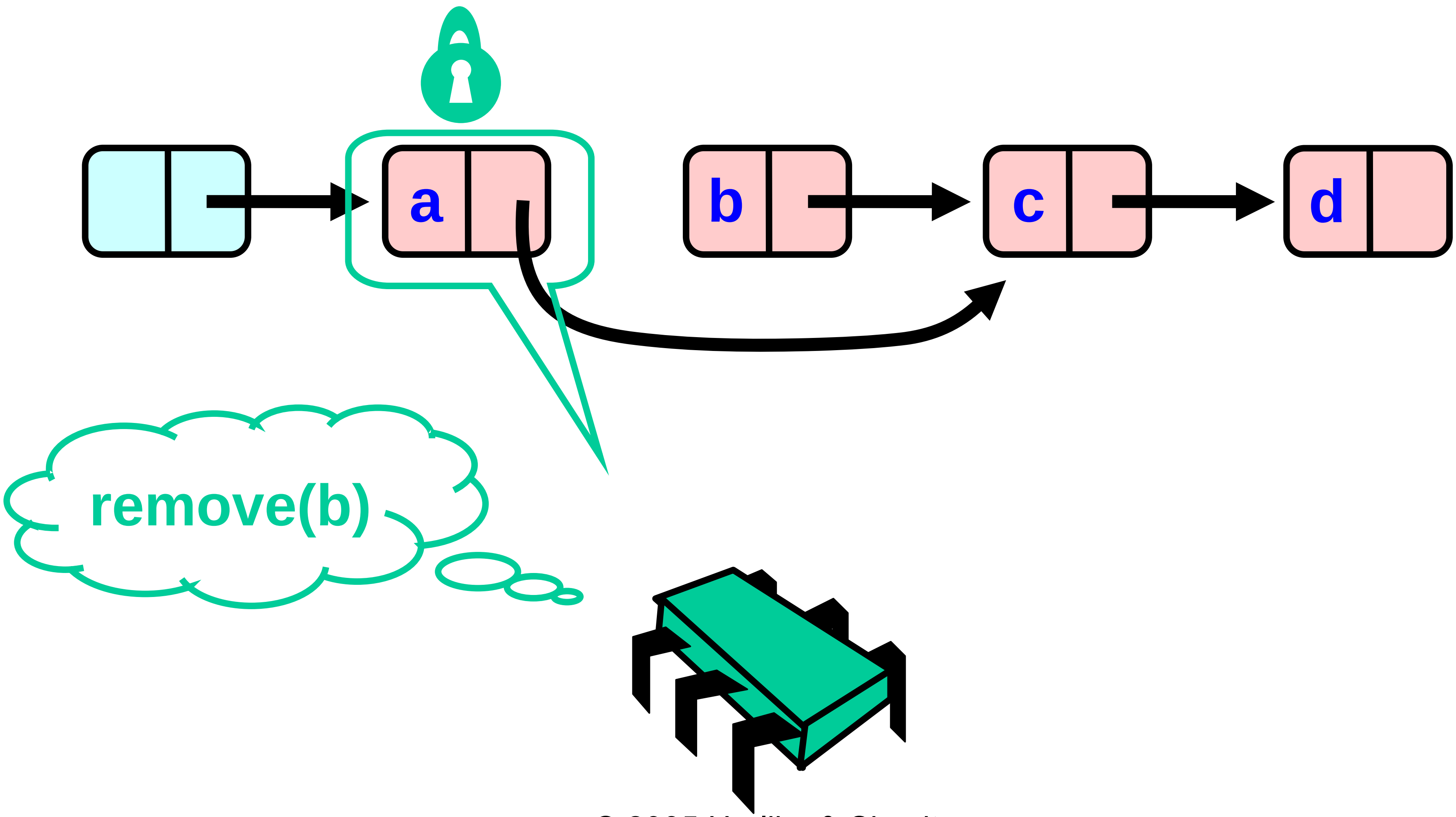
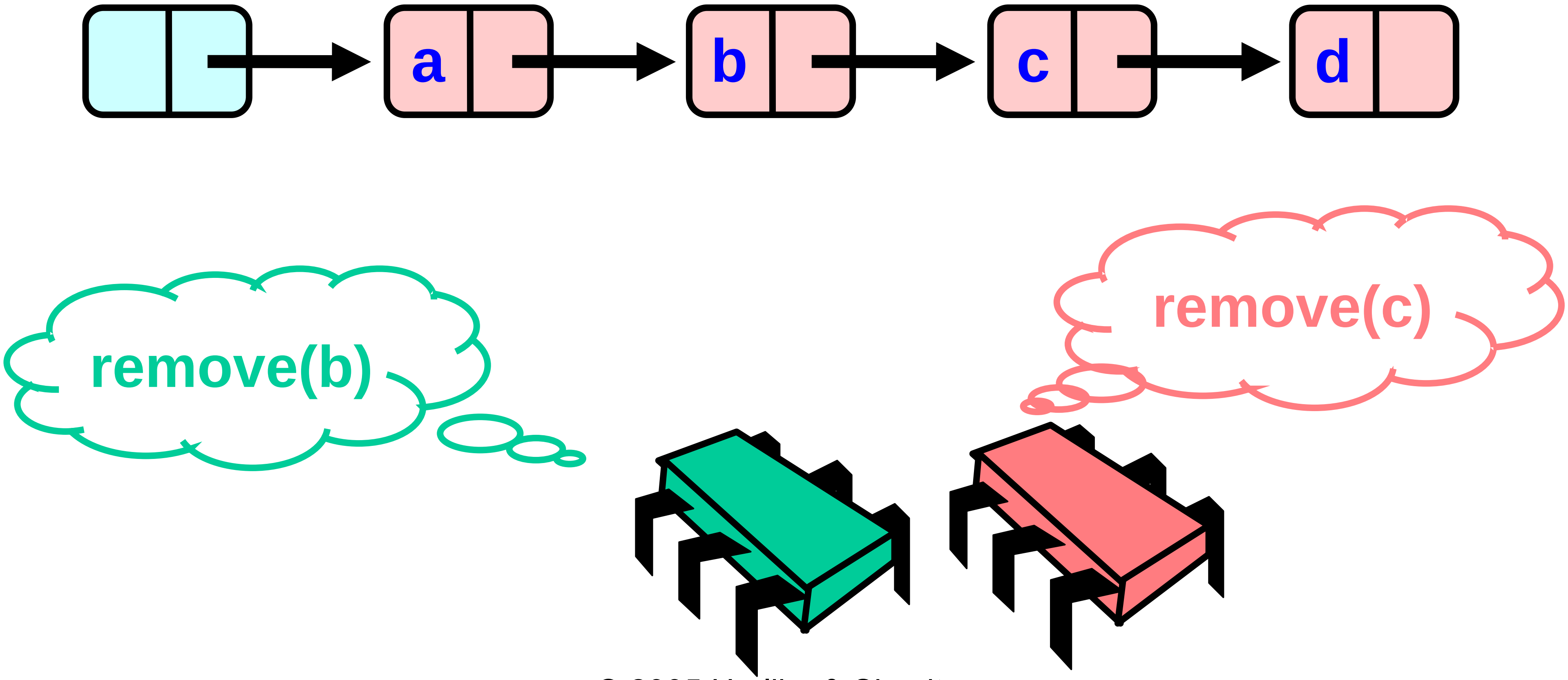


Removing Two Nodes



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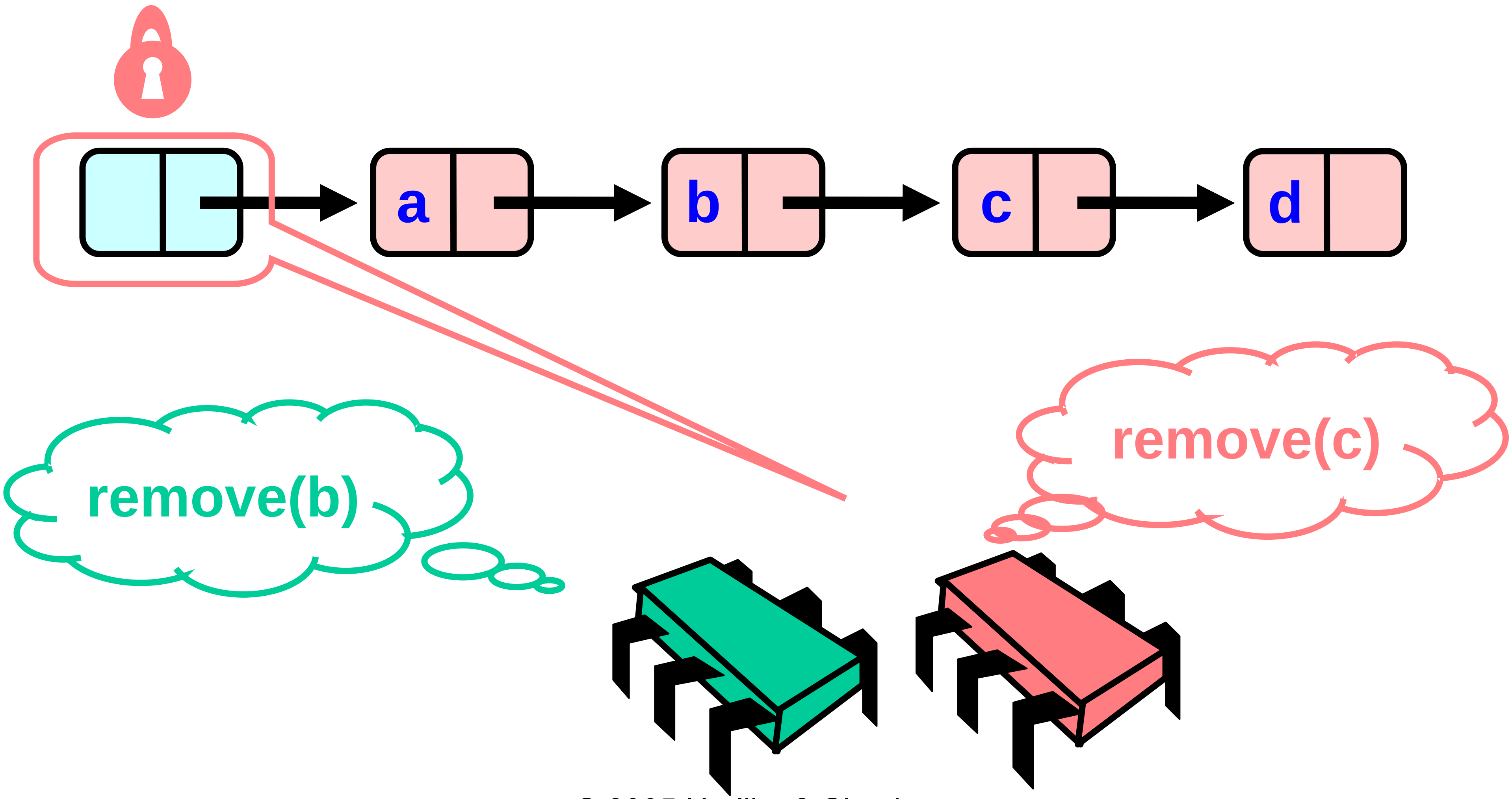

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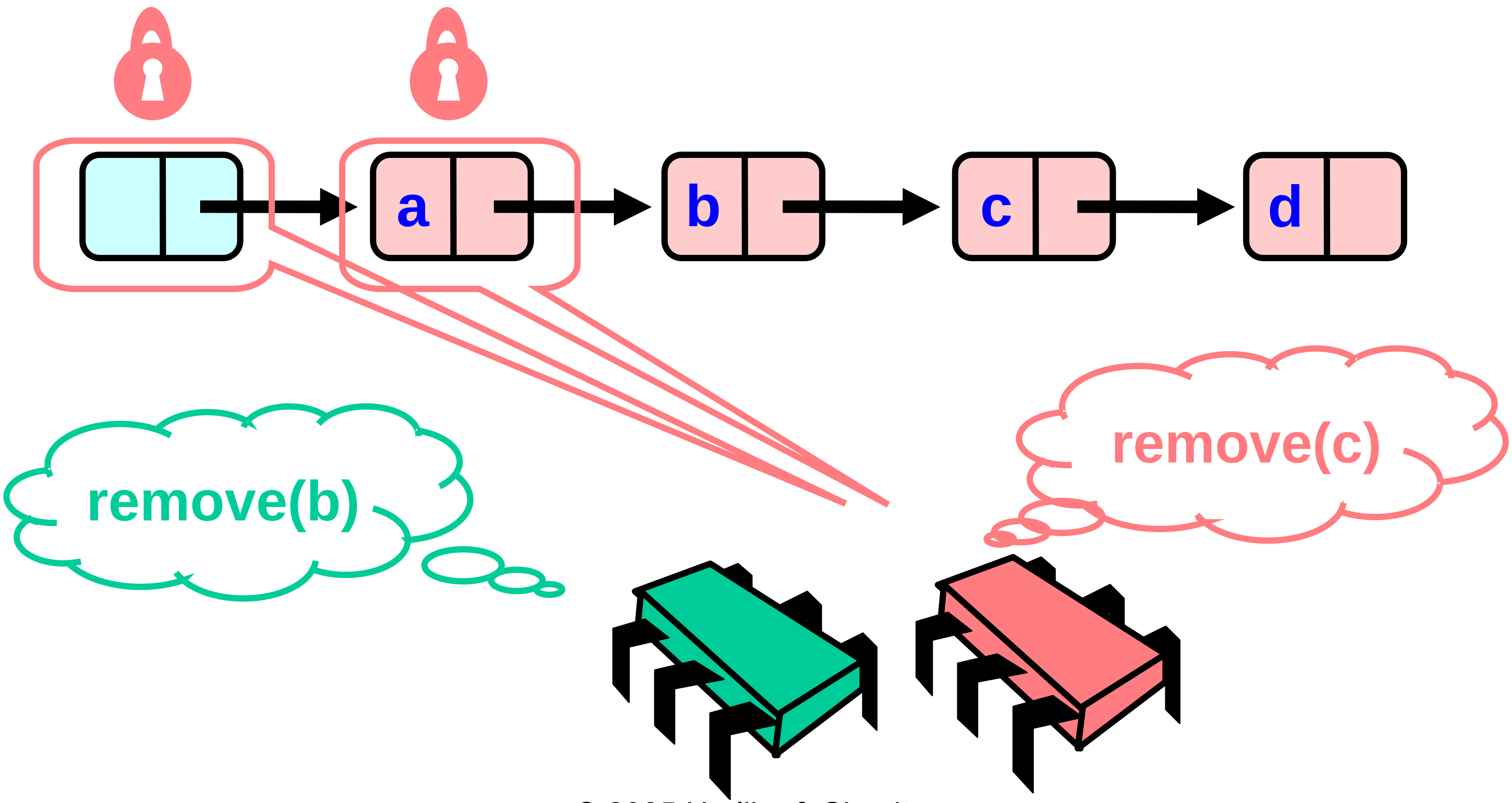

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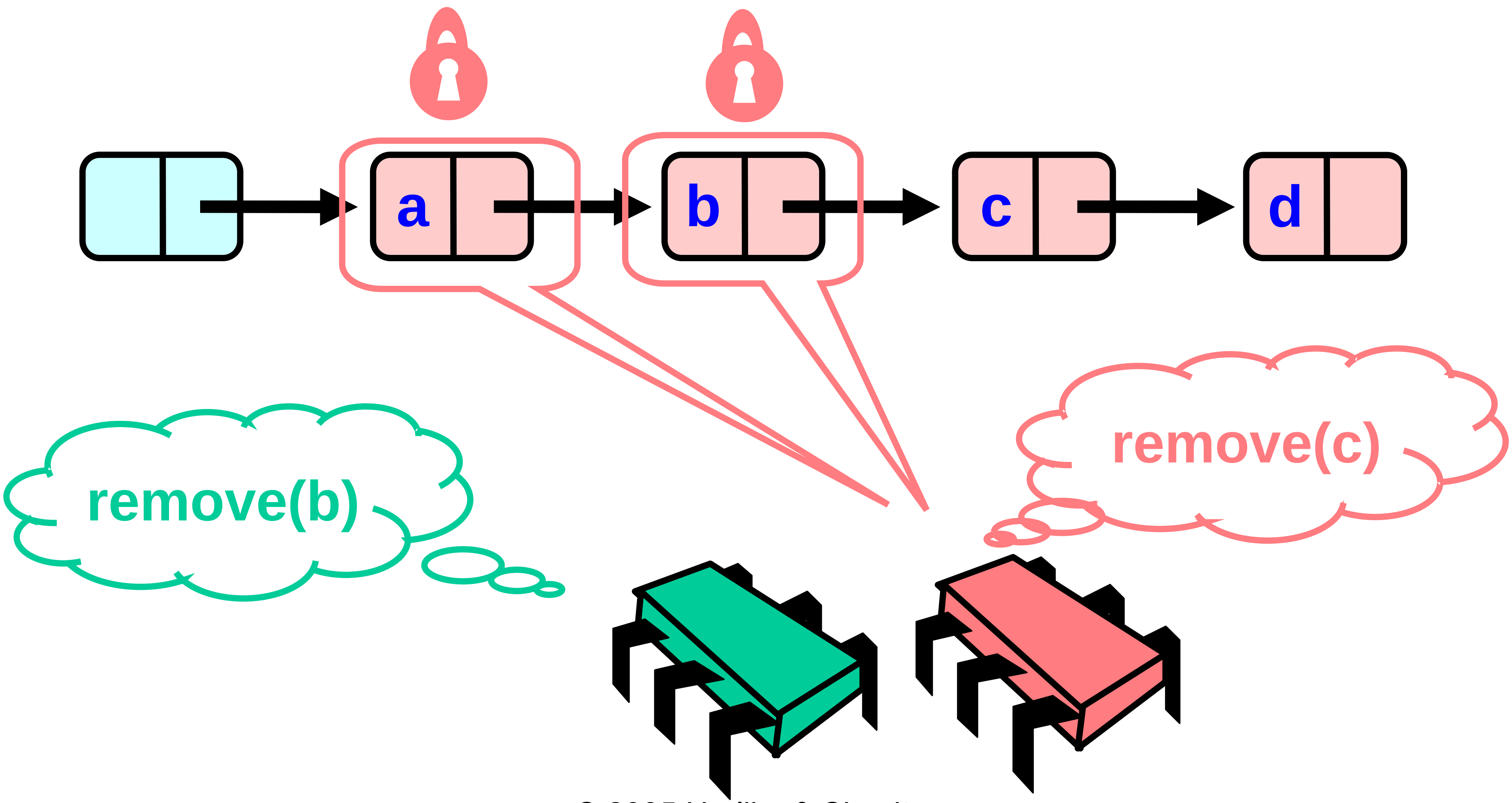

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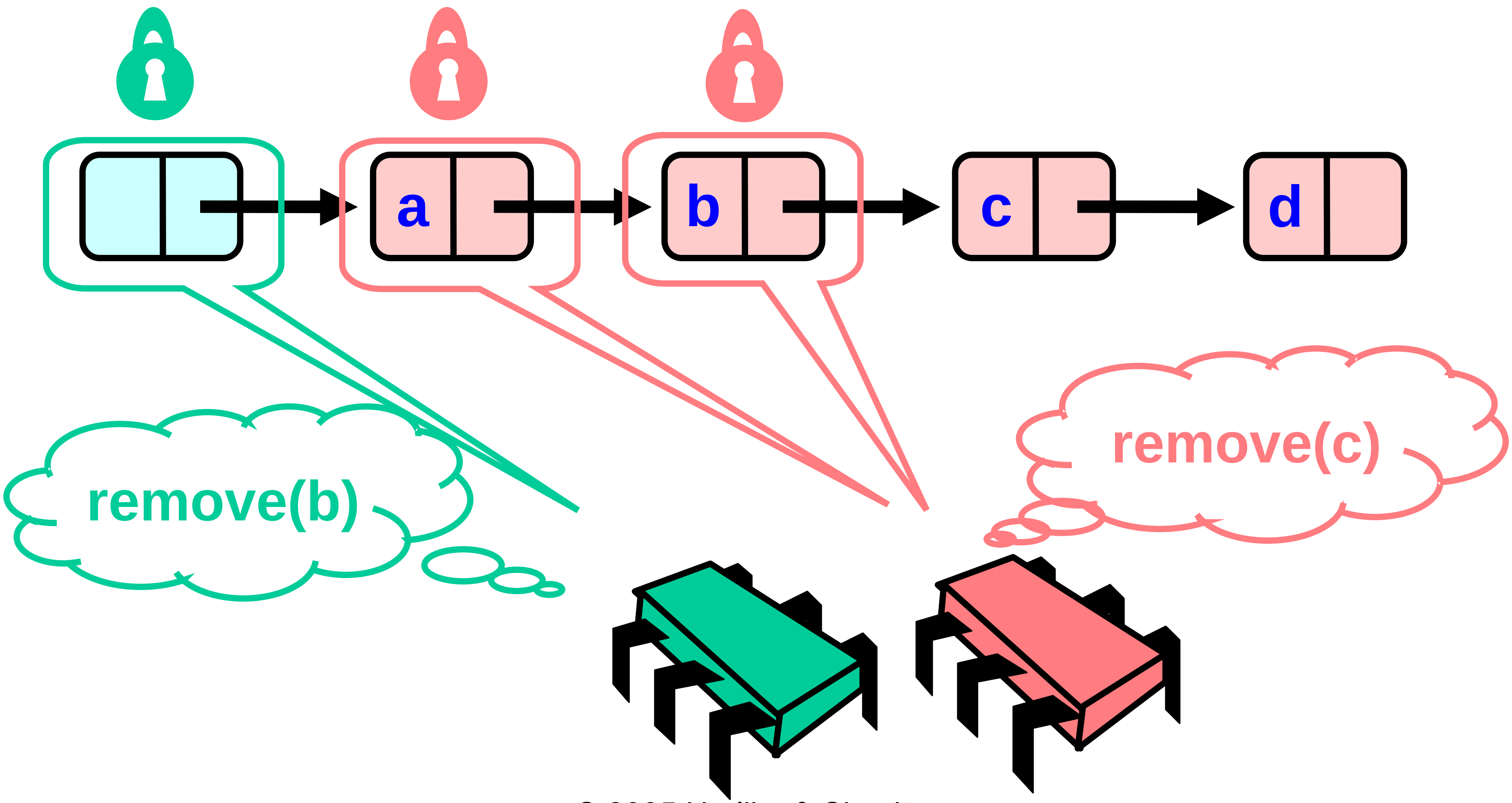

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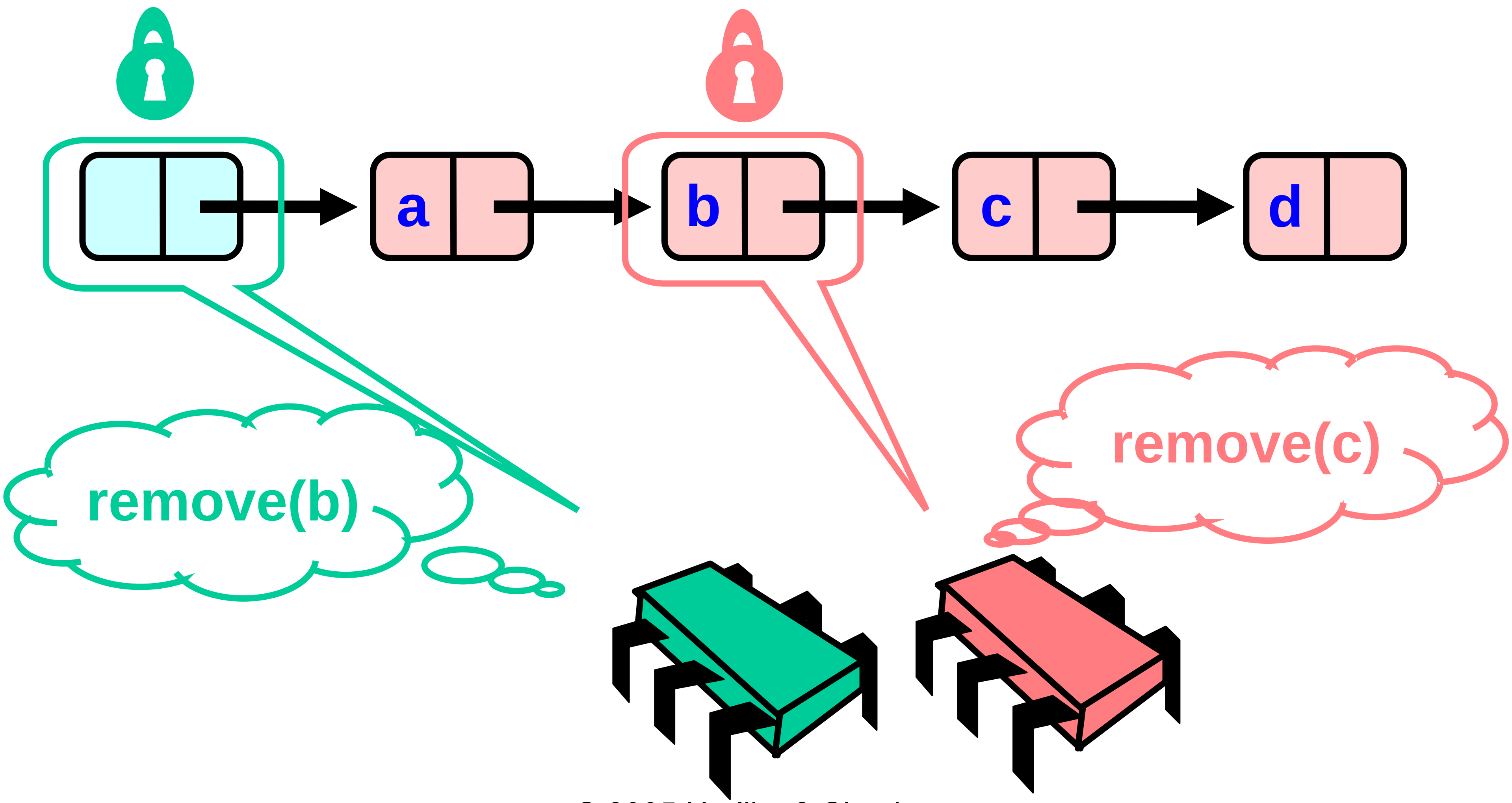

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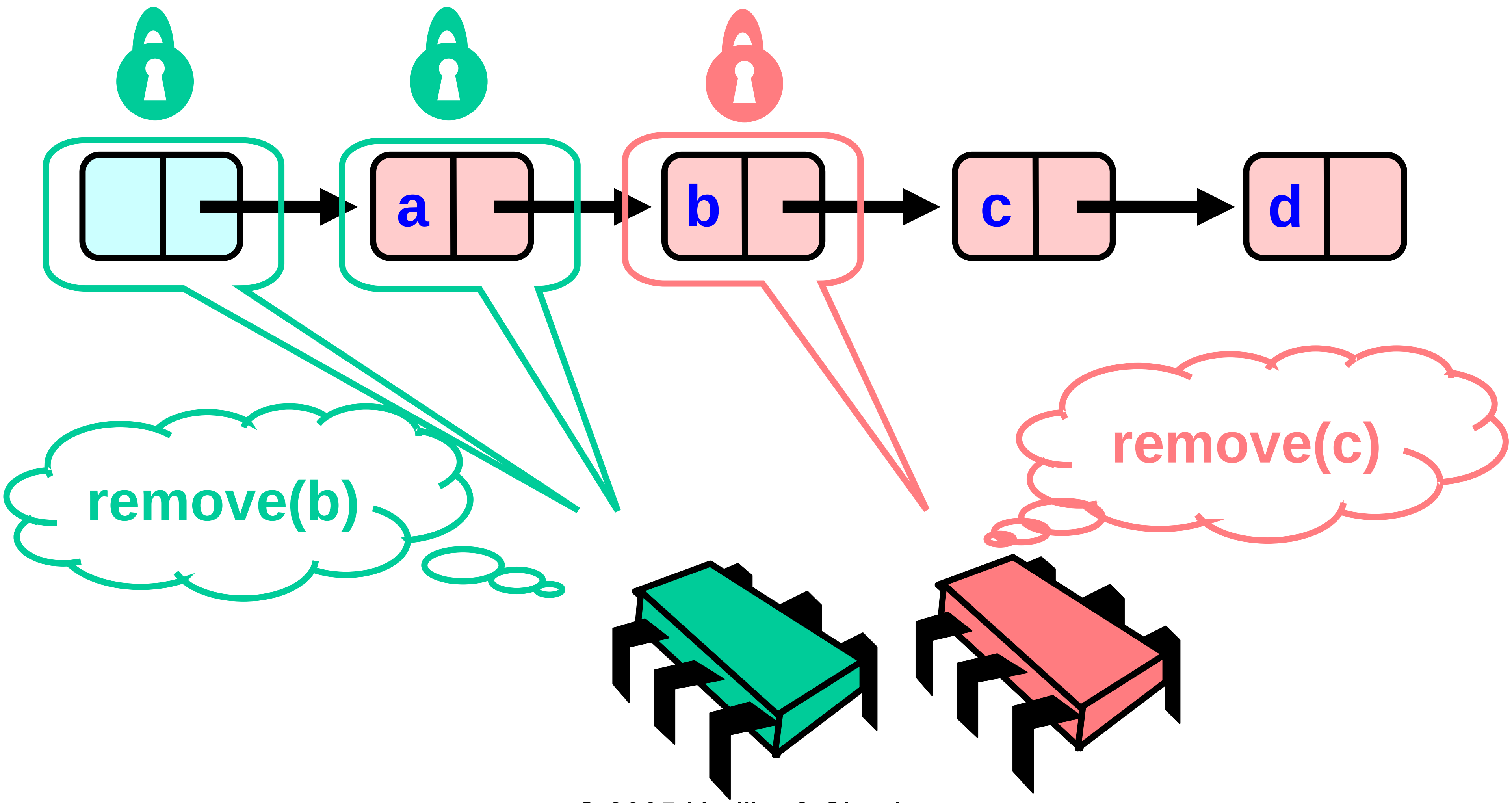

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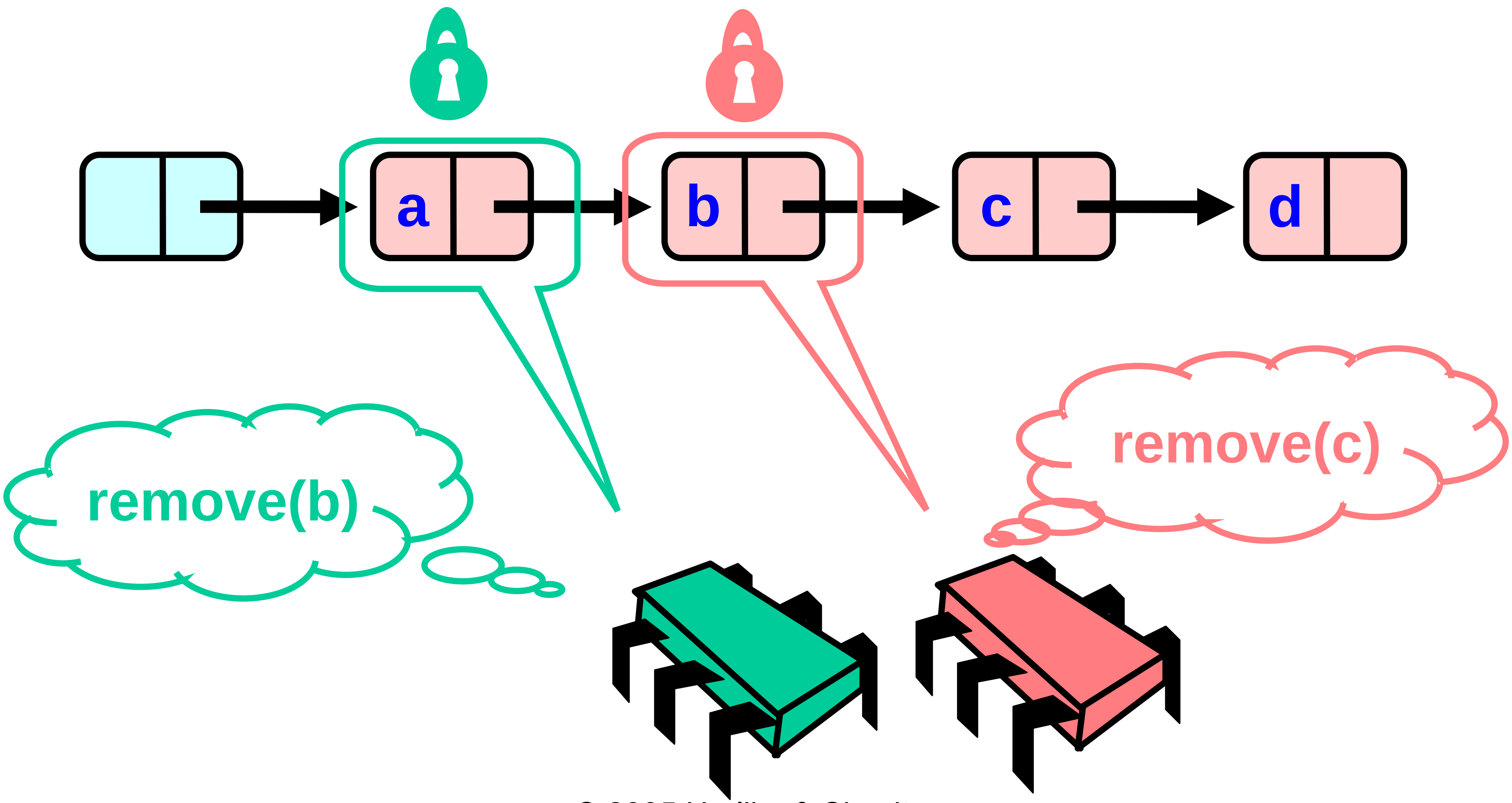

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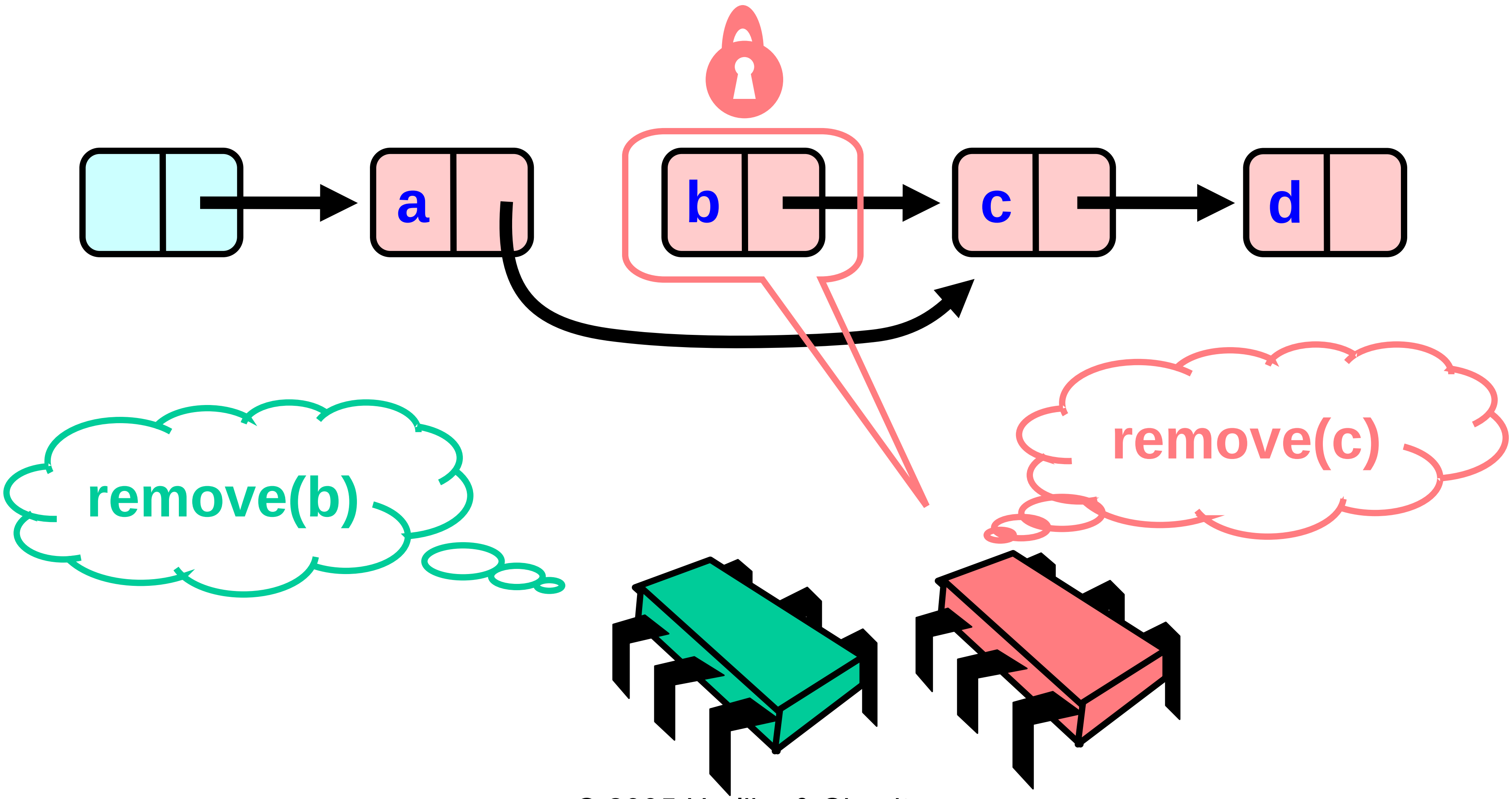

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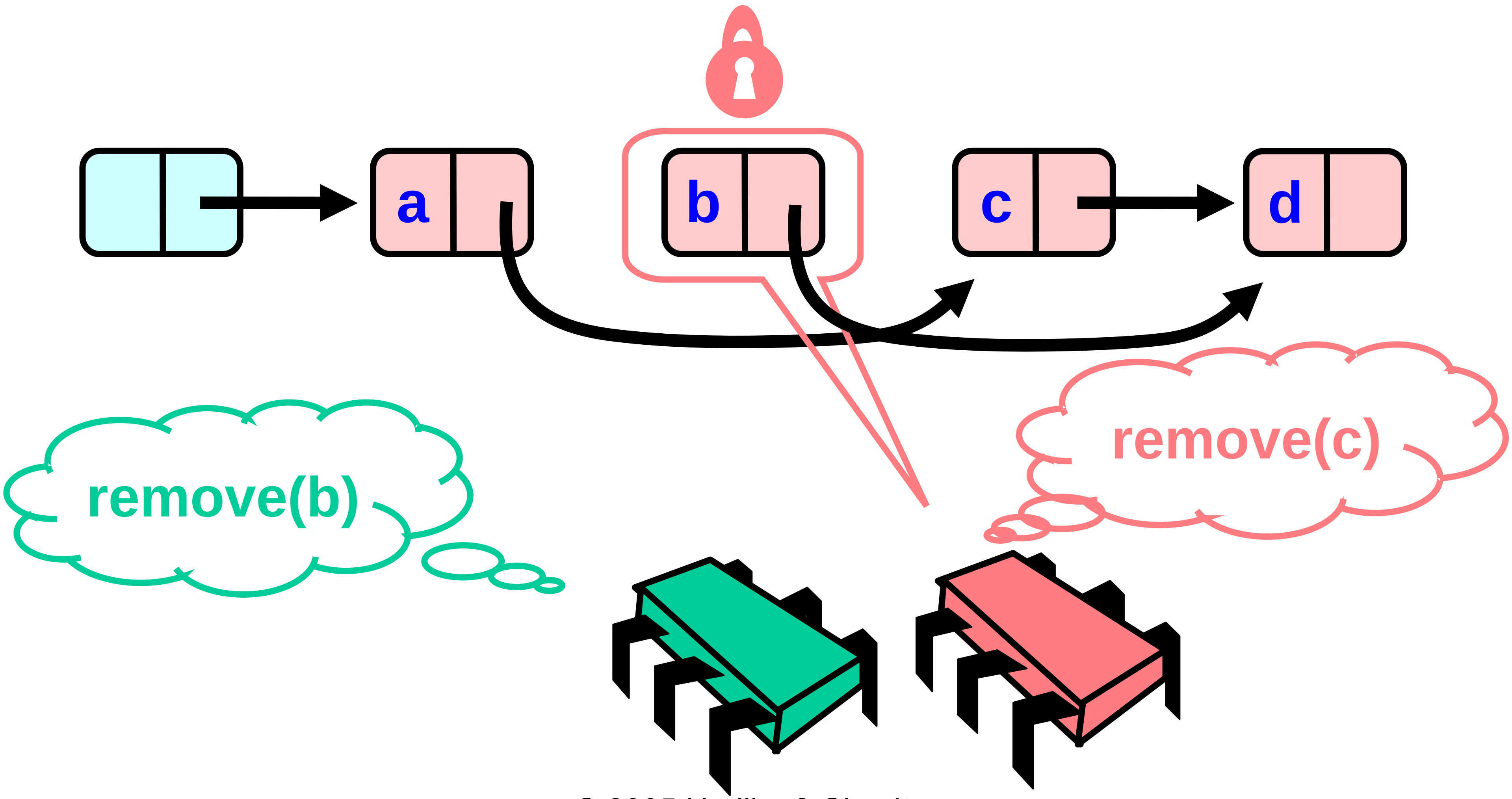

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Hand-over-hand locking

- add(x)
  - Lock hand-over-hand.
  - When adding new node, keep both predecessor and successor locked (HS Fig. 9.6).
  - We could actually release the lock on the successor before adding the new node.
- contains(x)
  - Lock hand-over-hand, can unlock everything before reading curr.key.

Hand-over-hand locking

```
S.add(x)
  pred := S.head
  pred.lock()
  curr := pred.next
  curr.lock()
  while (curr.key < x)
    pred.unlock()
    pred := curr
    curr := pred.next
    curr.lock()
  if curr.key = x then
    pred.unlock()
    curr.unlock()
    return false
  else
    node := new Node(x)
    node.next := curr
    pred.next := node
    pred.unlock()
    curr.unlock()
    return true
```

```
S.remove(x)
  pred := S.head
  pred.lock()
  curr := pred.next
  curr.lock()
  while (curr.key < x)
    pred.unlock()
    pred := curr
    curr := pred.next
    curr.lock()
  if curr.key = x then
    pred.next := curr.next
    pred.unlock()
    curr.unlock()
    return true
  else
    pred.unlock()
    curr.unlock()
    return false
```

```
S.contains(x)
  curr := S.head
  curr.lock()
  while (curr.key < x)
    temp := curr
    curr := curr.next
    curr.lock()
    temp.unlock()
  curr.unlock()
  if curr.key = x then
    return true
  else
    return false
```

Correctness

- Atomicity:
  - Similar to coarse-grained locking.
  - Forward simulation to canonical atomic set object: S = elements that are reachable in the list.
  - "perform" steps:
    - add(x):
      - If successful, then when pred.next := node.
      - Else any time the lock on the node already containing x is held.
    - remove(x):
      - If successful, then when pred.next := curr.next
      - Else any time the lock on the node seen to have a higher key is held.
    - contains(x): LTTR
      - If true, then any time the lock on the node containing x is held.
      - Else any time the lock on the node seen to have a higher key is held.
  - Invariant: Any locked node is reachable in the list.

Correctness

- Atomicity:
  - Forward simulation to canonical atomic set object:
    - S = elements that are reachable in the list.
- Liveness:
  - Guarantees progress, assuming that the locks do.
  - Guarantees lockout-freedom, assuming the locks do.
    - All processes compete for locks in the same order.
  - Blocking (not wait-free or lock-free).

Evaluation

- Problems:
  - Each operation must acquire O(|S|) locks.
  - Pipelining means that fast threads can get stuck behind slow threads.
  - Using reader/writer locks might help performance, but introduces new deadlock possibilities.
- Idea:
  - Can we examine the nodes first without locking, and then lock only the nodes we need?
  - Must ensure that the node we modify is still in list.
  - Optimistic locking.

Optimistic locking

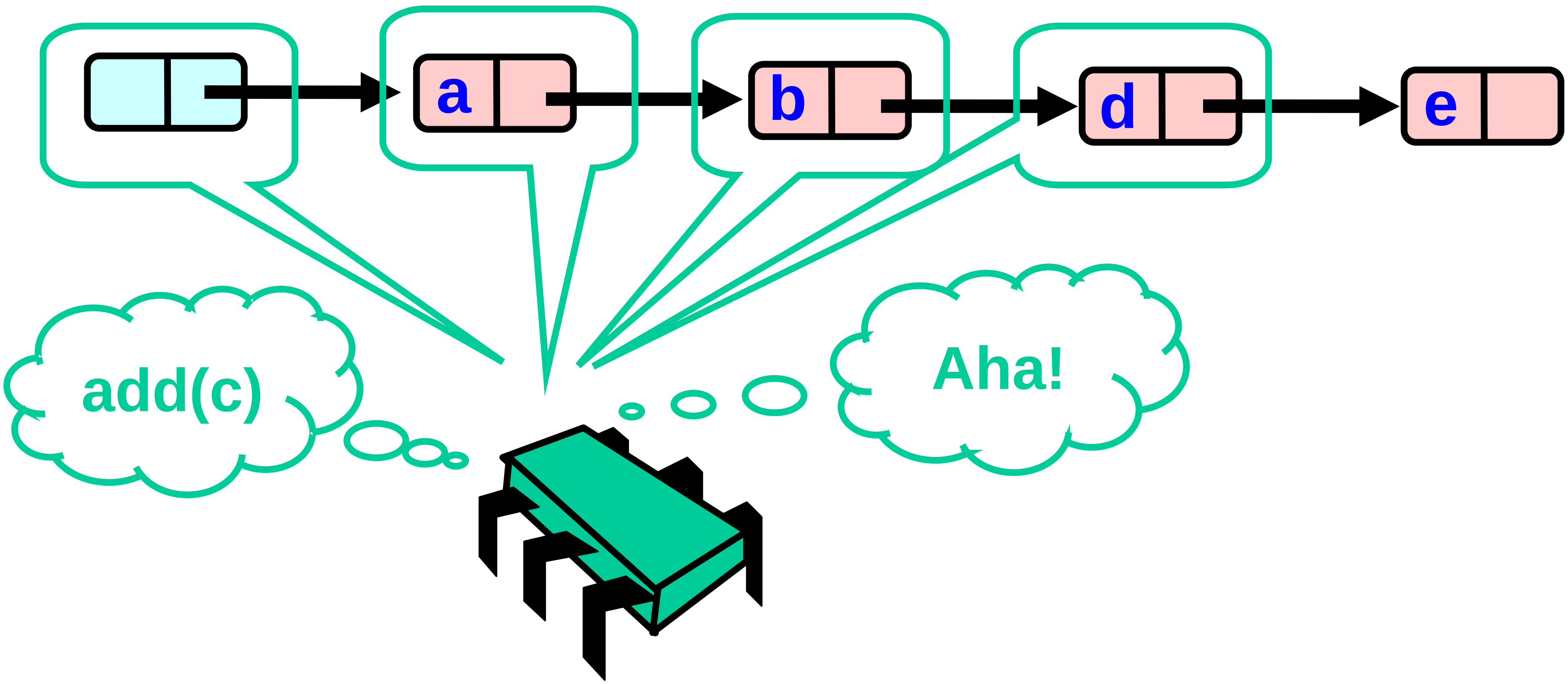
- Examine the nodes first without locking.
- Lock the nodes we need.
- Verify that the locked nodes are still in the list, before making modifications or determining results

Optimistic locking

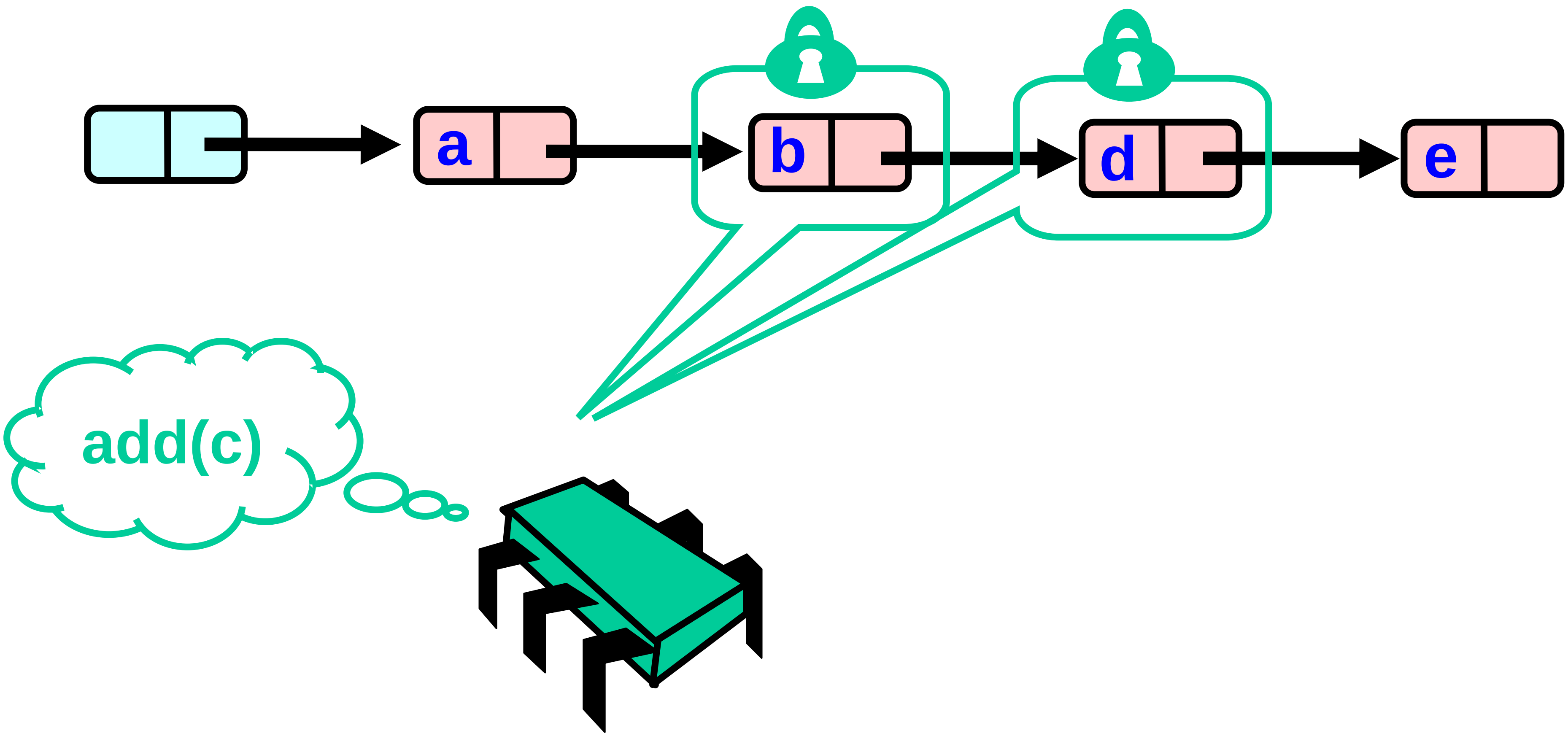
- add(x):
  - Traverse the list from the head, without locking, looking for the nodes we need (pred and curr).
  - Lock nodes pred and curr.
  - Validate that pred and curr are still in the list, and are still consecutive (pred.next = curr), by traversing the list once again.
  - If this works, then add the node and return true (or return false if it's already there).
  - If it doesn't work, start over.
- remove(x), contains(x): Similar.

- Better than hand-over-hand if
  - Traversing twice without locking is cheaper than once with locking.
  - Validation usually succeeds

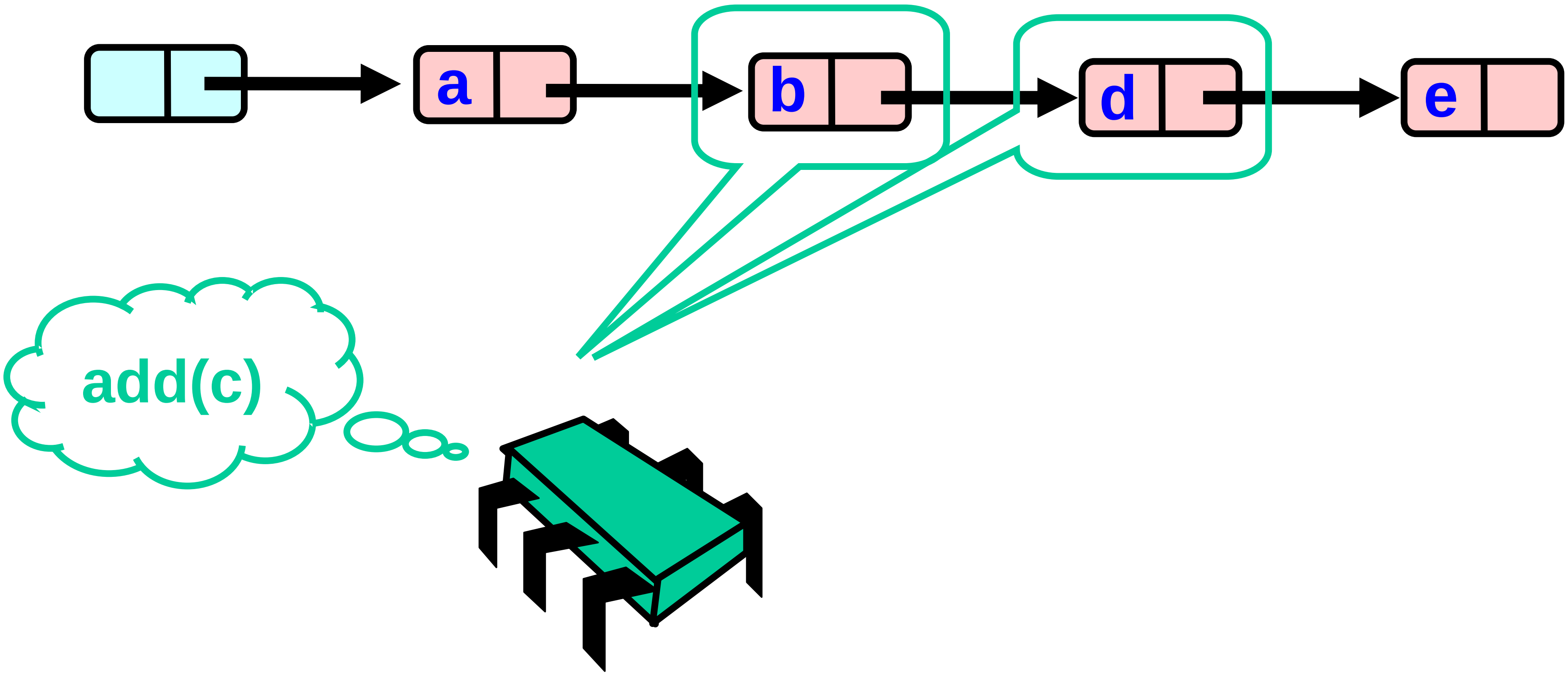
Optimistic locking



Optimistic locking



What can go wrong? (Part 1)



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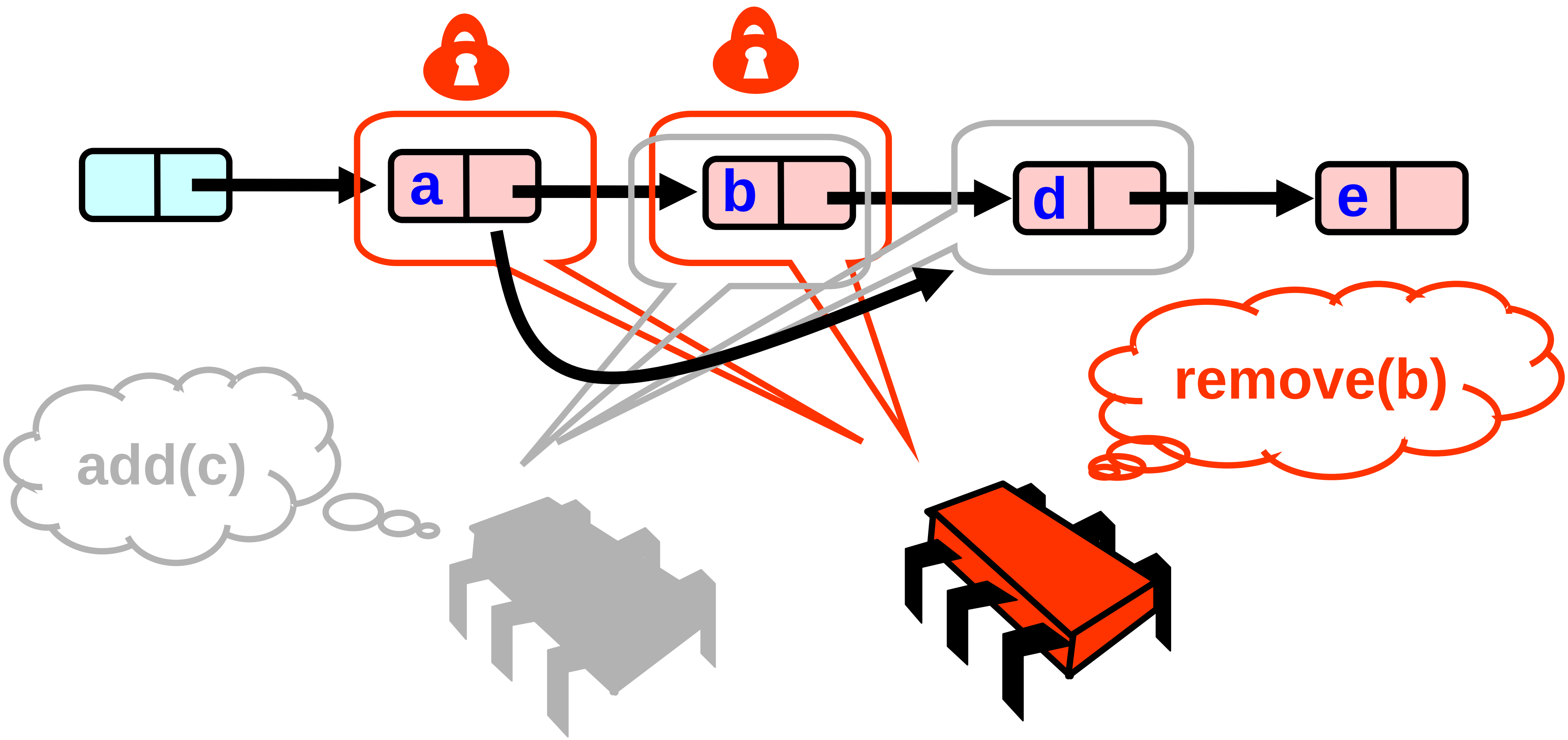

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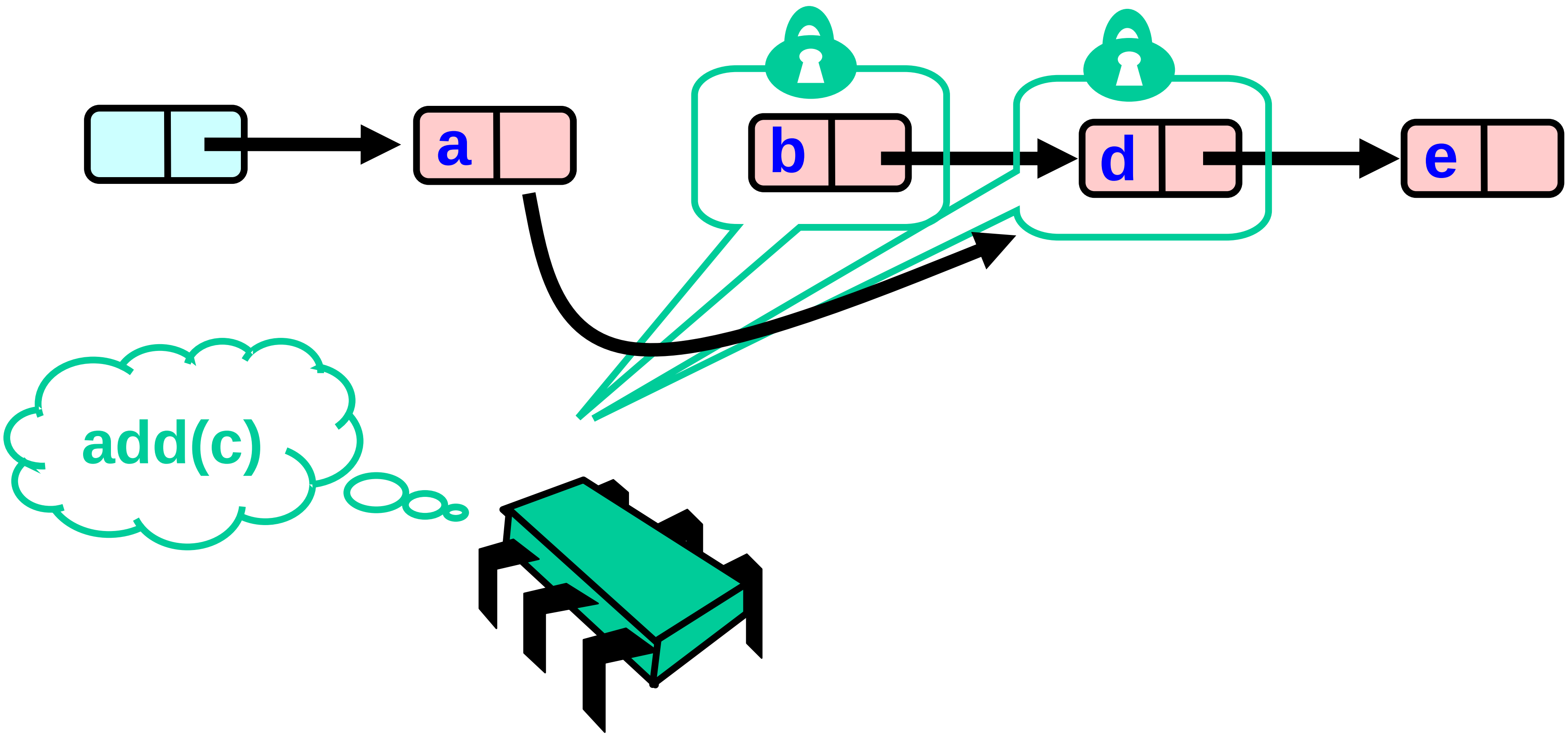
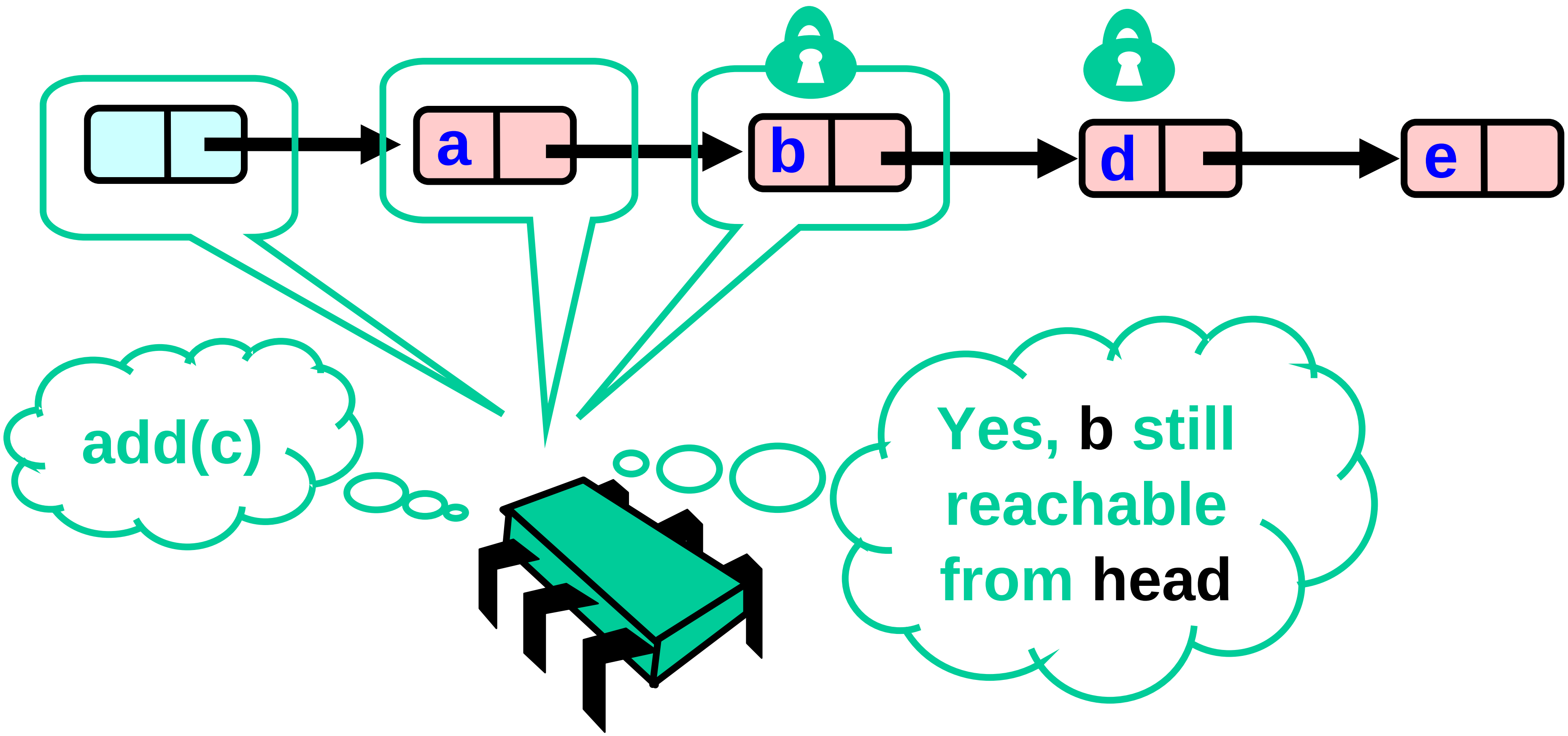
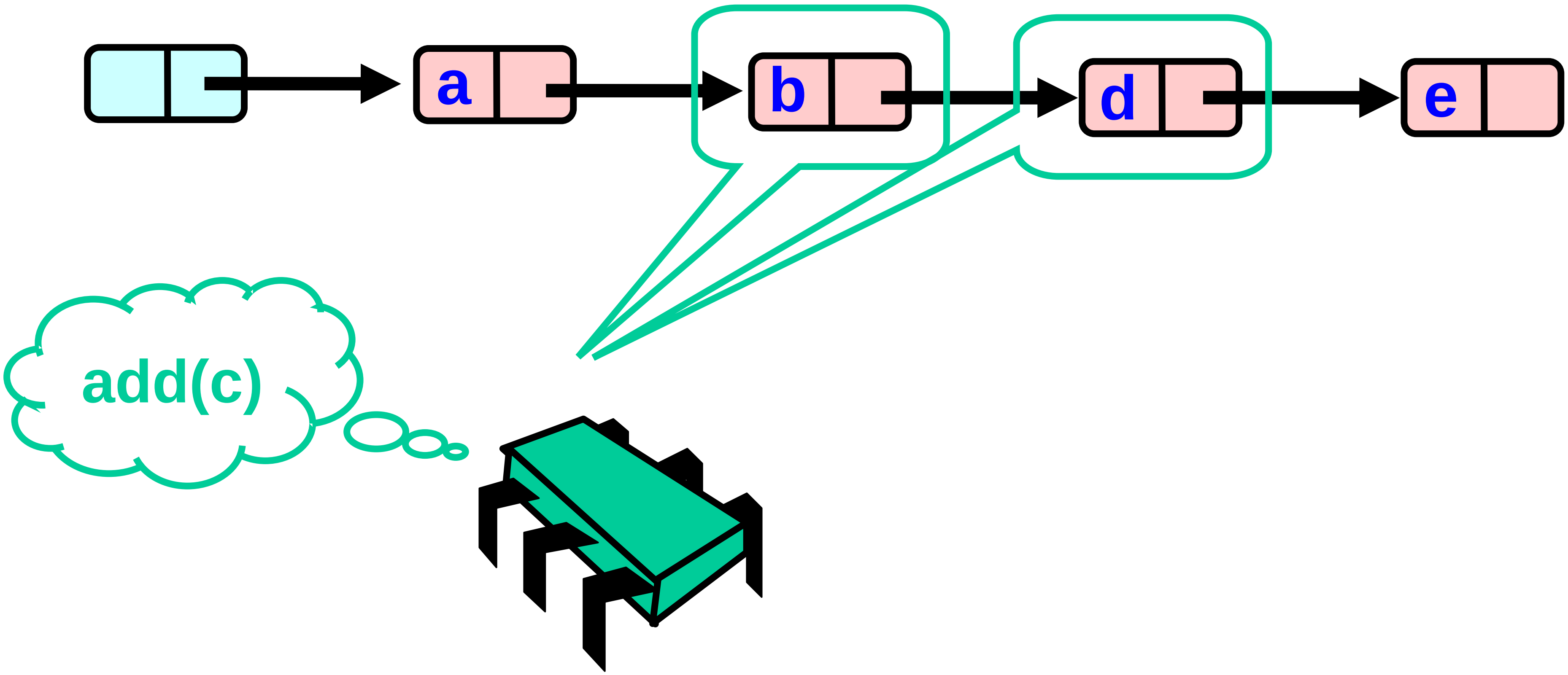


Validate (Part 1)



What can go wrong? (Part 2)



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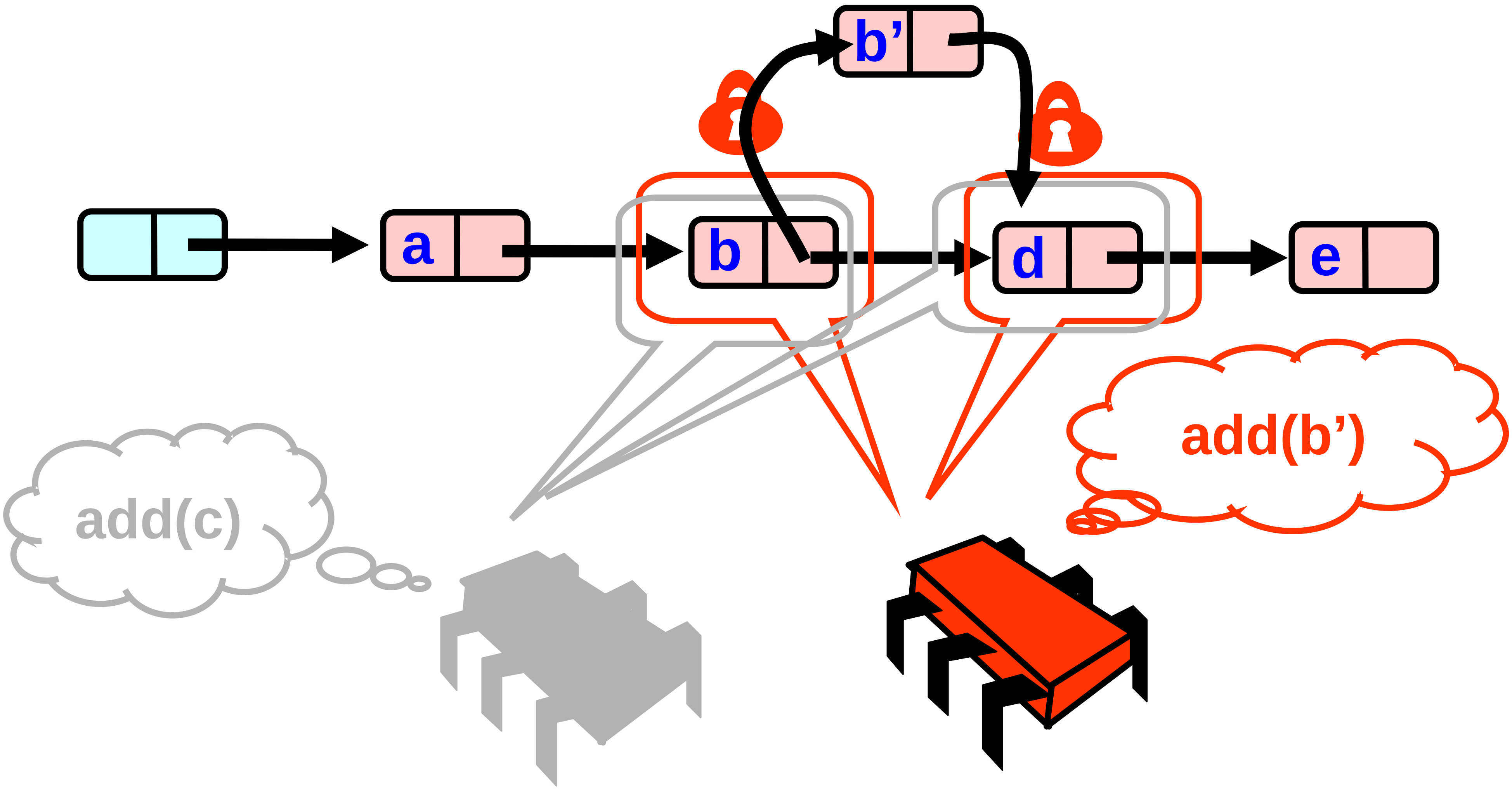

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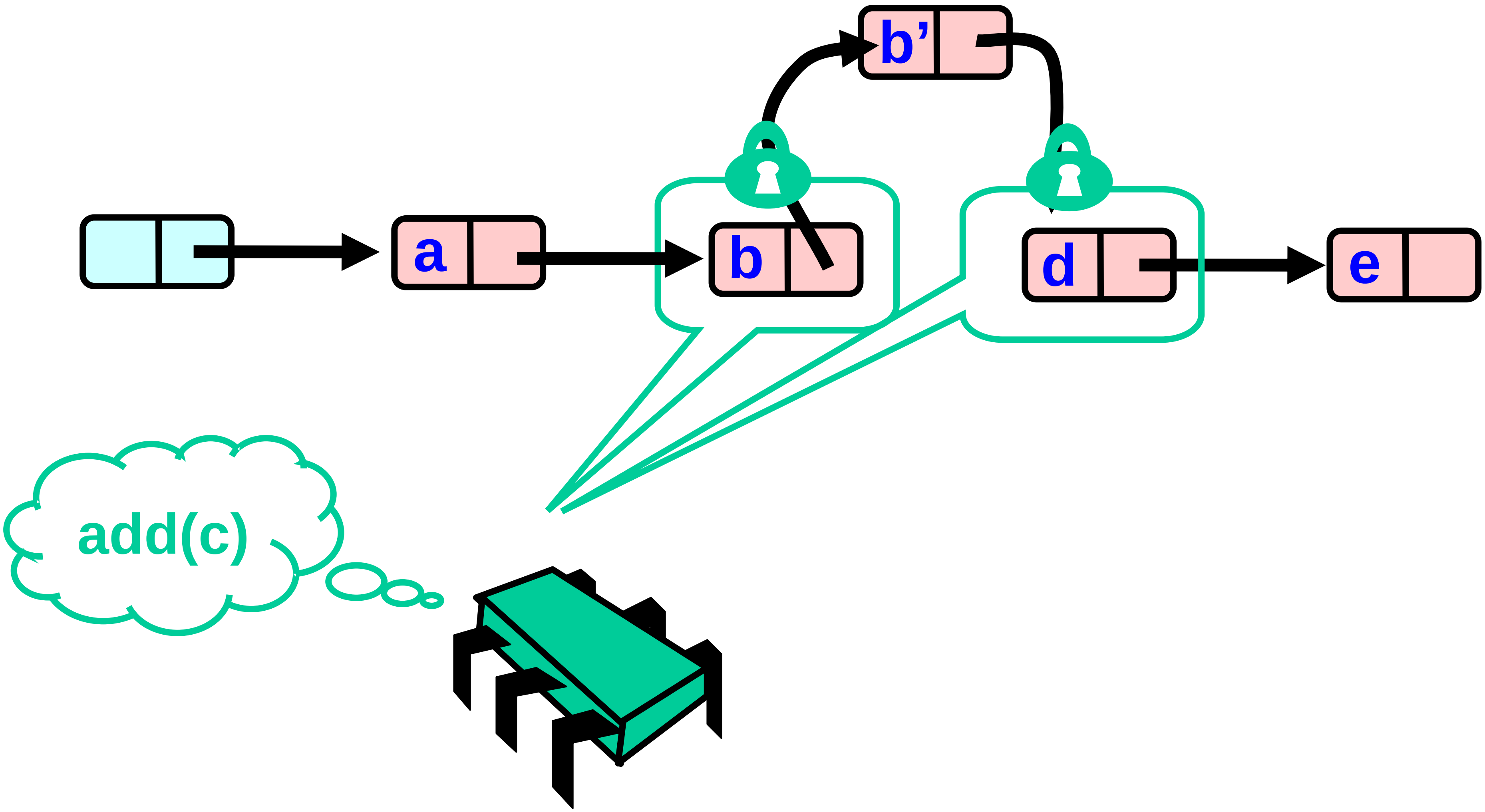

From *The Art of Multiprocessor Programming*, Maurice Herlihy and Nir Shavit.

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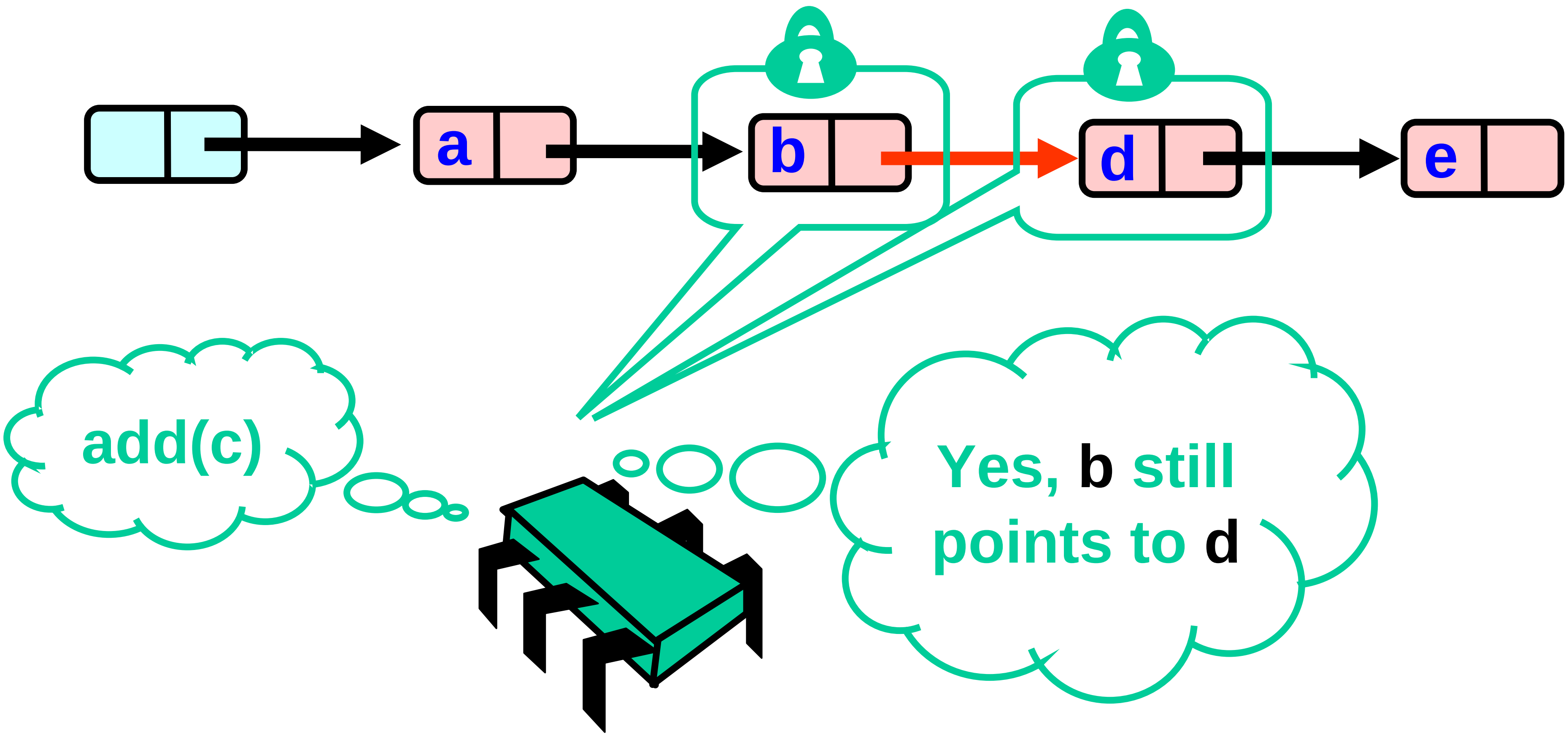
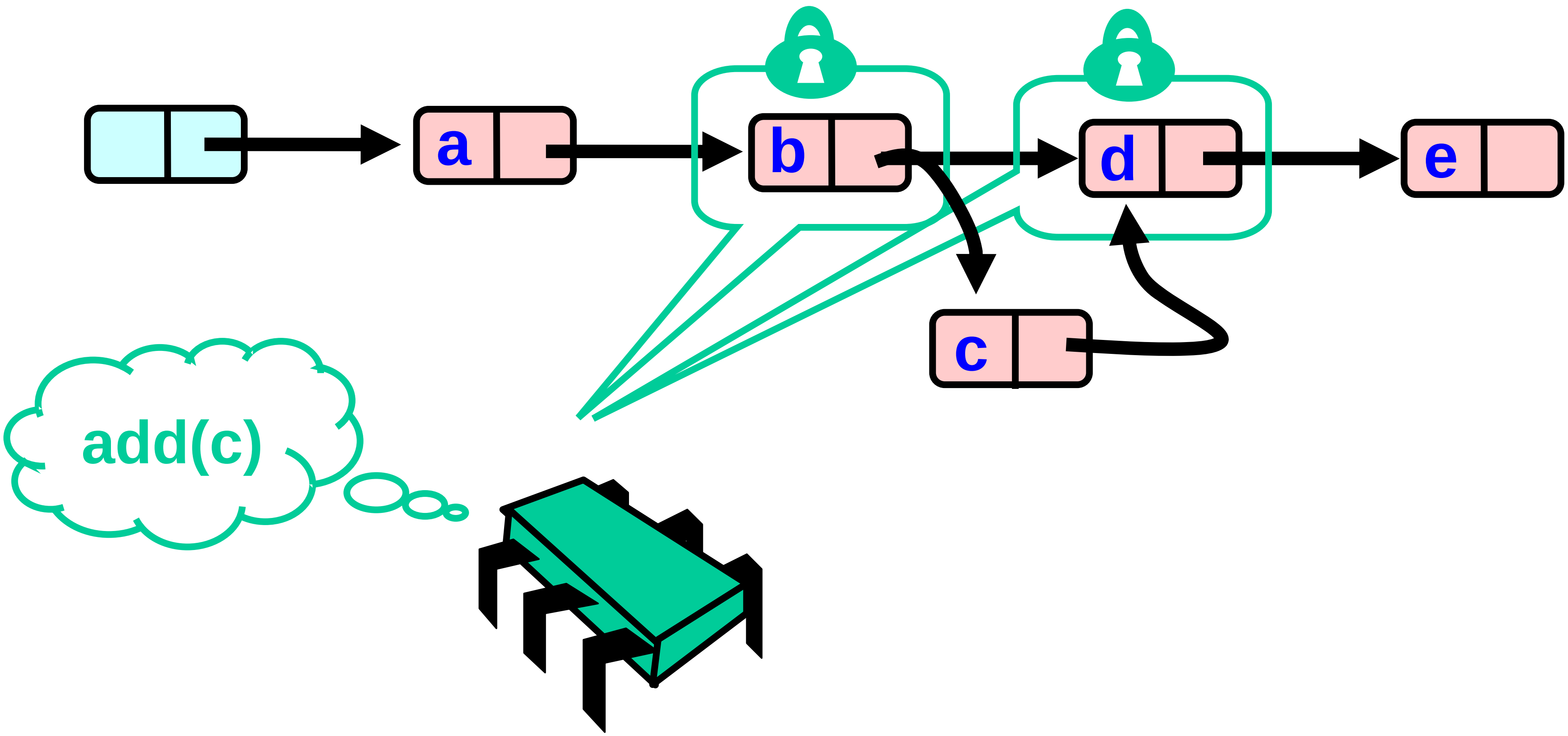


Optimistic locking



Correctness

- Atomicity: Similar to hand-over-hand locking.
  - Forward simulation to canonical atomic set object:
    - S = elements that are reachable in the list.
  - “perform” steps: As for hand-over-hand locking, but consider only the last attempt (for which validation succeeds).
- Liveness:
  - Guarantees progress, assuming the locks do.
  - Does not guarantee lockout-freedom (even if locks do).
  - Blocking (not wait-free or lock-free).

Evaluation

- Works well if lock-free traversal is fast, and contention is infrequent.
- Problems:
  - Repeated traversals.
  - Need to acquire locks.
    - Even contains() needs locks.
- Locks can cause problems:
  - Some operations take 1000x (or more) longer than others, due to page faults, descheduling, etc.
  - If this happens to anyone holding a lock, everyone else who wants to access that lock must wait.

- Q: Can we avoid locks?

Lock-free algorithm

- Avoids locks/blocking entirely.
- Instead, separates logical vs. physical node removal, marking nodes before deleting them.
- Operations help other operations by deleting marked nodes.

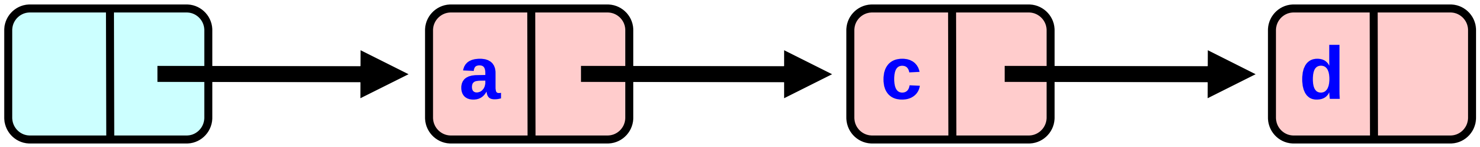
Lock-freedom

- If any process executing an operation does not stop then some operation completes.
- Weaker than wait-free: lockout is possible.
- Rules out a delayed process from blocking other processes indefinitely, and so, no locks.

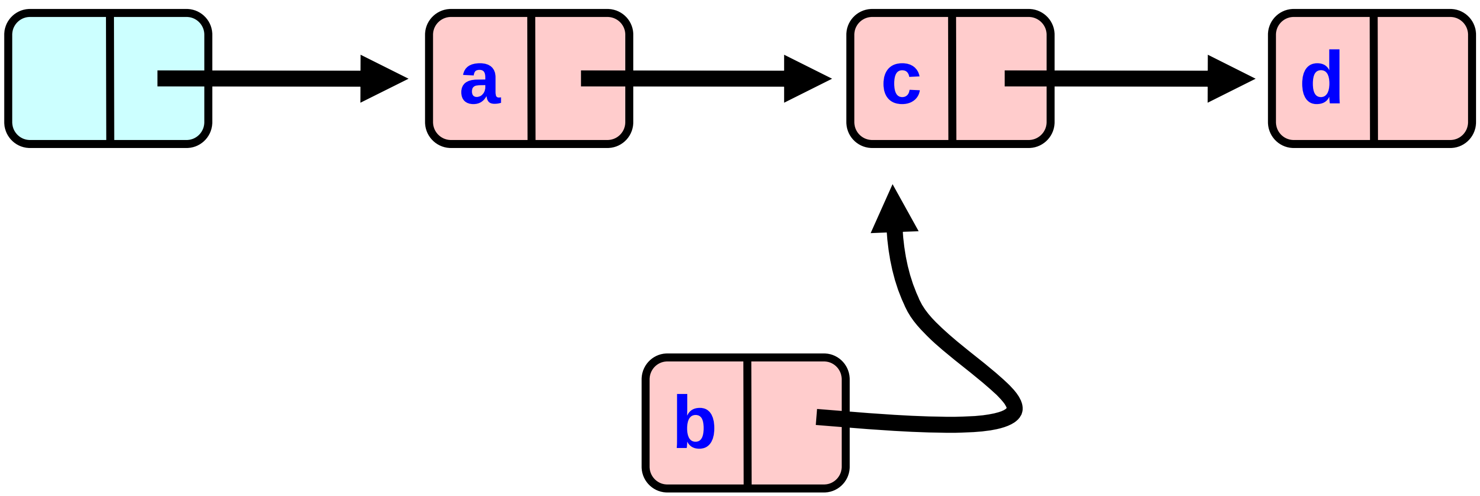
Lock-free list-based set

- Idea: Use CAS to change pred.next pointer.
- Make sure pred.next pointer hasn't changed since you read it.

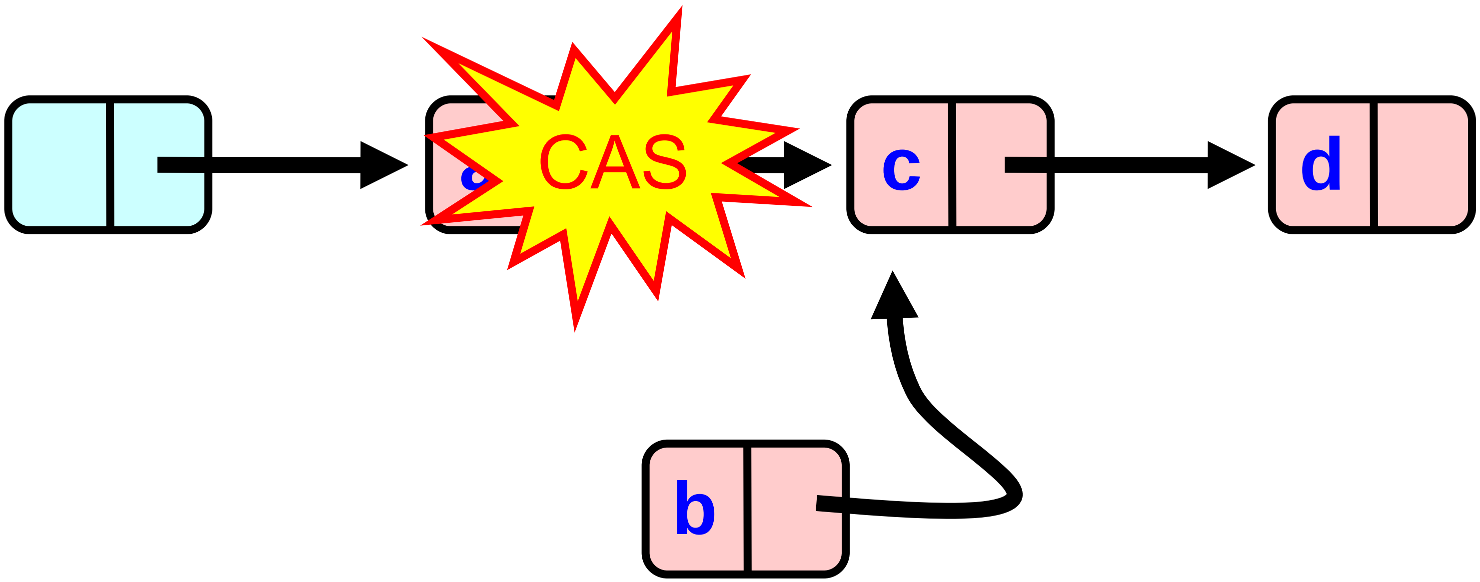
Adding a Node



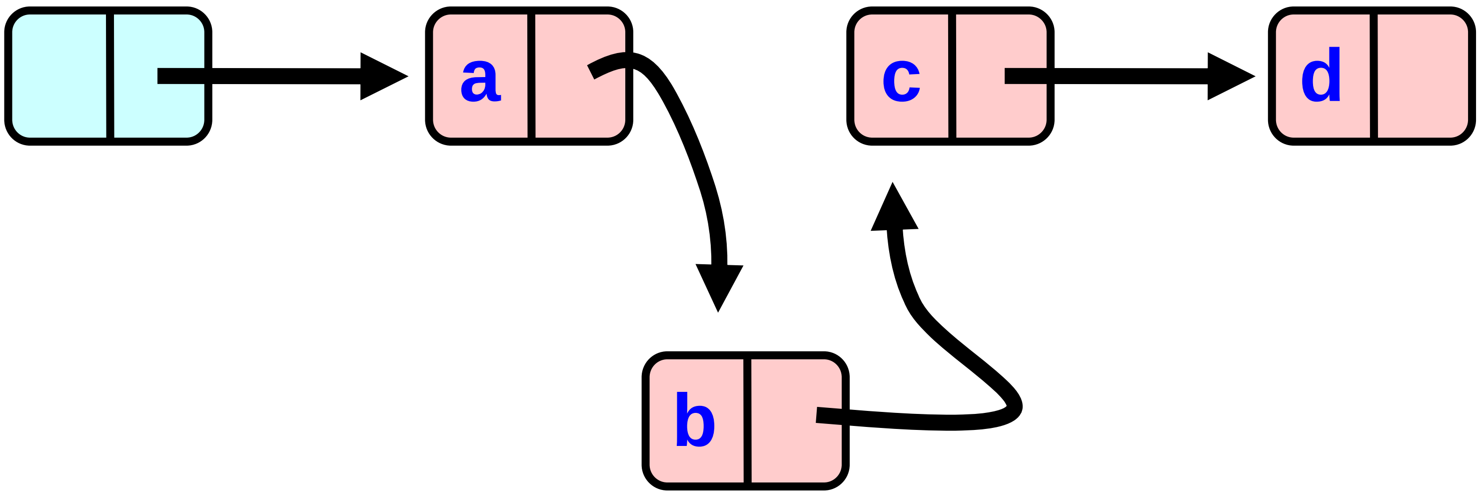
Adding a Node



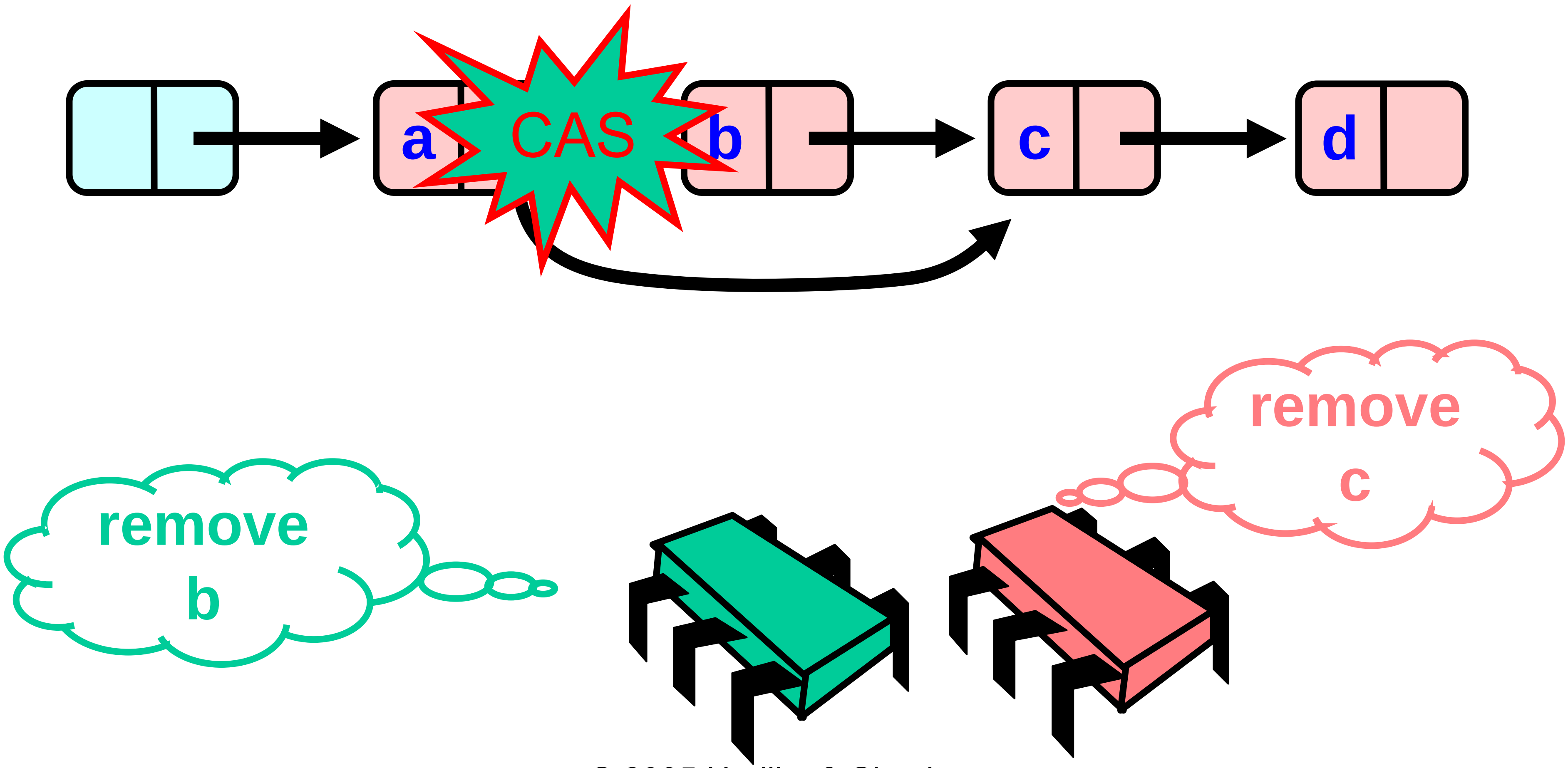
Adding a Node



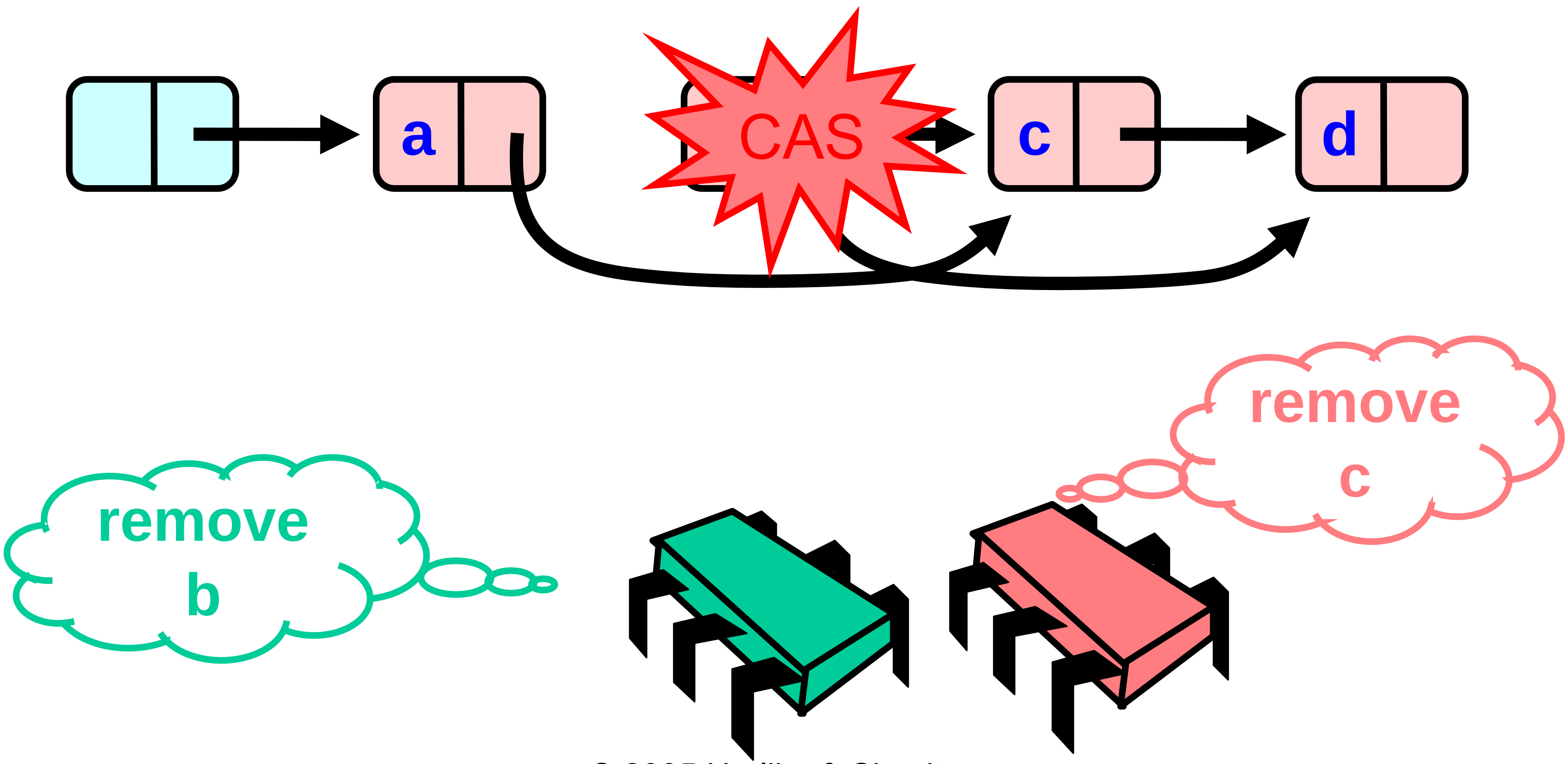
Adding a Node



Removing a Node

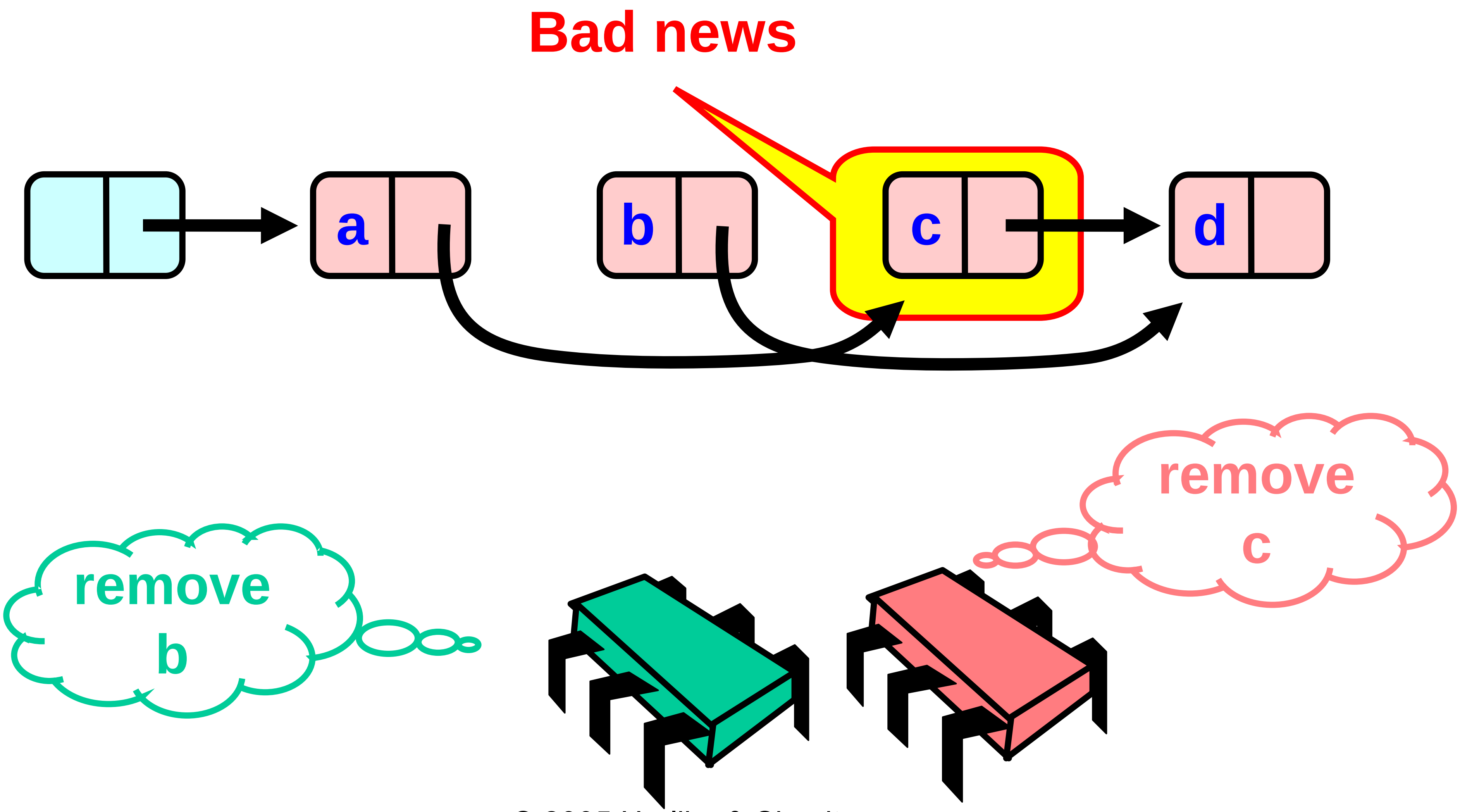


Removing a Node



From *The Art of Multiprocessor Programming*, Maurice Herlihy and Nir Shavit.

Look Familiar?



Lock-free list-based set

- Idea: Add “mark” bit to a node to indicate whether its key has been removed from the abstract set S.
  - If mark = true, then node’s key is not in the set.
  - When a node is first added to the list, its mark = false.
  - Set mark := true before physically removing node from list by detaching its incoming pointer.
  - Setting the mark logically removes the node’s key from the set: It is the serialization point of a successful remove operation.

- Simulation relation:
  - S is the set of values in reachable nodes with mark = false.
- Don't change next pointer of a marked node.
  - Mark and next pointer must be in the same word, change atomically.
  - “Steal” a bit from pointers.
  - Jave class AtomicMarkableReference (in Java concurrency library) supports techniques like those in this algorithm.

Lock-free list-based set

- To perform any operation, traverse the list, through marked and unmarked nodes, to find needed nodes.
- If needed nodes are marked, retry the operation.
- If needed nodes are unmarked then operate as follows:
  - For contains(x) or unsuccessful add/remove(x), return appropriate value as usual based on whether curr.key = x
  - For successful add(x), CAS pred's (curr, false) to (node, false).
  - For successful remove(x),
    - Logical removal: CAS curr's (next, false) to (next, true)
    - Physical removal: CAS pred's (curr, false) to (curr.next, false)
  - If any CAS except for the physical remove fails, retry the operation.

Helping

- Whenever an operation encounters marked nodes during traversal, it helps:
- If curr is marked:
  - CAS pred's (curr, false) to (curr.next, false).
  - If this CAS fails (because next is no longer curr or mark is now true), then retry the operation.

- Such helping is characteristic of lock-free and wait-free algorithms (not all have it, but most do).

- See HS Section 9.8.

Lock-free list: Find subroutine

Returns (pred, curr) such that at some point during execution, the following held simultaneously: pred.next = (curr, false), curr.next.mark = false, and pred.key < x ≤ curr.key.

```
S.find(x)
retry:
  pred := S.head; curr := pred.next.ref
  while (curr.key < x or curr.next.mark) do
    if curr.next.mark then
      if CAS(pred.next, (curr, false), (curr.next.ref, false)) then curr := pred.next.ref
      else
        if pred.next.mark then goto retry
        else curr := pred.next.ref
    else // It must be that curr.key < x.
      pred : = curr; curr := pred.next.ref
  return (pred, curr)
```

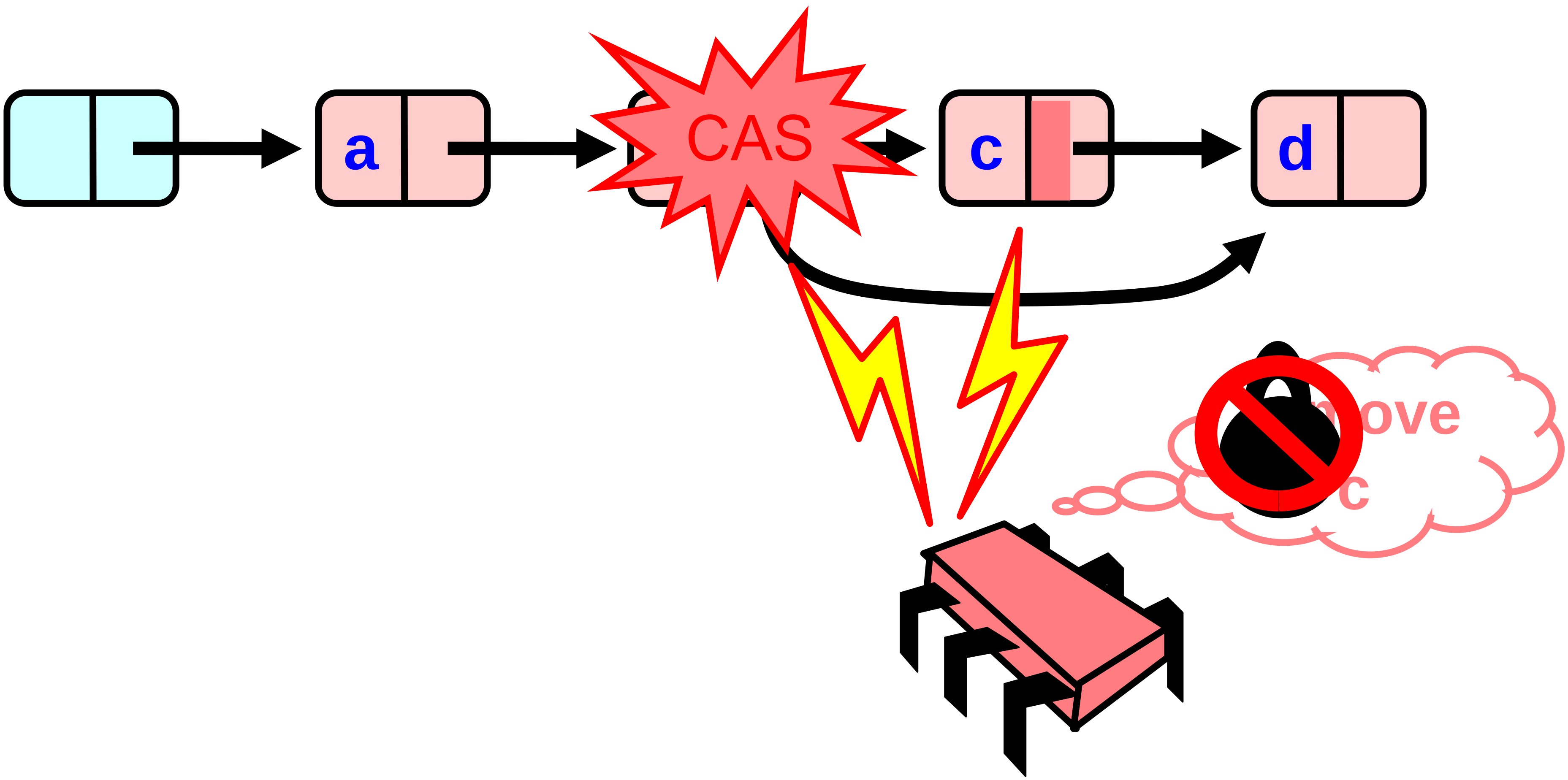
Lock-free list: Add

```
S.add(x)
retry:
    (pred, curr) := S.find(x)
    if curr.key = x then return false
    else
        node := new Node(x)
        node.next.ref := curr
        if CAS(pred.next, (curr, false), (node, false)) then return true
        else goto retry
```

Lock-free list: Remove and Contains

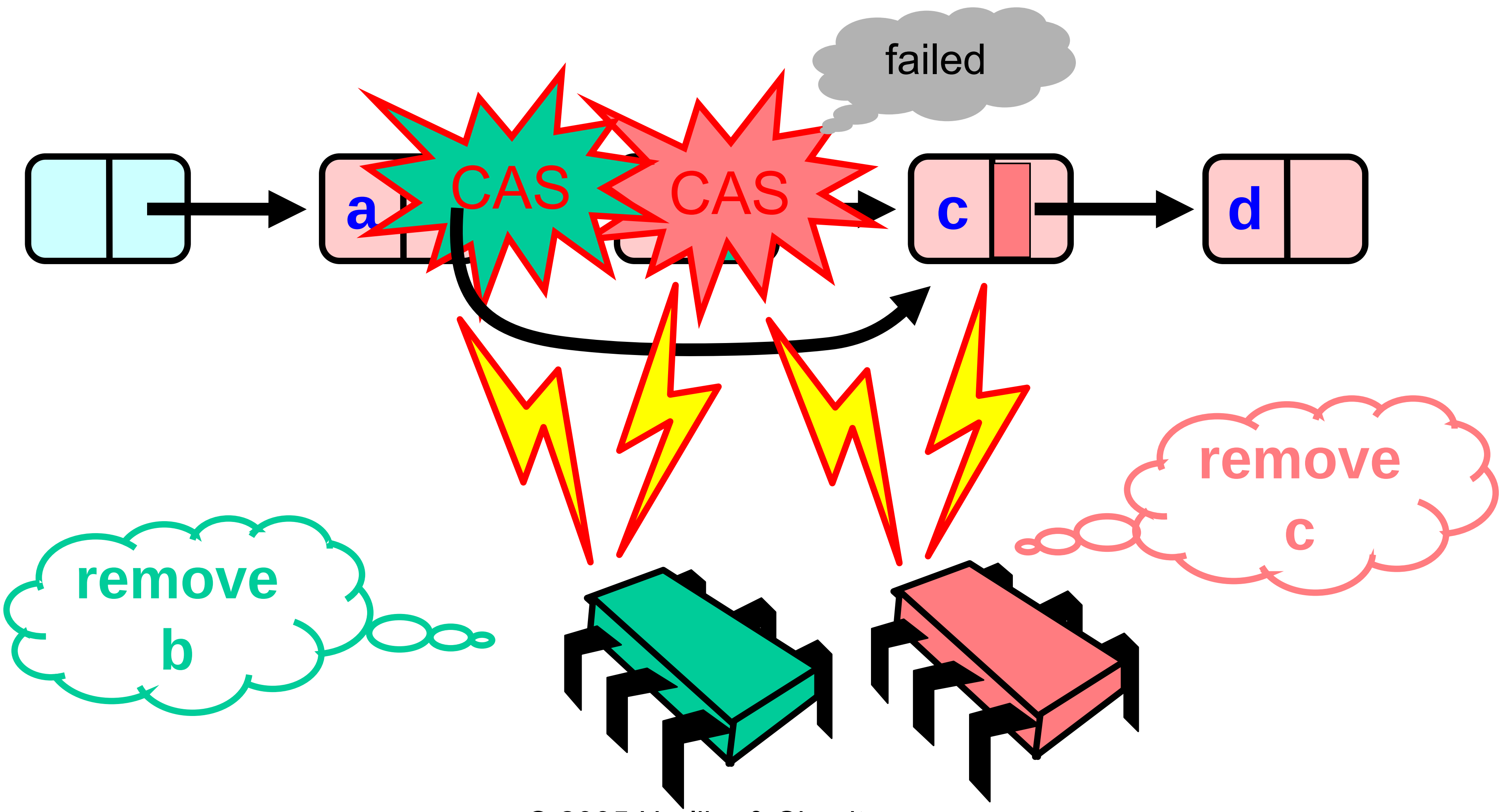
```
S.remove(x)
retry:
      (pred, curr) := S.find(x)
      if curr.key = x then
          next := curr.next.ref
          if CAS(curr.next, (next, false), (next, true)) then
              CAS(pred.next, (curr, false), (curr.next.ref, false))
              return true
          else goto retry
      else return false

S.contains(x)
    (pred, curr) := S.find(x)
    if curr.key = x then return true
    else return false
```

Removing a Node

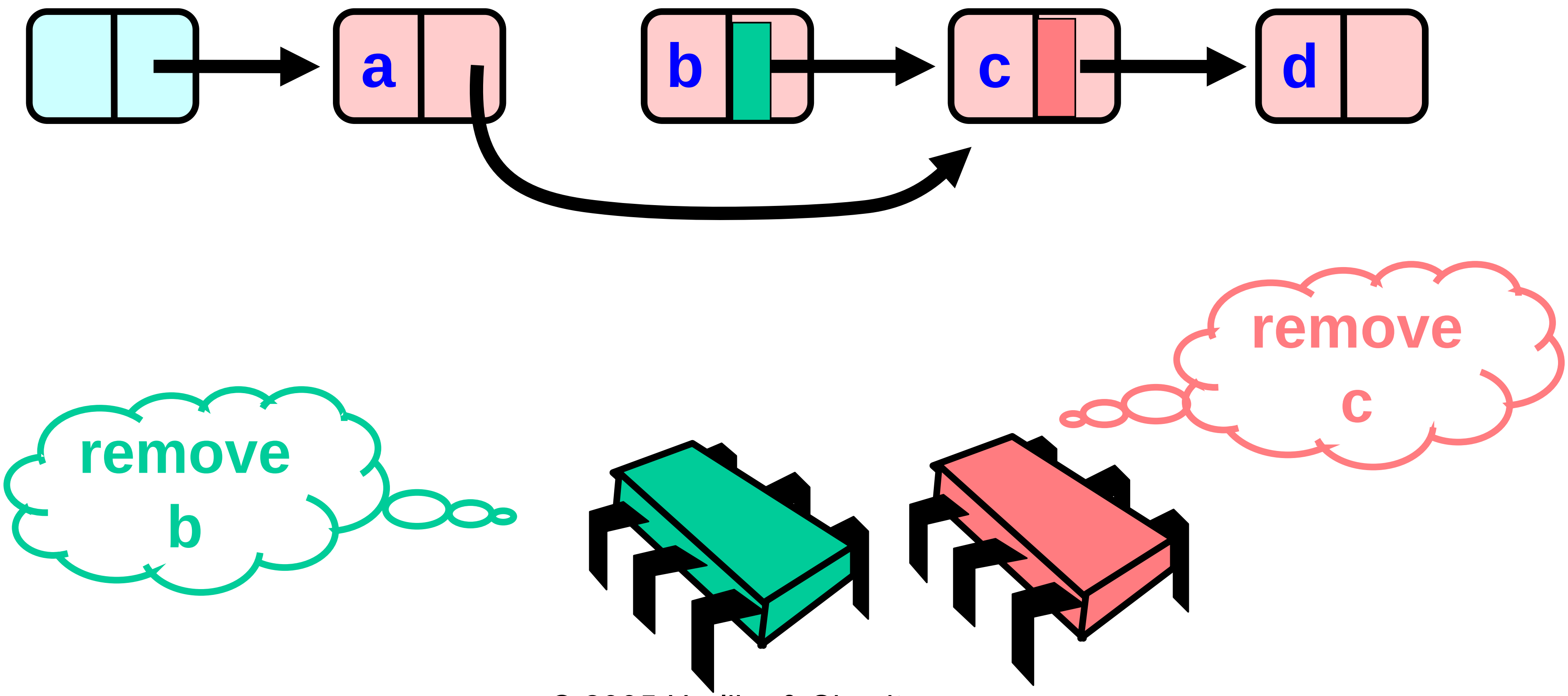


From *The Art of Multiprocessor Programming*, Maurice Herlihy and Nir Shavit.

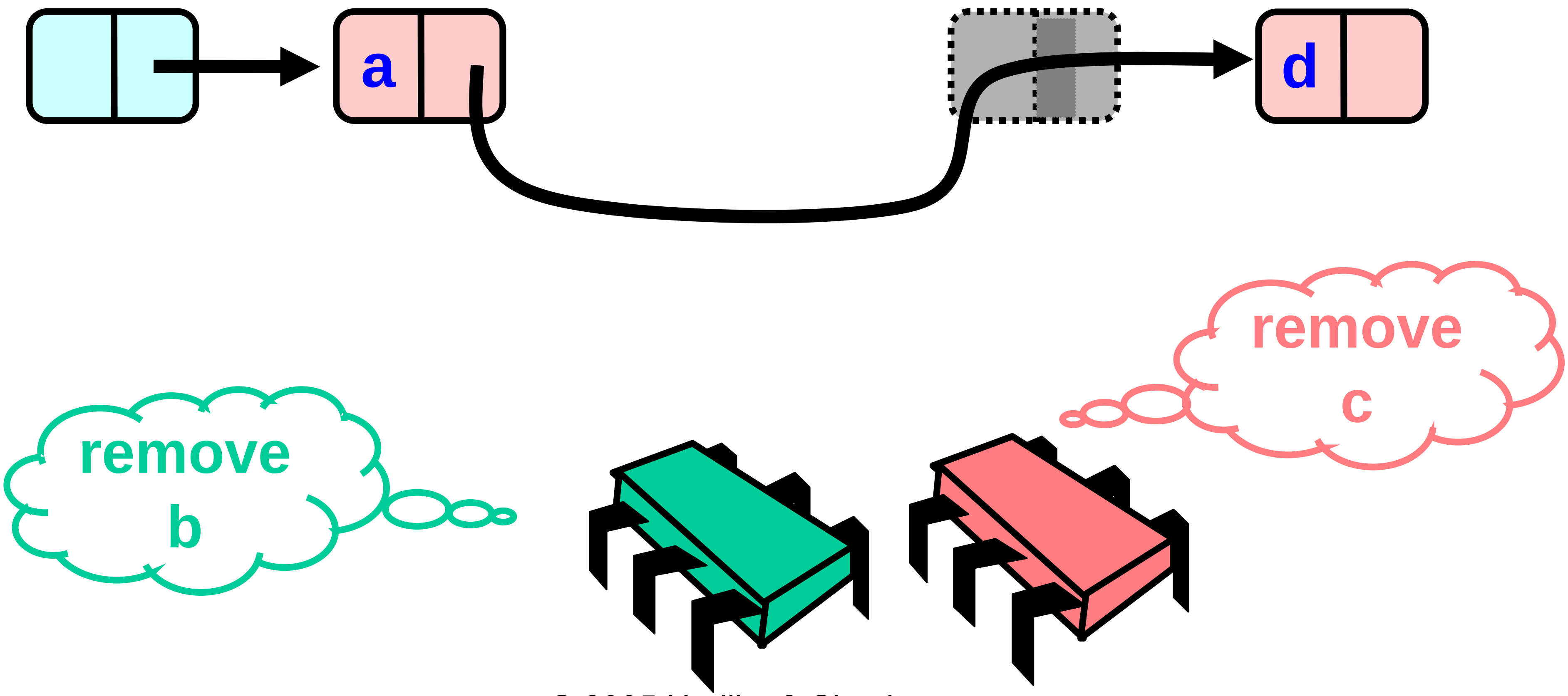
Removing a Node



Removing a Node



Removing a Node



Correctness

- Atomicity:
  - Forward simulation to canonical atomic set:
    - S = values in unmarked nodes that are reachable from the head via list pointers (through marked and unmarked nodes).
  - "perform" steps:
    - contains(x) or unsuccessful add(x) or remove(x): When curr is read from pred.next.
    - Successful add(x): When successful CAS sets pred.next := node.
    - Successful remove(x): When successful CAS marks node x (sets curr.mark := true).
  - Invariant: Any unmarked node encountered while traversing the list is reachable in the list.
- Liveness:
  - Nonblocking: lock-free
    - Operations may retry, but some must succeed.
  - Allows starvation (not lockout-free).

Evaluation

- No locks!
- Nonblocking, lock-free algorithm.
- But: Overhead for CAS and for helping.

Lazy algorithm

- Uses the marking trick as in the lock-free algorithm, removing nodes in two stages.
- Avoids CAS and helping.
- Instead, uses short-duration locks.

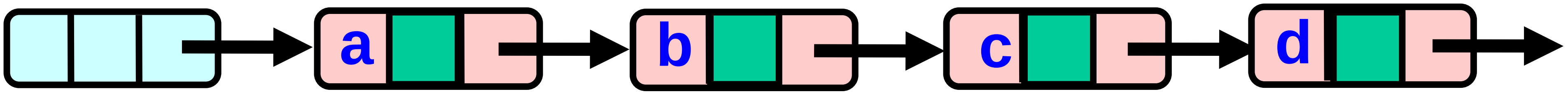
Lazy list algorithm

- Idea: Use mark as in lock-free list.
- "Lazy" removal: First mark node, then splice around it.
- Now mark can be separate from next pointer.
- No helping---assume each remove operation completes its own physical removal.

- Locks curr and pred nodes, with short-duration locks.
- Validation: Check locally that nodes are adjacent and unmarked; if not, retry the operation.

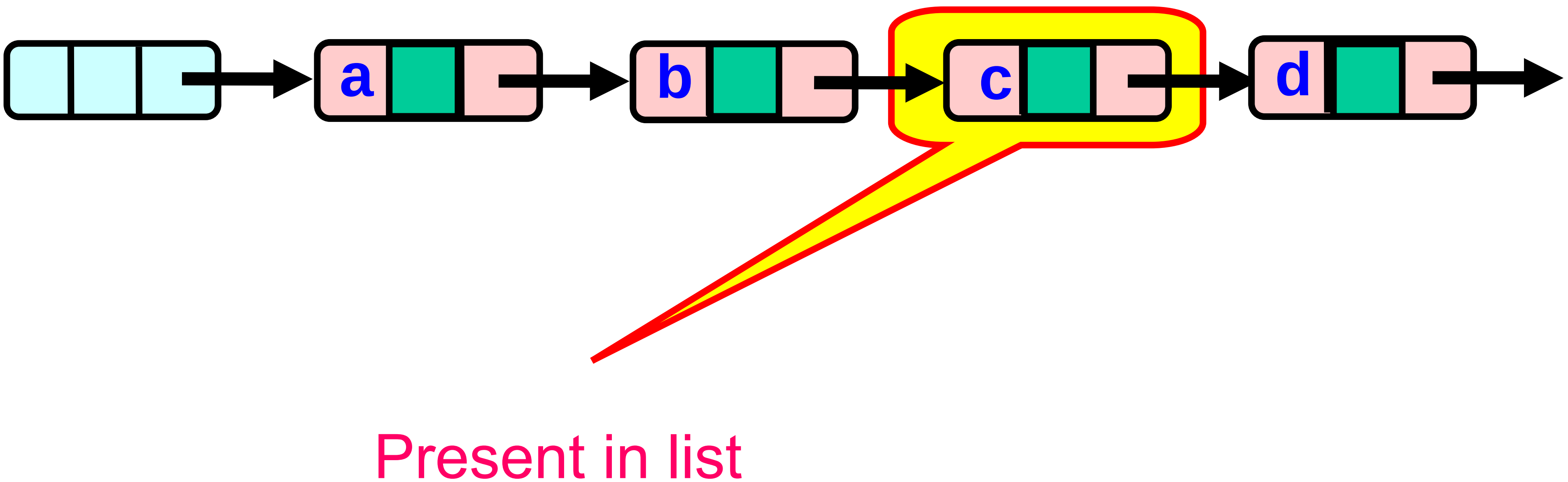
- See HS, Section 9.7.

Lazy Removal

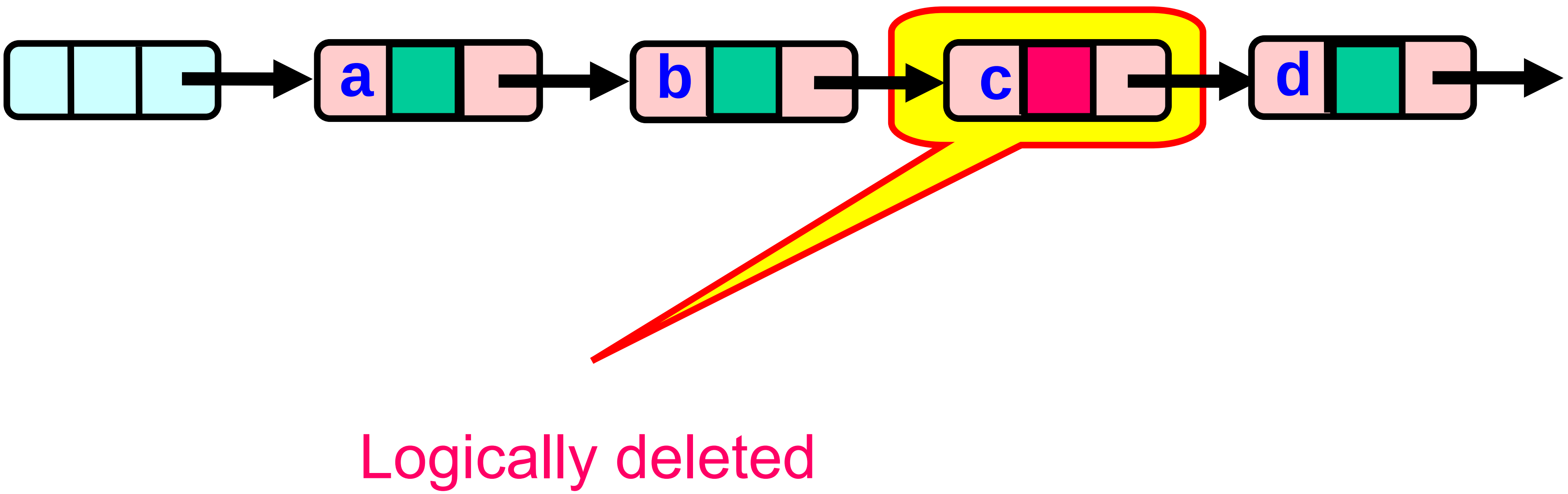


From *The Art of Multiprocessor Programming*, Maurice Herlihy and Nir Shavit.

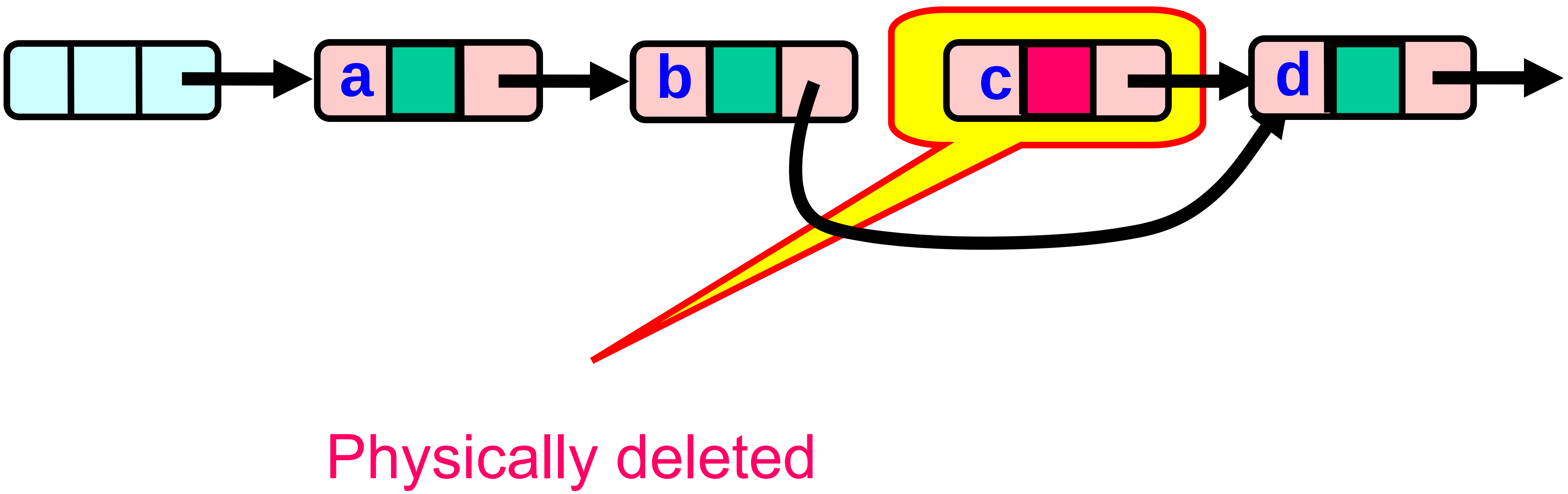
Lazy Removal



Lazy Removal



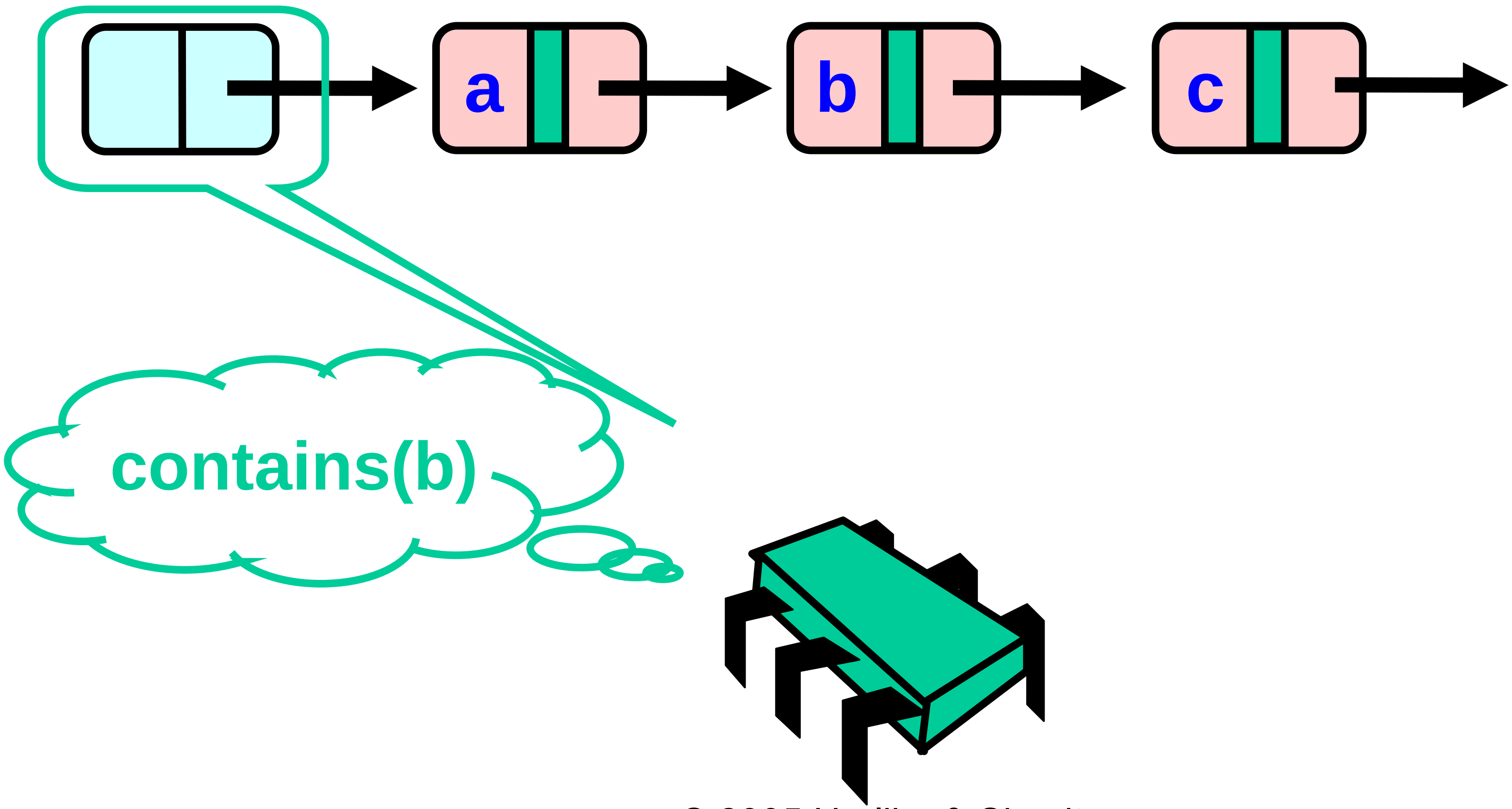
Lazy Removal



Lazy list algorithm

- Observation: contains(x) doesn't need to lock/validate.
- Just find first node with key ≥ x, return true iff key = x and unmarked.

Lazy list algorithm



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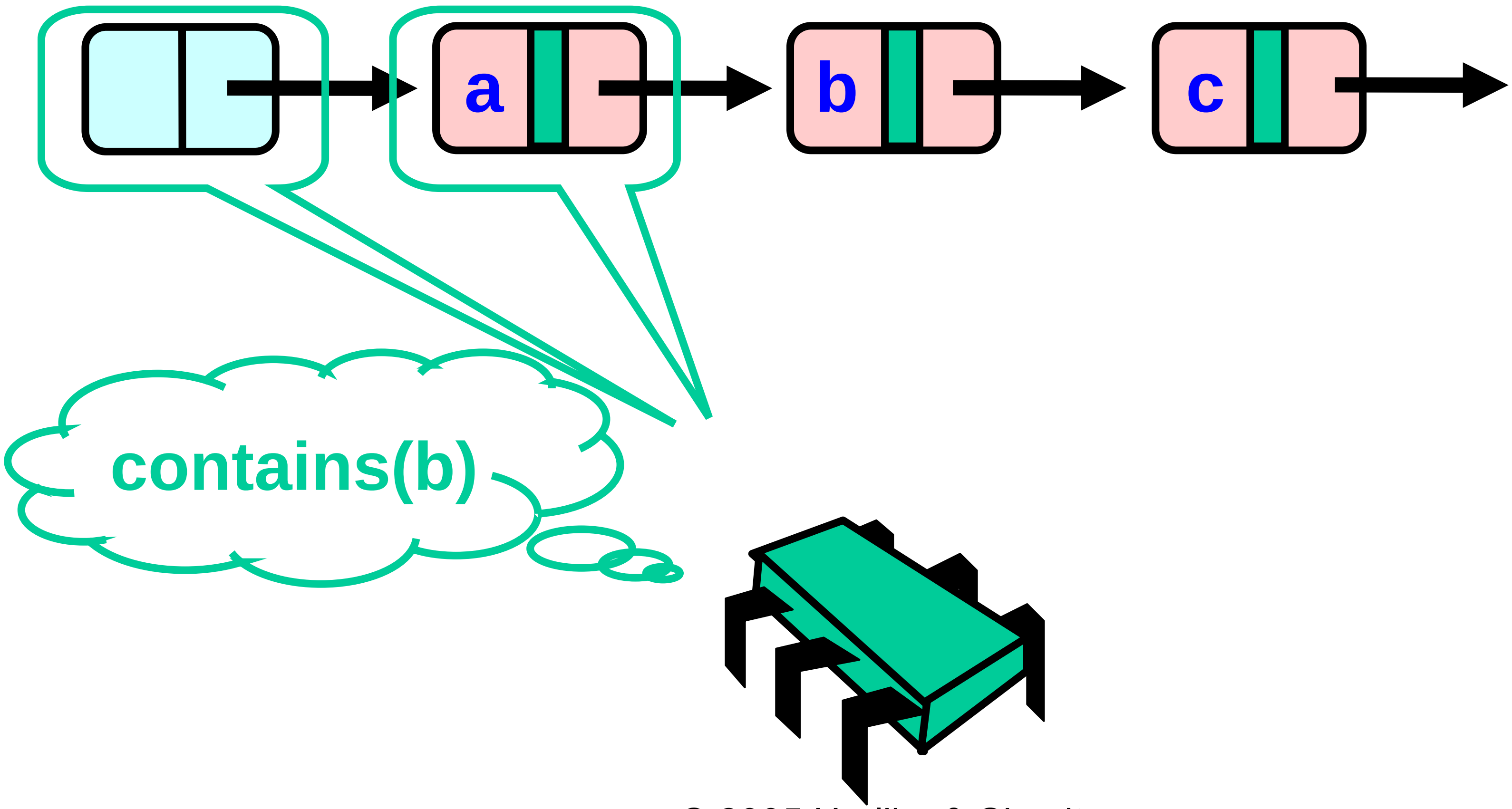
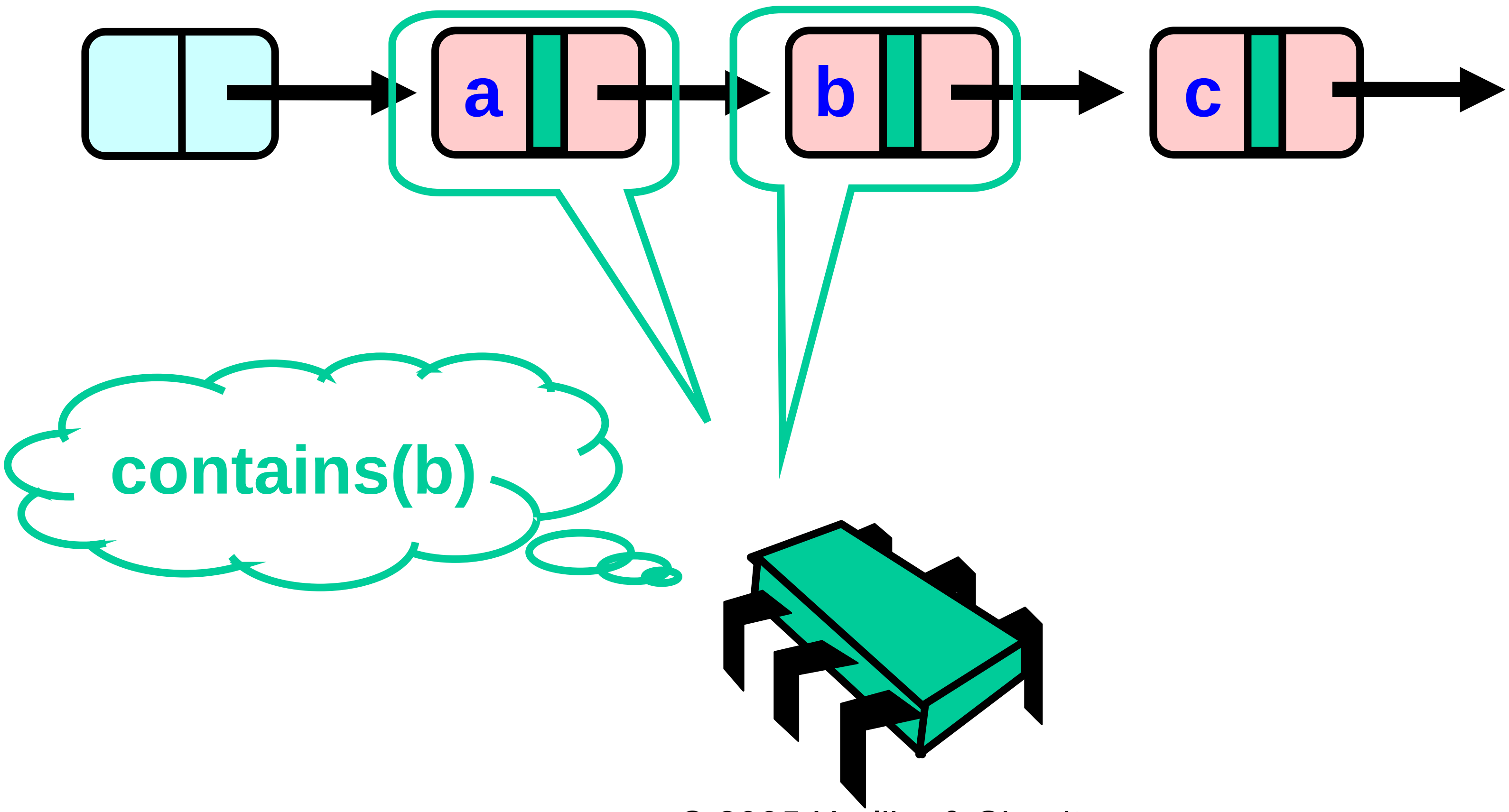
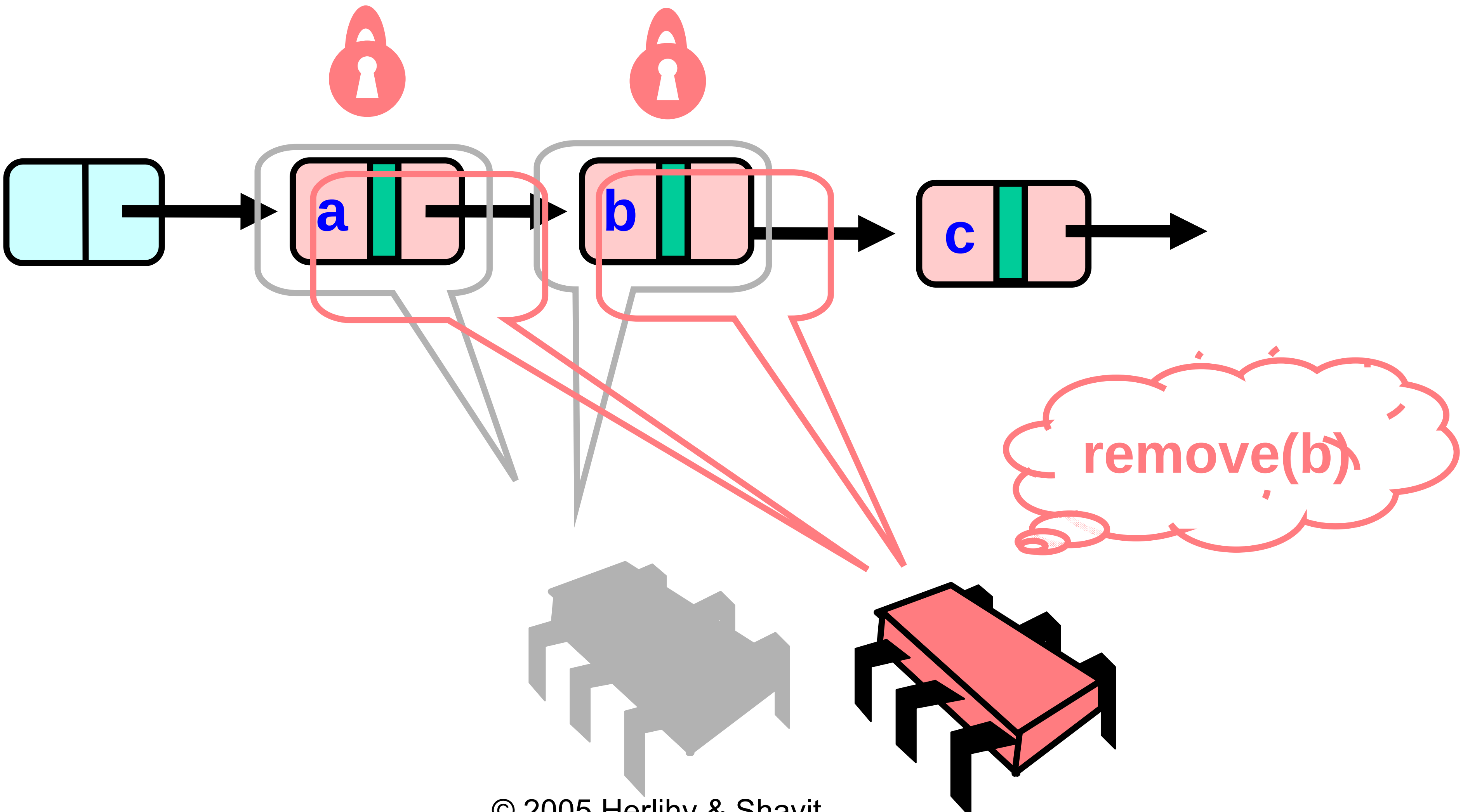
Lazy list algorithm



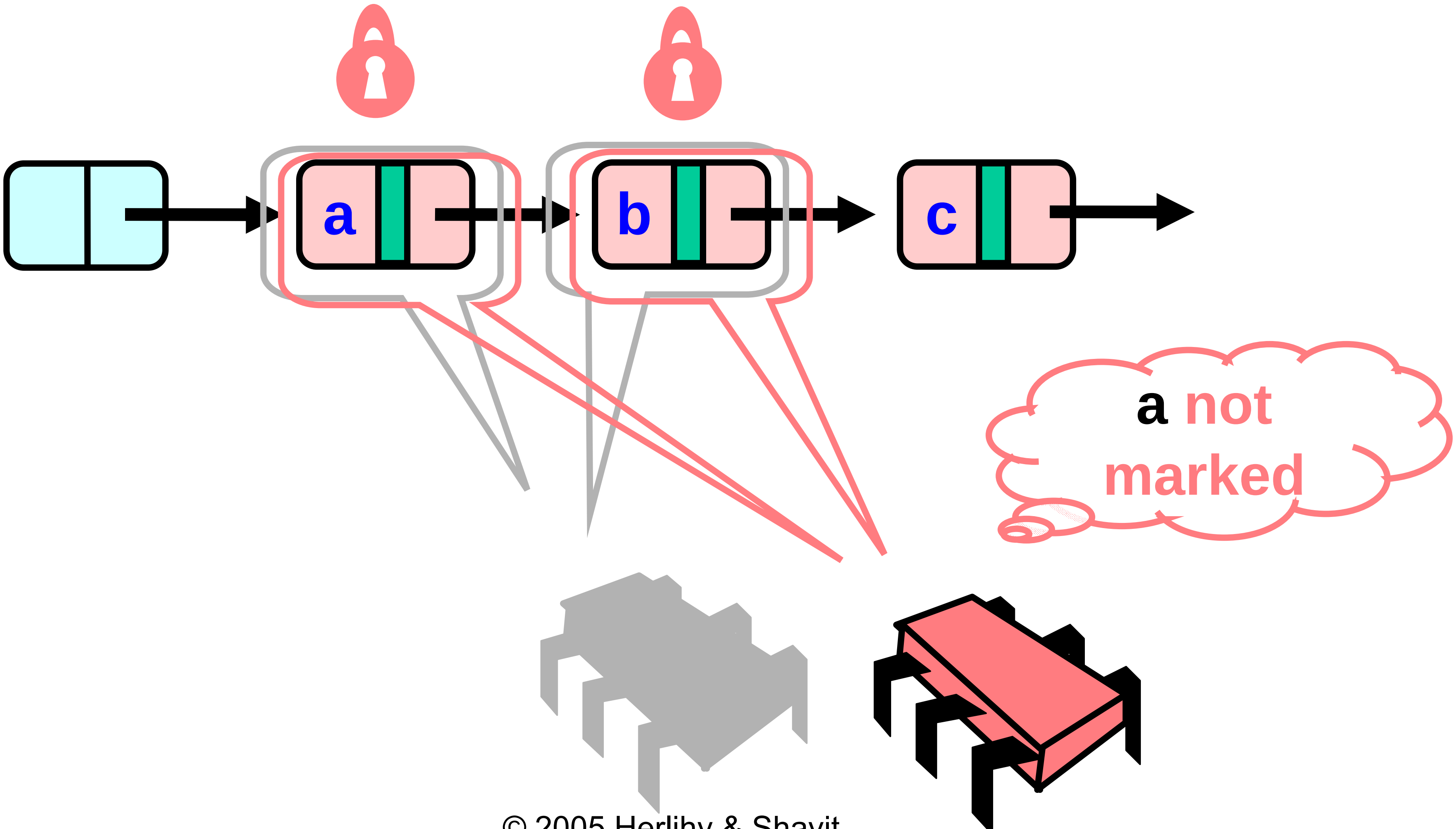
Lazy list algorithm



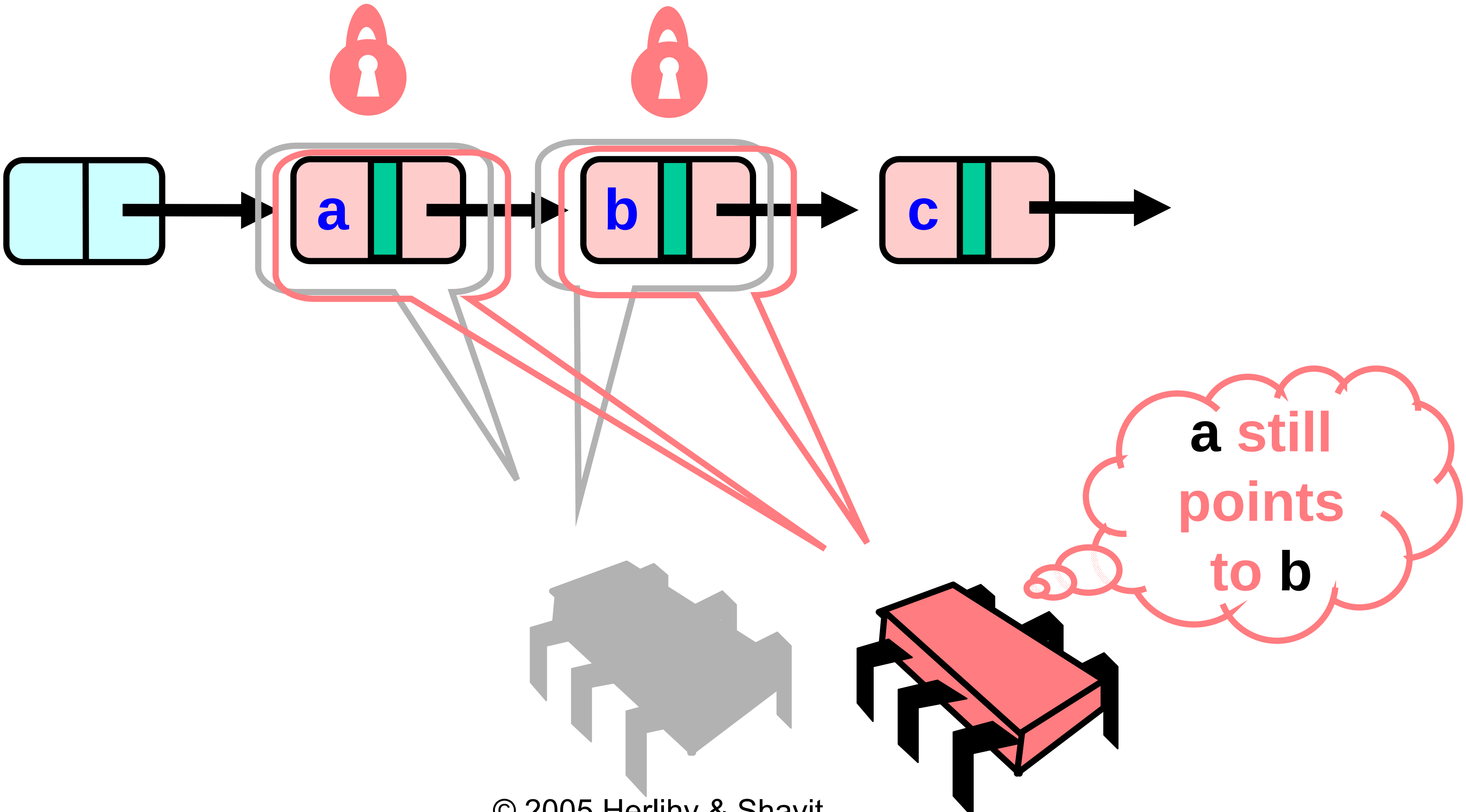
Lazy list algorithm



Lazy list algorithm



Lazy list algorithm



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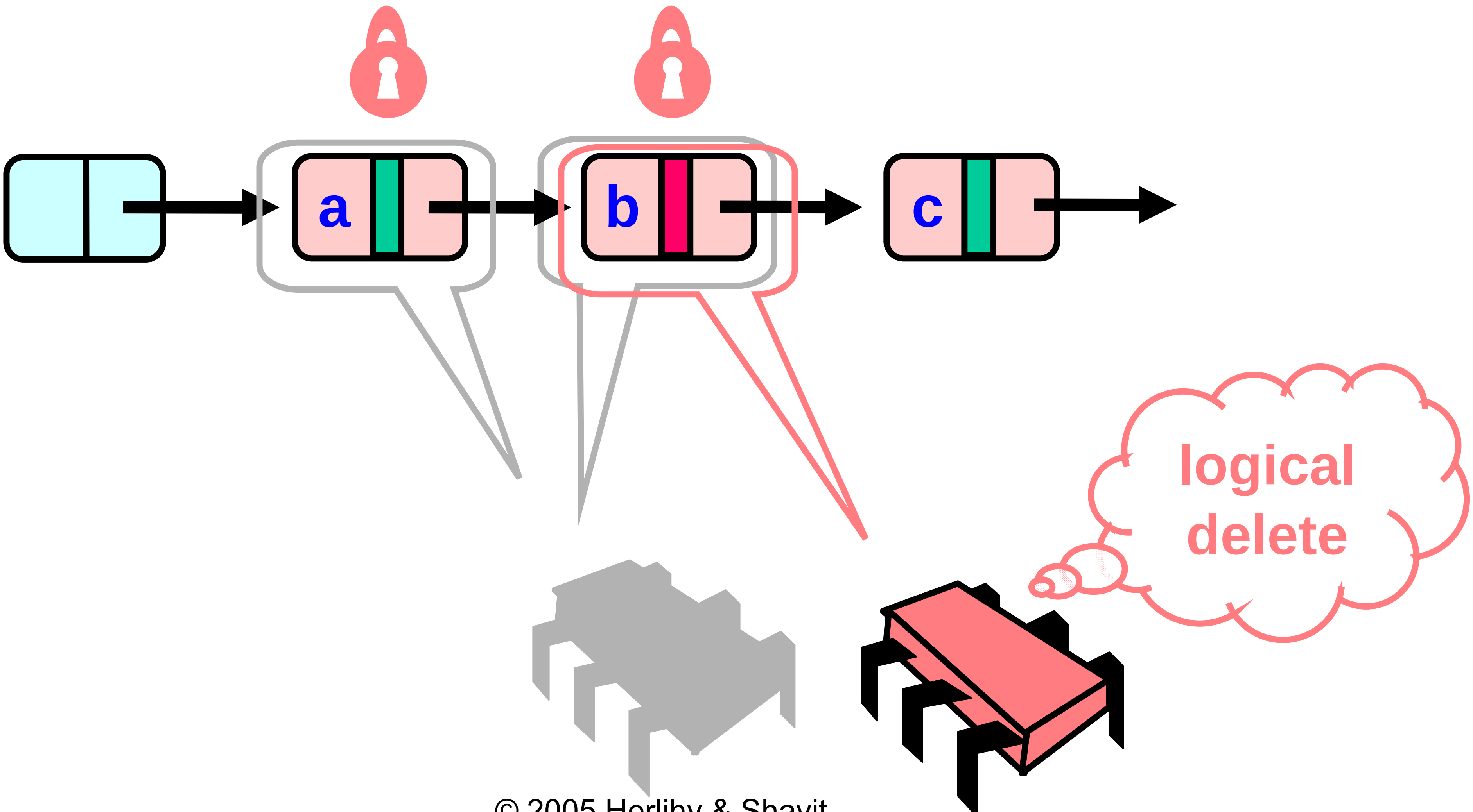
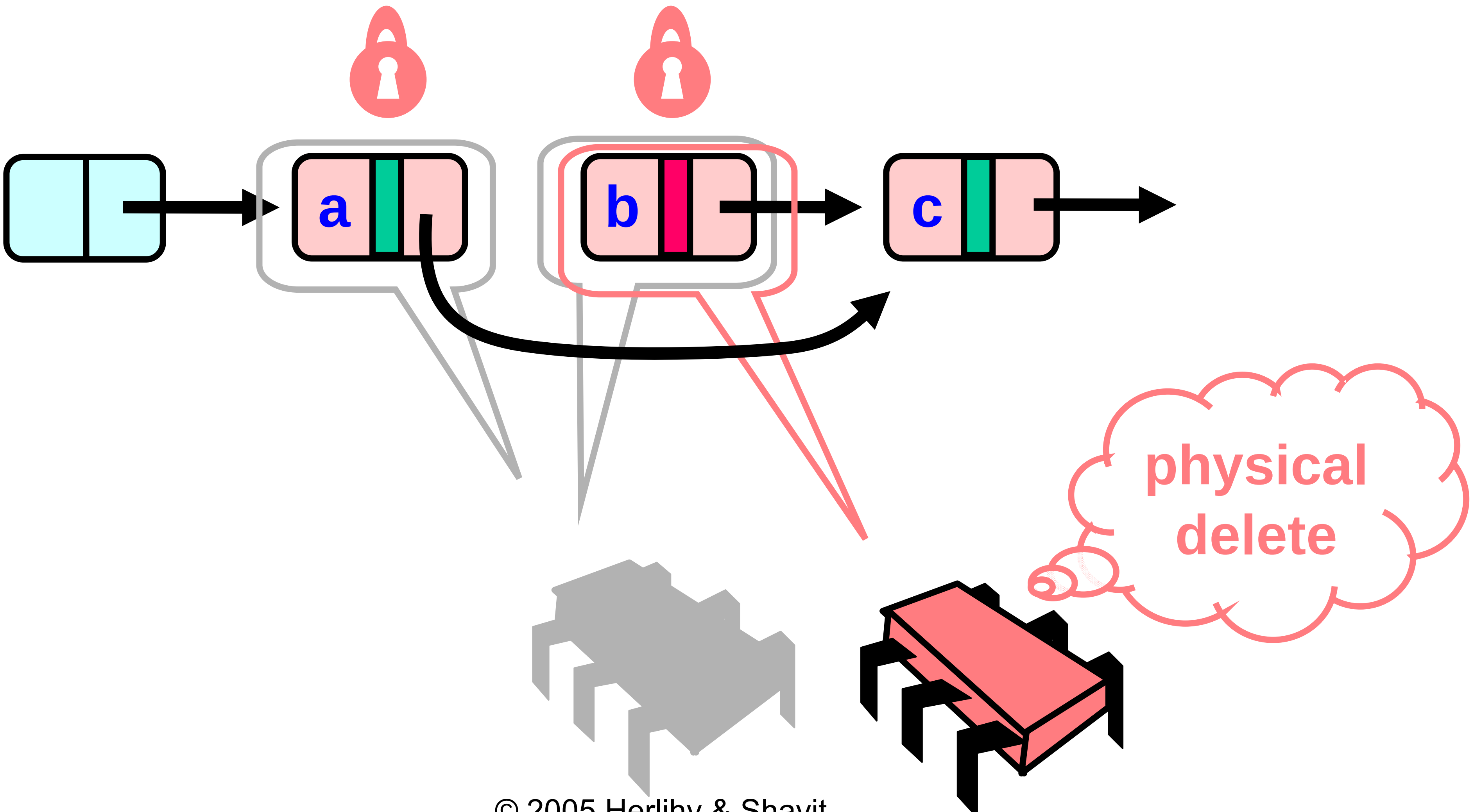
Lazy list algorithm

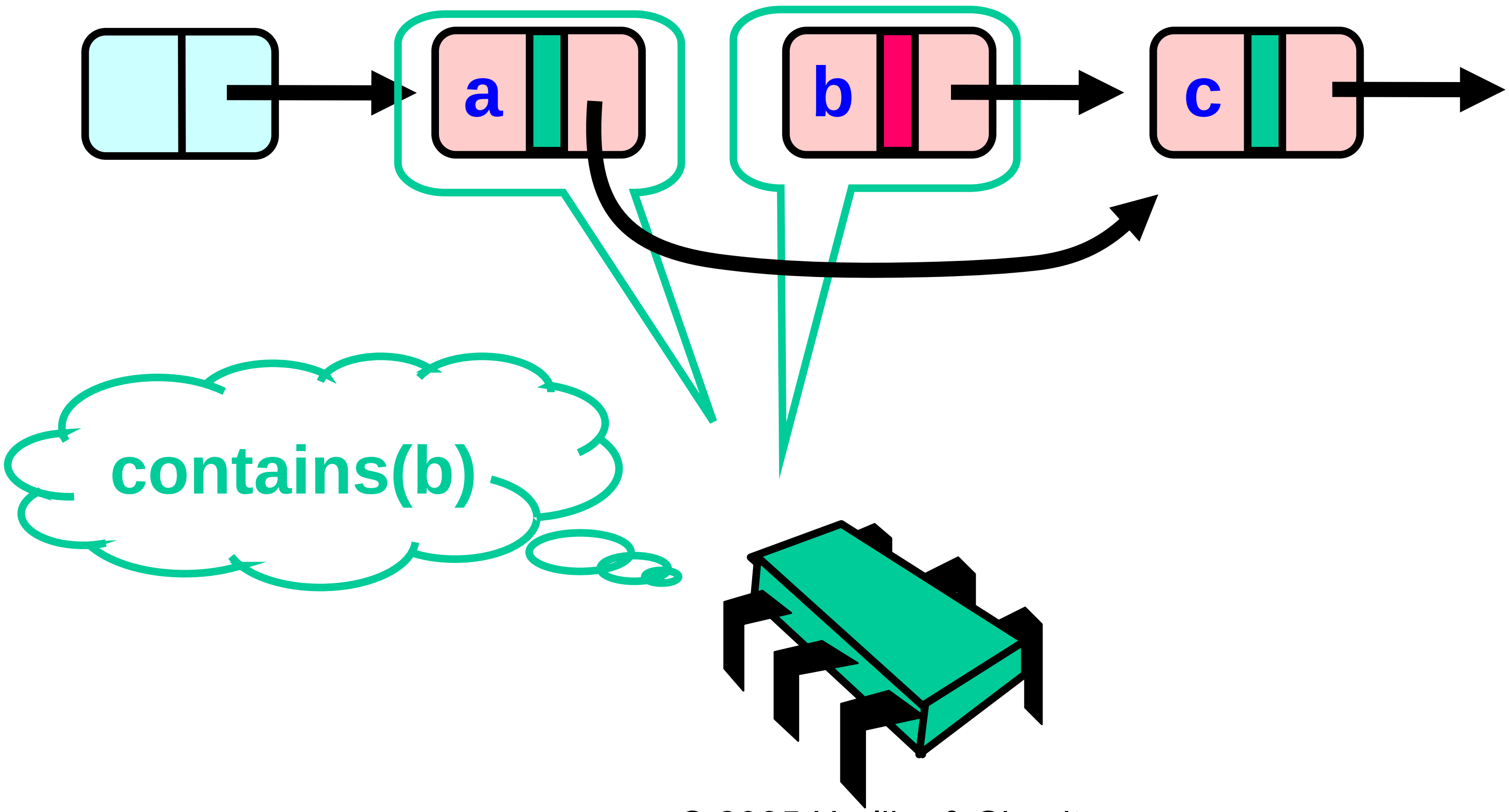


From *The Art of Multiprocessor Programming*, Maurice Herlihy and Nir Shavit.

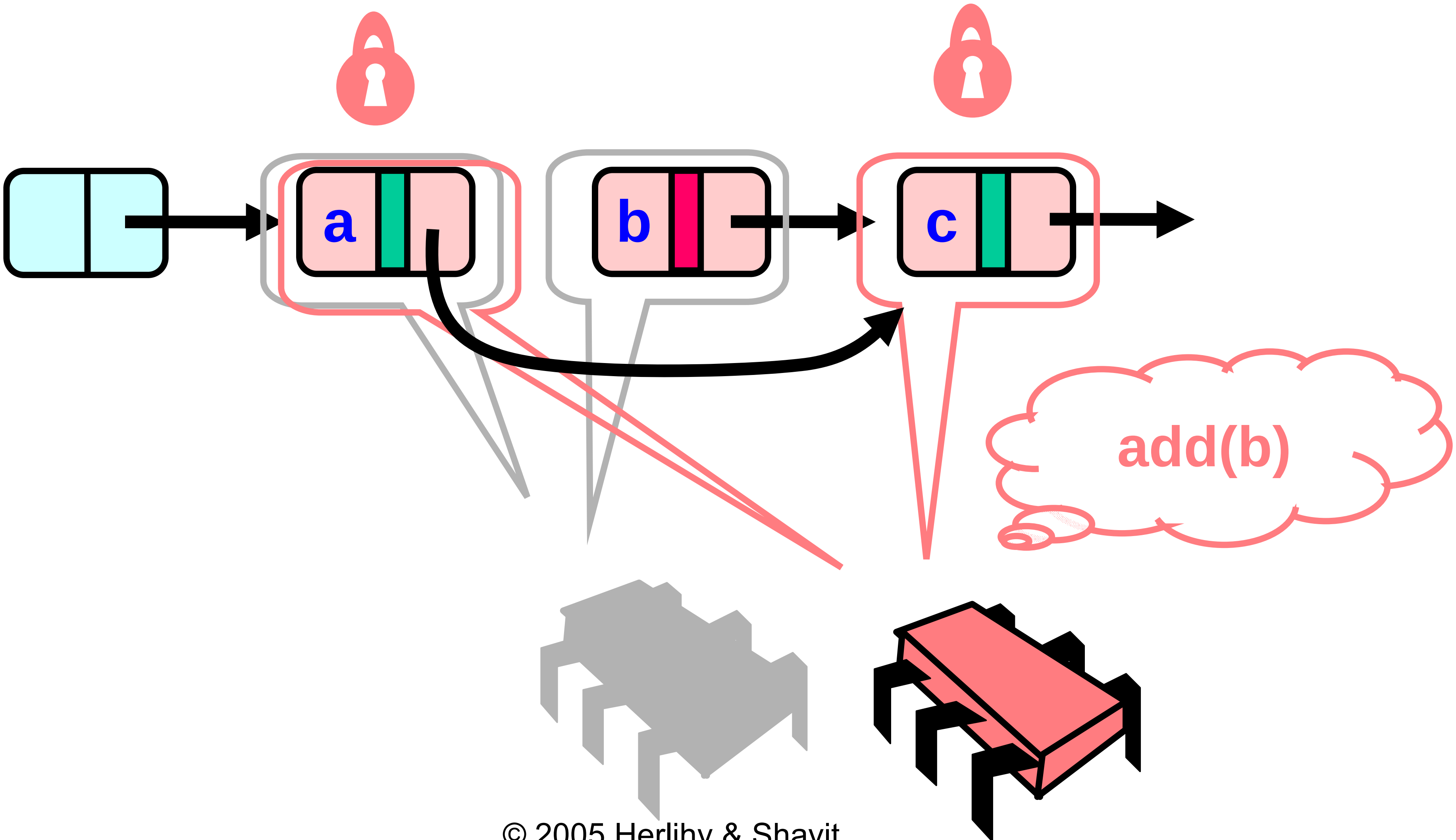
Lazy list algorithm



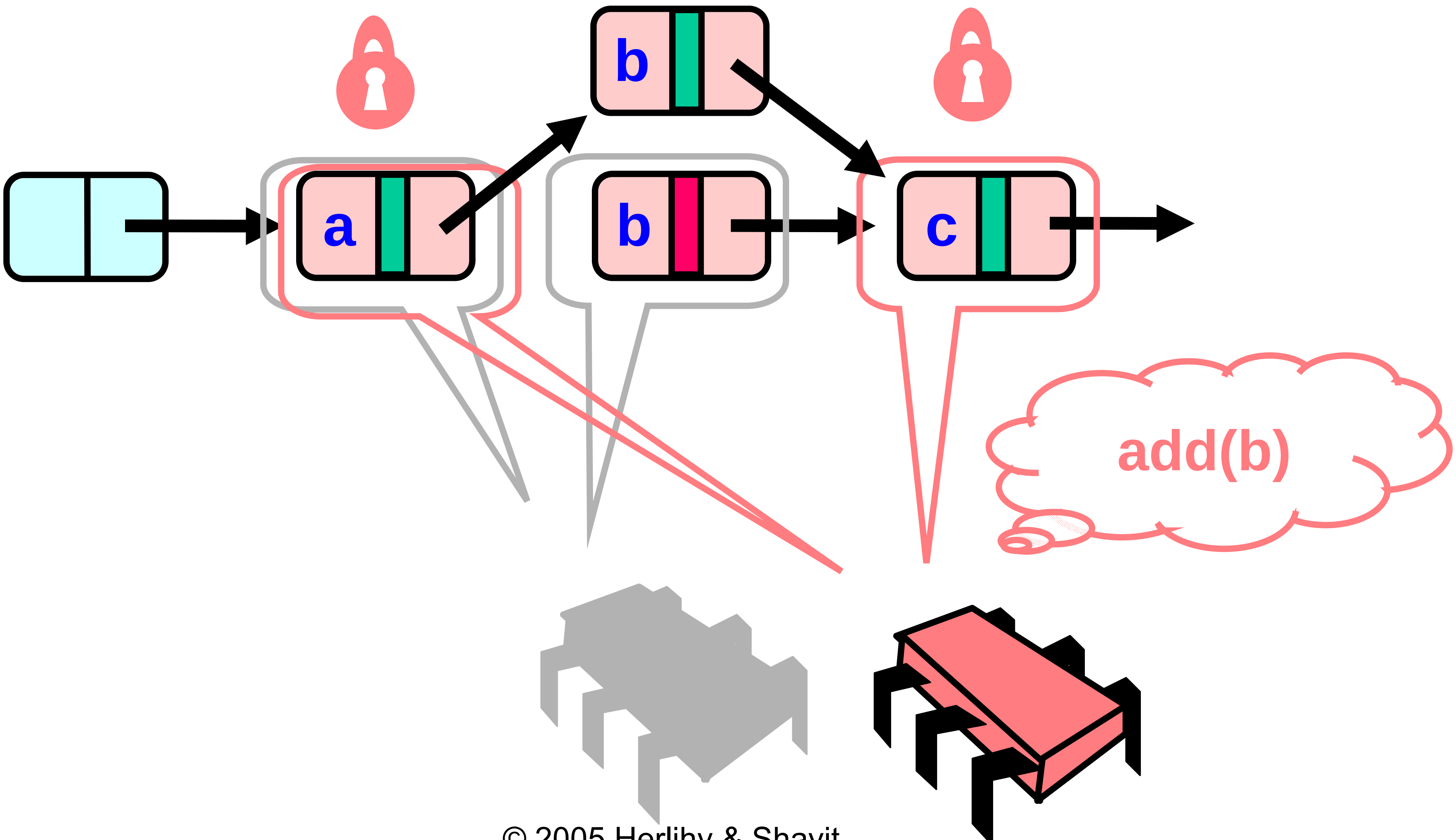
Lazy list algorithm



Lazy list algorithm



Lazy list algorithm



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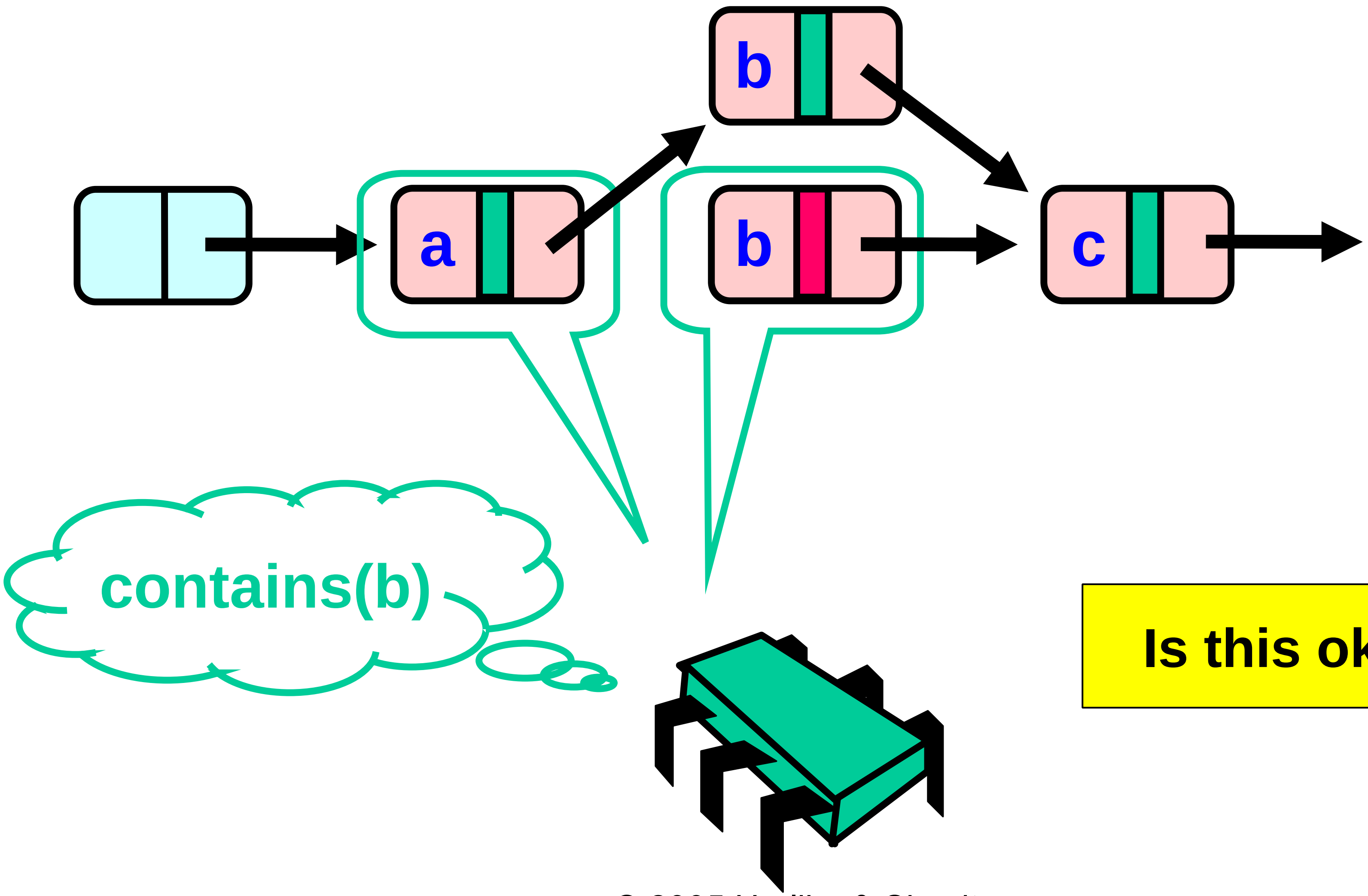
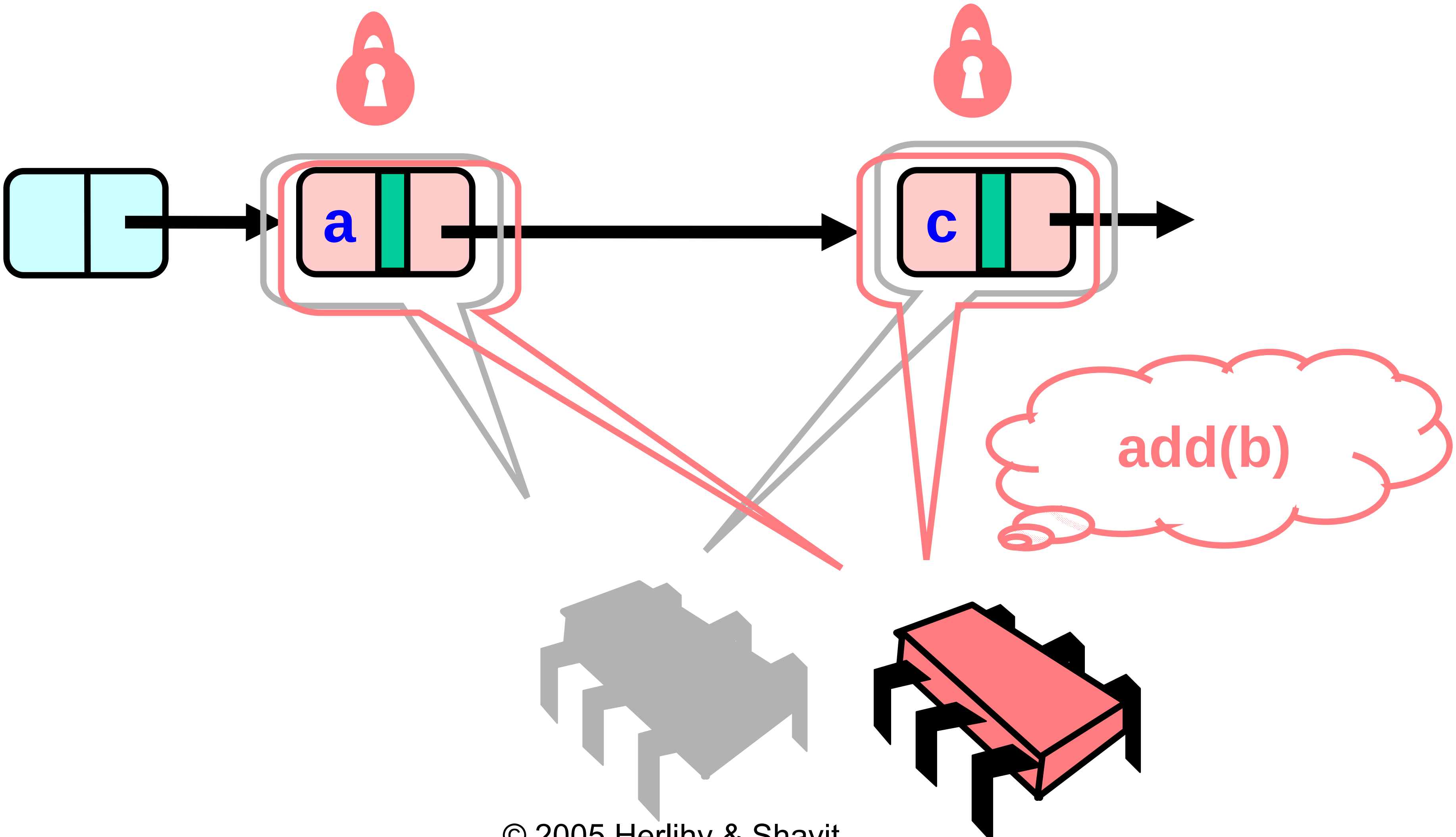
Lazy list algorithm

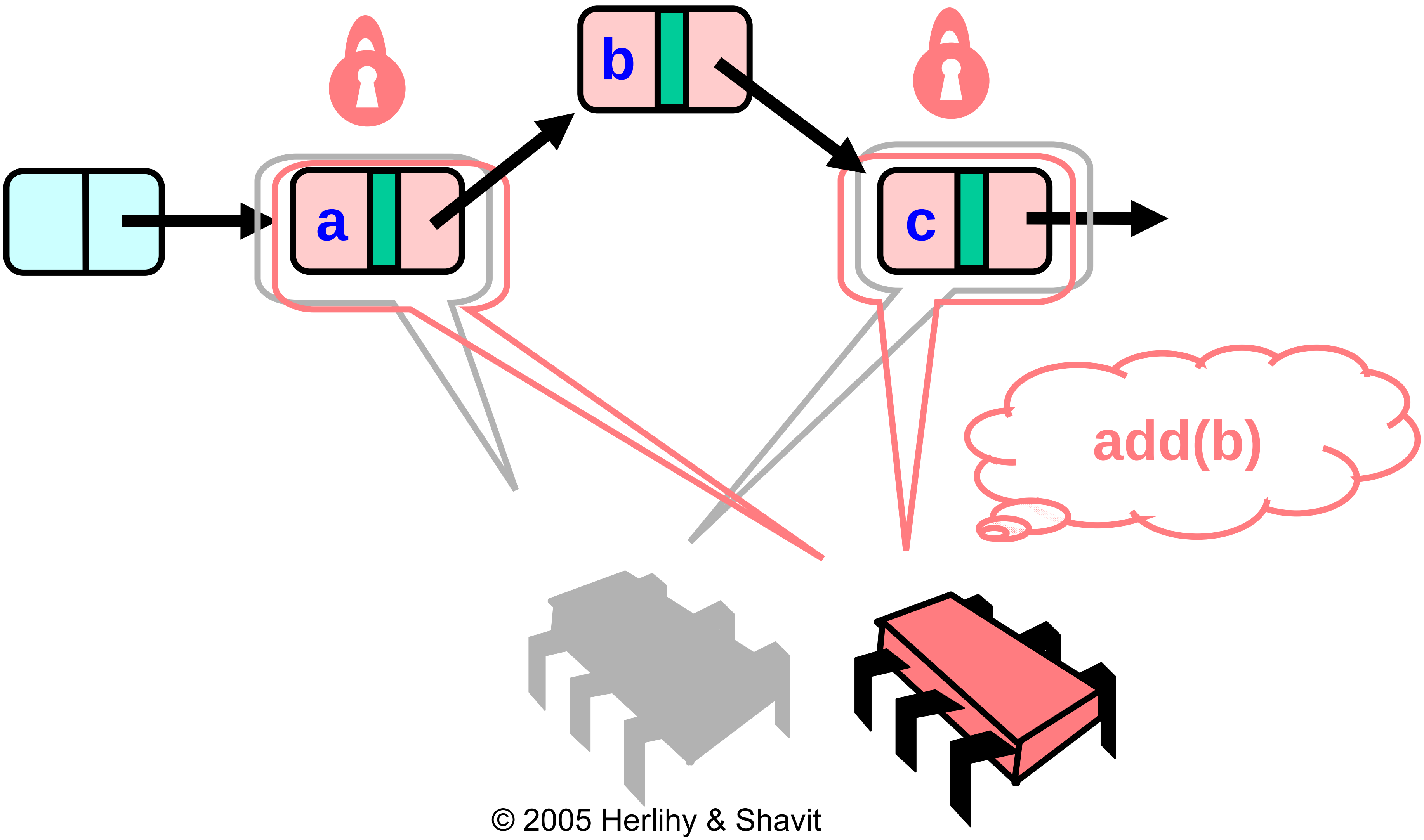


Is this okay?

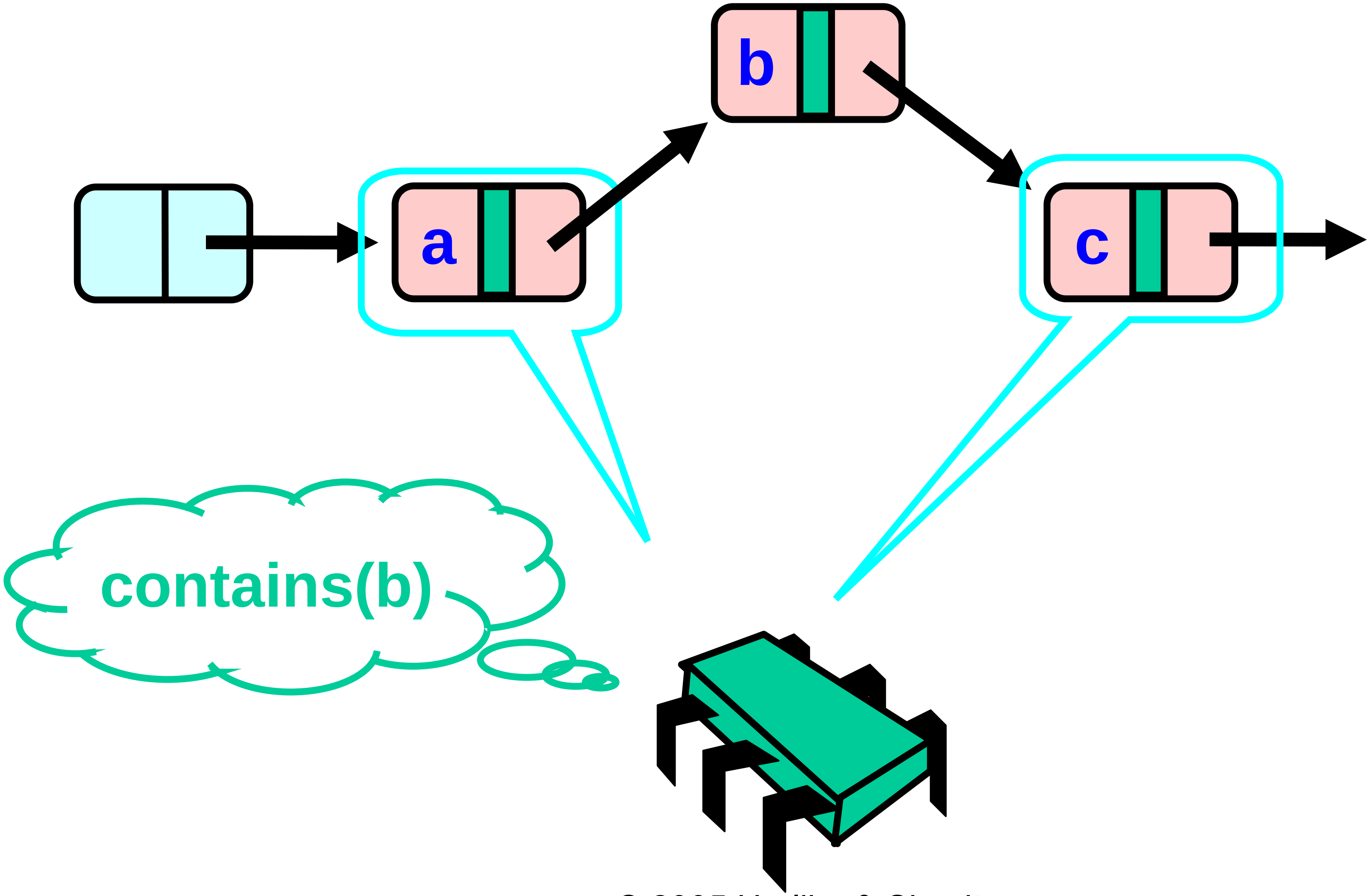
Lazy list algorithm



Lazy list algorithm



Lazy list algorithm



Lazy List: Add

Nodes have fields: key, next, mark.

```
S.add(x)
retry:
   pred := S.head; curr := pred.next
   while (curr.key < x) do pred := curr; curr := curr.next
   if (curr.key = x and curr.mark = false) then return false
   else
       pred.lock()
       if (pred.mark = false and pred.next = curr) then
         node := new Node(x)
         node.next := curr
         pred.next := node
         pred.unlock()
         return true
      else
         pred.unlock()
         goto retry
```

Lazy List: Remove

```
S.remove(x)
retry:
   pred := S.head; curr := pred.next
   while (curr.key < x) do pred := curr; curr := curr.next
   if (curr.key > x or curr.mark = true) then return false
   else
       pred.lock(); curr.lock()
       if (pred.mark = curr.mark = false and pred.next = curr) then
          curr.mark := true
          pred.next := curr.next
          pred.unlock(); curr.unlock()
          return true
       else
        pred.unlock(); curr.unlock()
        goto retry
```

Lazy List: Contains

```
S.contains(x)
curr := S.head.next
while (curr.key < x) do curr := curr.next
if (curr.key = x and curr.mark = false) then return true
else return false
```

Lazy list algorithm

- Serializing contains(x) that returns false
  - if node found has key > x
    - when node.key is read?
    - when pred.next is read?
    - when pred is marked (if it is marked)?
  - if node with key = x is marked
    - when mark is read?
    - when pred.next is read?
    - when mark is set?

Lazy list algorithm

- Serializing contains(x) that returns false
  - if node found has key > x
    - when node.key is read?
    - when pred.next is read?
    - when pred is marked (if it is marked)?
  - if node with key = x is marked
    - when mark is read?
    - when pred.next is read?
    - when mark is set?

Can we do this for the optimistic list?

Correctness

- Atomicity:
  - Forward simulation to canonical atomic set:
    - S = values in reachable unmarked nodes.
  - "perform" steps:
    - contains(x) or unsuccessful add(x) or remove(x): LTTR, based on some technical cases.
    - Successful add(x): When pred.next := node.
    - Successful remove(x): When curr.mark := true.
- Liveness:
  - contains is wait-free.
  - add, remove are blocking.
  - add, remove satisfy progress, but not lockout-freedom.

Lock-free list with wait-free contains()

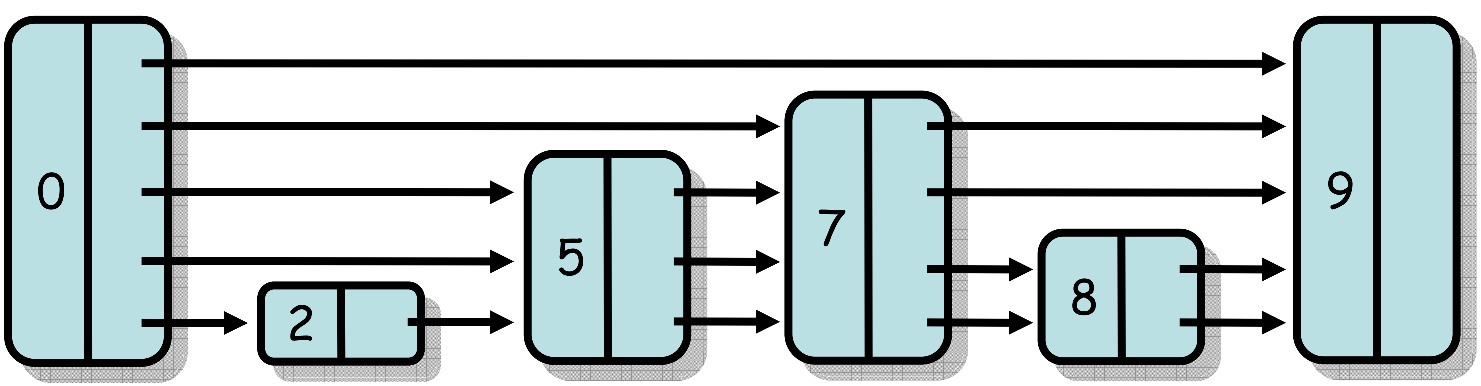
- Add and remove just like lock-free list.
- Contains() does not help, does not retry, just like in lazy list.

Evaluation/Comparison

- Lock-free list with wait-free contains():
  - contains() is wait-free
  - add() and remove() are nonblocking (lock-free)
  - Incurs overhead of CAS and of cleanup.
- Lazy list:
  - contains() is wait-free
  - add() and remove() are blocking, but use short lock durations.
  - Low overhead.

Application of list techniques

- Trees
- Skip lists
  - multiple layers of links
  - list at each layer is sublist of layer below
  - logarithmic expected search time if each list has half elements of next lower level
    - probabilistic guarantees



Next time

- Transactional memory
- Reading:
  - HS, Chapter 9
  - Guerraoui, Kapalka