6.852: Distributed Algorithms
Fall, 2009

Class 6

Today's plan

- f+1-round lower bound for stopping agreement, cont'd.
- Various other kinds of consensus problems in synchronous networks:
  - k-agreement
  - Approximate agreement (skip)
  - Distributed commit
- Reading:
  - [Aguilera, Toueg]
  - [Keidar, Rajsbaum]
  - Chapter 7 (skip 7.2)
- Next:
  - Modeling asynchronous systems
  - Chapter 8

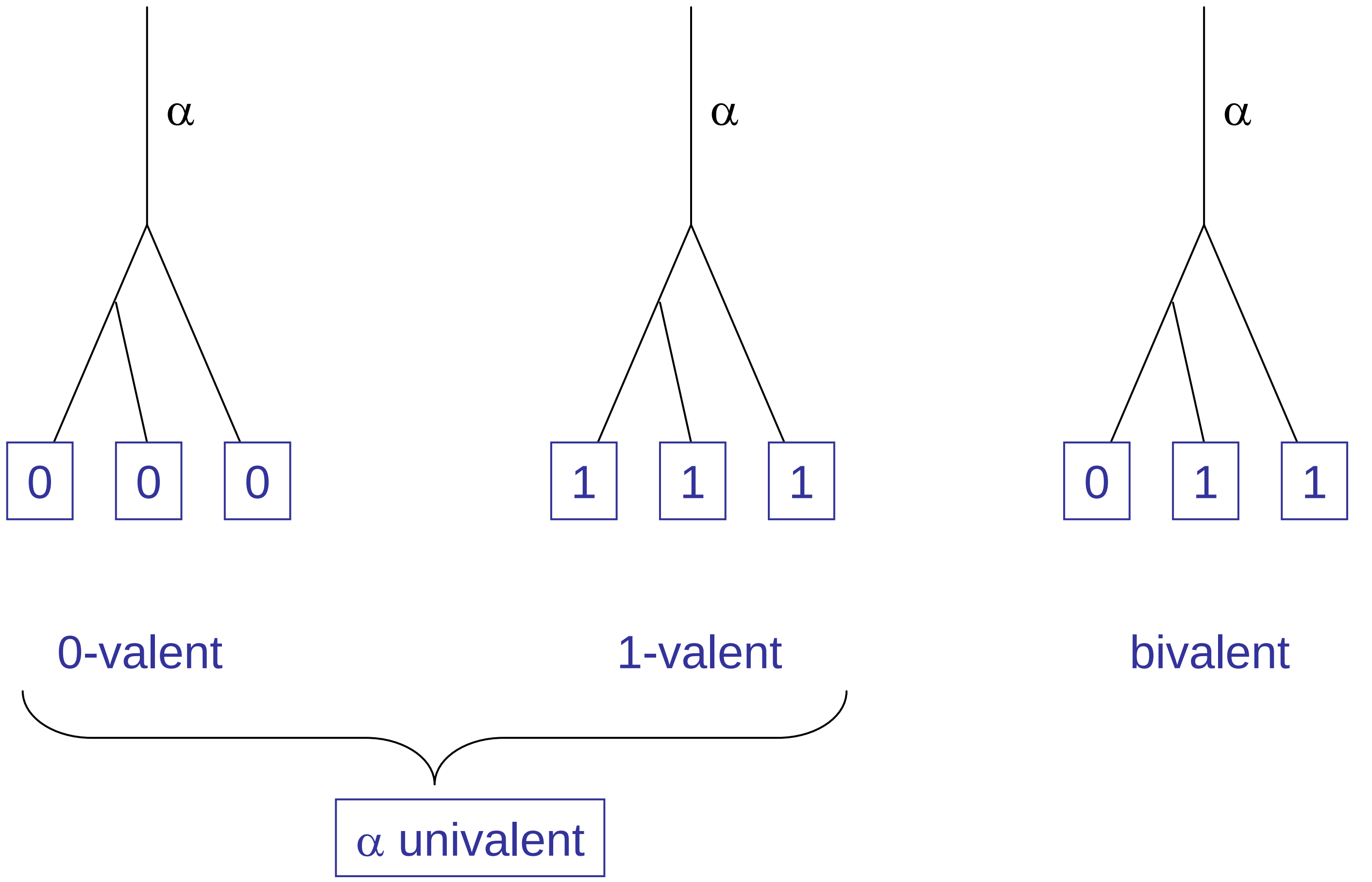
Lower Bound on Rounds

- Theorem 1: Suppose $n \geq f + 2$. There is no n-process f-fault stopping agreement algorithm in which nonfaulty processes always decide at the end of round f.
- Old proof: Suppose A exists.
  - Construct a chain of executions, each with at most f failures, where:
    - First has decision value 0, last has decision value 1.
    - Any two consecutive executions are indistinguishable to some process i that is nonfaulty in both.
  - So decisions in first and last executions are the same, contradiction.
  - Must fail f processes in some executions in the chain, in order to remove all the required messages, at all rounds.
  - Construction in book, LTTR.
- Newer proof [Aguilera, Toueg]:
  - Uses ideas from [Fischer, Lynch, Paterson], impossibility of asynchronous consensus.

[Aguilera, Toueg] proof

- By contradiction. Assume A solves stopping agreement for f failures and everyone decides after exactly f rounds.
- Consider only executions in which at most one process fails during each round.
- Recall failure at a round allows process to miss sending any subset of the messages, or to send all but halt before changing state.
- Regard vector of initial values as a 0-round execution.
- Defs (adapted from [FLP]): α, an execution that completes some finite number (possibly 0) of rounds, is:
  - 0-valent, if 0 is the only decision that can occur in any execution (of the kind we consider) that extends α.
  - 1-valent, if 1 is…
  - Univalent, if α is either 0-valent or 1-valent (essentially decided).
  - Bivalent, if both decisions occur in some extensions (undecided).

Univalence and Bivalence



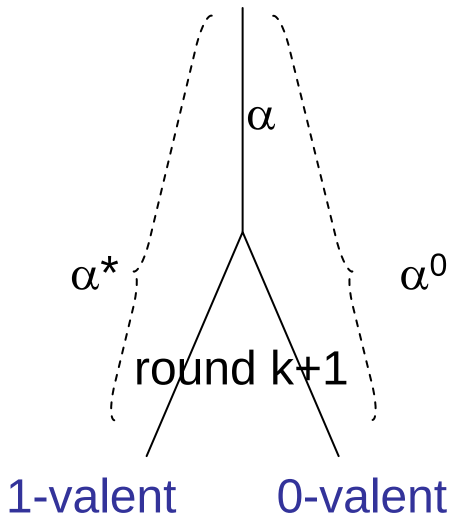
Initial bivalence

- Lemma 1: There is some 0-round execution (vector of initial values) that is bivalent.

- Proof (from [FLP]):
  - Assume for contradiction that all 0-round executions are univalent.
  - 000…0 is 0-valent.
  - 111…1 is 1-valent.
  - So there must be two 0-round executions that differ in the value of just one process, i, such that one is 0-valent and the other is 1-valent.
  - But this is impossible, because if i fails at the start, no one else can distinguish the two 0-round executions.

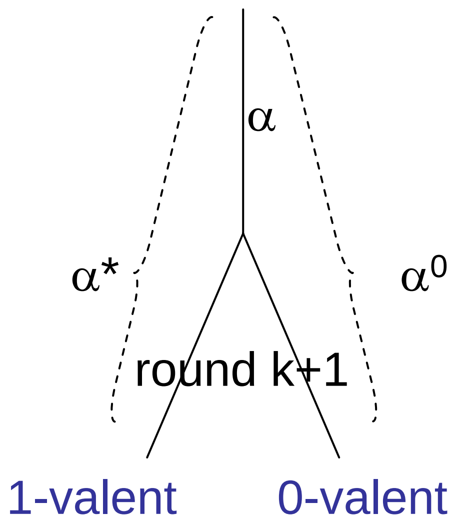
Bivalence through f-1 rounds

- Lemma 2: For every k, $0 \leq k \leq f-1$, there is a bivalent k-round execution.
- Proof: By induction on k.
  - Base: Lemma 1.
  - Inductive step: Assume for k, show for k+1, where k < f -1.
    - Assume bivalent k-round execution α.
    - Assume for contradiction that every 1-round extension of α (with at most one new failure) is univalent.
    - Let α^* be the 1-round extension of α in which no new failures occur in round k+1.
    - By assumption, α^* is univalent, WLOG 1-valent.
    - Since α is bivalent, there must be another 1-round extension of α, α^0, that is 0-valent.



Bivalence through f-1 rounds

- In α^0, some single process, say i, fails in round k+1, by not sending to some set of processes, say J = $\{j_1, j_2, \ldots j_m\}$.
- Define a chain of (k+1)-round executions, $\alpha^0, \alpha^1, \alpha^2, \ldots, \alpha^m$.
- Each α^l in this sequence is the same as α^0 except that i also sends messages to $j_1, j_2, \ldots j_l$.
  - Adding in messages from i, one at a time.
- Each α^l is univalent, by assumption.
- Since α^0 is 0-valent, either:
  - At least one of these is 1-valent, or
  - All are 0-valent.



Case 1: At least one α^l is 1-valent

- Then there must be some l such that α^{l-1} is 0-valent and α^l is 1-valent.
- But α^{l-1} and α^l differ after round k+1 only in the state of one process, j_l.
- We can extend both α^{l-1} and α^l by simply failing j_l at beginning of round k+2.
  - There is actually a round k+2 because we've assumed $k < f-1$, so $k+2 \leq f$.
- And no one left alive can tell the difference!
- Contradiction for Case 1.

Case 2: Every α^l is 0-valent

- Then compare:
  - α^m, in which i sends all its round k+1 messages and then fails, with
  - α^* , in which i sends all its round k+1 messages and does not fail.
- No other differences, since only i fails at round k+1 in α^m.
- α^m is 0-valent and α^* is 1-valent.
- Extend to full f-round executions:
  - α^m, by allowing no further failures,
  - α^*, by failing i right after round k+1 and then allowing no further failures.
- No one can tell the difference.
- Contradiction for Case 2.

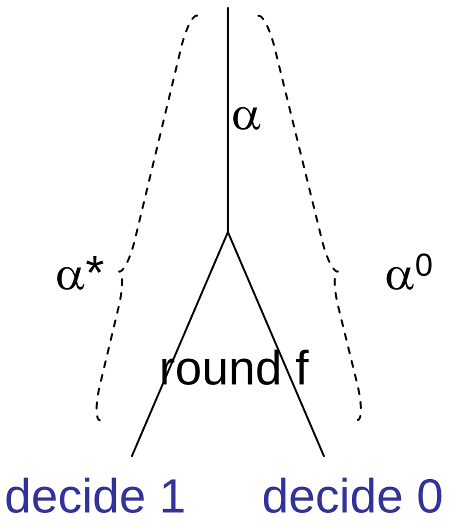
Bivalence through f-1 rounds

- So we've proved, so far:
- Lemma 2: For every k, $0 \leq k \leq f-1$, there is a bivalent k-round execution.

Disagreement after f rounds

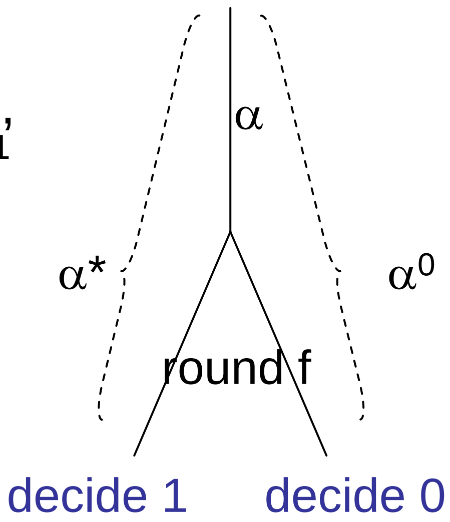
- Lemma 3: There is an f-round execution in which two nonfaulty processes decide differently.

- Proof:
  - Use Lemma 2 to get a bivalent (f-1)-round execution α with $\leq$ f-1 failures.
  - In every 1-round extension of α, everyone who hasn't failed must decide (and agree).
  - Let α^* be the 1-round extension of α in which no new failures occur in round f.
  - Everyone who is still alive decides after α^*, and they must decide the same thing. WLOG, say they decide 1.
  - Since α is bivalent, there must be another 1-round extension of α, say α^0, in which some nonfaulty process (and so, all nonfaulty processes) decide 0.



Disagreement after f rounds

- In α^0, some single process i fails in round f.
- Let j, k be two nonfaulty processes.
- Define a chain of three f-round executions, $\alpha^0, \alpha^1, \alpha^*$, where α^1 is identical to α^0 except that i sends to j in α^1 (it might not in α^0).



- Then $\alpha^1 \sim^k \alpha^0$.
- Since k decides 0 in α^0, k also decides 0 in α^1.
- Also, $\alpha^1 \sim^j \alpha^*$.
- Since j decides 1 in α^*, j also decides 1 in α^1.
- Yields disagreement in α^1, contradiction!

- So we've proved:
- Lemma 3: There is an f-round execution in which two nonfaulty processes decide differently.
- Which immediately yields the lower bound result.

Early-stopping agreement algorithms

- Tolerate f failures in general, but in executions with $f' < f$ failures, terminate faster.
- [Dolev, Reischuk, Strong 90] Stopping agreement algorithm in which all nonfaulty processes terminate in $\leq \min(f' + 2, f+1)$ rounds.
  - If $f' + 2 \leq f$, decide "early", within $f' + 2$ rounds; in any case decide within f+1 rounds.
- [Keidar, Rajsbaum 02] Lower bound of $f' + 2$ for early-stopping agreement.
  - Not just $f' + 1$. Early stopping requires an extra round.
- Theorem 2: Assume $0 \leq f' \leq f - 2$ and $f < n$. Every early-stopping agreement algorithm tolerating f failures has an execution with f' failures in which some nonfaulty process doesn't decide by the end of round $f' + 1$.

Just consider special case: $f' = 0$

- Theorem 3: Assume $2 \leq f < n$. Every early-stopping agreement algorithm tolerating f failures has a failure-free execution in which some nonfaulty process does not decide by the end of round 1.
- Definition: Let α be an execution that completes some finite number (possibly 0) of rounds. Then val(α) is the unique decision value in the extension of α with no new failures.
- Proof of Theorem 3:
  - Assume executions in which at most one process fails per round.
  - Identify 0-round executions with vectors of initial values.
  - Assume, for contradiction, that everyone decides by round 1, in all failure-free executions.
  - val(000…0) = 0, val(111…1) = 1.
  - So there must be two 0-round executions α^0 and α^1, that differ in the value of just one process i, such that val(α^0) = 0 and val(α^1) = 1.

Special case: f′ = 0

- 0-round executions α^0 and α^1, differing only in the initial value of process i, such that val(α^0) = 0 and val(α^1) = 1.
- In failure-free extensions of α^0, α^1, all processes decide in one round.
- Define:
  - β^0, 1-round extension of α^0, in which process i fails, sends only to j.
  - β^1, 1-round extension of α^1, in which process i fails, sends only to j.
- Then:
  - β^0 looks to j like ff extension of α^0, so j decides 0 in β^0 after 1 round.
  - β^1 looks to j like ff extension of α^1, so j decides 1 in β^1 after 1 round.
- β^0 and β^1 are indistinguishable to all processes except i, j.
- Define:
  - γ^0, infinite extension of β^0, in which process j fails right after round 1.
  - γ^1, infinite extension of β^1, in which process j fails right after round 1.
- By agreement, all nonfaulty processes must decide 0 in γ^0, 1 in γ^1.
- But γ^0 and γ^1 are indistinguishable to all nonfaulty processes, so they can’t decide differently, contradiction.

k-Agreement

k-agreement

- Usually called k-set agreement or k-set consensus.
- Generalizes ordinary stopping agreement by allowing k different decisions instead of just one.
- Motivation:
  - Practical:
    - Allocating shared resources, e.g., agreeing on small number of radio frequencies to use for sending/receiving broadcasts.
  - Mathematical:
    - Natural generalization of ordinary 1-agreement.
    - Elegant theory: Nice topological structure, tight bounds.

The k-agreement problem

- Assume:
  - n-node complete undirected graph
  - Stopping failures only
  - Inputs, decisions in finite totally-ordered set V (appear in state variables).
- Correctness conditions:
  - Agreement:
    - $\exists\, W \subseteq V$, $|W| = k$, all decision values in W.
    - That is, there are at most k different decision values.
  - Validity:
    - Any decision value is some process' initial value.
    - Like strong validity for 1-agreement.
  - Termination:
    - All nonfaulty processes eventually decide.

FloodMin k-agreement algorithm

- Algorithm:
  - Each process remembers the min value it has seen, initially its own value.
  - At each round, broadcasts its min value.
  - Decide after some generally-agreed-upon number of rounds, on current min value.
- Q: How many rounds are enough?
- 1-agreement: f+1 rounds
  - Argument like those for previous stopping agreement algorithms.
- k-agreement: $\lfloor f/k \rfloor + 1$ rounds.
- Allowing k values divides the runtime by k.

FloodMin correctness

- Theorem 1: FloodMin, for $\lfloor f/k \rfloor + 1$ rounds, solves k-agreement.

- Proof:

- Define M(r) = set of min values of active (not-yet-failed) processes after r rounds.
- This set can only decrease over time:

- Lemma 1: $M(r+1) \subseteq M(r)$ for every r, $0 \leq r \leq \lfloor f/k \rfloor$.
- Proof: Any min value after r+1 is someone's min value after r.

Proof of Theorem 1, cont'd

- Lemma 2: If at most d-1 processes fail during round r, then $|M(r)| \leq d$.
- E.g., for d = 1: If no one fails during round r then all have the same min value after r.
- Proof: Show contrapositive.
  - Suppose that $|M(r)| > d$, show at least d processes fail in round r.
  - Let m = max (M(r)).
  - Let $m' < m$ be any other element of M(r).
  - Then $m' \in M(r-1)$ by Lemma 1.
  - Let i be a process active after r-1 rounds that has m′ as its min value after r-1 rounds.
  - Claim i fails in round r:
    - If not, everyone would receive m; in round r.
    - Means that no one would choose $m > m'$ as its min, contradiction.
  - But this is true for every $m' < m$ in M(r), so at least d processes fail in round r.

Proof of Theorem 1, cont'd

- Validity: Easy
- Termination: Obvious
- Agreement: By contradiction.
  - Assume an execution with > k different decision values.
  - Then the number of min values for active processes after the full $\lfloor f/k \rfloor + 1$ rounds is > k.
  - That is, $|M(\lfloor f/k \rfloor + 1)| > k$.
  - Then by Lemma 1, $|M(r)| > k$ for every r, $0 \leq r \leq \lfloor f/k \rfloor + 1$.
  - So by Lemma 2, at least k processes fail in each round.
  - That's at least $(\lfloor f/k \rfloor + 1)\, k$ total failures, which is > f failures.
  - Contradiction!

Lower Bound (sketch)

- Theorem 2: Any algorithm for k-agreement requires $\geq \lfloor f/k \rfloor + 1$ rounds.

- Recall old proof for f+1-round lower bound for 1-agreement.
  - Chain of executions for assumed algorithm:

$$\alpha_0 \text{ ----- } \alpha_1 \text{ ----- } \ldots\text{-----}\alpha_j \text{ -----}\alpha_{j+1} \text{ ----- } \ldots\text{-----}\alpha_m$$

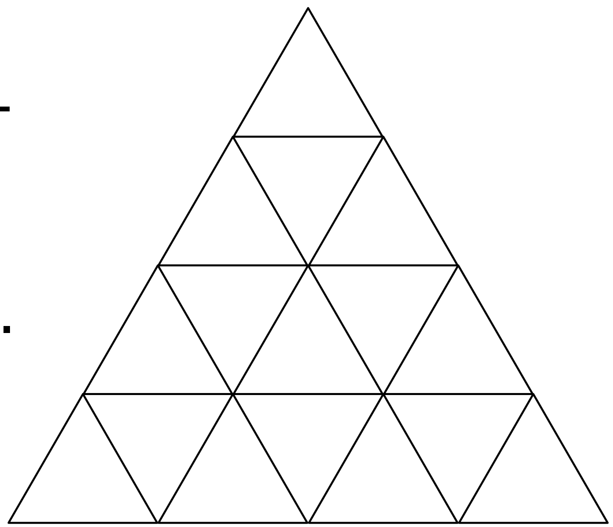
  - Each execution has a unique decision value.
  - Executions at ends of chain have specified decision values.
  - Two consecutive executions look the same to some nonfaulty process, who (therefore) decides the same in both.

- Argument doesn't extend immediately to k-agreement:
  - Can't assume a unique value in each execution.
  - Example: For 2-agreement, could have 3 different values in 2 consecutive executions without violating agreement.
- Instead, use a k-dimensional generalized chain.

Lower bound

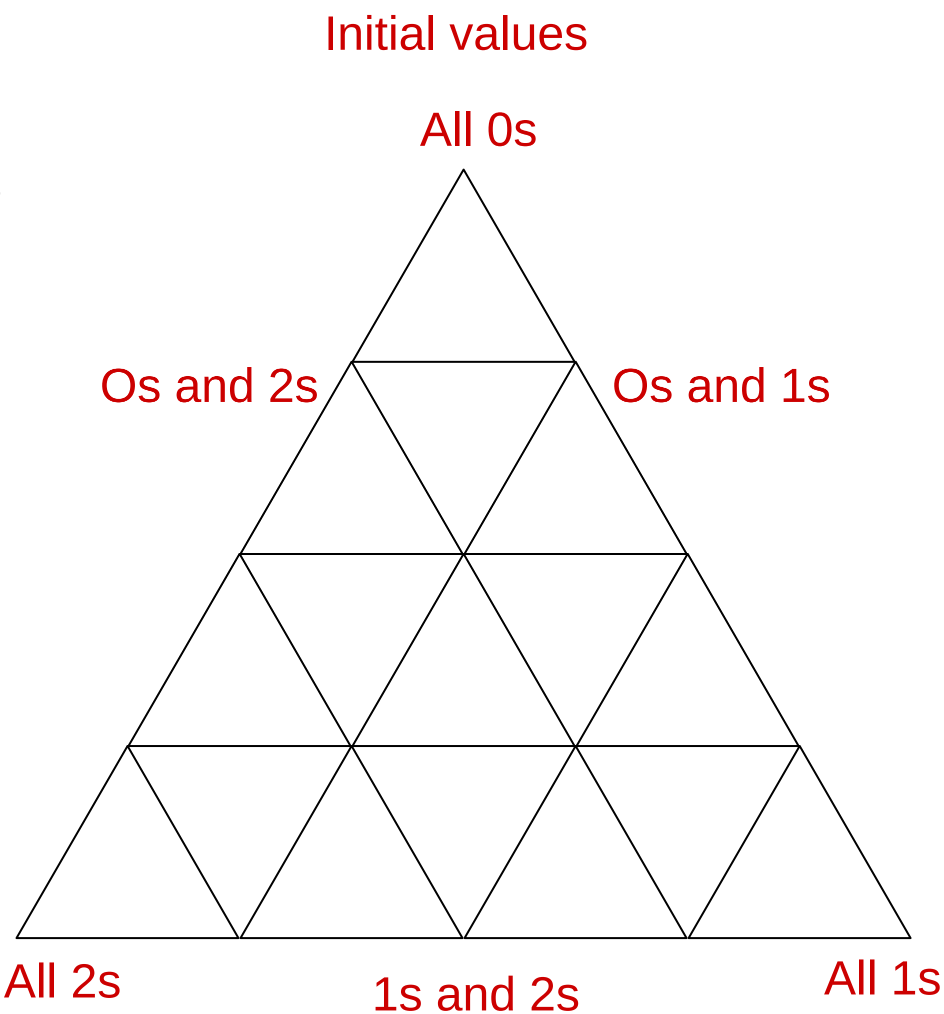
- Assume, for contradiction:
  - n-process k-agreement algorithm tolerating f failures.
  - All processes decide just after round r, where $r \leq \lfloor f/k \rfloor$.
  - All-to-all communication at all rounds.
  - $n \geq f + k + 1$ (so each execution we consider has at least k+1 nonfaulty processes)
  - $V = \{0,1,\ldots,k\}$, k+1 values.

- Get contradiction by proving existence of an execution with $\geq k + 1$ different decision values.
- Use k-dimensional collection of executions rather than 1-dimensional.
  - k = 2: Triangle
  - k = 3: Tetrahedron, etc.



Labeling nodes with executions

- Bermuda Triangle (k = 2): Any algorithm vanishes somewhere in the interior.
- Label nodes with executions:
  - Corner: No failures, all have same initial value.
  - Boundary edge: Initial values chosen from those of the two endpoints
  - For k > 2, generalize to boundary faces.
  - Interior: Mixture of inputs
- Label so executions "morph gradually" in all directions:
- Difference between two adjacent executions along an outer edge:
  - Remove or add one message, to a process that fails immediately.
  - Fail or recover a process.
  - Change initial value of failed process.



Labeling nodes with process names

- Also label each node with the name of a process that is nonfaulty in the node's execution.
- Consistency: For every tiny triangle (simplex) T, there is a single execution β, with at most f faults, that is "compatible" with the executions and processes labeling the corners of T:
  - All the corner processes are nonfaulty in β.
  - If (α', i) labels some corner of T, then α' is indistinguishable by i from β.
- Formalizes the "gradual morphing" property.
- Proof by laborious construction.
- Can recast chain arguments for 1-agreement in this style:

$$\begin{array}{cccccccc} & & & & \beta & & & \\ \alpha_0 \text{ ----- } & \alpha_1 \text{ ----- } & \ldots\text{-----} & \alpha_j & \text{ ----- } & \alpha_{j+1} \text{ ----- } & \ldots\text{-----} & \alpha_m \\ p_0 & p_1 & \ldots_. & p_j & & p_{j+1} & & p_m \end{array}$$

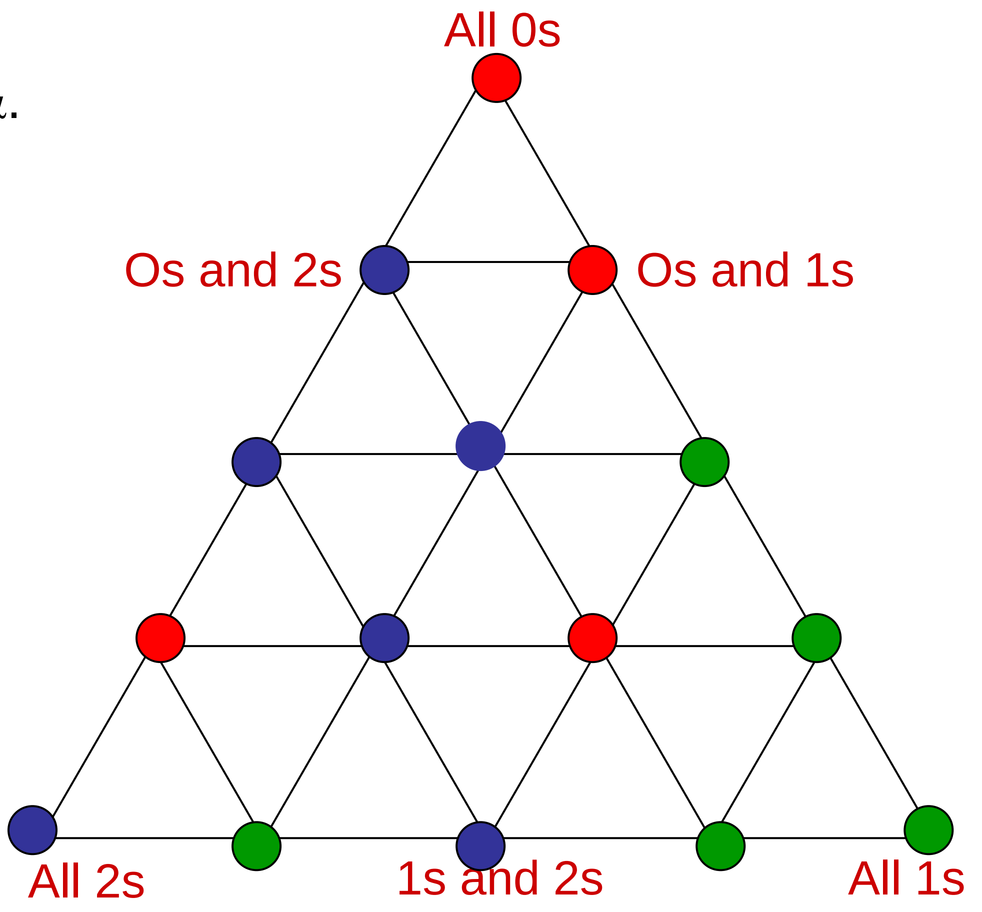
  - β indistinguishable by p_j from α_j
  - β indistinguishable by p_{j+1} from α_{j+1}

Bound on rounds

- This labeling construction uses the assumption $r \leq \lfloor f / k \rfloor$, that is, $f \geq r\,k$.
- How:
  - We are essentially constructing chains simultaneously in k directions (2 directions, in the Bermuda Triangle).
  - We need r failures (one per round) to construct the "chain" in each direction.
  - For k directions, that's r k total failures.
- Details LTTR (see book, or paper [Chaudhuri, Herlihy, Lynch, Tuttle])

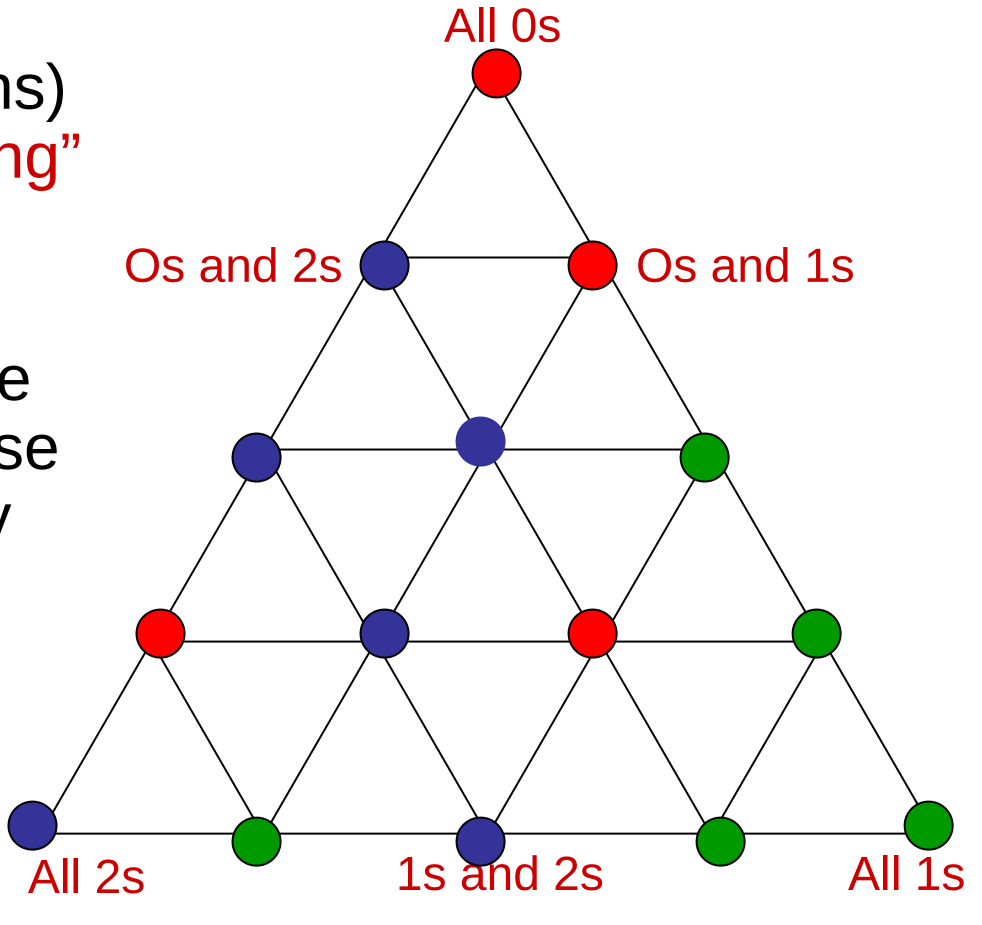
Coloring the nodes

- Now color each node v with a "color" in {0,1,…,k}:
  - If v is labeled with (α,i) then color(v) = i's decision value in α.
- Properties:
  - Colors of the major corners are all different.
  - Color of each boundary edge node is the same as one of the endpoint corners.
  - For k > 2, generalize to boundary faces.
- Coloring properties follow from Validity, because of the way the initial values are assigned.



Sperner Colorings

- A coloring with the listed properties (suitably generalized to k dimensions) is called a "Sperner Coloring" (in algebraic topology).
- Sperner's Lemma: Any Sperner Coloring has some tiny triangle (simplex) whose k+1 corners are colored by all k+1 colors.
- Find one?



Applying Sperner's Lemma

- Apply Sperner's Lemma to the coloring we constructed.
- Yields a tiny triangle (simplex) T with k+1 different colors on its corners.
- Which means k+1 different decision values for the executions and processes labeling its corners.
- But consistency for T yields a single execution β, with at most f faults, that is "compatible" with the executions and processes labeling the corners of T:
  - All the corner processes are nonfaulty in β.
  - If (α',i) labels some corner of T, then α' is indistinguishable by i from β.
- So all the corner processes behave the same in β as they do in their own corner executions, and decide on the same values as in those executions.
- That's k+1 different decision values in one execution with at most f faults.
- Contradicts k-agreement.

Approximate Agreement

Approximate Agreement problem

- Agreement on real number values:
  - Readings of several altimeters on an aircraft.
  - Values of approximately-synchronized clocks.
- Consider with Byzantine participants, e.g., faulty hardware.
- Abstract problem:
  - Inputs, outputs are reals
  - Agreement: Within ε.
  - Validity: Within range of initial values of nonfaulty processes.
  - Termination: Nonfaulty eventually decide.
- Assumptions: Complete n-node graph, $n > 3f$.
- Could solve by exact BA, using f+1 rounds and lots of communication.
- But better algorithms exist:
  - Simpler, cheaper
  - Extend to asynchronous settings, whereas BA is unsolvable in asynchronous networks.

Approximate agreement algorithm [Dolev, Lynch, Pinter, Stark, Weihl]

- Use convergence strategy, successively narrowing the interval of guesses of the nonfaulty processes.
  - Take an average at each round.
  - Because of Byzantine failures, need fault-tolerant average.

- Maintain val, latest estimate, initially initial value.
- At every round:
  - Broadcast val, collect received values into multiset W.
  - Fill in missing entries with any values.
  - Calculate W′ = reduce(W), by discarding f largest and f smallest elements.
  - Calculate W″ = select(W′), by choosing the smallest value in W′ and every f’th value thereafter.
  - Reset val to mean(W″).

Example: n = 4, f = 1

- Initial values: 1, 2, 3, 4
- Process 3 faulty, sends:

  proc 1: 2   proc. 2: 100   proc 3: -100
- Process 1:
  - Receives (1, 2, 2, 4), reduces to (2, 2), selects (2, 2), mean = 2.
- Process 2:
  - Receives (1, 2, 100, 4), reduces to (2, 4), selects (2, 4), mean = 3.
- Process 4:
  - Receives (1, 2, -100, 4), reduces to (1, 2), selects (1, 2), mean = 1.5.

One-round guarantees

- Lemma 1: Any nonfaulty process' val after the round is in the range of nonfaulty processes' vals before the round.
- Proof: All elements of reduce(W) are in this range, because there are at most f faults, and we discard the top and bottom f values.

- Lemma 2: Let d be the range of nonfaulty processes' vals just before the round. Then the range of nonfaulty processes' vals after the round is at most $d / (\lfloor (n - (2f+1)) / f \rfloor + 1)$.
- That is:
  - If $n = 3f + 1$, then the new range is $d / 2$.
  - If $n = kf + 1$, $k \geq 3$, then the new range is $d / (k - 1)$.
- Proof: Calculations, in book.

- Example: $n = 4$, $f = 1$
  - Initial vals: 1, 2, 3, 4, range is 3.
  - Process 3 faulty, sends 2 to proc 1, 100 to proc 2, -100 to proc 3.
  - New vals of nonfaulty processes: 2, 3, 1.5
  - New range is 1.5.

The complete algorithm

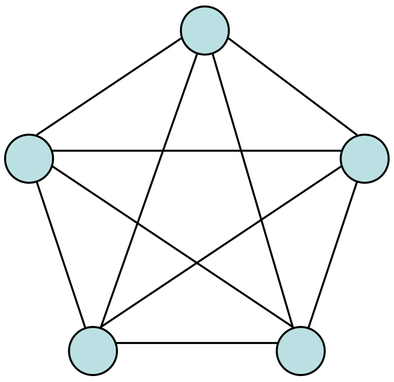
- Just run the 1-round algorithm repeatedly.
- Termination: Add a mechanism, e.g.:
  - Each node individually determines a round by which it knows that the vals of nonfaulty processes are all within ε.
    - Collect first round vals, predict using known convergence rate.
  - After the determined round, decide locally.
  - Thereafter, send the decision value.
    - Upsets the convergence calculation.
    - But that doesn't matter because the vals are already within ε.

- Remarks:
  - Convergence rate can be improved somewhat by using 2-round blocks [Fekete].
  - Algorithm extends easily to asynchronous case, using an "asynchronous round" structure we'll see later.

Distributed Commit

Distributed Commit

- Motivation: Distributed database transaction processing
  - A database transaction performs work at several distributed sites.
  - Transaction manager (TM) at each site decides whether it would like to “commit” or “abort” the transaction.
    - Based on whether the transaction’s work has been successfully completed at that site, and results made stable.
  - All TMs must agree on whether to commit or abort.
- Assume:
  - Process stopping failures only.
  - n-node, complete, undirected graph.

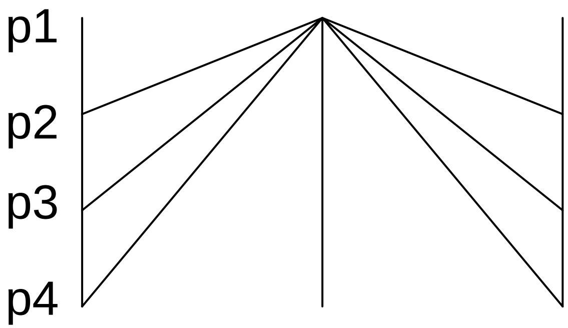


- Require:
  - Agreement: No two processes decide differently (faulty or not, uniformity)
  - Validity:
    - If any process starts with 0 (abort) then 0 is the only allowed decision.
    - If all start with 1 (commit) and there are no faulty processes then 1 is the only allowed decision.

Correctness Conditions for Commit

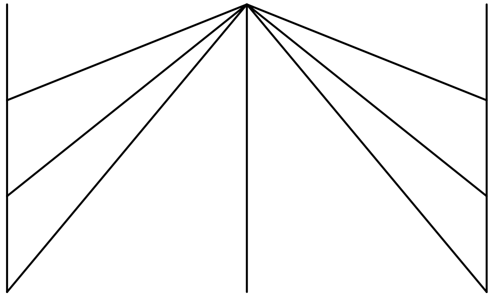
- Agreement: No two processes decide differently.
- Validity:
  - If any process starts with 0 then 0 is the only allowed decision.
  - If all start with 1 and there are no faulty processes then 1 is the only allowed decision.
  - Note the asymmetry: Guarantee abort (0) if anyone wants to abort; guarantee commit (1) if everyone wants to commit and no one fails (best case).
- Termination:
  - Weak termination: If there are no failures then all processes eventually decide.
  - Strong termination (non-blocking condition): All nonfaulty processes eventually decide.

2-Phase Commit

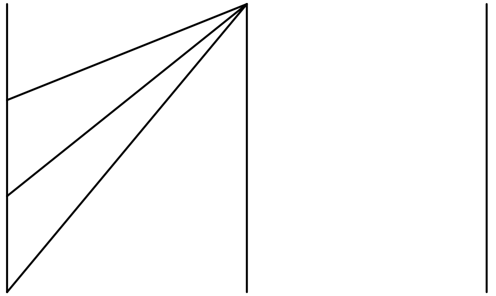


- Traditional, blocking algorithm (guarantees weak termination only).
- Assumes distinguished process 1, acts as "coordinator" (leader).
- Round 1: All send initial values to process 1, who determines the decision.
- Round 2: Process 1 sends out the decision.
- Q: When can each process actually decide?
- Anyone with initial value 0 can decide at the beginning.
- Process 1 decides after receiving round 1 messages:
  - If it sees 0, or doesn't hear from someone, it decides 0; otherwise decides 1.
- Everyone else decides after round 2.

Correctness of 2-Phase Commit

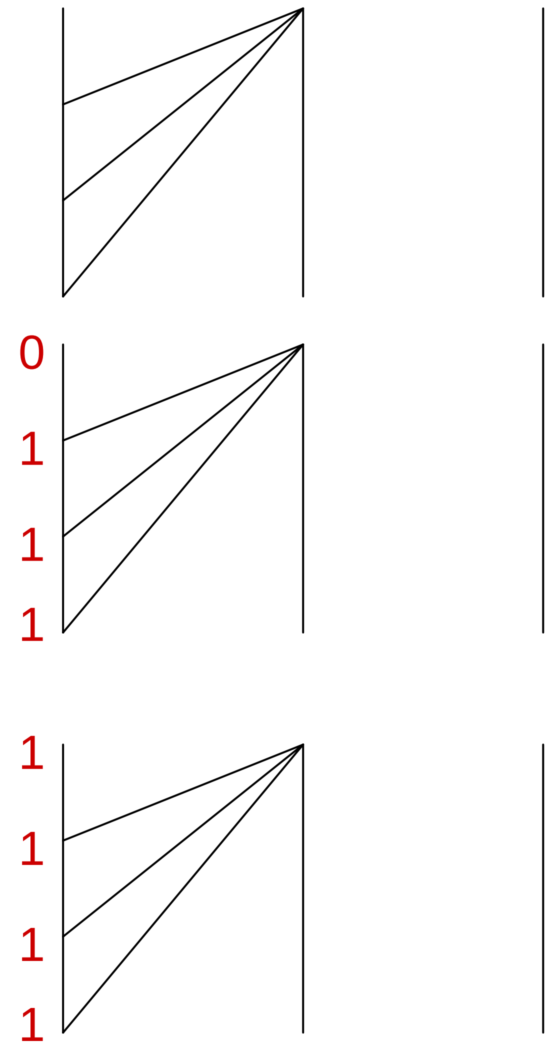


- Agreement:
  - Because decision is centralized (and consistent with any individual initial decisions).
- Validity:
  - Because of how the coordinator decides.
- Weak termination:
  - If no one fails, everyone terminates by end of round 2.
- Strong termination?
  - No: If coordinator fails before sending its round 2 messages, then others with initial value 1 will never terminate.



Add a termination protocol?

- We might try to add a termination protocol: other processes try to detect failure of coordinator and finish agreeing on their own.
- But this can't always work:
  - If initial values are 0,1,1,1, then by validity, others must decide 0.
  - If initial values are 1,1,1,1 and process 1 fails just after deciding, and before sending out its round 2 messages, then:
    - By validity, process 1 must decide 1.
    - By agreement, others must decide 1.
  - But the other processes can't distinguish these two situations.

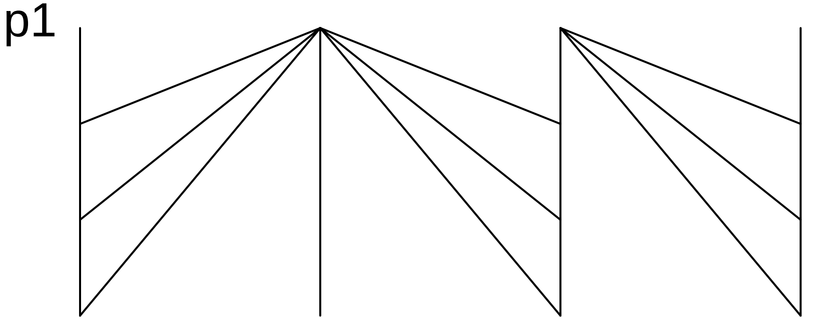


Complexity of 2-phase commit

- Time:
  - 2 rounds
- Communication:
  - At most 2n messages

3-Phase Commit [Skeen]

- Yields strong termination.
- Trick: Introduce intermediate stage, before actually deciding.
- Process states classified into 4 categories:
  - dec-0: Already decided 0.
  - dec-1: Already decided 1.
  - ready: Ready to decide 1 but hasn't yet.
  - uncertain: Otherwise.
- Again, process 1 acts as "coordinator".
- Communication pattern:

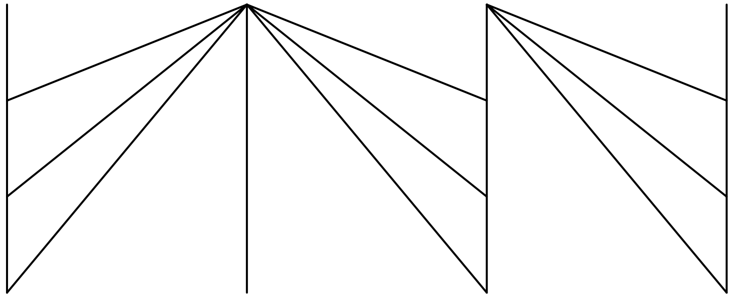


3-Phase Commit

- All processes initially uncertain.
- Round 1:
  - All other processes send their initial values to p1.
  - All with initial value 0 decide 0 (and enter dec-0 state)
  - If p1 receives 1s from everyone and its own initial value is 1, p1 becomes ready, but doesn't yet decide.
  - If p1 sees 0 or doesn't hear from someone, p1 decides 0.

- Round 2:
  - If p1 has decided 0, broadcasts "decide 0", else broadcasts "ready".
  - Anyone else who receives "decide 0" decides 0.
  - Anyone else who receives "ready" becomes ready.
  - Now p1 decides 1 if it hasn't already decided.

- Round 3:
  - If p1 has decided 1, bcasts "decide 1".
  - Anyone else who receives "decide 1" decides 1.



3-Phase Commit

- Key invariants (after 0, 1, 2, or 3 rounds):
  - If any process is in ready or dec-1, then all processes have initial value 1.
  - If any process is in dec-0 then:
    - No process is in dec-1, and no non-failed process is ready.
  - If any process is in dec-1 then:
    - No process is in dec-0, and no non-failed process is uncertain.

- Proof: LTTR.
  - Key step: Third condition is preserved when p1 decides 1 after round 2.
  - In this case, p1 knows that:
    - Everyone's input is 1.
    - No one decided 0 at the end of round 1.
    - Every other process has either become ready or has failed (without deciding).
  - Implies third condition.

- Note critical use of synchrony here:
  - p1 infers that non-failed processes are ready just because round 2 is completed.
  - Without synchrony, would need positive acknowledgments.

Correctness conditions (so far)

- Agreement and validity follow, for these three rounds.
- Weak termination holds
- Strong termination:
  - Doesn’t hold yet---must add a termination protocol.
  - Allow process 2 to act as coordinator, then 3,…
  - “Rotating coordinator” strategy

3-Phase Commit

- Round 4:
  - All processes send current decision status (dec-0, uncertain, ready, or dec-1) to p2.
  - If p2 receives any dec-0's and hasn't already decided, then p2 decides 0.
  - If p2 receives any dec-1's and hasn't already decided, then p2 decides 1.
  - If all received values, and its own value, are uncertain, then p2 decides 0.
  - Otherwise (all values are uncertain or ready and at least one is ready), p2 becomes ready, but doesn't decide yet.

- Round 5 (like round 2):
  - If p1 has (ever) decided 0, broadcasts "decide 0", and similarly for 1.
  - Else broadcasts "ready".
  - Any undecided process who receives "decide()" decides accordingly.
  - Any process who receives "ready" becomes ready.
  - Now p2 decides 1 if it hasn't already decided.

- Round 6 (like round 3):
  - If p2 has decided 1, broadcasts "decide 1".
  - Anyone else who receives "decide 1" decides 1.

- Continue with subsequent rounds for p3, p4,…

Correctness

- Key invariants still hold:
  - If any process is in ready or dec-1, then all processes have initial value 1.
  - If any process is in dec-0 then:
    - No process is in dec-1, and no non-failed process is ready.
  - If any process is in dec-1 then:
    - No process is in dec-0, and no non-failed process is uncertain.
- Imply agreement, validity
- Strong termination:
  - Because eventually some coordinator will finish the job (unless everyone fails).

Complexity

- Time until everyone decides:
  - Normal case 3
  - Worst case 3n
- Messages until everyone decides:
  - Normal case O(n)
    - Technicality: When can processes stop sending messages?
  - Worst case $O(n^2)$

Practical issues for 3-phase commit

- Depends on strong assumptions, which may be hard to guarantee in practice:
  - Synchronous model:
    - Could emulate with approximately-synchronized clocks, timeouts.
  - Reliable message delivery:
    - Could emulate with acks and retransmissions.
    - But if retransmissions add too much delay, then we can't emulate the synchronous model accurately.
    - Leads to unbounded delays, asynchronous model.
  - Accurate diagnosis of process failures:
    - Get this "for free" in the synchronous model.
    - E.g., 3-phase commit algorithm lets process that doesn't hear from another process i at a round conclude that i must have failed.
    - Very hard to guarantee in practice: In Internet, or even a LAN, how to reliably distinguish failure of a process from lost communication?
- Other consensus algorithms can be used for commit, including some that don't depend on such strong timing and reliability assumptions.

Paxos consensus algorithm

- A more robust consensus algorithm, could be used for commit.
- Tolerates process stopping and recovery, message losses and delays,…
- Runs in partially synchronous model.
- Based on earlier algorithm [Dwork, Lynch, Stockmeyer].
- Algorithm idea:
  - Processes use unreliable leader election subalgorithm to choose coordinator, who tries to achieve consensus.
  - Coordinator decides based on active support from majority of processes.
  - Does not assume anything based on not receiving a message.
  - Difficulties arise when multiple coordinators are active---must ensure consistency.

- Practical difficulties with fault-tolerance in the synchronous model motivate moving on to study the asynchronous model (next time).

Next time…

- Modeling asynchronous systems
- Reading: Chapter 8

6.852J / 18.437J Distributed Algorithms
Fall 2009