Massachusetts Institute of Technology Handout 4
6.852: Distributed Algorithms
Prof. Nancy Lynch September 10, 2009

Problem Set 1, Part a

Due: Thursday, September 24, 2009
Problem sets will be collected in class. Please hand in each problem on a separate page.
Students who agree to let us hand out their writeups can help us by writing elegant and concise solutions and formatting them using LaTeX.

Reading:

Chapters 1 and 2 of *Distributed Algorithms.*
Sections 3.1-3.5

Reading for next week: Section 3.6
Chapter 4 (skip 4.5.3)

Problems:

Note: In all homework problems, you are free to invoke theorems proved in the book without re-proving them.

1. (Based on Exercise 3.2) For the LCR algorithm,
   (a) Give a UID assignment for which $\Omega(n^2)$ messages are sent.
   (b) Give a UID assignment for which only $O(n)$ messages are sent.
   (c) Show that the average number of messages sent is $O(n \log n)$, where this average is taken over all possible orderings of the processes on the ring, each assumed to be equally likely.
   (d) Give a specific UID assignment for which $\Theta(n \log n)$ messages are sent.

2. (Based on Exercise 3.8) Consider modifying the HS algorithm so that the processes only send tokens in one direction rather than both.
   (a) Show that the most straightforward modification to the algorithm in the text does not yield $O(n \log n)$ communication complexity. What is an upper bound for the communication complexity?
   (b) Add a little more cleverness to the algorithm in order to restore the $O(n \log n)$ complexity bound.

3. (Based on Exercise 3.9) Design a unidirectional leader-election algorithm that works with unknown ring size, and only uses $O(n \log n)$ messages in the worst case. Your algorithm should manipulate the UIDs using comparisons only. Prove correctness and prove the complexity bound. (Note: You may find this hard.)

4. Consider the problem of electing a leader in a synchronous ring of known size n, where the processes all have UIDs, but where the only operations available to the processes for manipulating these UIDs are (equals, unequals) comparisons. Observe that UIDs are abstract objects, and thus you should not assume they are a subset of the integers, or anything similar. Is this problem solvable or unsolvable? Prove carefully.

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Fall 2009