

In this lecture, we'll see the best binary search tree we know, in the sense of achieving the best known competitive ratio of  $O(\lg \lg n)$  compared to the offline optimal. To analyze these "Tango trees", we compare against a lower bound. Specifically, we describe a Signed Greedy algorithm that, for a given access sequence, computes a number of node touches that every BST in the world must perform when given that access sequence. Indeed, the Signed Greedy algorithm adds points row by row just like the Greedy algorithm we saw last lecture. The only catch is that Signed Greedy doesn't actually satisfy the point set, so it doesn't correspond to a BST... yet it is tantalizingly close to Greedy, which we saw last lecture corresponds to an online BST, suggesting that the two bounds are within a constant factor of each other (making Greedy dynamically optimal). While this gap hasn't been closed yet, we can still use the lower bound to analyze Tango trees, and get the exponential improvement over the trivial  $O(\lg n)$  competitive ratio held by any balanced binary search tree.

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