

# MIT 6.849

## Geometric Folding Algorithms

Prof. Erik Demaine

### Lecture 6: Origami Art and Design

Guest Lecturer:  
Jason Ku

September 27, 2010

|

Jason Ku  
President of OrigamiMIT  
Mechanical Engineering Bachelor's, MIT '09  
PhD student in Mechanical Engineering working in folding on the micro and nano scales

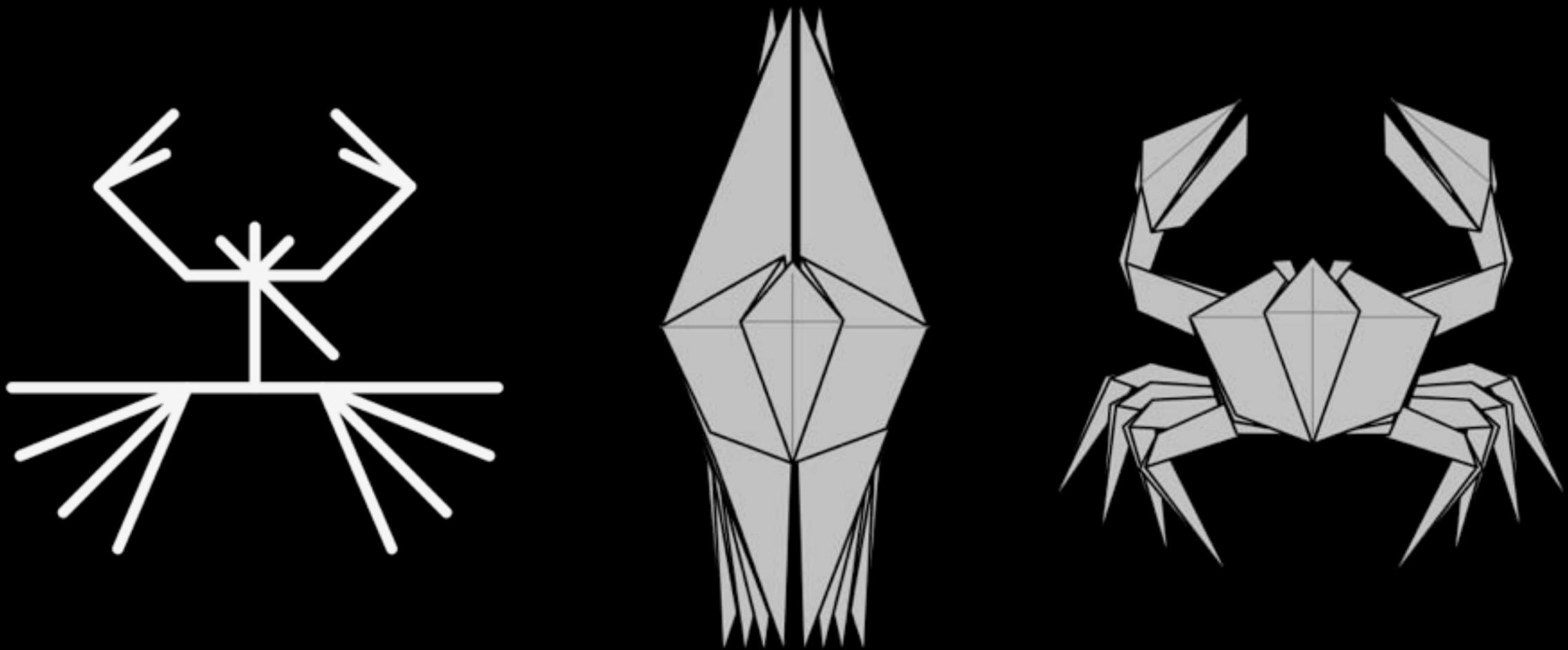
Courtesy of Jason Ku. Used with permission.

# Origami Art

- Akira Yoshizawa <http://www.origami.vancouver.bc.ca/>
- Hideo Komatsu <http://www.origami.gr.jp/~komatsu/>
- Takashi Hojyo <http://origami.gr.jp/~hojyo>
- David Brill <http://www.brilliantorigami.com/>
- Michael LaFosse <http://www.origamido.com>
- Eric Joisel <http://www.ericjoisel.com>
- Robert Lang <http://www.langorigami.com/>
- Brian Chan <http://chosetec.darkclan.net/origami/>
- Satoshi Kamiya <http://www.folders.jp/>
- Jason Ku <http://scripts.mit.edu/~jasonku/>

Websites where photos in presentation come from  
Origami Art comparison with Music  
Representational origami = traditionally represent living things

# Tree Theory Review

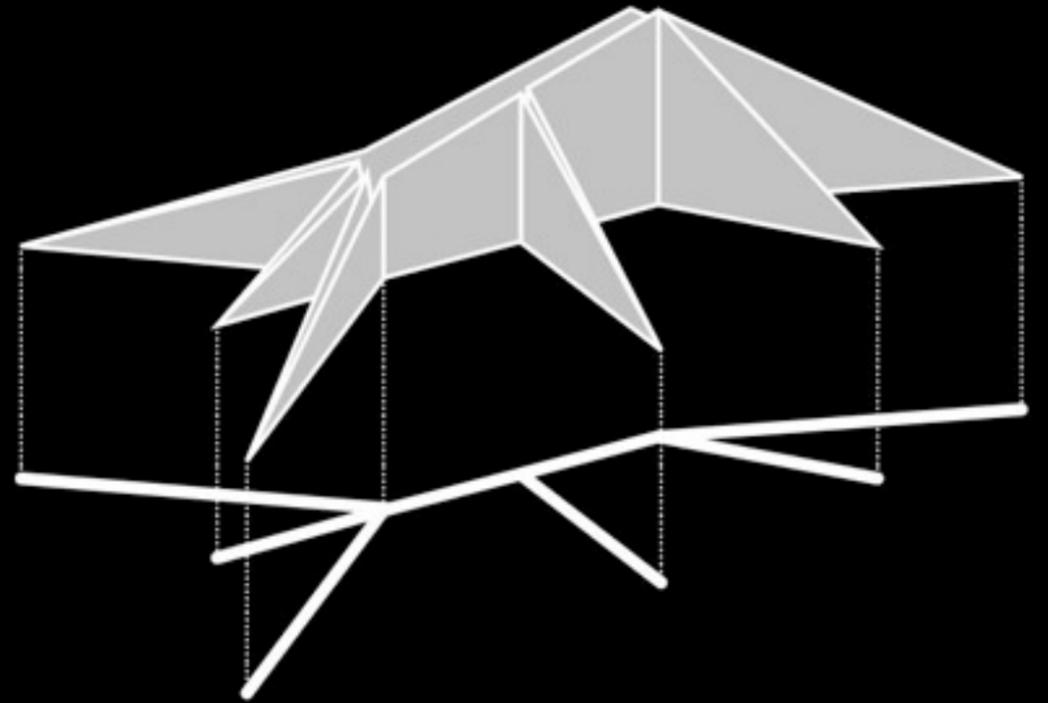


Review of Tree Theory thought process

- 1) Start with object
- 2) Draw tree
- 3) Change tree into uniaxial base
- 4) Shape uniaxial base

Courtesy of Jason Ku. Used with permission.

# Uniaxial Bases



1. in  $z \geq 0$  half plane
2. intersection with  $z=0$  plane = projection onto the plane
3. partition of faces into flaps, each projecting to a line segment
4. hinge crease shared by two flaps project to a point
5. graph of flap projections as edges is a tree
6. only one point of paper folds to each leaf

Previous definition of uniaxial bases

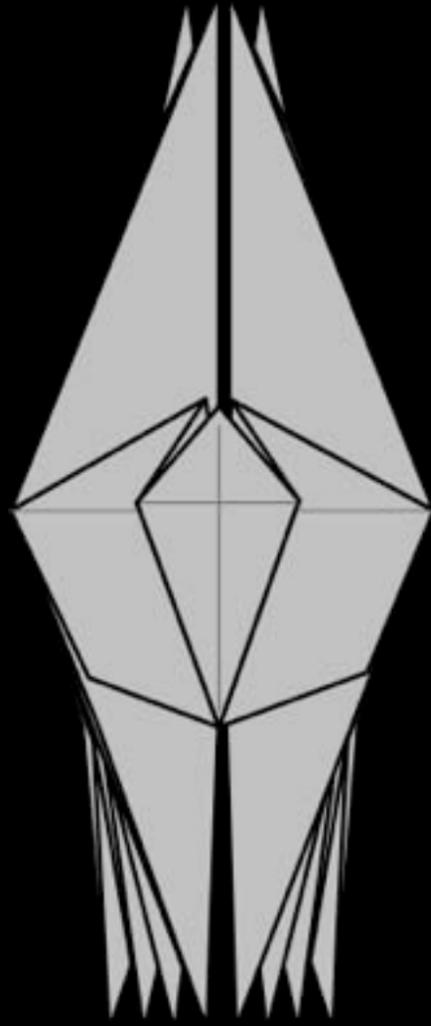
(6) not necessary but convenient

Why would it be useful to have the end of a leaf node map to more than one point on paper? Ans: flap thickness at end

What does this really mean?

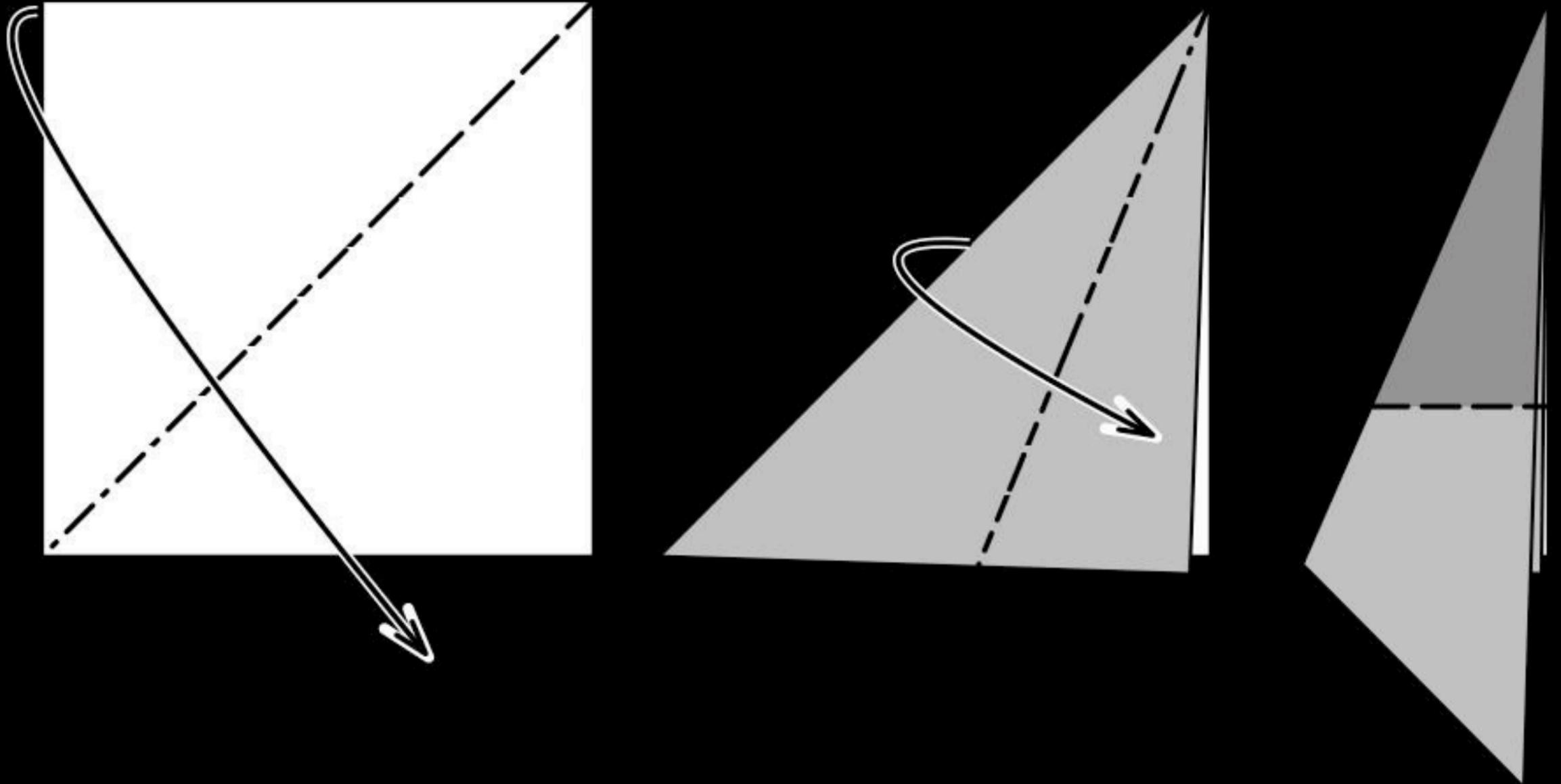
Courtesy of Jason Ku. Used with permission.

# Uniaxial Bases



1. flaps lie along or straddle a single line (the axis)
2. flaps hinge perpendicular to the axis
3. can thin to stick figure (tree)

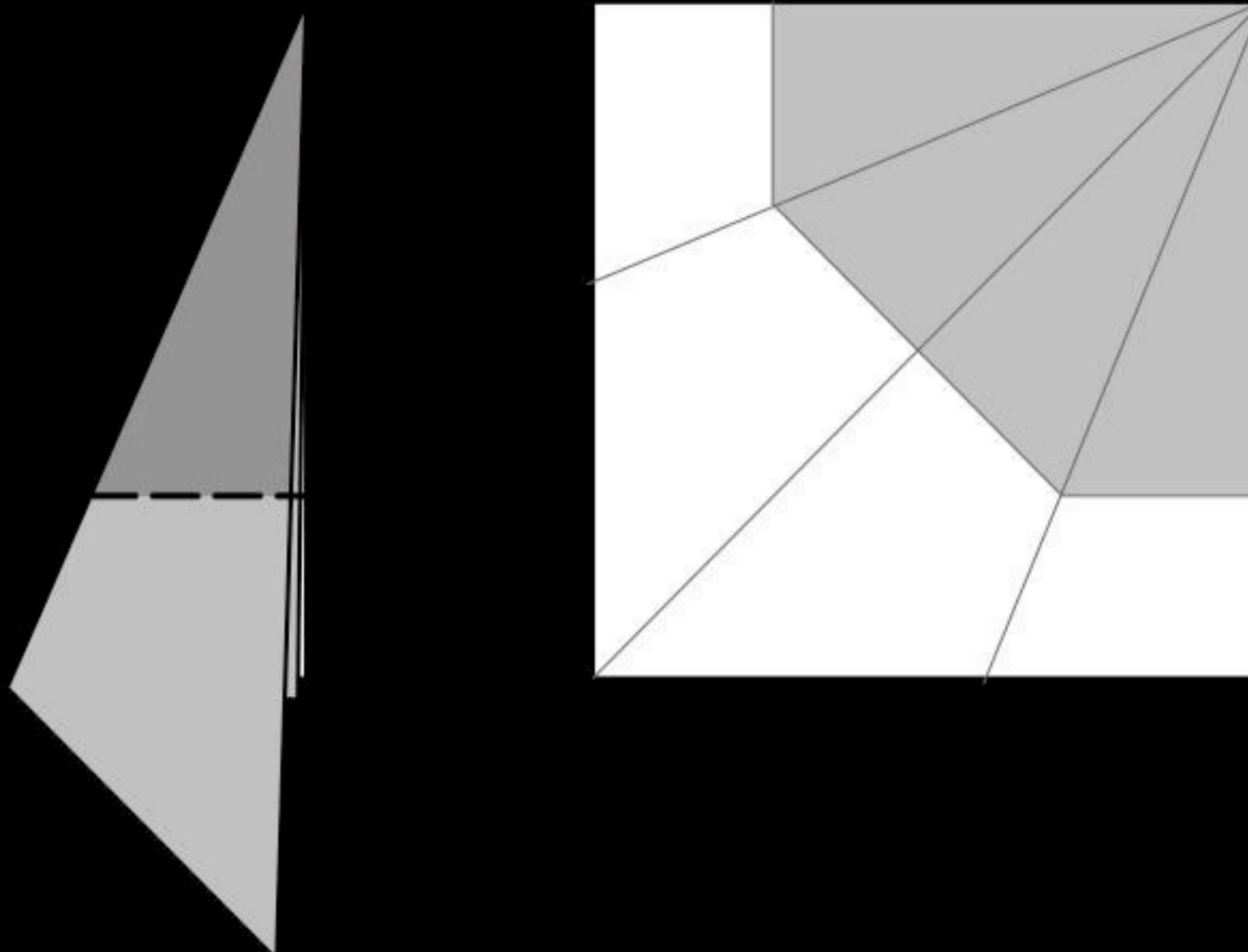
# Flaps



Modeling a flap

Courtesy of Jason Ku. Used with permission.

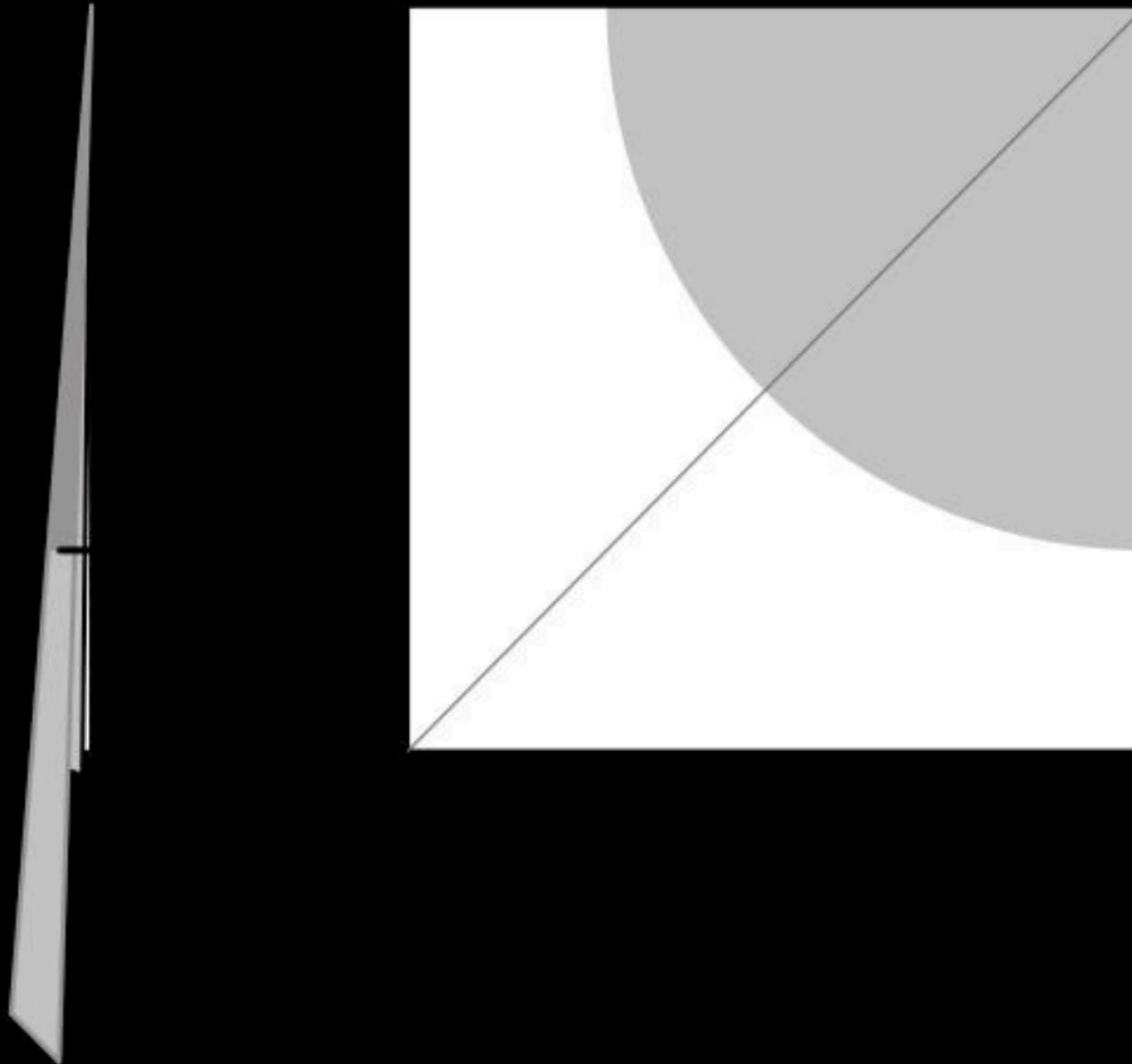
# Flaps



Idea of 'elevation' on a flap/tree edge  
Rivers separate two parts of a tree with strip of constant width  
Circle limiting case of river separating single point from rest  
Splitting a leaf edge into a leaf and brach creates a redundant node

Courtesy of Jason Ku. Used with permission.

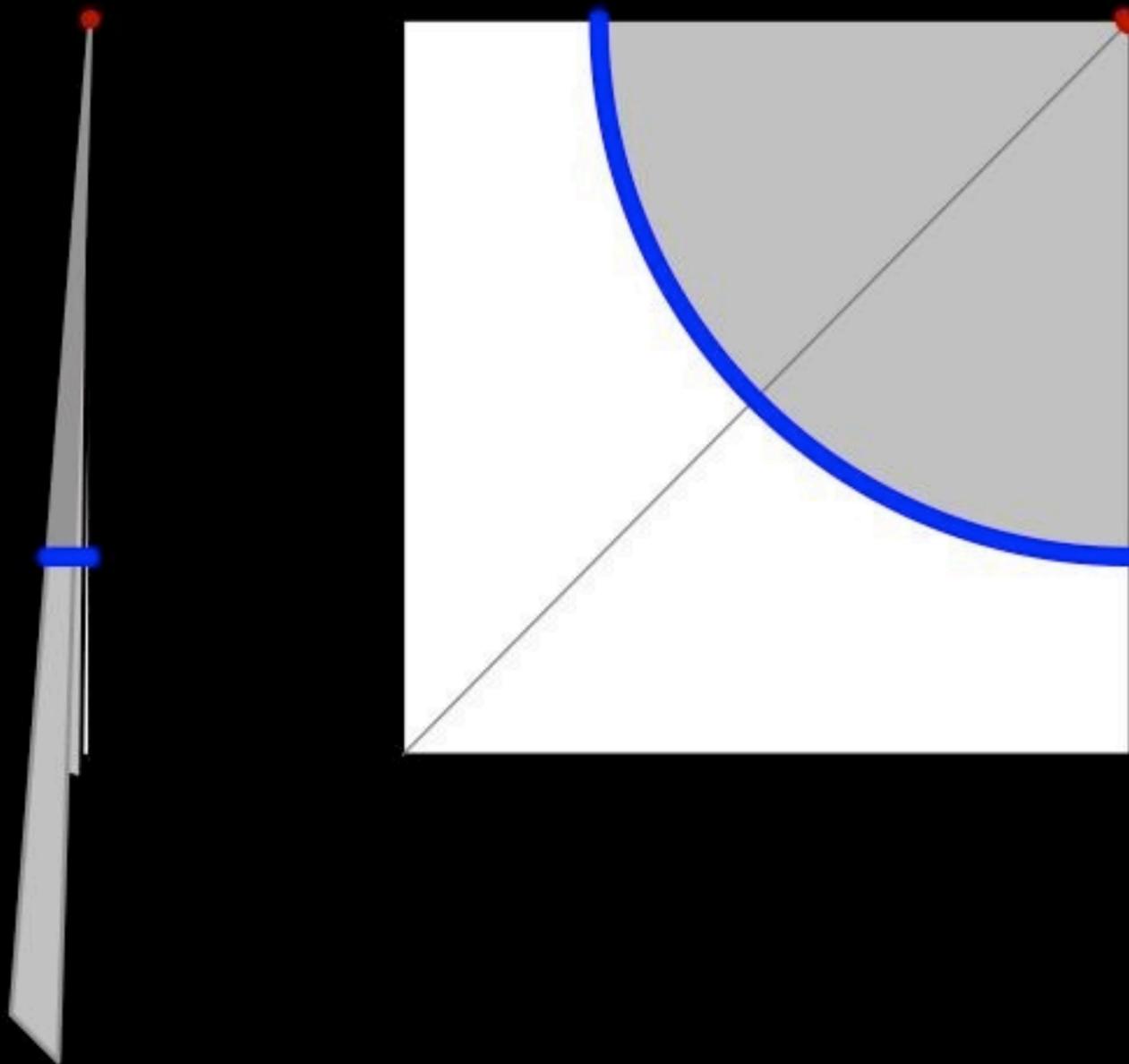
# Flaps



Idea of 'elevation' on a flap/tree edge  
Rivers separate two parts of a tree with strip of constant width  
Circle limiting case of river separating single point from rest  
Splitting a leaf edge into a leaf and brach creates a redundant node

Courtesy of Jason Ku. Used with permission.

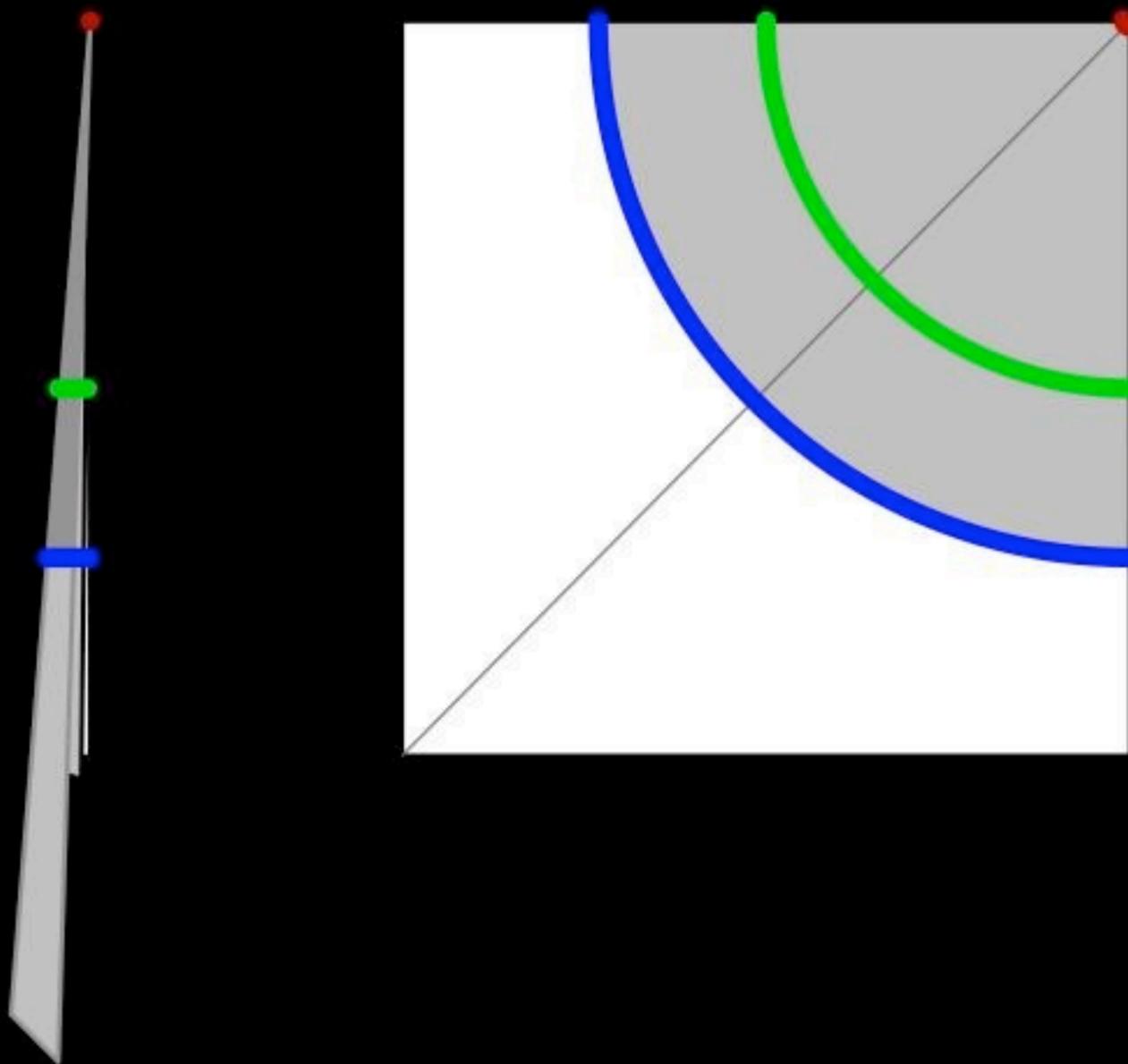
# Flaps



Idea of 'elevation' on a flap/tree edge  
Rivers separate two parts of a tree with strip of constant width  
Circle limiting case of river separating single point from rest  
Splitting a leaf edge into a leaf and branch creates a redundant node

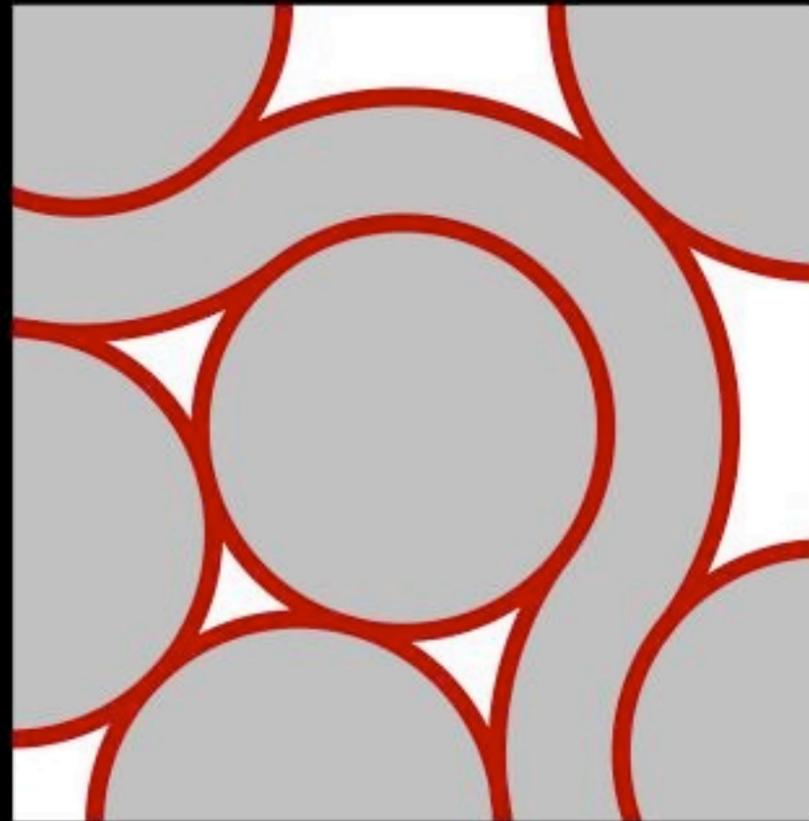
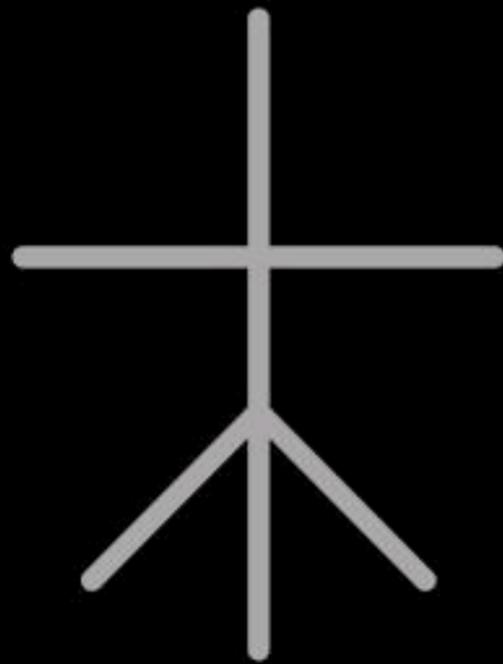
Courtesy of Jason Ku. Used with permission.

# Flaps



Idea of 'elevation' on a flap/tree edge  
Rivers separate two parts of a tree with strip of constant width  
Circle limiting case of river separating single point from rest  
Splitting a leaf edge into a leaf and brach creates a redundant node

# Flaps



Circle/River Packing (CRP) as a space allocation

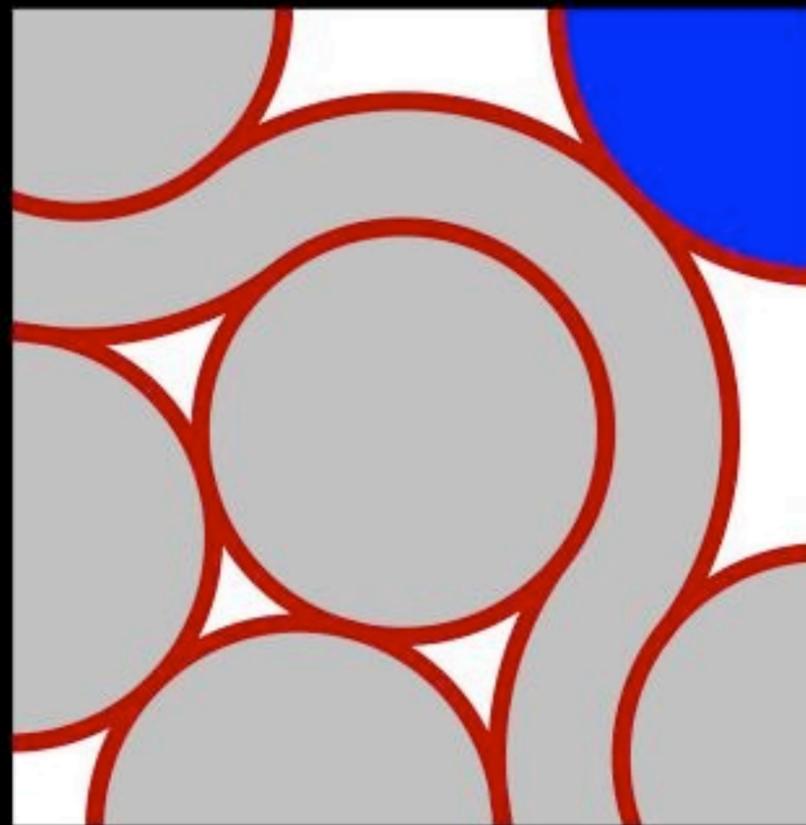
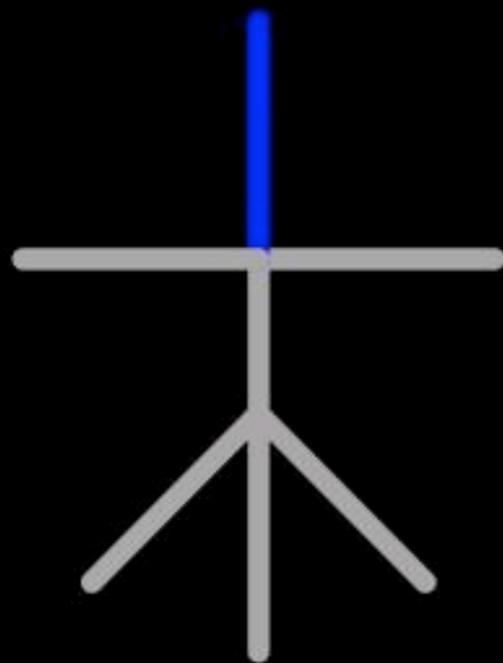
Uniquely defines a tree

Tree edges can be oriented anyway we like because if uniaxial base is infinitely thinned, base is actually stick figure

Space between circles is wasted paper and maps to a single tree node

Courtesy of Jason Ku. Used with permission.

# Flaps



Circle/River Packing (CRP) as a space allocation

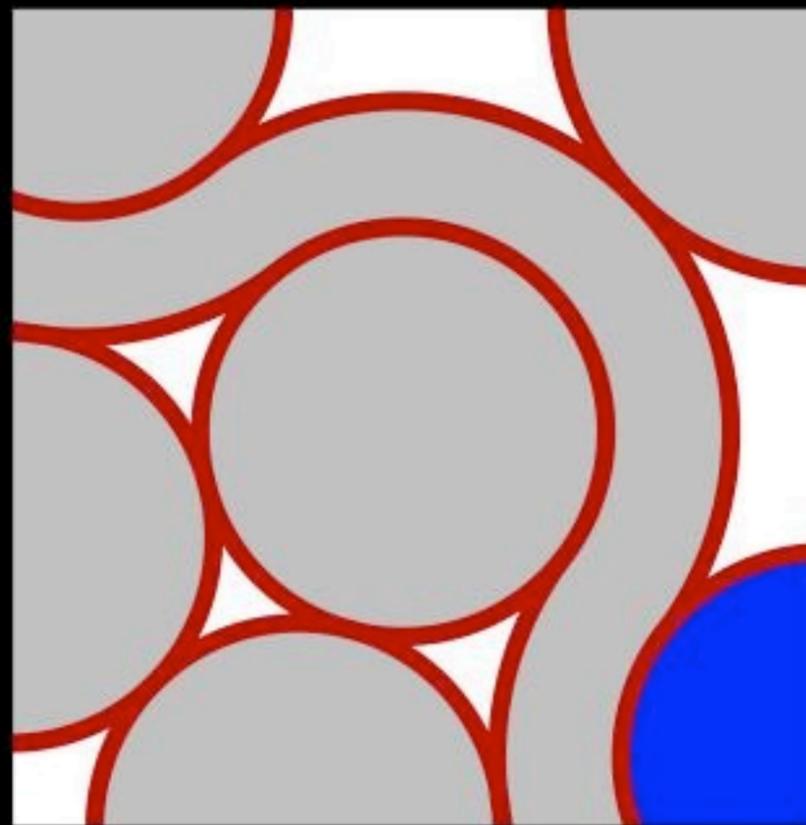
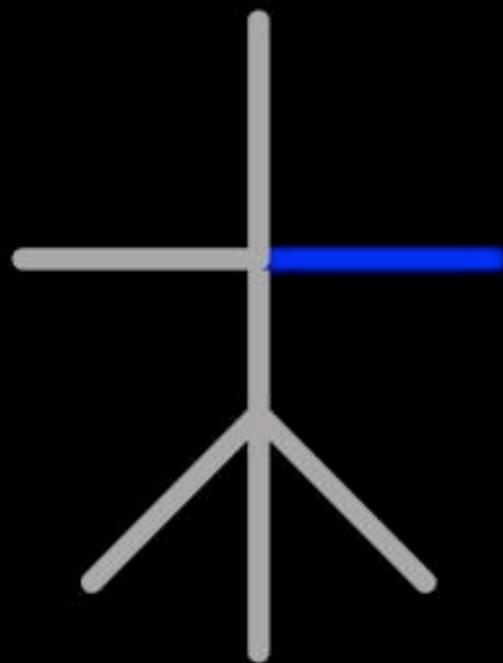
Uniquely defines a tree

Tree edges can be oriented anyway we like because if uniaxial base is infinitely thinned, base is actually stick figure

Space between circles is wasted paper and maps to a single tree node

Courtesy of Jason Ku. Used with permission.

# Flaps



Circle/River Packing (CRP) as a space allocation

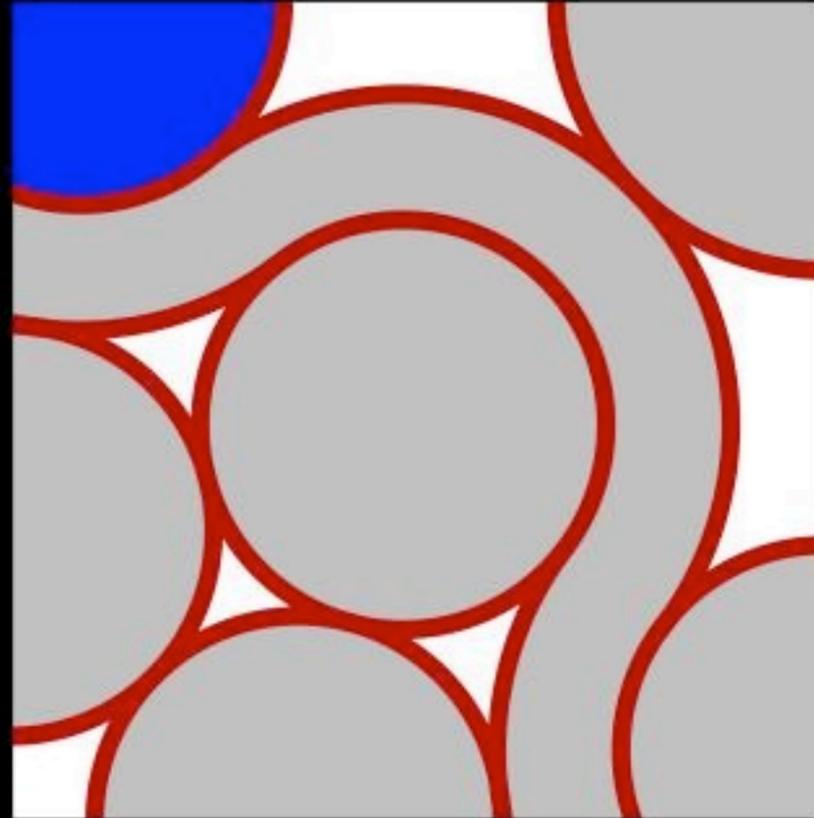
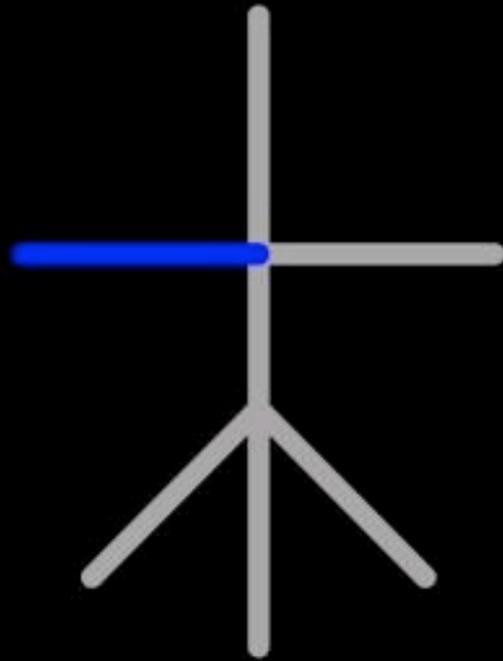
Uniquely defines a tree

Tree edges can be oriented anyway we like because if uniaxial base is infinitely thinned, base is actually stick figure

Space between circles is wasted paper and maps to a single tree node

Courtesy of Jason Ku. Used with permission.

# Flaps



Circle/River Packing (CRP) as a space allocation

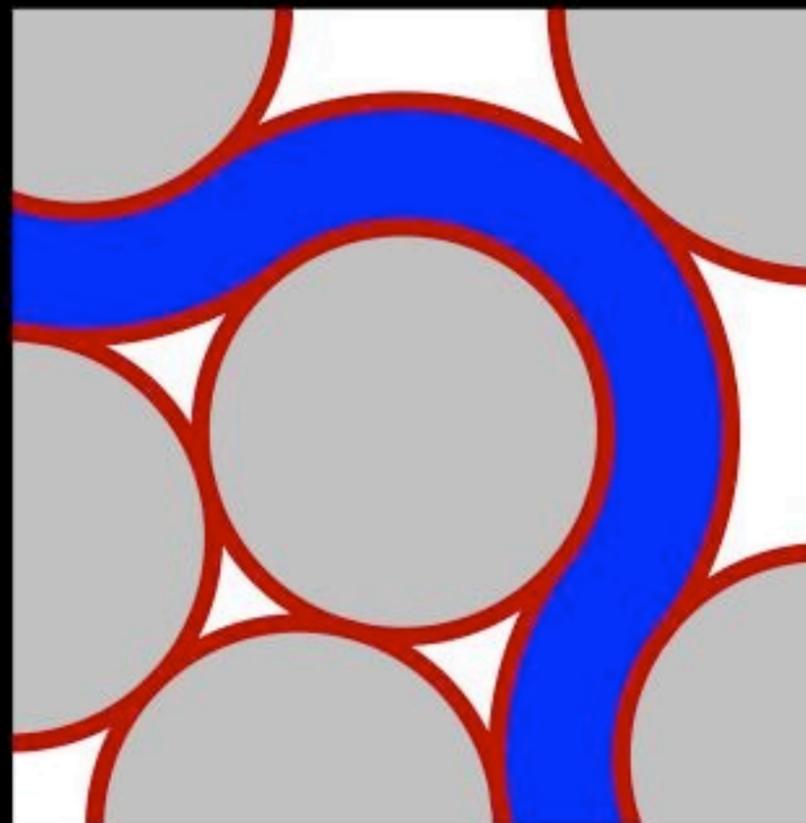
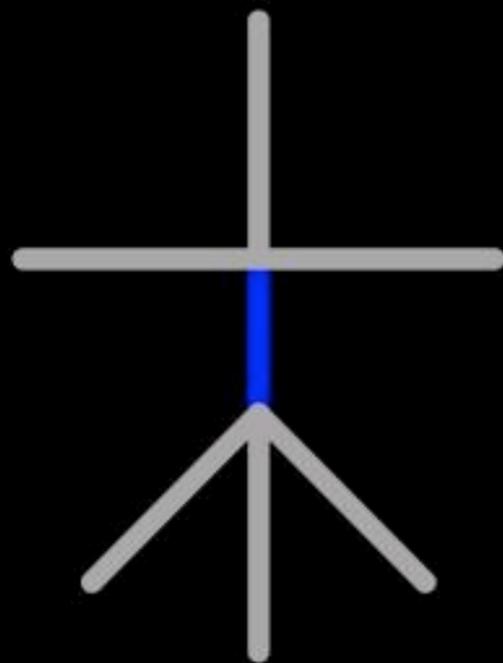
Uniquely defines a tree

Tree edges can be oriented anyway we like because if uniaxial base is infinitely thinned, base is actually stick figure

Space between circles is wasted paper and maps to a single tree node

Courtesy of Jason Ku. Used with permission.

# Flaps



Circle/River Packing (CRP) as a space allocation

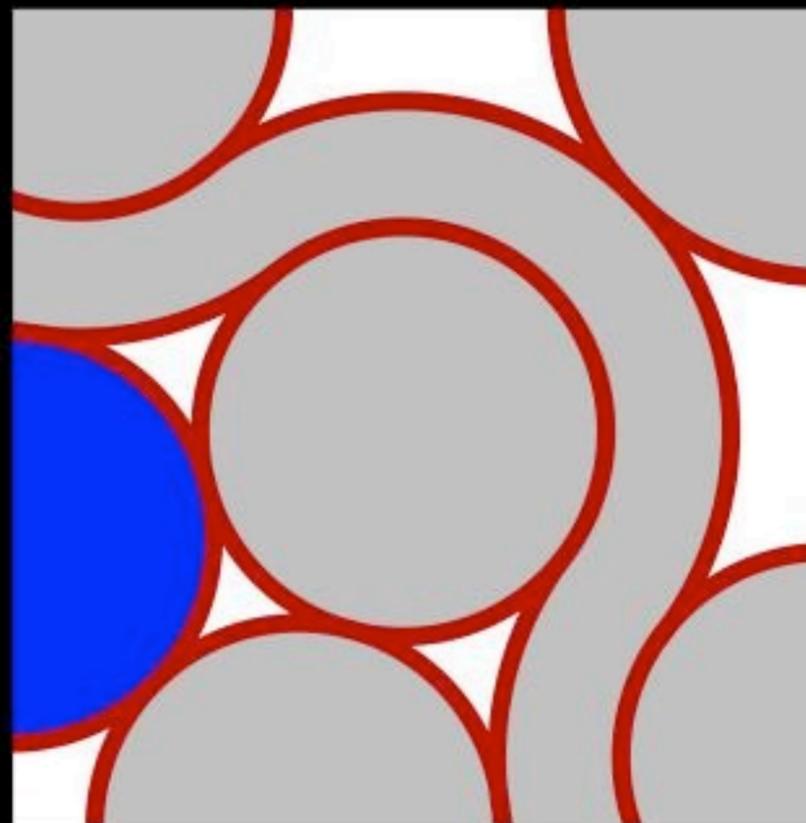
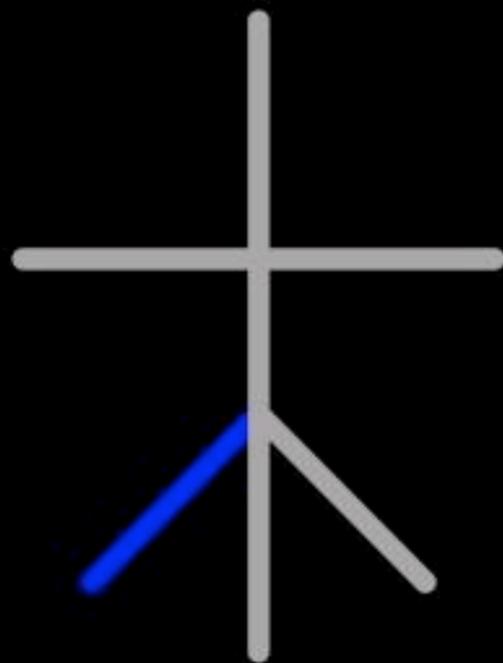
Uniquely defines a tree

Tree edges can be oriented anyway we like because if uniaxial base is infinitely thinned, base is actually stick figure

Space between circles is wasted paper and maps to a single tree node

Courtesy of Jason Ku. Used with permission.

# Flaps



Circle/River Packing (CRP) as a space allocation

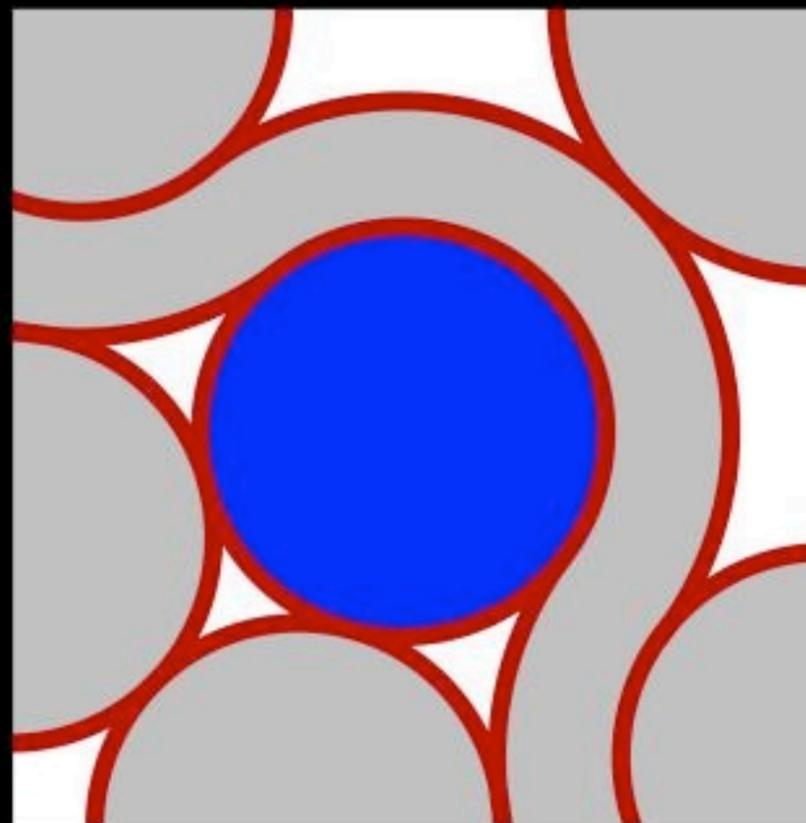
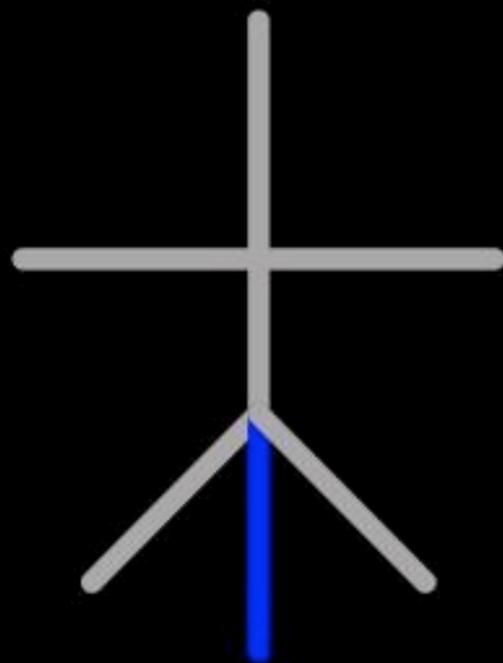
Uniquely defines a tree

Tree edges can be oriented anyway we like because if uniaxial base is infinitely thinned, base is actually stick figure

Space between circles is wasted paper and maps to a single tree node

Courtesy of Jason Ku. Used with permission.

# Flaps



Circle/River Packing (CRP) as a space allocation

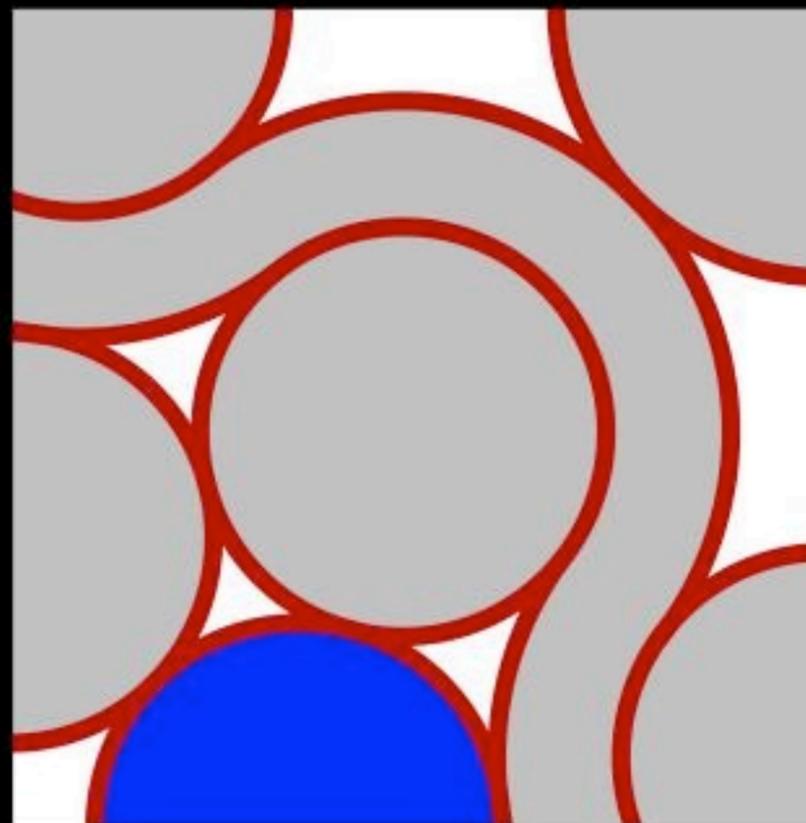
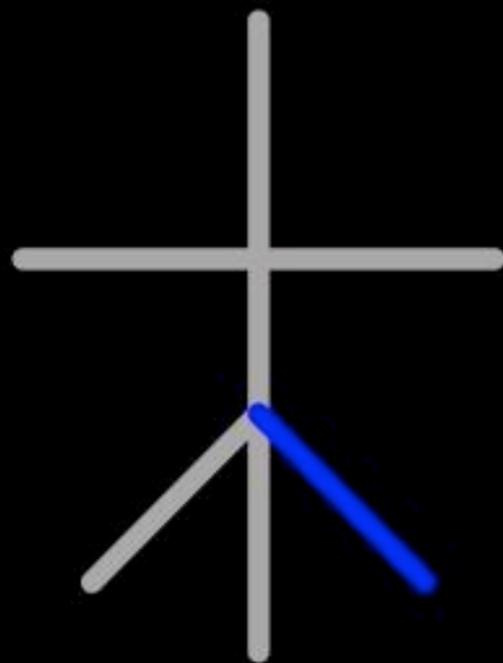
Uniquely defines a tree

Tree edges can be oriented anyway we like because if uniaxial base is infinitely thinned, base is actually stick figure

Space between circles is wasted paper and maps to a single tree node

Courtesy of Jason Ku. Used with permission.

# Flaps



Circle/River Packing (CRP) as a space allocation

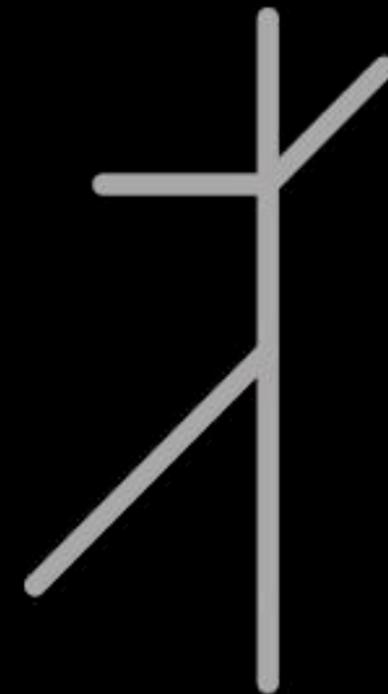
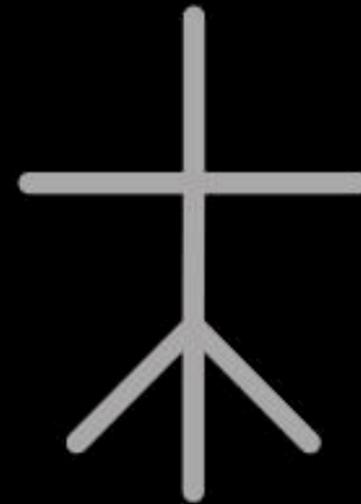
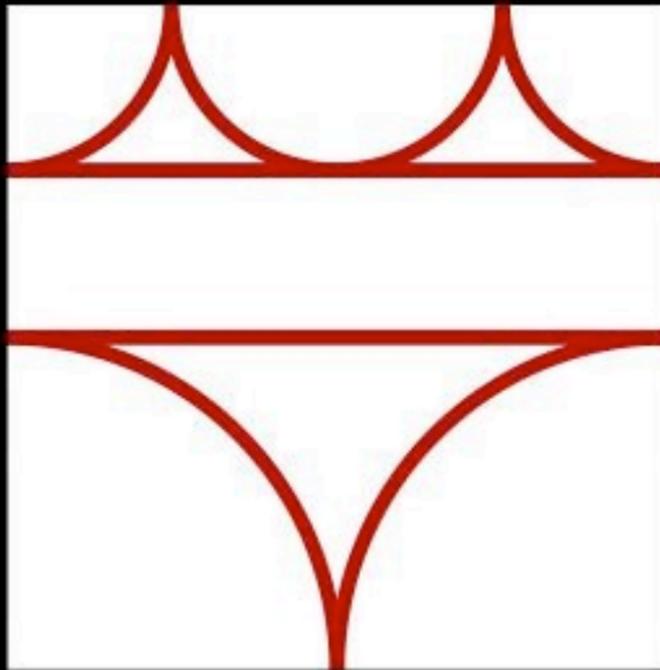
Uniquely defines a tree

Tree edges can be oriented anyway we like because if uniaxial base is infinitely thinned, base is actually stick figure

Space between circles is wasted paper and maps to a single tree node

Courtesy of Jason Ku. Used with permission.

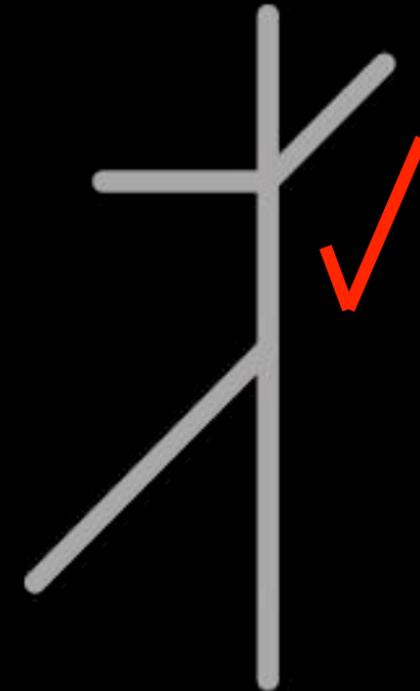
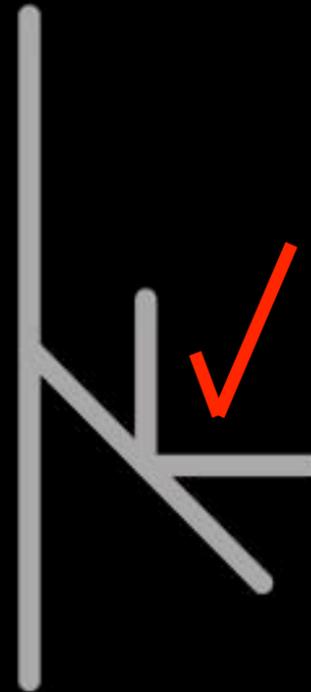
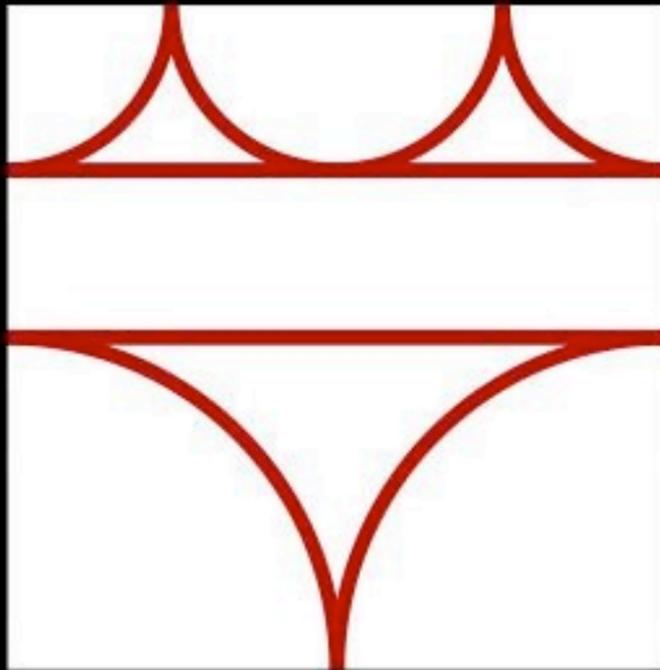
# Practice!



Which trees represent the given CRP?

Courtesy of Jason Ku. Used with permission.

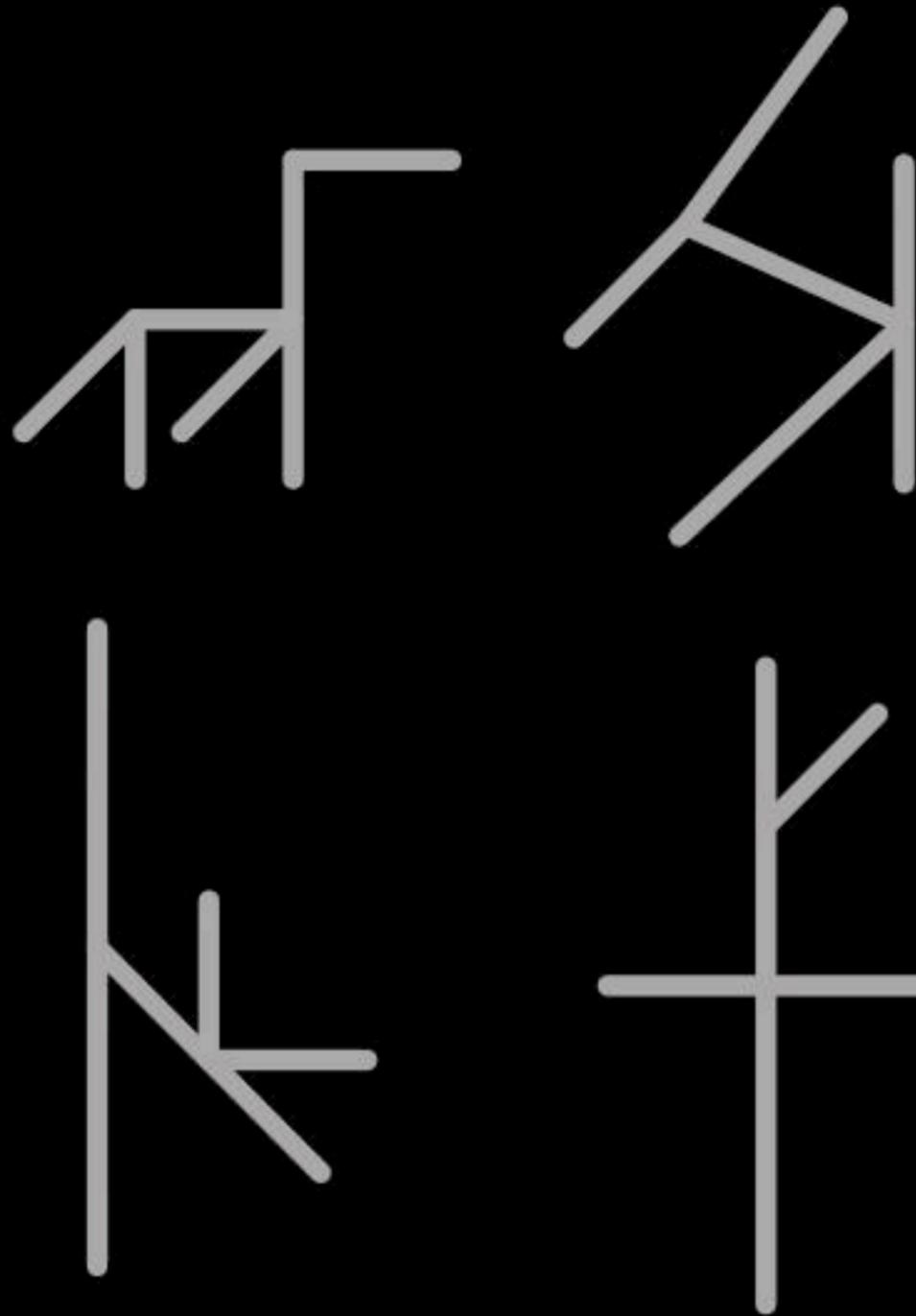
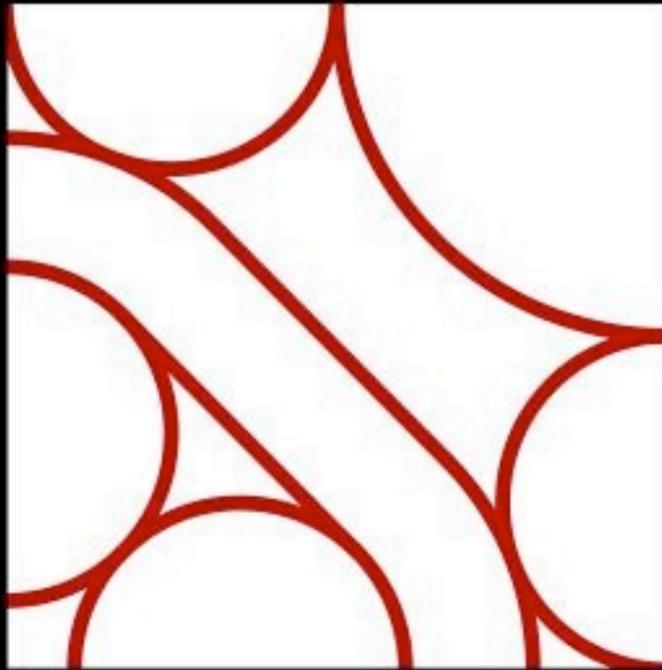
# Practice!



Which trees represent the given CRP?

Courtesy of Jason Ku. Used with permission.

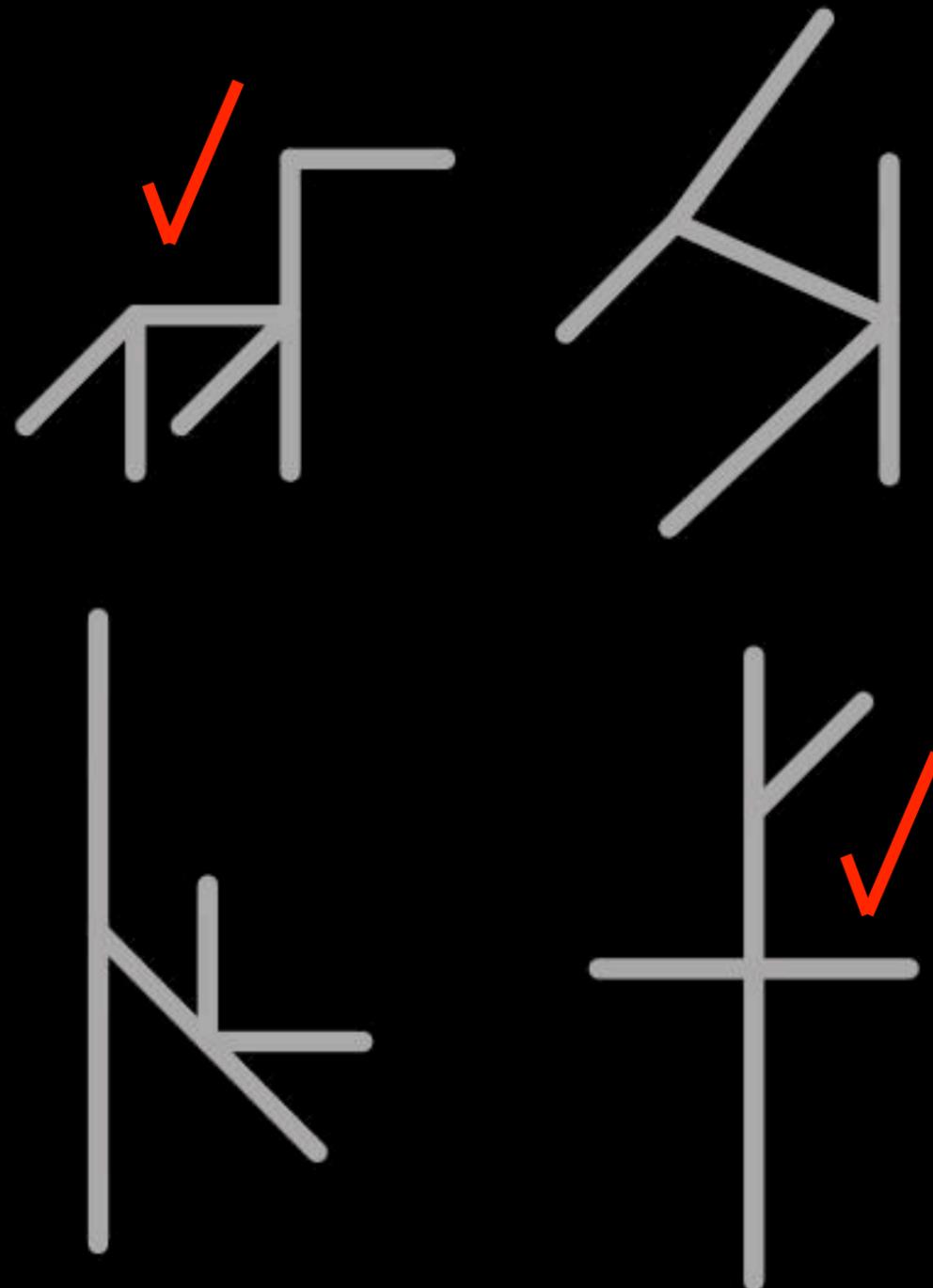
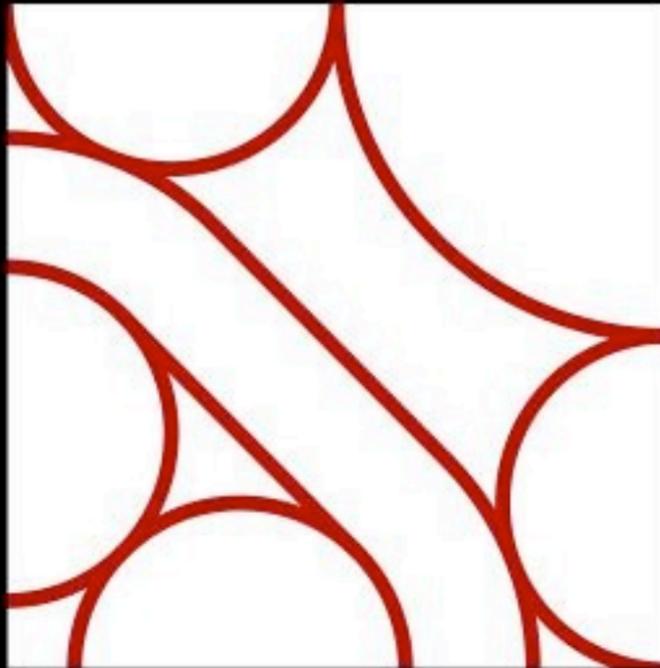
# Practice!



Which trees represent the given CRP?

Courtesy of Jason Ku. Used with permission.

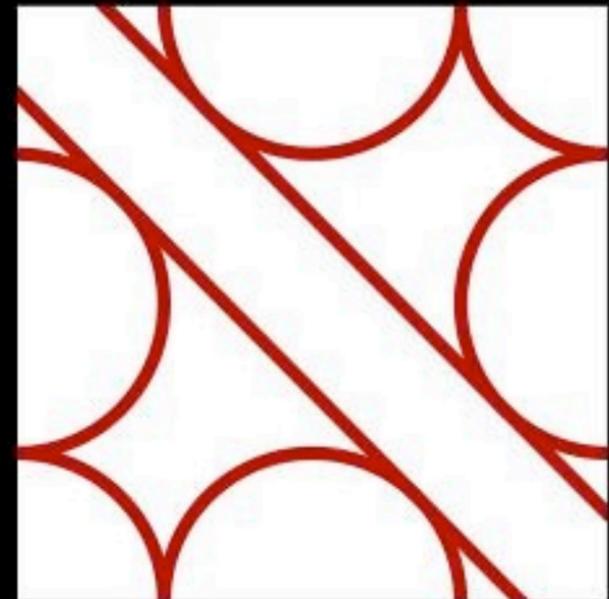
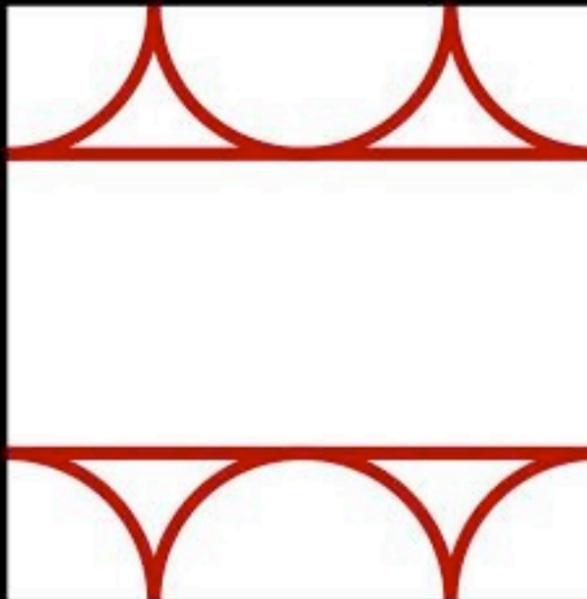
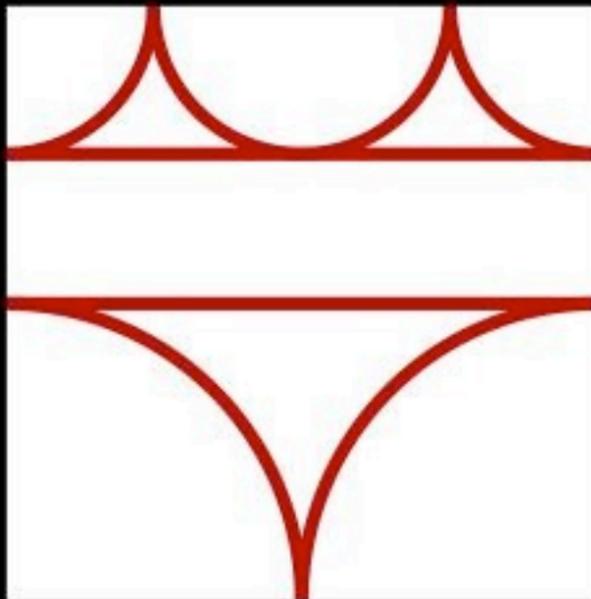
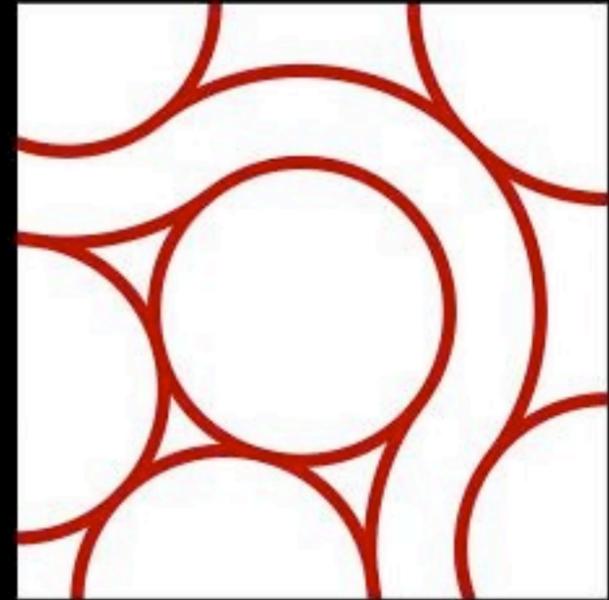
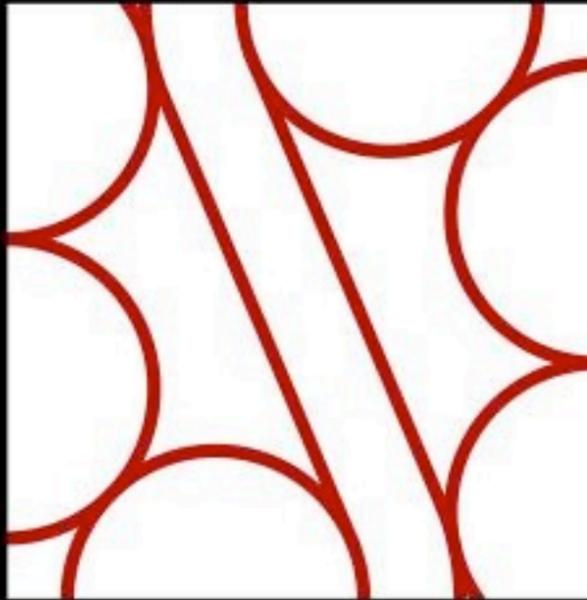
# Practice!



Which trees represent the given CRP?

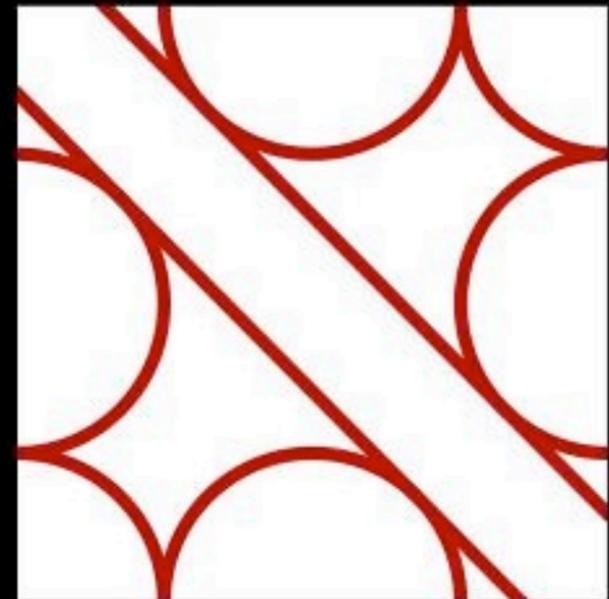
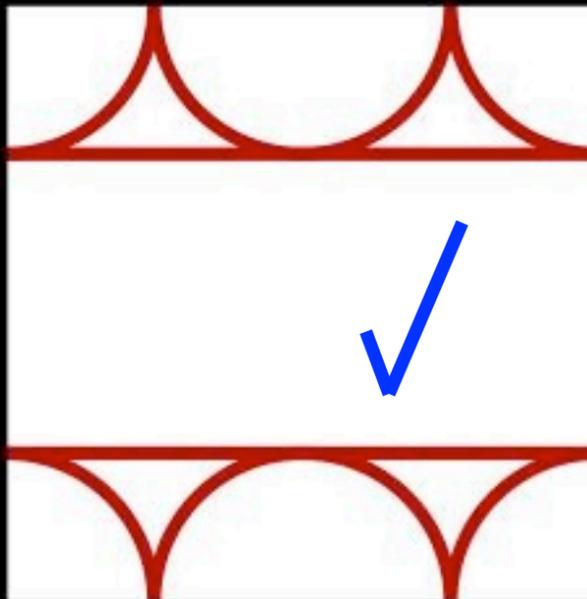
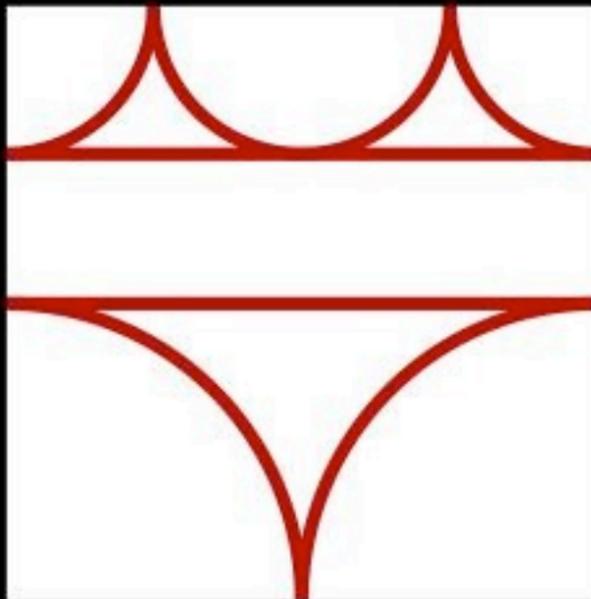
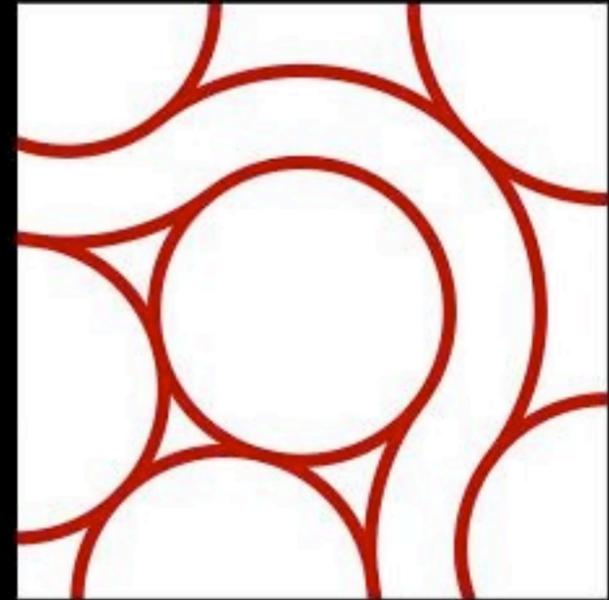
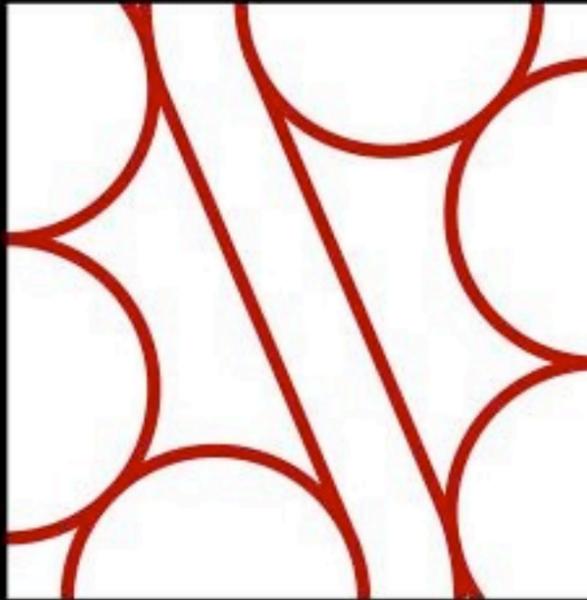
Courtesy of Jason Ku. Used with permission.

# Practice!



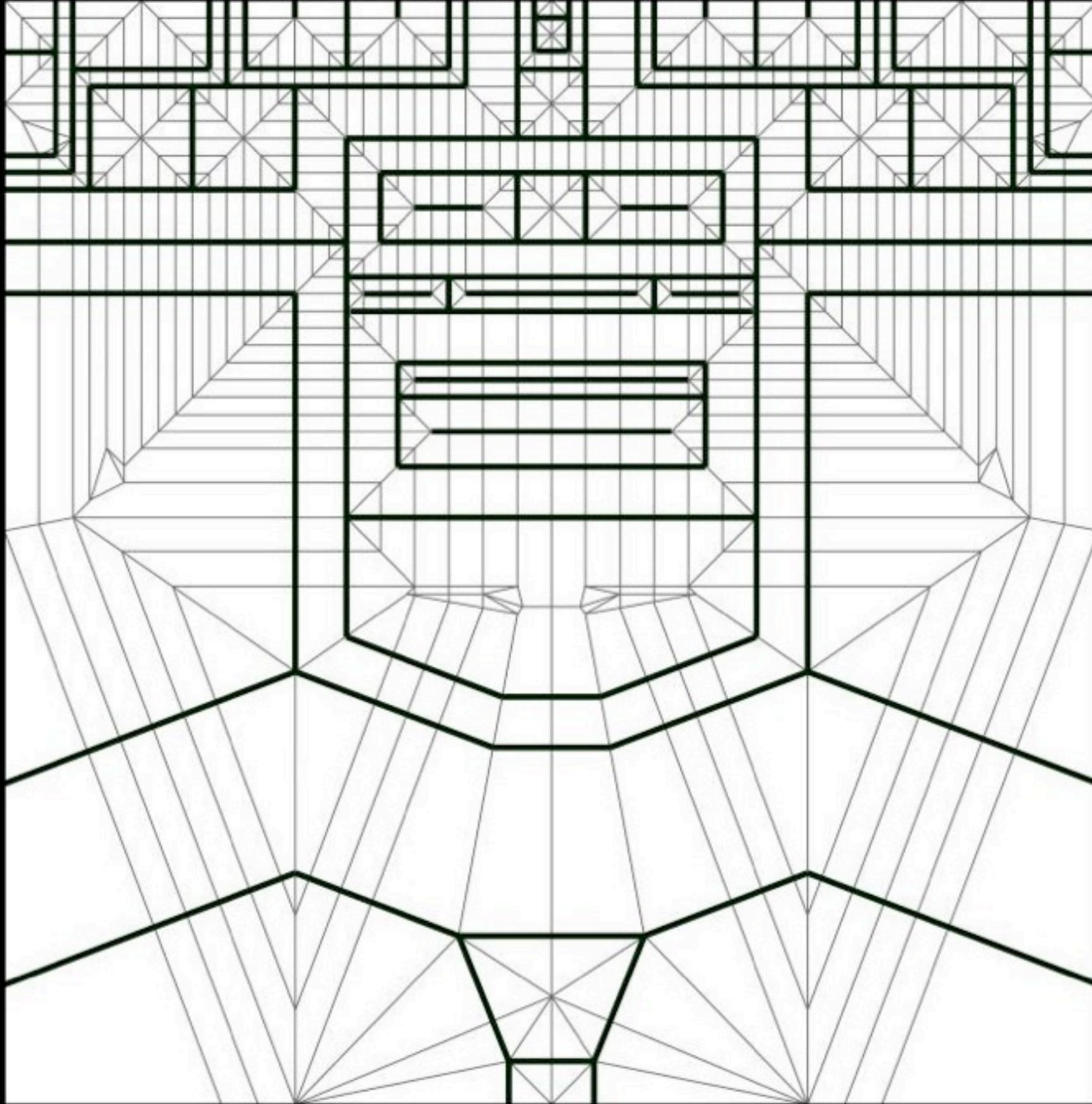
Which CRP correspond to the given tree?  
CRP 1, 2, and 5 have similar trees, but different space allocation  
(CRP  $\Rightarrow$  Tree) = unique  
(Tree  $\Rightarrow$  CRP) = non-unique

# Practice!



Which CRP correspond to the given tree?  
CRP 1, 2, and 5 have similar trees, but different space allocation  
(CRP  $\Rightarrow$  Tree) = unique  
(Tree  $\Rightarrow$  CRP) = non-unique

# Model vs. Reality

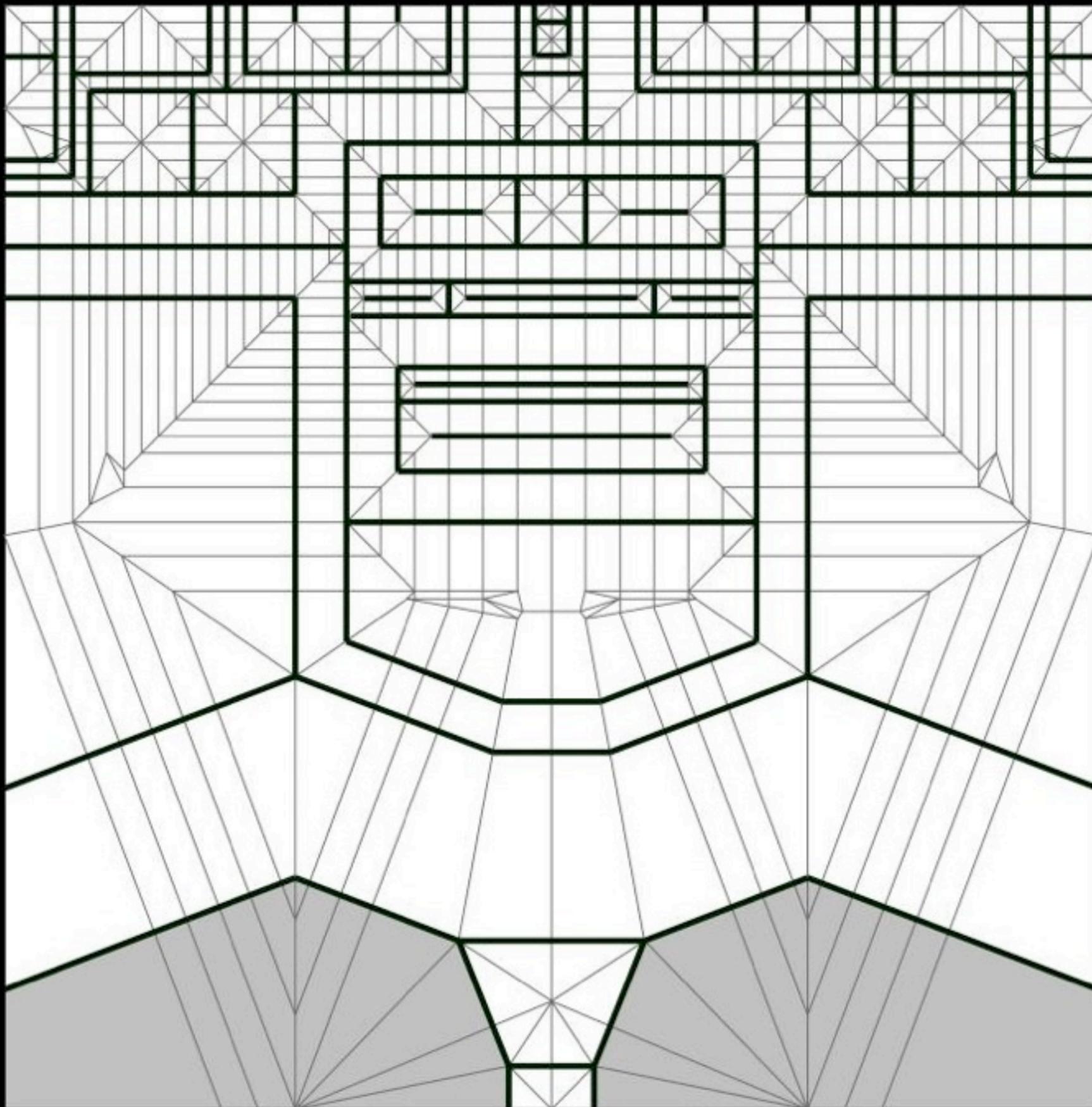


25

In reality, CRP is an idealization  
By definition, locus of all possible hinge creases represents something topologically similar to a CRP  
Can read off tree as before

Courtesy of Jason Ku. Used with permission.

# Model vs. Reality

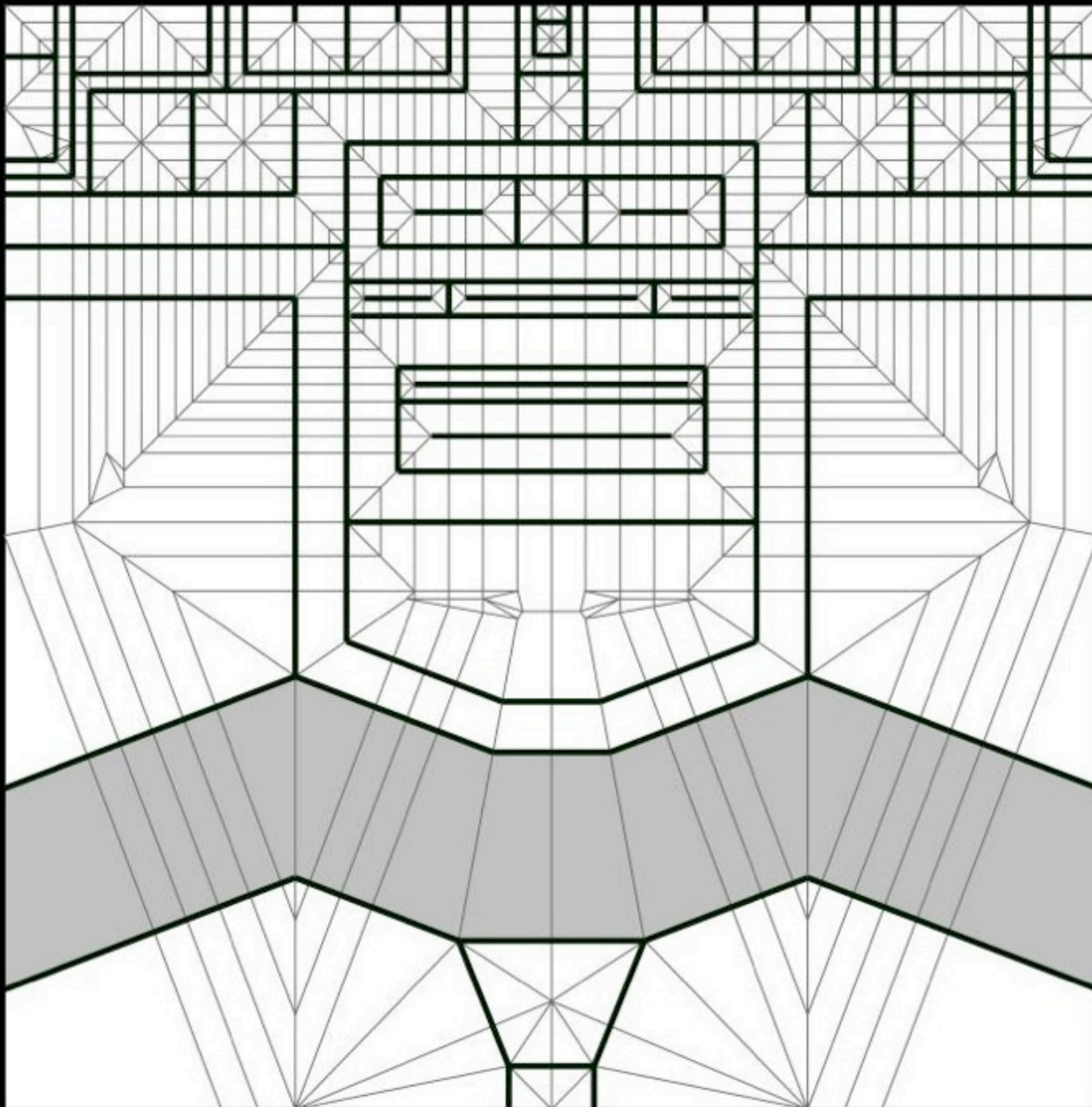


26

In reality, CRP is an idealization  
By definition, locus of all possible hinge creases represents something topologically similar to a CRP  
Can read off tree as before

Courtesy of Jason Ku. Used with permission.

# Model vs. Reality

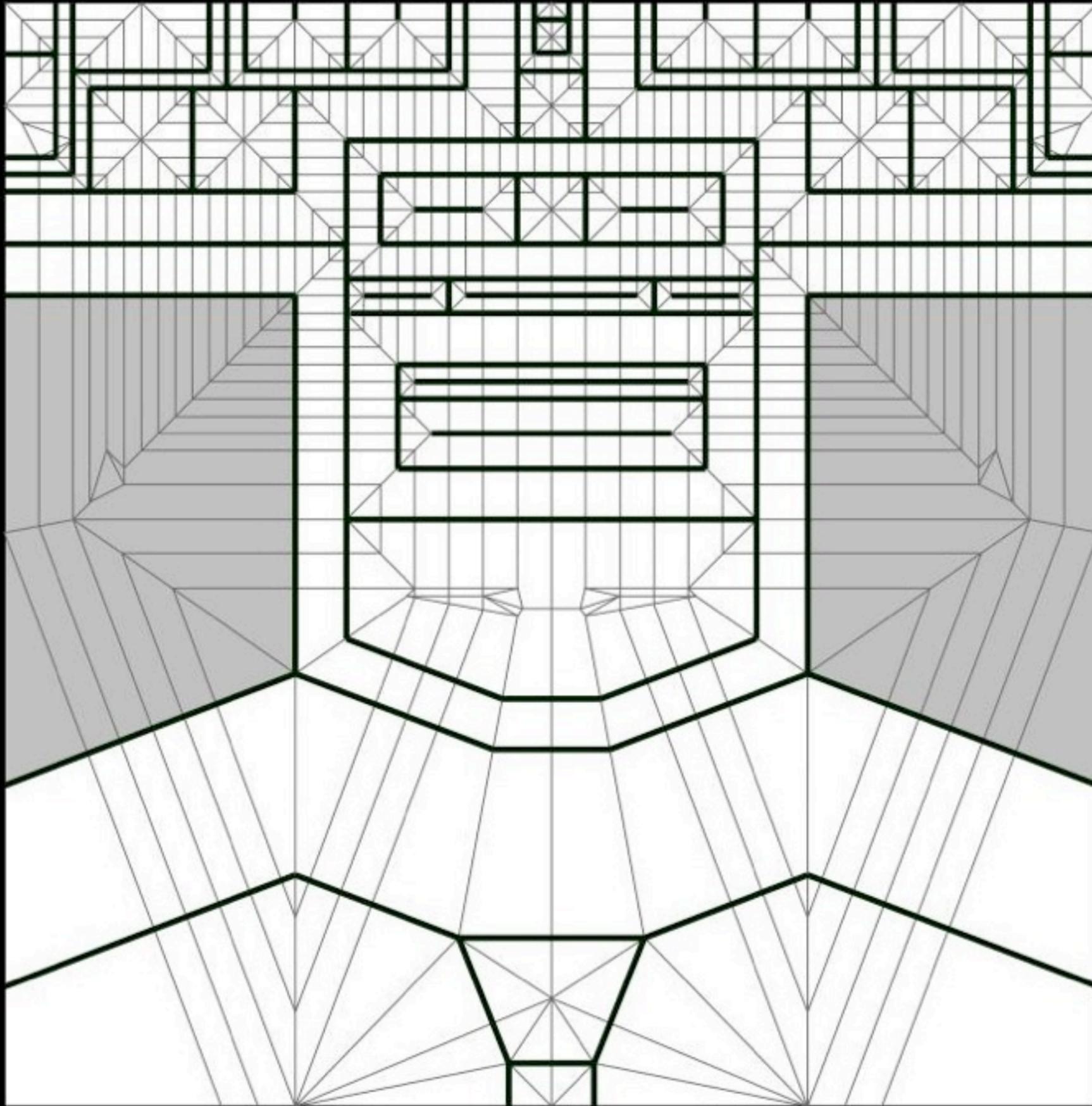


27

In reality, CRP is an idealization  
By definition, locus of all possible hinge creases represents something topologically similar to a CRP  
Can read off tree as before

Courtesy of Jason Ku. Used with permission.

# Model vs. Reality

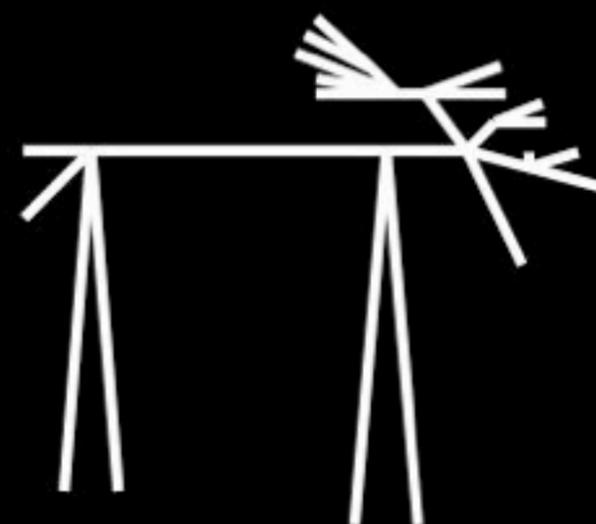
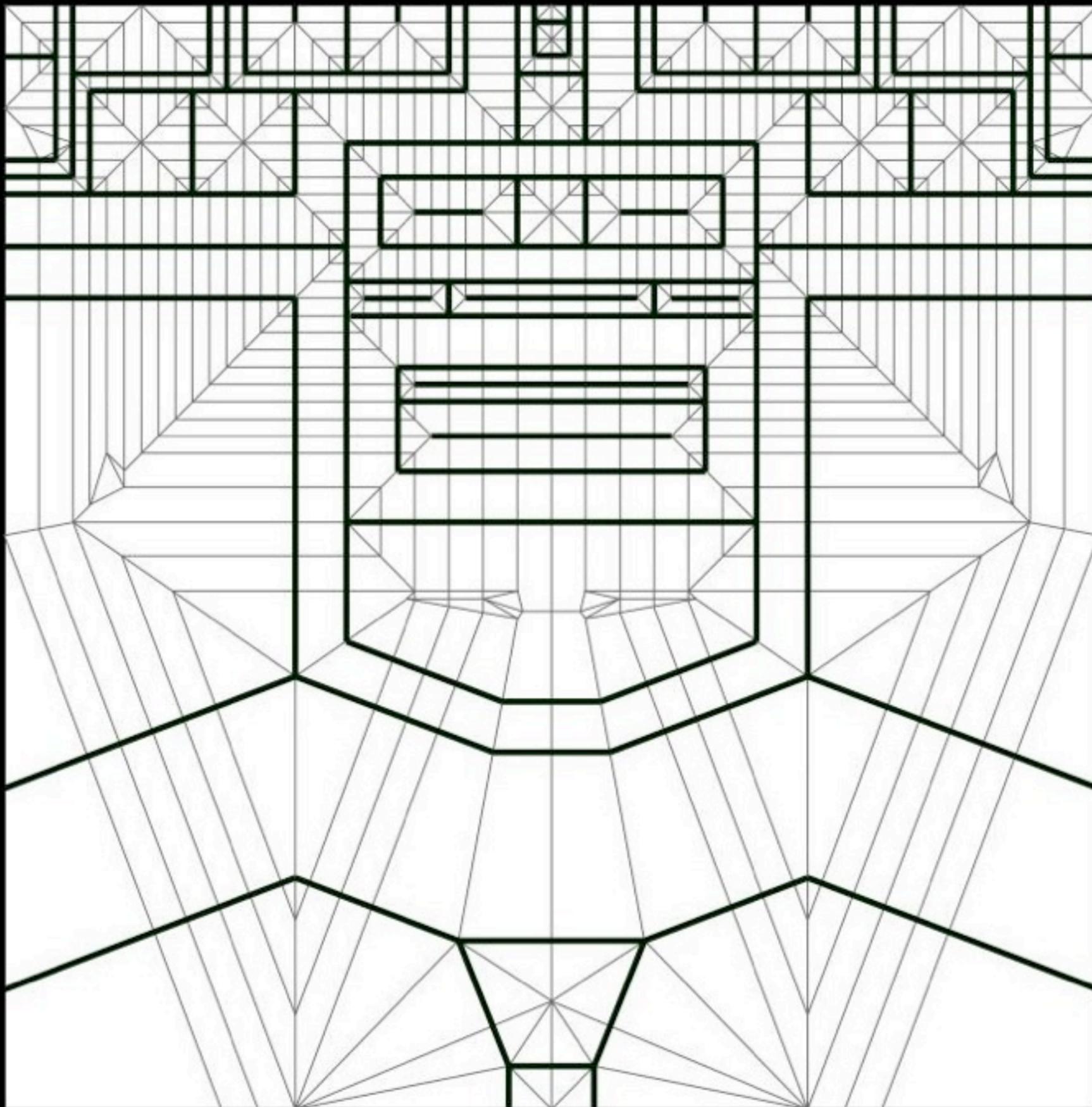


28

In reality, CRP is an idealization  
By definition, locus of all possible hinge creases represents something topologically similar to a CRP  
Can read off tree as before

Courtesy of Jason Ku. Used with permission.

# Model vs. Reality



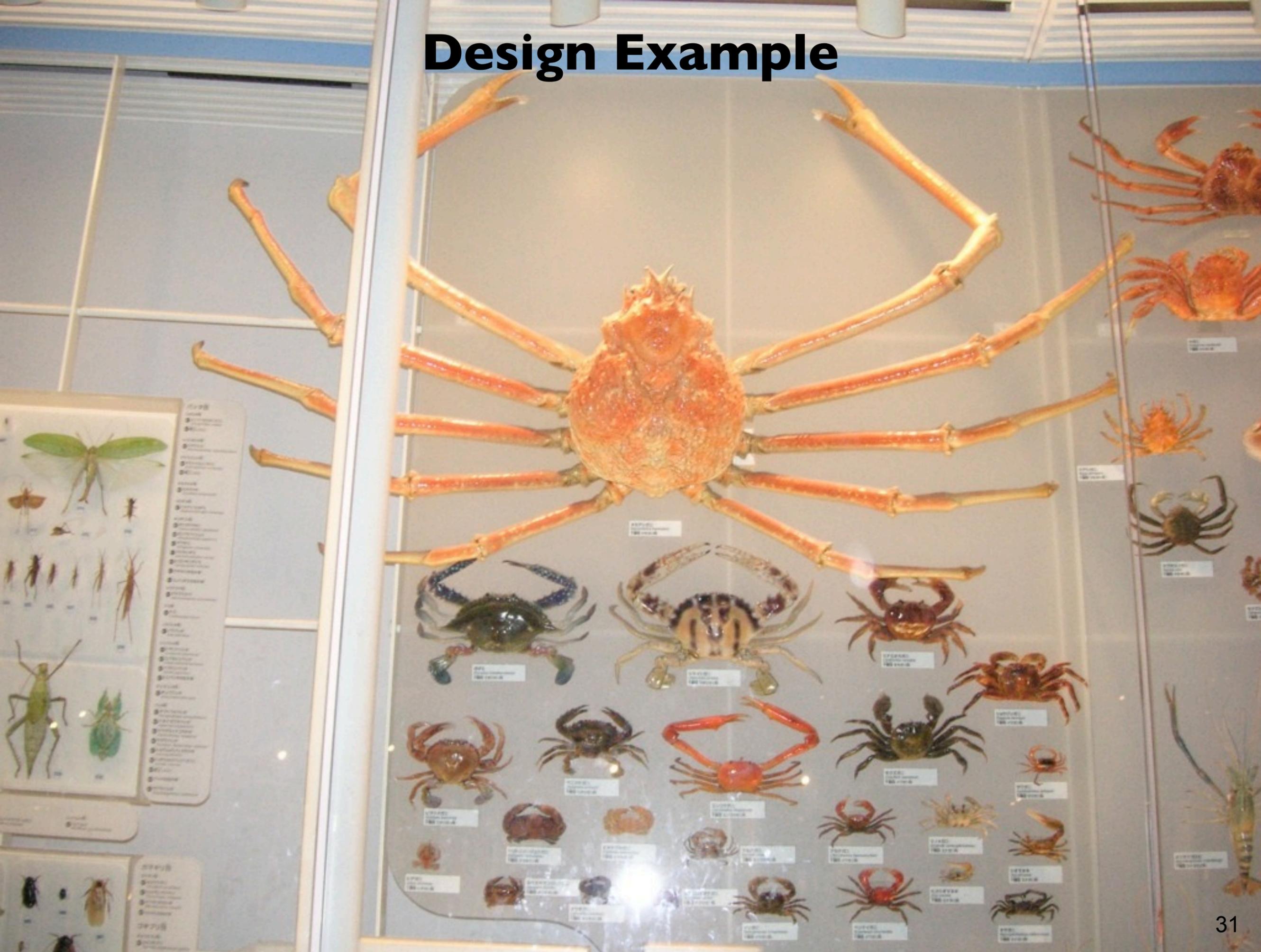
29

In reality, CRP is an idealization  
By definition, locus of all possible hinge creases represents something topologically similar to a CRP  
Can read off tree as before

Courtesy of Jason Ku. Used with permission.



# Design Example



Modeling a crab  
First draw tree (blackboard)

Courtesy of Jason Ku. Used with permission.

# TreeMaker Example

Symmetry (book/diagonal)  
Identifying/fixing unconstrained nodes with local strain  
Triangulation of creasepattern (need three degrees of freedom)  
View Settings

Courtesy of Jason Ku. Used with permission.

# Useful Features in TreeMaker

## Conditions

- axis of symmetry conditions
- force paths to be active or at specific angles
- force nodes to edge/corner/specific locations

## Tree manipulation

- adding local strain (Menu/Action/Scale Selection/)
- triangulation (Menu/Edit/Stub/Triangulate Tree/)

## Views

- Menu/View/Show View Settings/ very useful
- Can view just locus of hinge creases by turning off all but (Creases/Minor Creases) and (Creases/Lines)

# Possible Problems in Optimization

**Problem:** A polygon bounded by active paths is concave

**Solution:** add extra leaf node in interior & expand  
(split polygon into multiple convex polygons)

**Problem:** A polygon bounded by active paths contains an unconstrained node

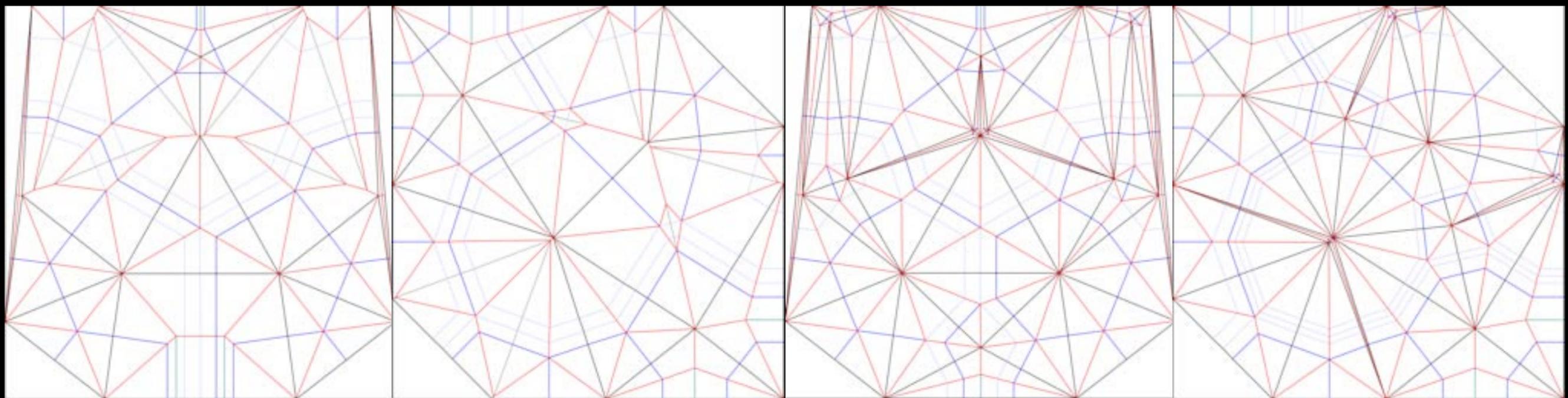
**Solution:** add local strain to interior node to create additional active paths

**Problem:** Optimizer can not find a solution due to trying to optimize under too many constraints

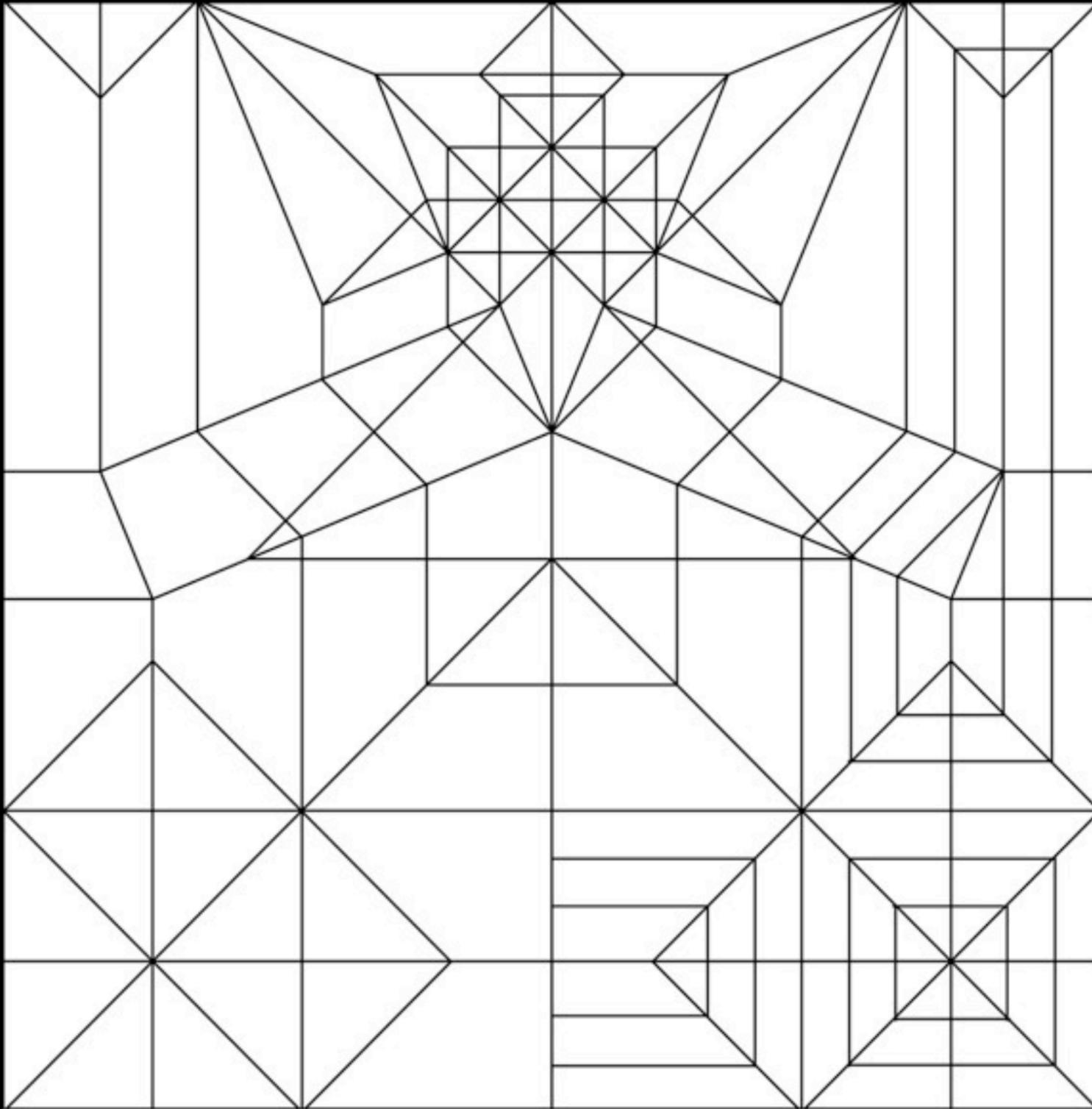
**Solution:** decrease the number of additional constraints

# Example Files

- `crab_book.tmd5` = crab with book symmetry
- `crab_diag.tmd5` = crab with diagonal symmetry
- `crab_book_tri.tmd5` = triangulated version of book
- `crab_diag_tri.tmd5` = triangulated version of diagonal



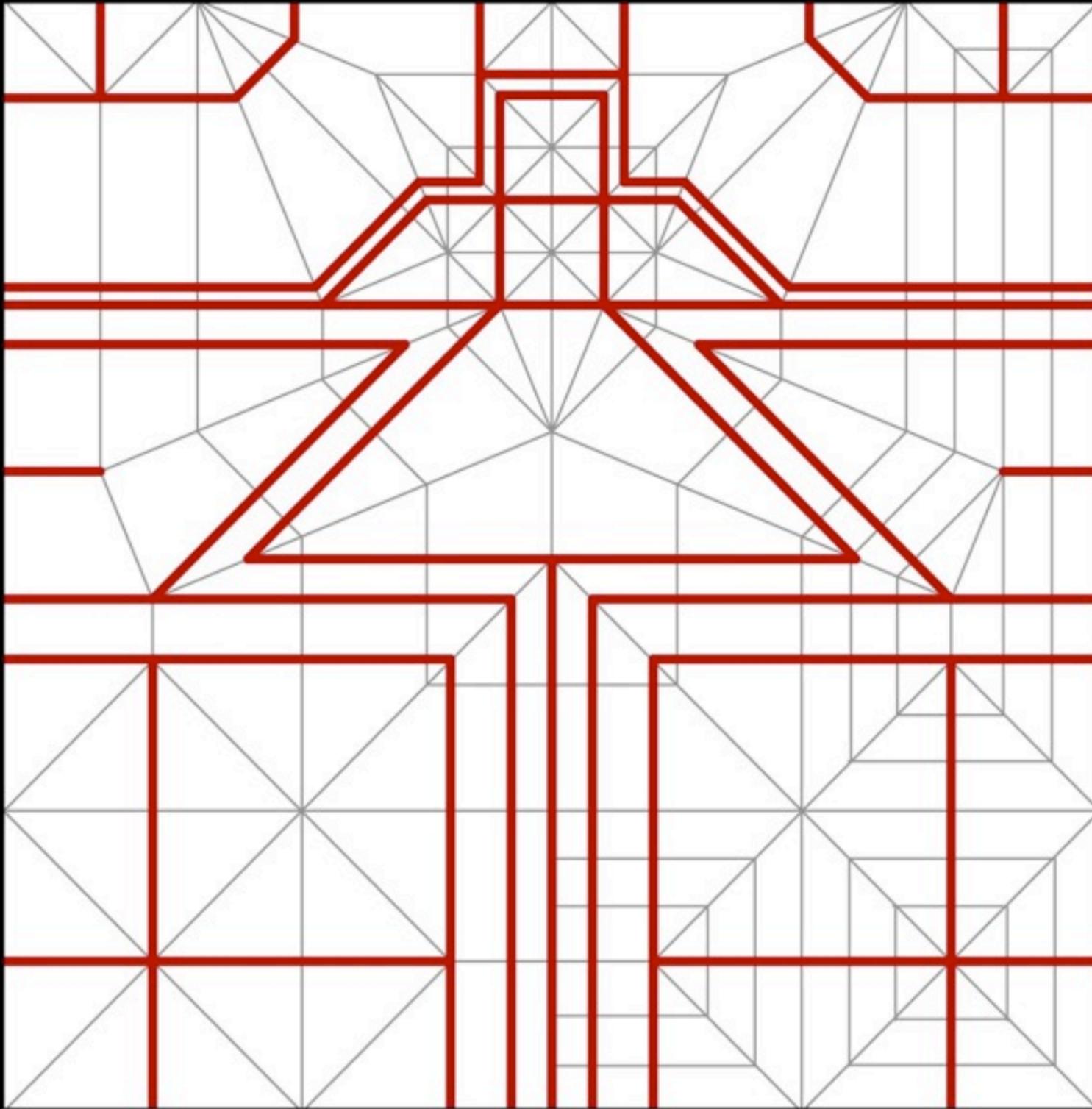
# Non-TreeMaker Example



22.5 degree folding  
Constrained under back geometry  
Taking thickness into account  
Non-uniaxial in ultimate folded form  
Texture

Courtesy of Jason Ku. Used with permission.

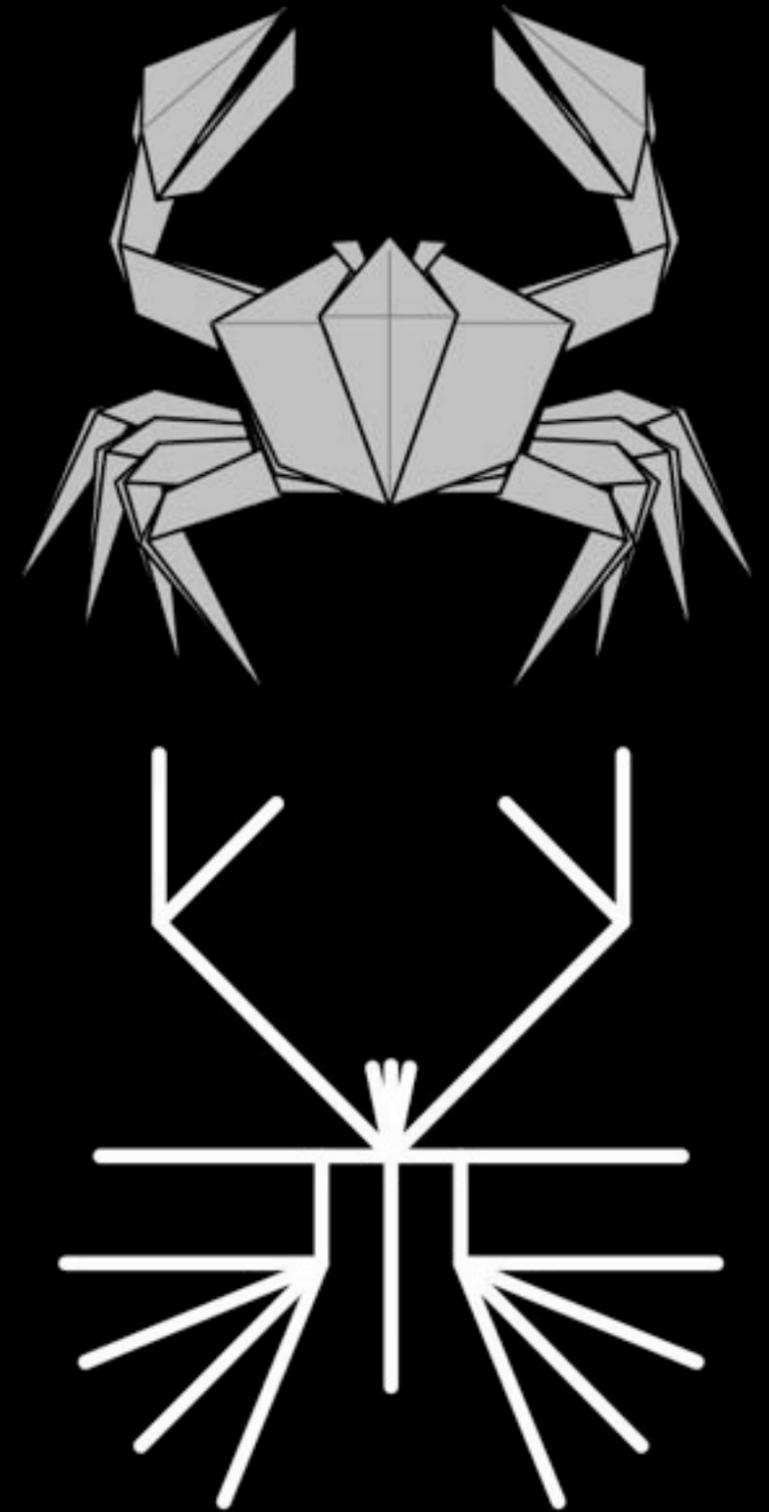
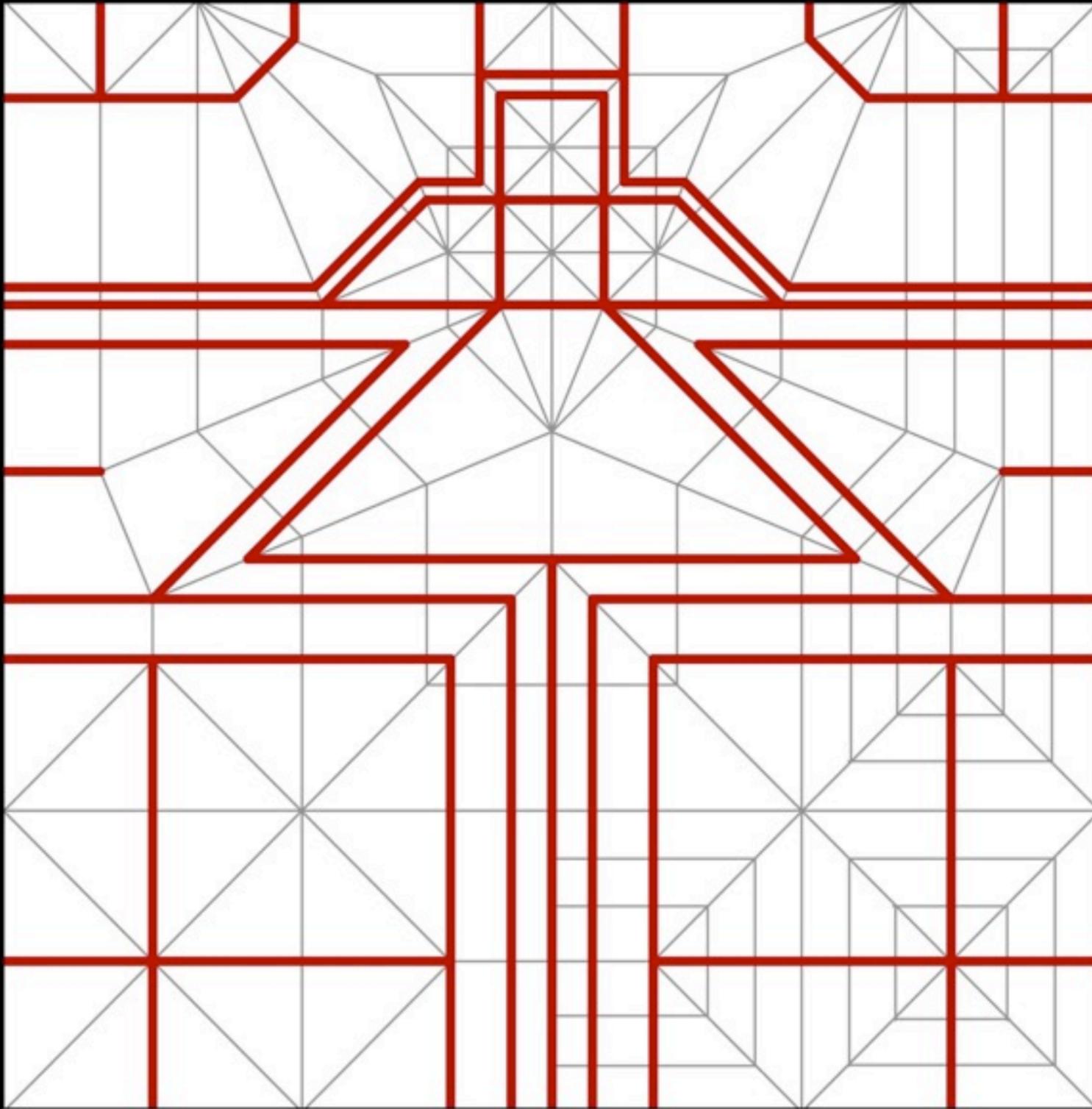
# Non-TreeMaker Example



22.5 degree folding  
Constrained under back geometry  
Taking thickness into account  
Non-uniaxial in ultimate folded form  
Texture

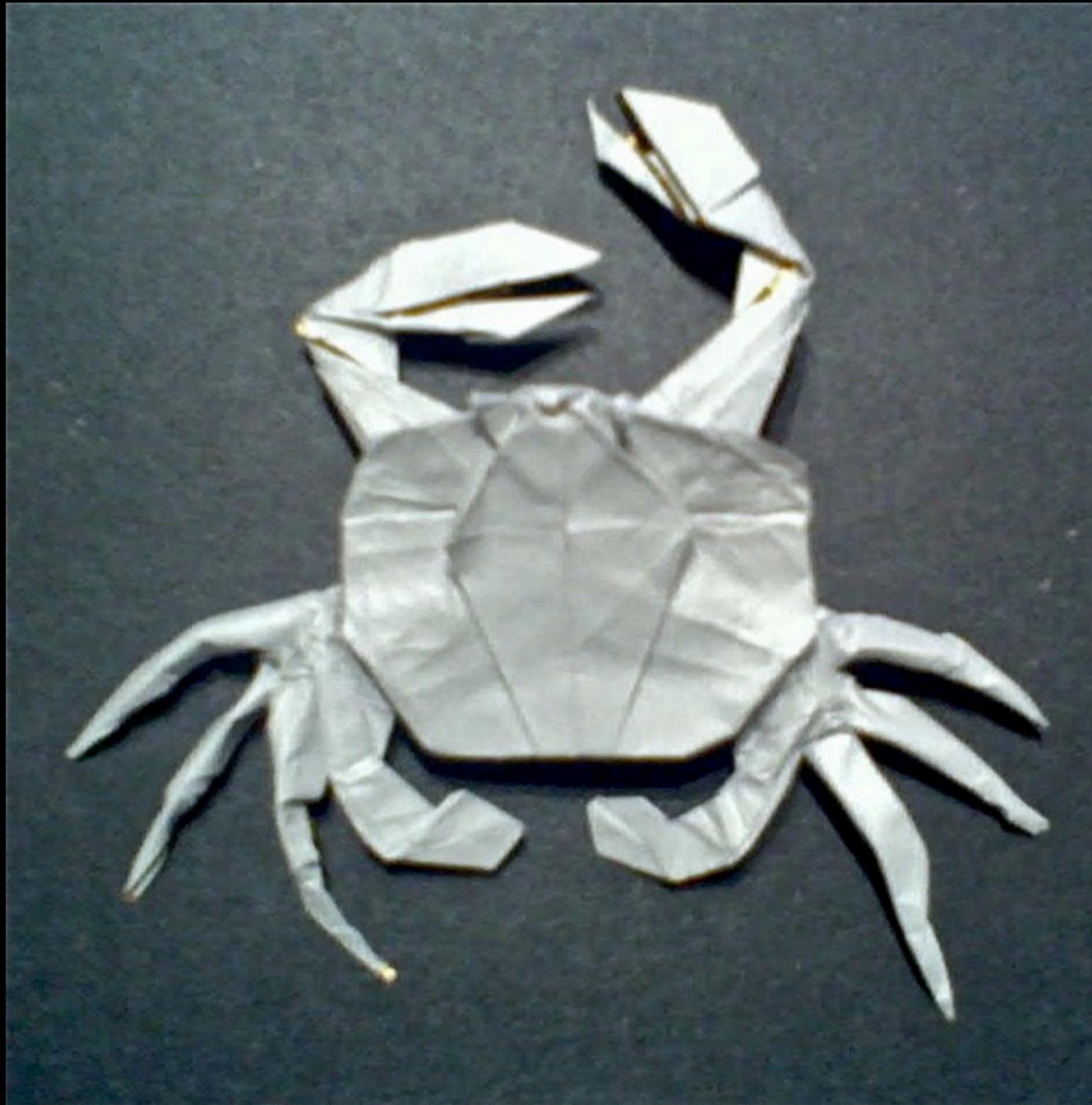
Courtesy of Jason Ku. Used with permission.

# Non-TreeMaker Example



22.5 degree folding  
Constrained under back geometry  
Taking thickness into account  
Non-uniaxial in ultimate folded form  
Texture

# Non-TreeMaker Example



# Origami Forum

The screenshot shows a web browser window titled "The Origami Forum :: Index" with the URL <http://www.thekhans.me.uk/forum/>. The browser's search bar contains "mens et manus". The website has a dark red header with the word "ORIGAMI" in large white letters and the tagline "The Ancient Art of Paperfolding" below it. To the right of the header is a white origami crane. Below the header is a navigation menu with links: LOGOUT, PROFILE, MESSAGES, FAQ, SEARCH, MEMBERLIST, and USERGROUPS. A status bar indicates the user's last visit and current time. The main content area features a table of forum topics with columns for Forum, Topics, Posts, and Last Post.

Forum	Topics	Posts	Last Post
<b>About the Origami Forum</b>			
<b>Using the Forum</b> Everything you need to know about the Origami Forum. Moderator: Moderator-Team	27	174	Fri Aug 27, 2010 10:16 am DavidW
<b>Origami</b>			
<b>Diagrams &amp; Crease Patterns</b> Need help with folding a model? Ask here. Moderator: Moderator-Team	1988	19276	Sun Sep 26, 2010 8:49 pm kamliya
<b>General Origami Talk</b> General discussion about Origami, Papers, Diagramming, ... Moderator: Moderator-Team	1723	19297	Sun Sep 26, 2010 5:34 pm thedeasmellbad
<b>Origami Galleries</b> A forum to exhibit your Origami work. Moderator: Moderator-Team	189	16257	Sun Sep 26, 2010 11:40 pm DavidW
<b>Origami Clubs and Websites</b> Useful Information about Origami Societies, Meetings and Websites. Moderator: Moderator-Team	210	1710	Sun Sep 26, 2010 10:56 am Nick

<http://www.thekhans.me.uk/forum/>

For more information on all things origami...

Courtesy of Jason Ku. Used with permission.



# MIT's Origami Club

Weekly Meetings  
Sundays 2-4pm  
Student Center

<http://origamit.scripts.mit.edu>

MIT OpenCourseWare  
<http://ocw.mit.edu>

6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra  
Fall 2012

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.