

Session 15 (In preparation for Class 15, students are asked to view Lecture 15.)

Topics for Class 15

Polyhedron unfolding: Handles, holes, ridge trees; sun unfolding; zipper unfolding; more ununfoldable polyhedra; NP-completeness of edge unfolding; band unfolding; continuous blooming.

Detailed Description of Class 15

This class covers five types of unfoldings:

- **Sun unfolding:** A new generalization of the source unfolding that also incorporates elements of the star unfolding. It probably doesn't overlap. (Non)overlap of a dual version, which generalizes the star unfolding, remains an open problem.
- **Zipper unfolding:** A new type of unfolding where the cutting forms a path. We have several specific examples and counterexamples of zipper edge unfoldings, but it remains open whether all convex polyhedra have general zipper unfoldings.
- **Edge-ununfoldability:** We show several other edge-ununfoldable examples, the smallest of which has 13 faces, which is conjectured optimal. More generally, deciding whether a topologically convex orthogonal polyhedron has an edge unfolding is NP-complete. Pepakura's brute-force heuristics do decently in practice.
- **Band unfolding:** A brief overview of how the side band of a prismatoid unfolds (even continuously blooms) without overlap. Attaching the top and bottom faces remains an open problem.
- **Continuous blooming:** We cover two ways to continuously unfold any convex polyhedra. First, any unfolding can be refined and then bloomed, by making its dual Hamiltonian and sequentially unrolling. Second, the source unfolding blooms as is, by following a postorder traversal. Many open problems remain.

The class also address a few specific questions:

- What's the formal definition of a handle?
- Why can't unfoldings of convex polyhedra have holes? (**Gauss-Bonnet Theorem**)
- Why are polyhedron vertices leaves of the ridge tree?

Topics for Lecture 15

Polyhedron unfolding: Edge unfoldings, general unfoldings, big questions, curvature, general unfolding of convex polyhedra, source unfolding, star unfolding, edge-unfolding special convex polyhedra, fewest nets, edge-ununfoldable nonconvex polyhedra.

Detailed Description of Lecture 15

This lecture begins our coverage of polyhedron (un)folding. Specifically, we'll talk about how to cut open a polyhedral surface into one piece without overlap so that that piece can be cut out of sheet material and assembled into the 3D surface. What distinguishes this field from origami is that we're not allowed

to cover any part of the surface more than once. Two of the biggest open problems in geometric folding are here:

1. Does every convex polyhedron have an edge unfolding? This is also the oldest problem, implicit back to 1525.
2. Does every polyhedron (without boundary) have a general unfolding? This is one of my favorite open problems.

I'll describe the various attacks and partial results toward solving the first problem, as well as the reverse situations where we know much more: general unfolding of convex polyhedra and edge unfolding of general polyhedra.

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6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra
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