

Session 11 *(In preparation for Class 11, students are asked to view Lecture 11.)*

Topics for Class 11

Rigidity theory: Pebble algorithms, rigid component decomposition, body-and-bar framework, angular rigidity, 5-connected double bananas.

Detailed Description of Class 11

This class focuses on one main question: how exactly and why does that pebble algorithm detect Laman's condition for minimal generic rigidity? We'll start with a simpler version of the algorithm that tests whether every k vertices induce at most $2k$ edges, and then extend to the needed $2k - 3$ condition. Then I'll mention several extensions:

- Decomposing any graph into minimally rigid components and redundant edges
- Body and bar (and hinge) frameworks
- Angular constraints between lines and planes, or between bodies

Finally, we'll see that the tricky 3D double bananas example, and indeed any graph in 3D, can be extended to be 5-connected while preserving Laman and generic flexibility.

Topics for Lecture 11

Rigidity theory: Rigidity, generic rigidity, minimal generic rigidity, Henneberg characterization, Laman characterization, polynomial-time algorithm, convex polyhedra.

Detailed Description of Lecture 11

This lecture is about rigidity theory, which is about telling when a linkage can fold at all. This field goes back to mechanical engineering of the 18th and 19th centuries, with applications to structural engineering and architecture (getting buildings and bridges to stand up), biology (understanding which parts of a protein still move after folding up), and linkage folding itself (beyond just “does it move at all?”, as we'll see in the next lecture).

We'll cover two main theorems characterizing “generically” rigid graphs in 2D. Henneberg's Theorem, from 1911, gives a nice and direct characterization, but it's hard to turn into an algorithm. Laman's Theorem, from 1970, is intuitively harder to work with, but turns into a fast (quadratic-time) algorithm.

Unfortunately, this is all just for 2D, and we don't know any good characterizations for generic rigidity in 3D. I'll briefly describe some nice theorems about the rigidity of convex polyhedra in 3D, though, which in particular explain why Buckminster Fuller's geodesic domes stand up.

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6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra
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