Session 10 (In preparation for Class 10, students are asked to view Lecture 10.)

Topics for Class 10

Folding motions: trouble with holes.

Linkages to sign your name: sliding joints, contraparallelogram bracing, higher dimensions, semi-

algebraic sets, splines.

Geometric construction: straight edge and compass, origami axioms, angle trisection, cube doubling.

Detailed Description of Class 10

This class addresses the following questions:

- Why paper with holes makes it hard to convert folded states to folding motions
- How to simulate sliding joints with regular linkages
- How the contraparallelogram gets braced (and why)
- Kempe Universality Theorem in higher dimensions
- Drawing semi-algebraic sets, e.g., splines, by taking intersections and unions of constructions

In addition, we cover the new topic of geometric construction via origami axioms, mentioned briefly at the end of Lecture 10. Single-fold origami can solve all cubic equations, unlike straight edge and compass which can solve (only) all quadratic equations. Two-fold origami can e.g. quintisect an angle, while *n*-fold origami can solve any polynomial.

We will also assemble our (nonexistent) square hypars from Class 9 and assemble them into a hyparhedron sculpture.

Topics for Lecture 10

Folding motions: folded states vs. motions; universal foldability of polygonal paper. **Linkages to sign your name:** graphs vs. linkages vs. configurations, configuration space; Kempe Universality Theorem, original proof, bug, corrections, generalizations and strengthenings.

<u>Detailed Description of Lecture 10</u>

This lecture kicks off the beginning of linkage folding. We'll segue from origami by thinking about folding motions of pieces of paper, and prove that it's always possible to reach any folded state of a polygonal piece of paper. How many extra creases do we need? This unsolved question leads to problems in the linkage-folding domain.

Then we'll focus on Kempe's Universality Theorem: there's a linkage to sign your name, or more mathematically, trace any polynomial curve. This proof introduces a bunch of cool gadgets, in particular to add angles and multiply (or divide) angles by constant factors.

6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra Fall 2012

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