

Session 9 *(In preparation for Class 9, students are asked to view Lecture 9.)*

Topics for Class 9

Pleat folding: triangulated hypars, smoothness, normals, mathematical vs. real paper, pleat folding algorithms, hypar folding.

Detailed Description of Class 9

This class consists of three parts: review, new material, and real folding.

For review, we cover some open problems about triangulated hypars, define smoothness (C^k), detail why uncreased polygonal regions of paper must be flat in 3D (normal arguments), and discuss how real paper might differ from mathematical paper.

For new material, we cover algorithms for optimally folding specified M/V patterns by a sequence of simple folds and unfolds. Specifically, we'll see that the patterns MMM... and MVMV... can be folded using $O(\lg^2 n)$ folds, and require $\Omega(\lg^2 n / \lg \lg n)$ folds, while most patterns require $\Theta(n / \lg n)$ folds.

For real folding, we will make (nonexistent) square hypars and assemble them into a [hyparhedron sculpture](#).

Topics for Lecture 9

Pleat folding: “Hyperbolic paraboloid”, circular pleat, self-folding origami, plastic deformation and elastic memory; triangulation, interval arithmetic; how paper folds between creases, ruled surfaces, torsal, straight creases stay straight, polygons stay flat; nonexistence.

Inflation: Teabag problem, inflating polyhedra.

Curved creases: Recreating David Huffman's curved-crease folding.

Detailed Description of Lecture 9

This lecture is primarily about pleat folding, specifically the so-called “hyperbolic paraboloid” model common in origami and originating at the Bauhaus in the late 1920s. Surprisingly, this origami model does not exist: it's impossible to fold anything nonflat with a crease pattern consisting of concentric squares and diagonals. We'll prove this, using a new theory for how paper folds in between creases, in particular showing that straight creases stay straight when folded, and interior polygons stay flat when folded. By contrast, we'll show how the “hyperbolic paraboloid” folds fine after adding specific diagonal creases to the trapezoidal faces.

Afterward, we'll briefly look at a couple of related topics: how to maximally inflate a teabag or other polyhedral surface by folding, and what kinds of foldings are possible with curved creases. On the latter topic, I'll overview a big ongoing project in which we're trying to reconstruct the mathematics underlying a beautiful series of sculptures folded by David Huffman from the 1970s–1990s.

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6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra
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