6.849	Lecture 1	Sept. 8, 2010

6.849: Geometric Folding Algorithms

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http://courses.csail.mit.edu/6.849/fall10/

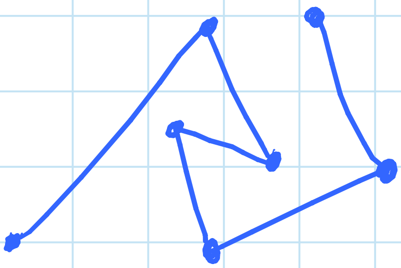
In general: Mathematics & algorithms behind (un)folding of geometric objects

Applications/connections to:

- robotics → arms, Transformers, programmable matter, ...
- graphics → morphing, animation, ...
- mechanics → steam engines, ...
- manufacturing → sheet-metal & tube bending, nanomanufacturing, optics, ...
- medical → stents, drug delivery, ...
- aerospace → telescope deployment, ...
- biology → protein folding & design, ...
- sculpture → origami, interactive sculpture, ...
- architecture → dynamic architecture, deployable/collapsible structures, ...

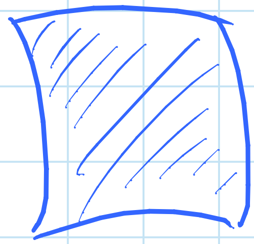
Geometric objects & rules for folding:

(I) linkage



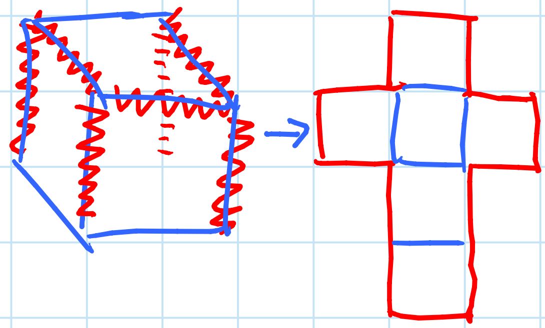
- ↳ rigid bars
- ↳ [don't cross]

(II) paper



- ↳ don't stretch
- ↳ don't tear
- ↳ don't cross

(III) polyhedron



- ↳ cut surface
- ↳ one piece
- ↳ no overlap

Questions:

- What structures can fold at all or in a particular way? — FOLDABILITY
- What shapes (or other properties) can be folded, and how? — DESIGN

Results:

- Everything is foldable! (& here's an algorithm to do it) — UNIVERSALITY
- Efficient algorithm to decide foldability — DECISION
- Computationally intractable to decide foldability — HARDNESS

The Class:

- lectures (mandatory attendence)
- problem sets (not a lot)
- project & presentation
  - implement algorithm/illustration/tool
  - sculpture/design
  - pose an open problem
  - survey a subfield
  - try to solve an open problem
- open problem session (optional)

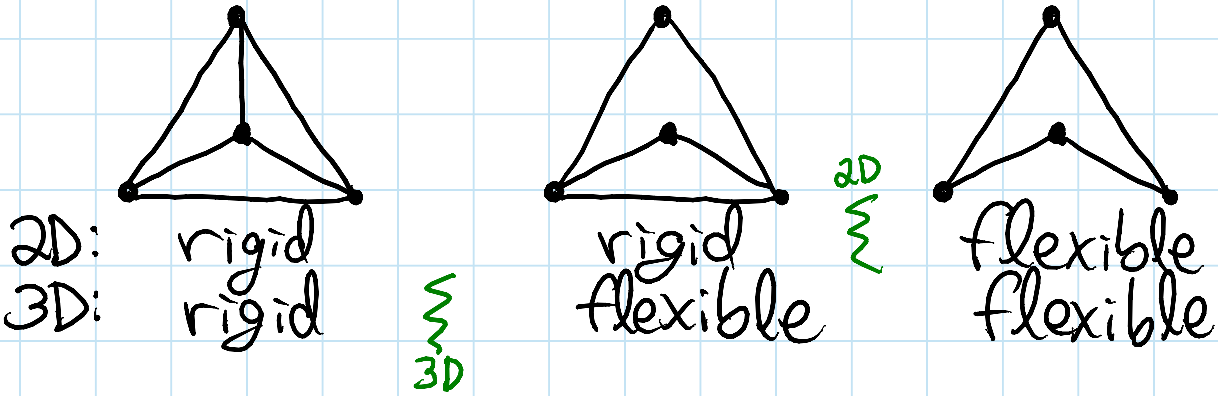
(I) LINKAGES: first, allowing intersection

Early motivation: convert linear motion ⇔ circular motion
(steam engines)
- Watt parallel motion [1784]
- Peaucellier inversor [1864]
- Kempe's *How To Draw a Straight Line* [1877]

Universality: [Kempe 1876; Kapovich & Millson 2002; King 1998; Abbott, Barton, Demaine, O'Rourke]
- there's a linkage signing your name (tracing any polynomial curve)

- OPEN: forbidding crossings?

Rigidity: which linkages fold at all?

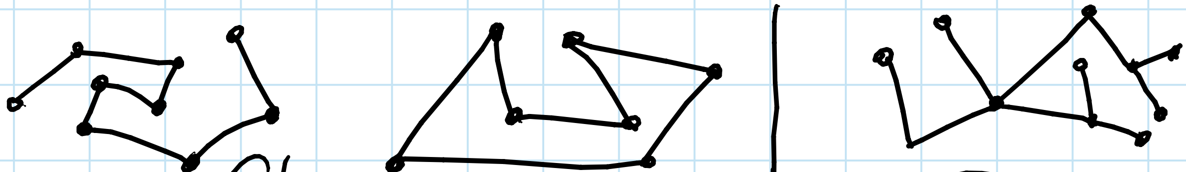


- efficient characterization in 2D
- OPEN: 3D

(I) LINKAGES: forbidding intersection

Reconfiguration: fold from config. A to config. B

- PSPACE-complete in general
  polynomial space – likely exponential time
  [Canny et al.; Alt, Knauer, Rote, Whitesides 2004]
- universality for special linkages:



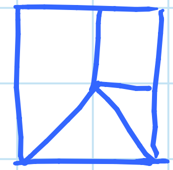
	Chains	Trees
2D	Always [Connelly, Demaine, Rote 2000]	Not [Biedl et al. 1998]
3D	Not [Cantarella & Johnston 1998]	Not
$4D^+$	Always [Cocan & O'Rourke 2001]	Always [ditto]

+ algorithms
[Streinu 2000; Cantarella, Demaine, Iben, O'Brien 2004]

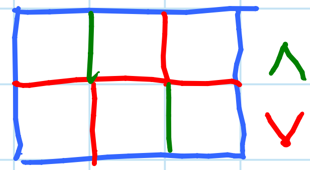
- OPEN: which 3D chains / 2D trees have locked configurations
  (vs. can be folded between all configs.)
  - whereas A → B problem is PSPACE-comp.
- protein folding leads to many cool problems

(II) PAPER:

Foldability: which crease patterns fold flat?

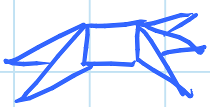


- NP-hard [Bern & Hayes 1996]
- efficient characterization for single vertex [Kawasaki; Justin; Hull 2003]
- OPEN: $2 \times n$ maps



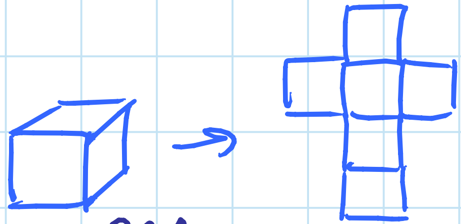
Design: what shapes can be folded?

- any 2D polygon, 3D polyhedron, 2-color pattern (inefficiently) [Demaine, Demaine, Mitchell 2001]
- practically: Origamizer [Tachi & Demaine]
- efficient "stick figures" for origami bases: TreeMaker [Lang 2003; Lang, Demaine, Demaine]
  - NP-hard [Demaine, Fekete, Lang 2010]



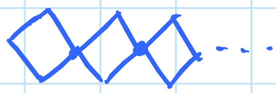
- fold & cut: any set of line segments can be aligned by flat folding [Demaine, Demaine, Lubiw 1998; Bern, Demaine, Eppstein, Hayes 1998]
- curved creases [Huffman; Resch; Demaine, Demaine, Koschitz]
- universal "hinge patterns" (possible creases) [Benbernou, Demaine, Demaine, Ovadya 2010]
  ↪ self-folding sheets [Hawkes, An, Benbernou, Tanaka, Kim, Demaine, Rus, Wood – PNAS 2010]

III Polyhedra:



Unfolding:

- OPEN: edge-unfolding convex polyhedra [Dürer 1525]
  - no conjectured counterexamples [Schlickenrieder 1997; Lucier 2006]
  - every attempted algorithm fails
  - false for triangulated nonconvex [Bern et al. 2001]
- general unfolding (can cut anywhere)
  - possible for convex [Agarwal et al. 1997; Sharir & Schorr 1986]
  - possible for orthogonal [Damian, Flatland, O'Rourke 2006]
  - OPEN in general
- vertex unfolding



  - possible for triangulated [Demaine, Eppstein, Erickson, Hart, O'Rourke 2002]
  - OPEN for convex

Folding: glue polygon boundary to make convex polyhed.

- exponential algorithm to list all gluings (there can be that many) [Demaine, Demaine, Lubiw, O'Rourke 2002]
- poly. time for bounded sharpness
- OPEN: poly. time decision / 1 shape? [Lubiw & O'Rourke 1998]
  - possible for "edge-to-edge" gluing
- reconstructing resulting 3D shapes [Bobenko & Izmestiev 2006; Kane, Price, Demaine 2009]

(IV) HINGED DISSECTIONS

- any finite set of polygons of same area can be folded from one chain of polygons (without collision)
  [Abbott, Abel, Charlton, Demaine, Demaine, Kominers 2008]