6.837 Introduction to Computer Graphics
Quiz 1

Thursday October 19, 2006 2:40-4pm
One sheet of notes (2 pages) allowed

Name:

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1 Transformations [/10]

1.1 Linearity [/3]

What does it mean for a transformation or an operator to be linear? [**/3**]

1.2 Homogeneous coordinates and IFS [/7]

Consider the 2D IFS (Iterated Function System) defined in 2D by

$$A = \cup f_i(A)$$

That is, this fractal is the set of points A that is equal to the union of its transformed versions by the transformations f_i. The f_i are described by the following matrices in homogeneous coordinates

$$f_0 = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad f_1 = \begin{pmatrix} 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad f_2 = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{pmatrix}$$

Explain the effect of each of the three transformations (e.g. translation by something followed by a rotation by another thing). [**/4**]

What is the resulting fractal? The name is not necessary, you can roughly draw it. Advice: draw the first few iterations starting with the unit square from $(0, 0)$ to $(1, 1)$. [**/3**]

2 Curves and surfaces [/22]

In class, we have focused on cubic Bézier splines. However, one can similarly define quadratic Bézier splines using the Bernstein polynomials:

$$B_1(t) = (1-t)^2 \quad B_2(t) = 2t(1-t) \quad B_3(t) = t^2$$

How many control points do we need for a quadratic Bézier spline? [**/1**]

Prove that the weights defined by these basis functions always sum to one. [**/4**]

Why is it critical for splines that the weights sum to one? [**/4**]

Does the curve approximate or interpolate the control points? The answer can be different for the various points... [**/3**]

What is the derivative (tangent) at the two extremities as a function of the control point? [**/4**]

What does it mean geometrically? That is, how is the geometric tangent at the extremities related to the control points? [**/3**]

How many control points do we need for a tensor-product quadratic Bézier patch? [**/3**]

3 Animation [/18]

3.1 Particles [/10]

Consider a simplified 1D version of the spring equation: $\frac{d^2x}{dt^2} = -kx$ where x is a scalar function. The initials conditions are $x(0) = d$ and $\frac{dx}{dt}(0) = 0$.

What is the rest length of this spring? [**/1**]

Describe the system after one step of Euler integration with time step h (that is, give the values of x and $\frac{dx}{dt}(0) = 0$, which you can note v). [**/3**]

Describe the system after two step of Euler integration with time step h. [**/3**]

For which value of h does the length of x increase after two iterations compared to the initial length? That is, when do we have $|x_2| > |x_0|$, where x_i is the value after i iterations. [**/3**]

3.2 Quaternion [/8]

Let q_1 and q_2 be two unit quaternions. Prove that $(q_1 q_2)^* = q_2^* q_1^*$.

First, prove this using quaternion algebra. Recall that $(d; \vec{u})^* = (d; \vec{-u})$ and $(d, \vec{u})(d', \vec{u}') = (dd' - \vec{u}.\vec{u}'; d\vec{u}' + d'\vec{u} + \vec{u} \times \vec{u}')$. [**/5**]

Second, give a geometric or matrix argument. [**/3**]

4 EXTRA CREDIT

4.1 Easy extra credit

Consider a general multivariable linear first-order ODE of the form $\frac{dX}{dt} = MX$ where X is an n-dimensional vector and M is an $n \times n$ matrix.

Derive the implicit Euler integration for this case. That is, express $X(t + h)$.

4.2 Harder extra credit

What limits the stability of the method, that is, how does the maximum stable time step h relate to properties of the matrix?

4.3 Even more fun extra credit

Now consider a general first-order multivariate ODE of the type $\frac{dX}{dt} = f(X)$ where f is an arbitrary smooth function. How do you adapt the above implicit integration scheme to this situation?

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