Particle Systems and ODEs

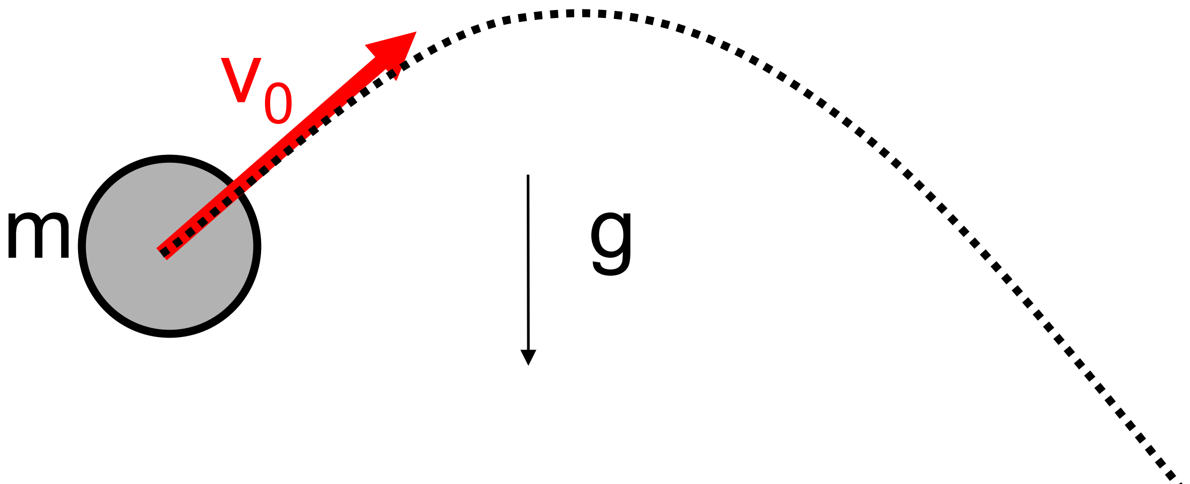
Image removed due to copyright restrictions.

Types of Animation

- Keyframing
- Procedural
- Physically-based
  - Particle Systems: **TODAY**
    - Smoke, water, fire, sparks, etc.
    - Usually heuristic as opposed to simulation, but not always
    - Mass-Spring Models (Cloth) **NEXT CLASS**
  - Continuum Mechanics (fluids, etc.), finite elements
    - Not in this class
  - Rigid body simulation
    - Not in this class

Types of Animation: Physically-Based

- Assign physical properties to objects
  - Masses, forces, etc.
- Also procedural forces (like wind)
- Simulate physics by solving equations of motion
  - Rigid bodies, fluids, plastic deformation, etc.
- Realistic but difficult to control



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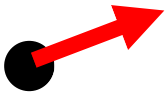
Types of Dynamics

- Point



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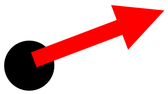
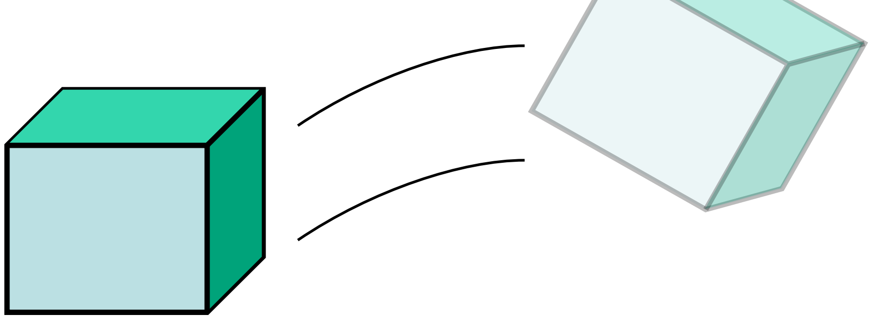


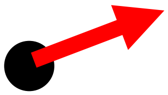
- Rigid body



Animation by Mark Carlson

Types of Dynamics

- Point



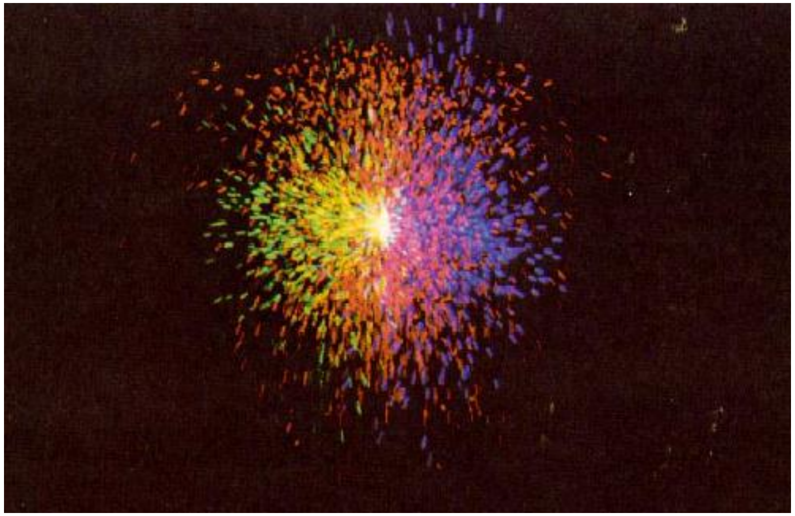
- Rigid body

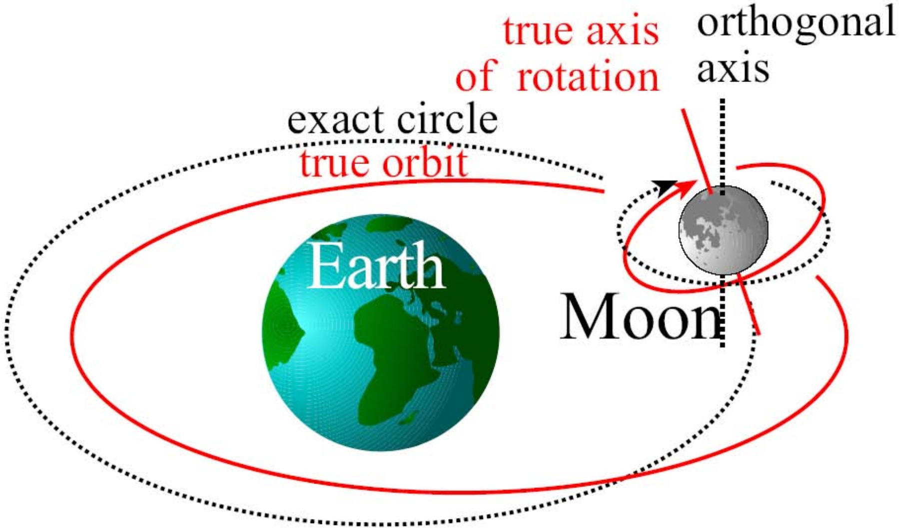
- Deformable body
  (include clothes, fluids, smoke, etc.)

Mark Carlson

Today We Focus on Point Dynamics

- Lots of points!
- Particles systems
  – Borderline between procedural and physically-based





Particle Systems Overview

- **Emitters** generate tons of “particles”
  - Sprinkler, waterfall, chimney, gun muzzle, exhaust pipe, etc.

Images of particle systems removed due to copyright restrictions.

http://www.particlesystems.org/

Particle Systems Overview

- **Emitters** generate tons of "particles"
- Describe the external **forces** with a force field
  - E.g., gravity, wind

http://www.particlesystems.org/

Particle Systems Overview

- **Emitters** generate tons of "particles"
- Describe the external **forces** with a force field
- **Integrate** the laws of mechanics (ODEs)
  - Makes the particles move

Images of particle systems removed due to copyright restrictions.

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Particle Systems Overview

- **Emitters** generate tons of “particles”
- Describe the external **forces** with a force field
- **Integrate** the laws of mechanics (ODEs)
- In the simplest case, each particle is **independent**

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Particle Systems Overview

- **Emitters** generate tons of “particles”
- Describe the external **forces** with a force field
- **Integrate** the laws of mechanics (ODEs)
- In the simplest case, each particle is **independent**
- If there is enough **randomness** (in particular at the emitter) you get nice effects
  - sand, dust, smoke, sparks, flame, water, …

Images of particle systems removed due to copyright restrictions.

http://www.particlesystems.org/

Sprinkler

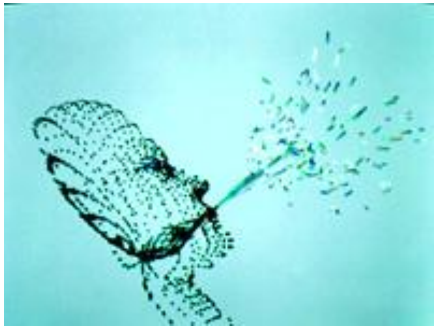
- http://www.youtube.com/watch?v=rhvH12nC6_Q

Fire

- http://www.youtube.com/watch?v=6hG00etwRBU

Generalizations

- More advanced versions of behavior
  - flocks, crowds
- Forces between particles
  - Not independent any more



See http://www.red3d.com/cwr/boids/ for discussion on how to do flocking.

Image removed due to copyright restrictions.

We'll come back to this a little later.

http://www.blendernation.com/2008/01/05/simulating-flocks-herds-and-swarms-using-experimental-blender-boids-particles/

Generalizations – Next Class

- Mass-spring and deformable surface dynamics
  - surface represented as a set of points
  - forces between neighbors keep the surface coherent

Image Witkin & Baraff

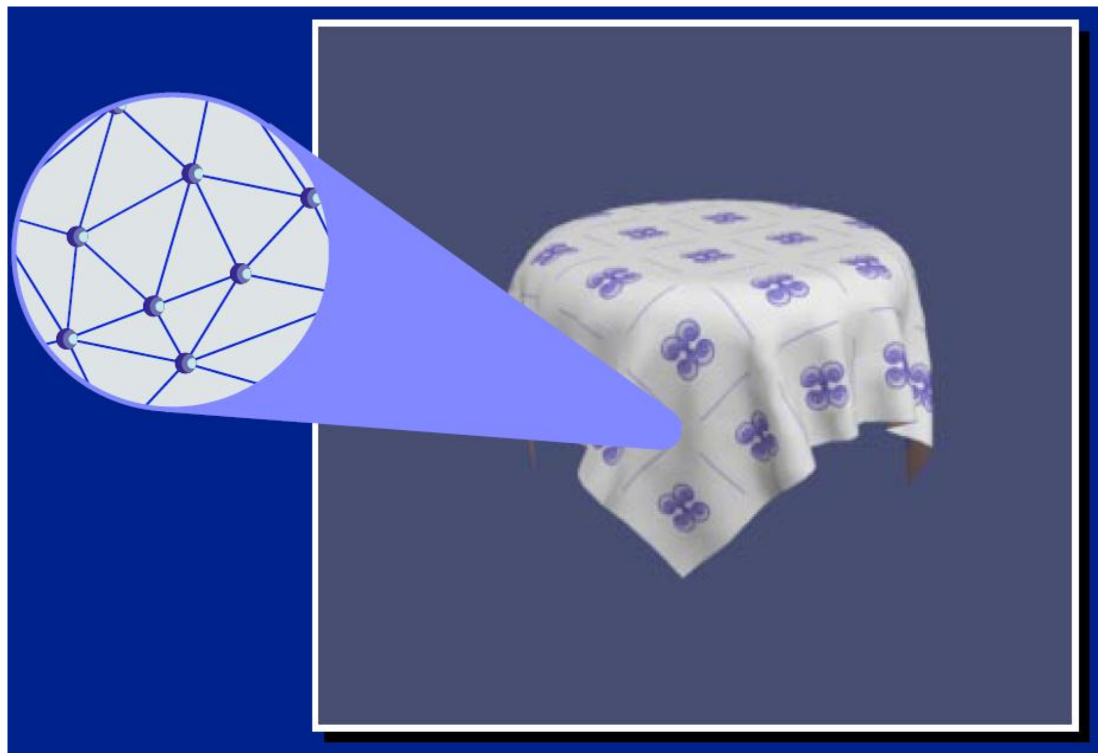


Image Michael Kass

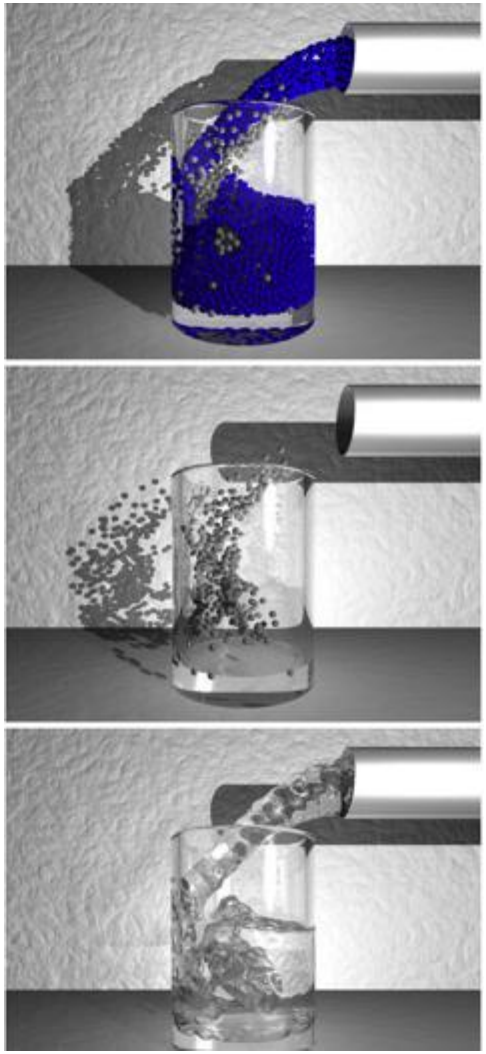
Cloth Video

Selle. A, Su, J., Irving, G. and Fedkiw, R., "Robust High-Resolution Cloth Using Parallelism, History-Based Collisions, and Accurate Friction," IEEE TVCG 15, 339-350 (2009).

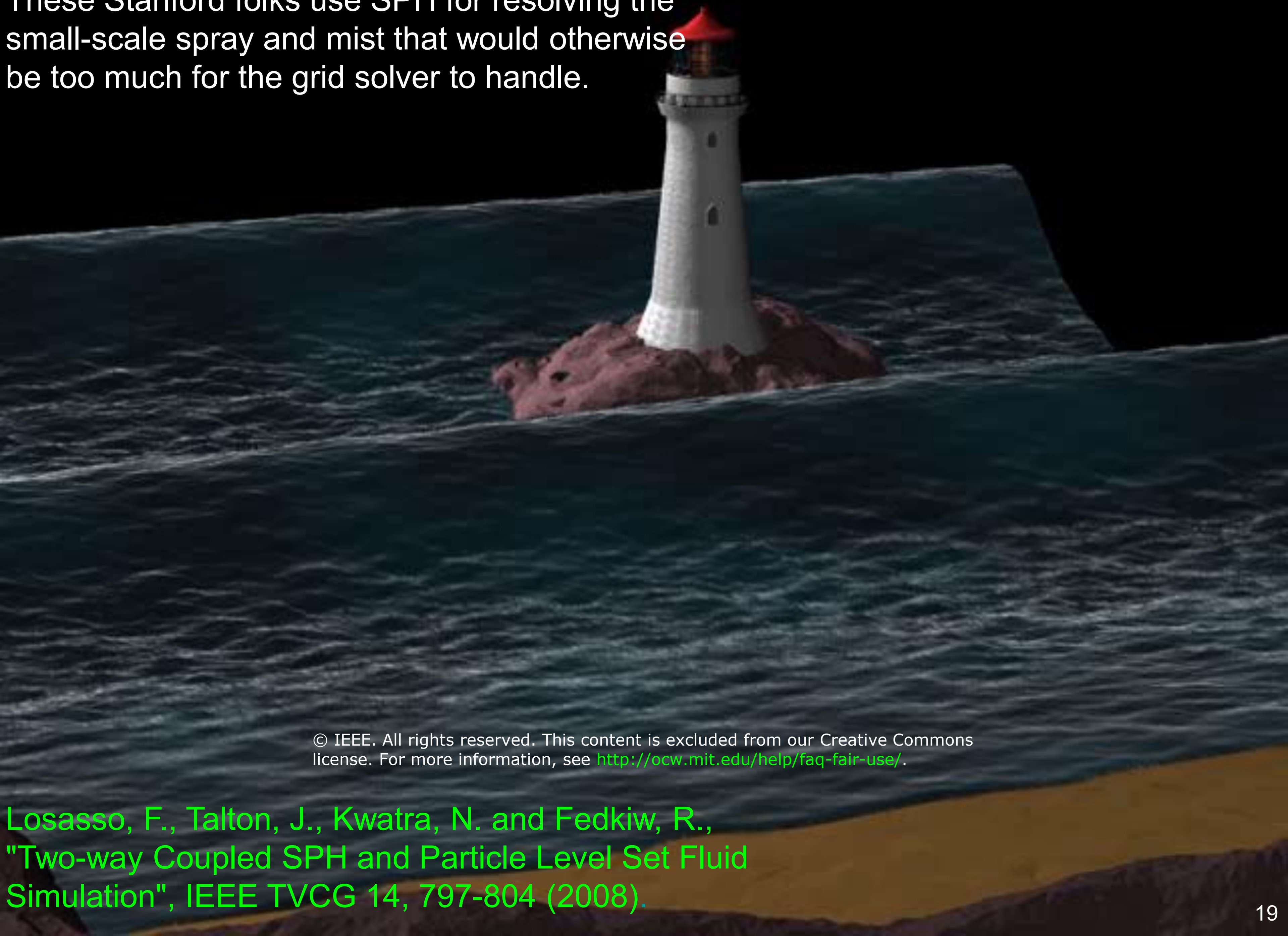
Generalizations

Müller et al. 2005

- It's not all hacks: Smoothed Particle Hydrodynamics (SPH)
  - A family of "real" particle-based fluid simulation techniques.
  - Fluid flow is described by the Navier-Stokes Equations, a nonlinear partial differential equation (PDE)
    - SPH discretizes the fluid as small packets (particles!), and evaluates pressures and forces based on them.



These Stanford folks use SPH for resolving the small-scale spray and mist that would otherwise be too much for the grid solver to handle.



Losasso, F., Talton, J., Kwatra, N. and Fedkiw, R., "Two-way Coupled SPH and Particle Level Set Fluid Simulation", IEEE TVCG 14, 797-804 (2008).

Real-time particles in games

- http://www.youtube.com/watch?v=6DicVajK2xQ

EA Fight Night 4 Physics Trailer

Take-Home Message

- Particle-based methods can range from pure heuristics (hacks that happen to look good) to “real” simulation

Andrew Selle et al.

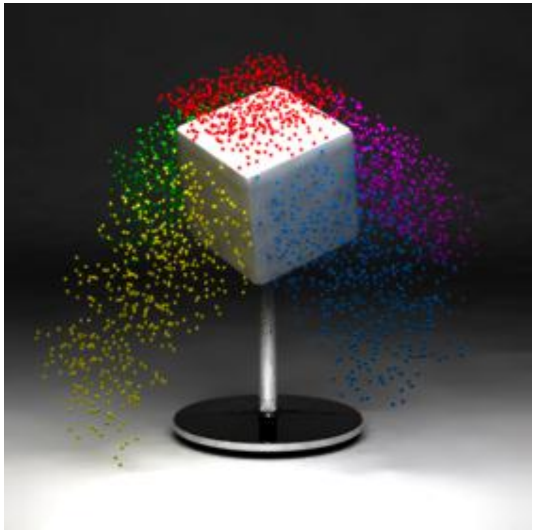
- Basics are the same: **Things always boil down to integrating ODEs!**
  - Also in the case of grids/computational meshes

Questions?

http://www.cs.columbia.edu/cg/ESIC/esic.html

What is a Particle System?

- Collection of many small simple pointlike things
  - Described by their current state: position, velocity, age, color, etc.
- Particle motion influenced by external force fields and internal forces between particles
- Particles created by **generators** or **emitters**
  - With some randomness
- Particles often have lifetimes
- Particles are often independent
- Treat as points for dynamics, but rendered as anything you want



Simple Particle System: Sprinkler

PL: linked list of particle = empty;

Image by Jeff Lander removed due to copyright restrictions.

Image Jeff Lander

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spread=0.1;*//how random the initial velocity is*

colorSpread=0.1; *//how random the colors are*

Image Jeff Lander

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For each time step

Image Jeff Lander

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PL: linked list of particle = empty;
spread=0.1;//how random the initial velocity is
colorSpread=0.1; //how random the colors are
For each time step
   Generate k particles
      p=new particle();
      p->position=(0,0,0);
      p->velocity=(0,0,1)+spread*(rnd(), rnd(), rnd());
      p.color=(0,0,1)+colorSpread*(rnd(), rnd(),rnd());
      PL->add(p);
```


Image Jeff Lander

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        p.color=(0,0,1)+colorSpread*(rnd(), rnd(),rnd());
        PL->add(p);
    For each particle p in PL
        p->position+=p->velocity*dt; //dt: time step
        p->velocity-=g*dt; //g: gravitation constant
        glColor(p.color);
        glVertex(p.position);
```


Image Jeff Lander

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Image Jeff Lander

Demo with Processing

- http://processing.org/learning/topics/simpleparticlesystem.html

Questions?

Image removed due to copyright restrictions.

Path forward

- Basic particle systems are simple hacks
- Extend to physical simulations, e.g., clothes
- For this, we need to understand numerical integration
- This lecture: point particles
- Next lecture: mass-spring and clothes

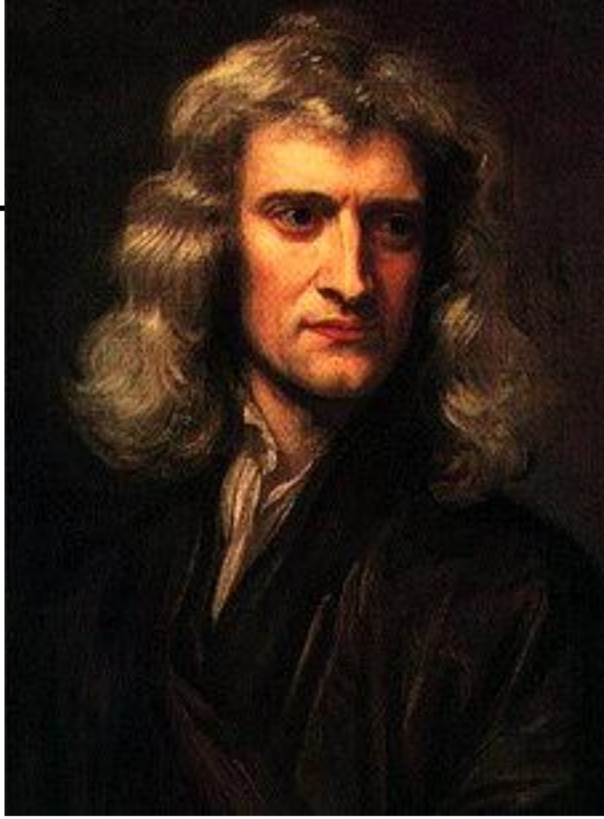
Ordinary Differential Equations

$$\frac{d\,\mathbf{X}(t)}{dt} = f(\mathbf{X}(t), t)$$

- Given a function $f(\mathbf{X},t)$ compute $\mathbf{X}(t)$
- Typically, *initial value problems*:
  - Given values $\mathbf{X}(t_0)=\mathbf{X}_0$
  - Find values $\mathbf{X}(t)$ for $t > t_0$

- We can use lots of standard tools

Newtonian Mechanics



- Point mass: 2nd order ODE

$$\vec{F} = m\vec{a} \quad \text{or} \quad \vec{\boldsymbol{F}} = m\frac{d^2\vec{\boldsymbol{x}}}{dt^2}$$

- Position ***x*** and force ***F*** are vector quantities
  - We know ***F*** and *m*, want to solve for ***x***
- You have all seen this a million times before

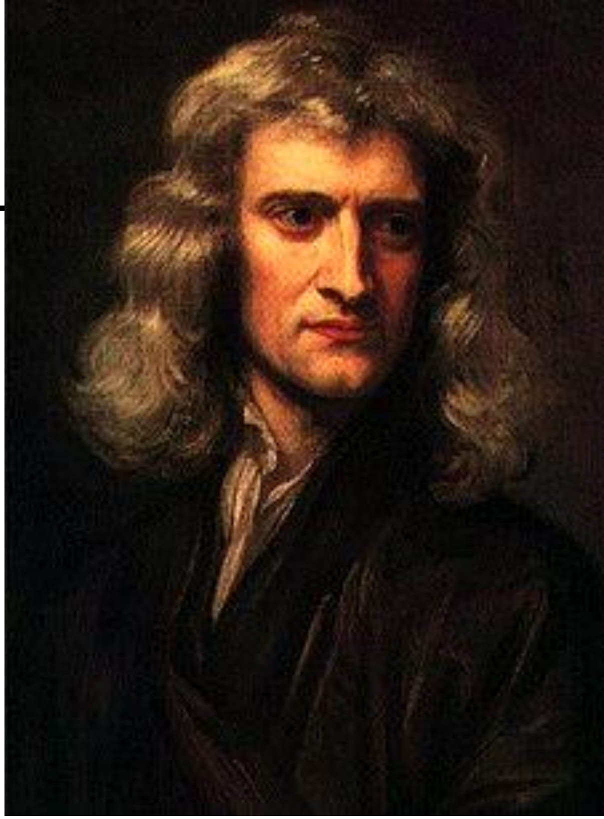
Reduction to 1st Order

- Point mass: 2nd order ODE

$$\vec{F} = m\vec{a} \quad \text{or} \quad \vec{F} = m\frac{d^2\vec{x}}{dt^2}$$

- Corresponds to system of first order ODEs

$$\begin{cases} \frac{d}{dt}\vec{x} = \vec{v} \\ \frac{d}{dt}\vec{v} = \vec{F}/m \end{cases}$$



2 unknowns (**x**, **v**) instead of just **x**

Reduction to 1st Order

$$\begin{cases} \frac{d}{dt}\vec{\boldsymbol{x}} = \vec{\boldsymbol{v}} \\ \frac{d}{dt}\vec{\boldsymbol{v}} = \vec{\boldsymbol{F}}/m \end{cases}$$

2 variables (**x**, **v**)
instead of just one

- Why reduce?

Reduction to 1st Order

$$\begin{cases} \frac{d}{dt}\vec{\boldsymbol{x}} = \vec{\boldsymbol{v}} \\ \frac{d}{dt}\vec{\boldsymbol{v}} = \vec{\boldsymbol{F}}/m \end{cases}$$

2 variables (**x**, **v**)
instead of just one

- Why reduce?
  - Numerical solvers grow more complicated with increasing order, can just write one 1st order solver and use it
  - Note that this doesn't mean it would always be easy :-)

Notation

- Let's stack the pair (**x**, **v**) into a bigger state vector **X**

$$\boldsymbol{X} = \begin{pmatrix} \vec{\boldsymbol{x}} \\ \vec{\boldsymbol{v}} \end{pmatrix}$$

For a particle in 3D, state vector **X** has 6 numbers

$$\frac{d}{dt}\boldsymbol{X} = f(\boldsymbol{X}, t) = \begin{pmatrix} \vec{\boldsymbol{v}} \\ \vec{\boldsymbol{F}}(x, v)/m \end{pmatrix}$$

Now, Many Particles

- We have N point masses
  - Let's just stack all **x**s and **v**s in a big vector of length 6N

$$\boldsymbol{X} = \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{v}_1 \\ \vdots \\ \boldsymbol{x}_N \\ \boldsymbol{v}_N \end{pmatrix} \qquad f(\boldsymbol{X}, t) = \begin{pmatrix} \boldsymbol{v}_1 \\ \boldsymbol{F}^1(\boldsymbol{X}, t) \\ \vdots \\ \boldsymbol{v}_N \\ \boldsymbol{F}^N(\boldsymbol{X}, t) \end{pmatrix}$$

Now, Many Particles

- We have N point masses
  - Let's just stack all **x**s and **v**s in a big vector of length 6N
  - $\mathbf{F}^i$ denotes the force on particle *i*
    - When particles don't interact, $\mathbf{F}^i$ only depends on $\mathbf{x}_i$ and $\mathbf{v}_i$.

$$\boldsymbol{X} = \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{v}_1 \\ \vdots \\ \boldsymbol{x}_N \\ \boldsymbol{v}_N \end{pmatrix} \qquad f(\boldsymbol{X}, t) = \begin{pmatrix} \boldsymbol{v}_1 \\ \boldsymbol{F}^1(\boldsymbol{X}, t) \\ \vdots \\ \boldsymbol{v}_N \\ \boldsymbol{F}^N(\boldsymbol{X}, t) \end{pmatrix}$$

***f* gives d/dt X, remember!**

Path through a Vector Field

- ***X(t)***: path in multidimensional <u>phase space</u>

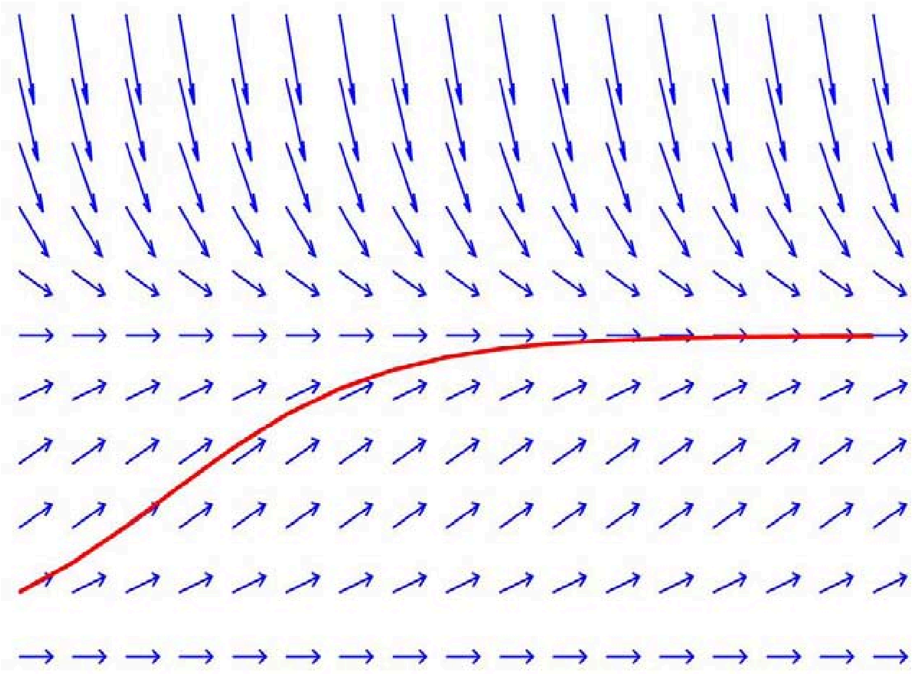


Image by MIT OpenCourseWare.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = f(\boldsymbol{X}, t)$$

"When we are at state **X** at time *t*, where will **X** be after an infinitely small time interval d*t* ?"

Path through a Vector Field

- ***X****(t)*: path in multidimensional <u>phase space</u>

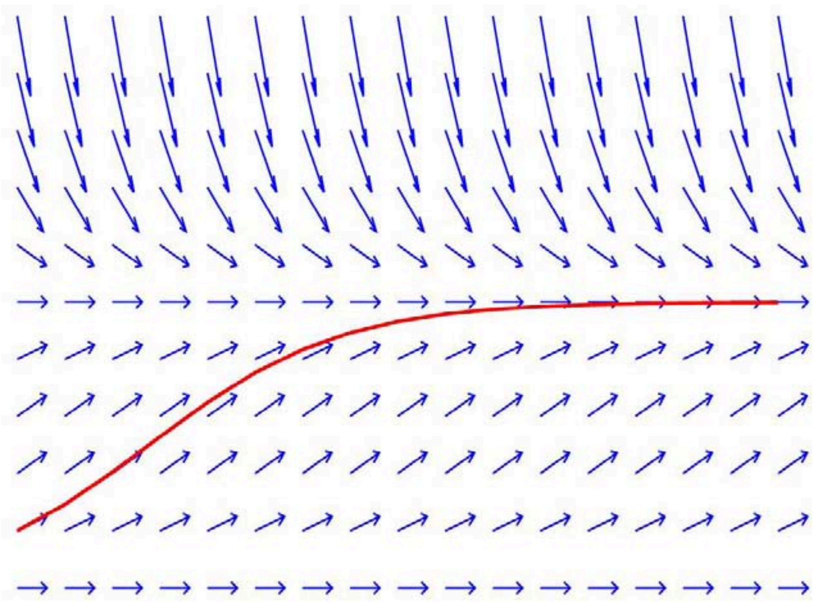


Image by MIT OpenCourseWare.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = f(\boldsymbol{X}, t)$$

"When we are at state **X** at time *t*, where will **X** be after an infinitely small time interval d*t* ?"

- *f*=d/d*t* ***X*** is a vector that sits at each point in phase space, pointing the direction.

Questions?

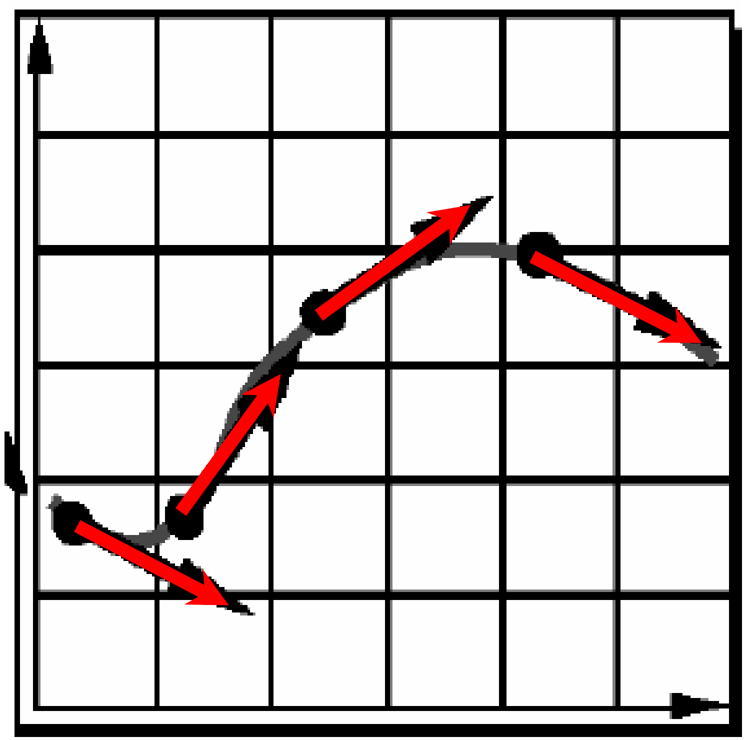
http://vimeo.com/14597952

Numerics of ODEs

- Numerical solution is called “integration of the ODE”
- Many techniques
  - Today, the simplest one
  - Thursday and next week we’ll look at some more advanced techniques

Intuitive Solution: Take Steps

- Current state **X**
- Examine f(**X**,t) at (or near) current state
- Take a step to new value of **X**



$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = f(\boldsymbol{X}, t)$$

$$\Rightarrow \text{“}\mathrm{d}\boldsymbol{X} = \mathrm{d}t\, f(\boldsymbol{X}, t)\text{”}$$

f = d/d*t* ***X*** is a vector that sits at each point in phase space, pointing the direction.

Euler's Method

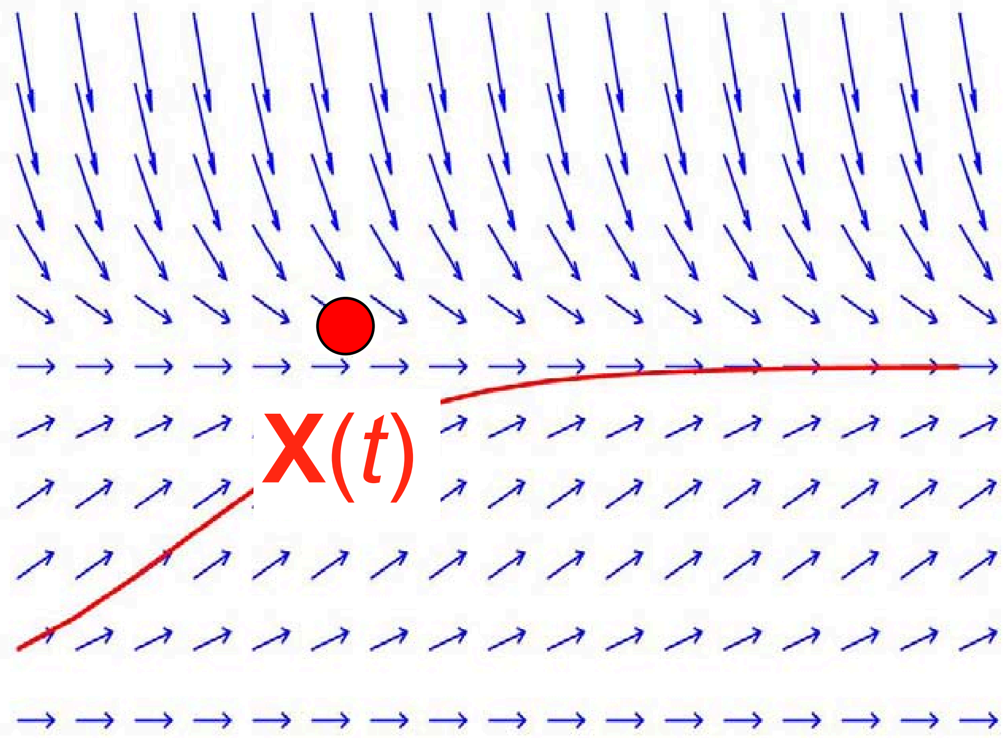
- Simplest and most intuitive
- Pick a **step size** *h*
- Given $\mathbf{X}_0=\mathbf{X}(t_0)$, take step:

$$t_1 = t_0 + h$$
$$\mathbf{X}_1 = \mathbf{X}_0 + h\,f(\mathbf{X}_0, t_0)$$

- Piecewise-linear approximation to the path
- **Basically, just replace d*t* by a small but finite number**

Euler, Visually

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = f(\boldsymbol{X}, t)$$



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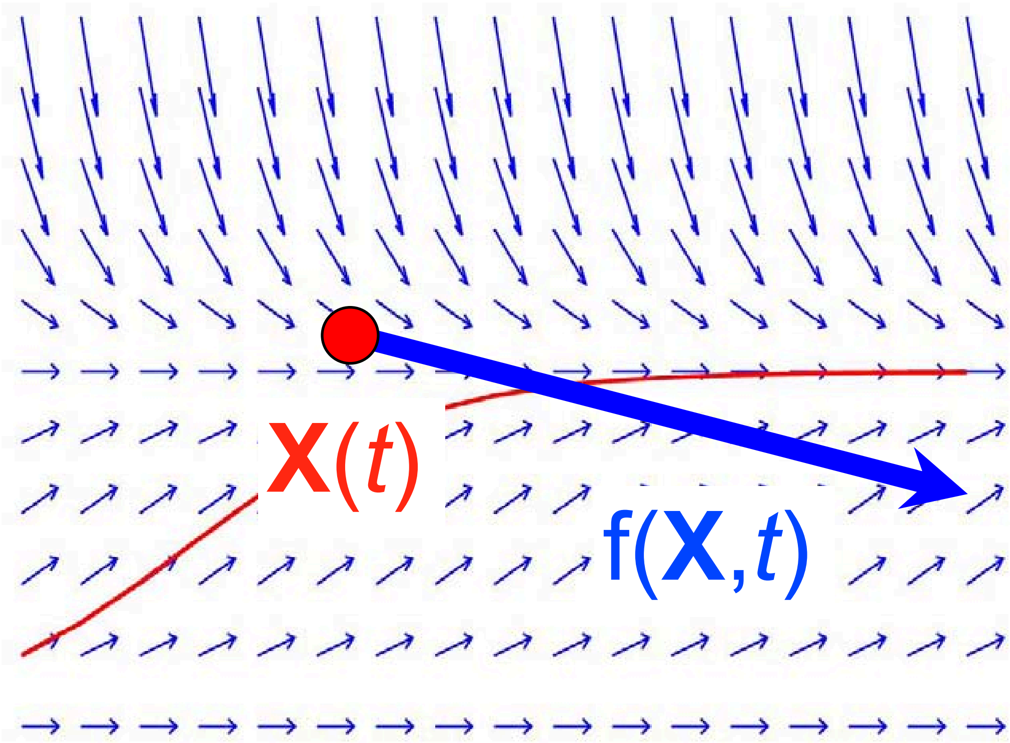

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Euler, Visually

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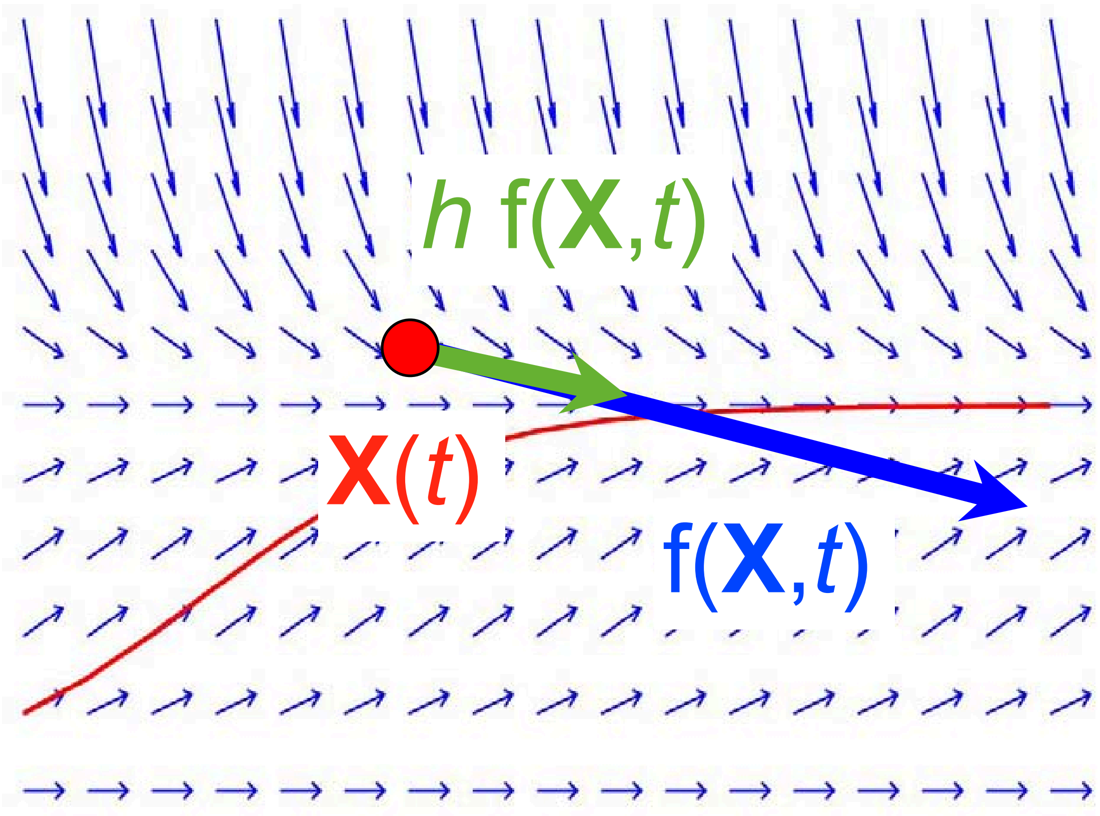

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Euler, Visually

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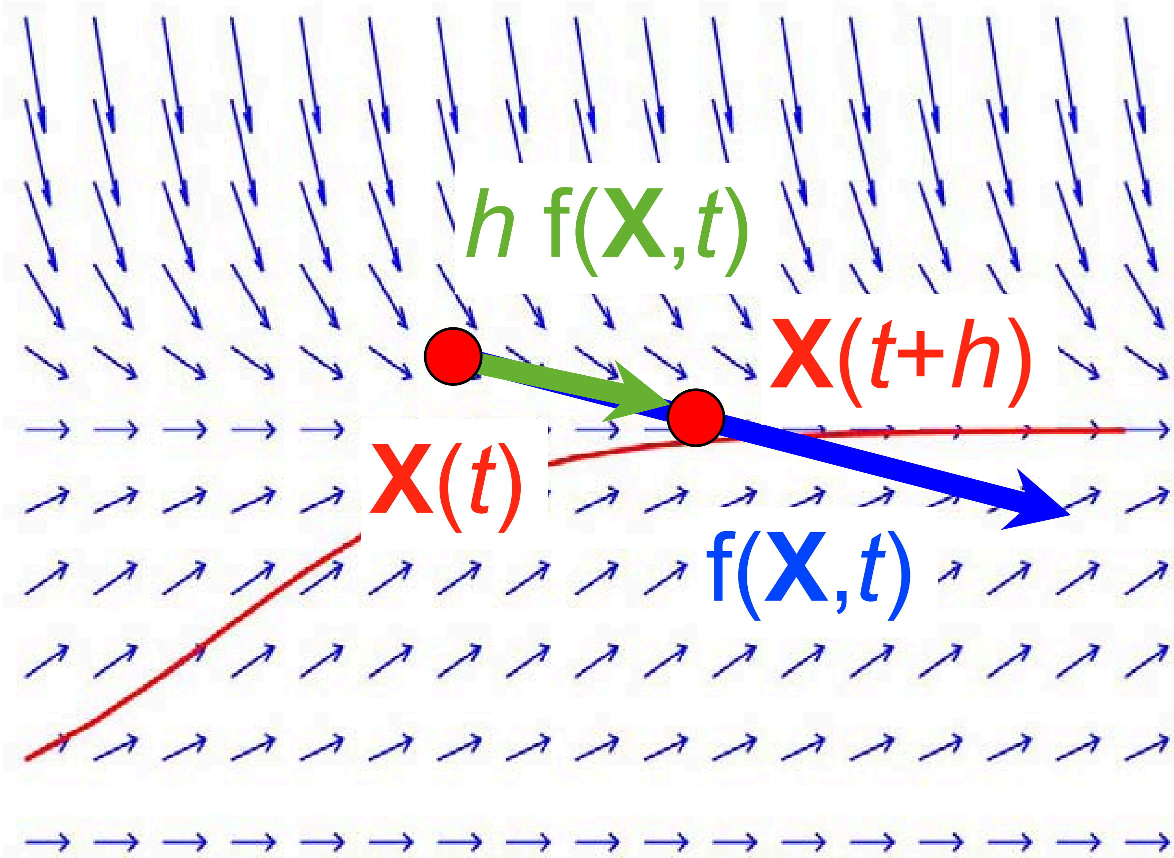

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Questions?

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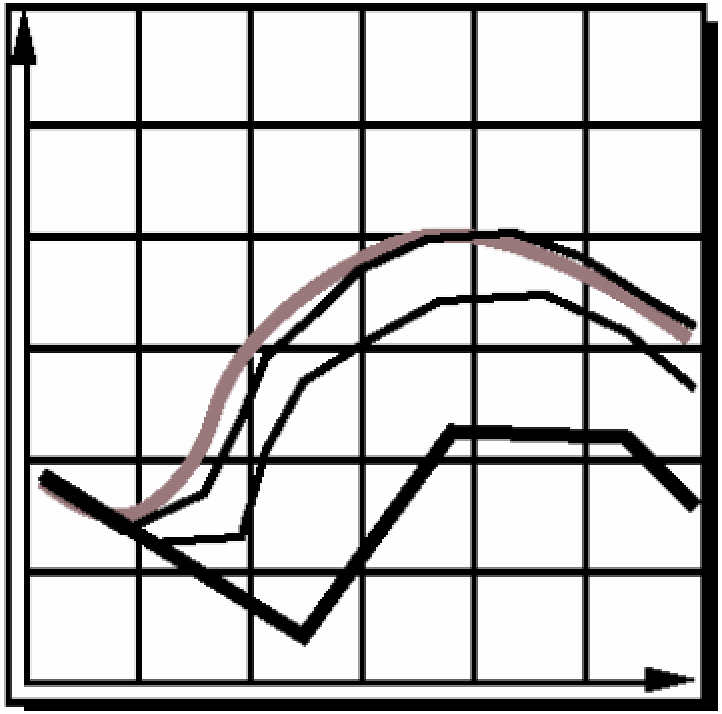
Effect of Step Size

- Step size controls accuracy
- Smaller steps more closely follow curve
  - May need to take many small steps per frame
  - Properties of $f(\mathbf{X}, t)$ determine this (more later)



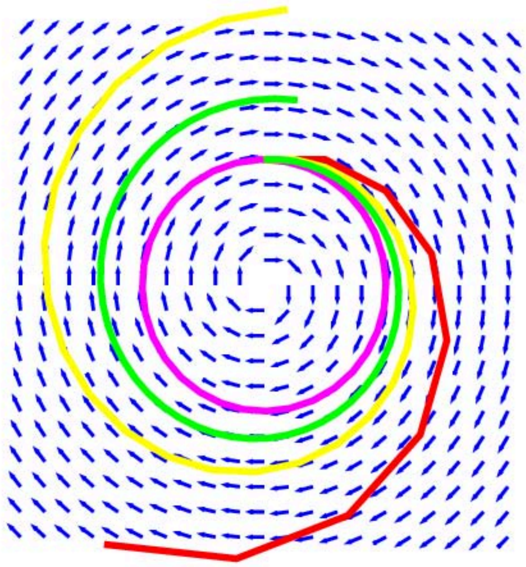
Euler's method: Inaccurate

- Moves along tangent; can leave solution curve, e.g.:

$$f(\mathbf{X},t) = \begin{pmatrix} -y \\ x \end{pmatrix}$$

- Exact solution is circle:

$$\mathbf{X}(t) = \begin{pmatrix} r\cos(t+k) \\ r\sin(t+k) \end{pmatrix}$$



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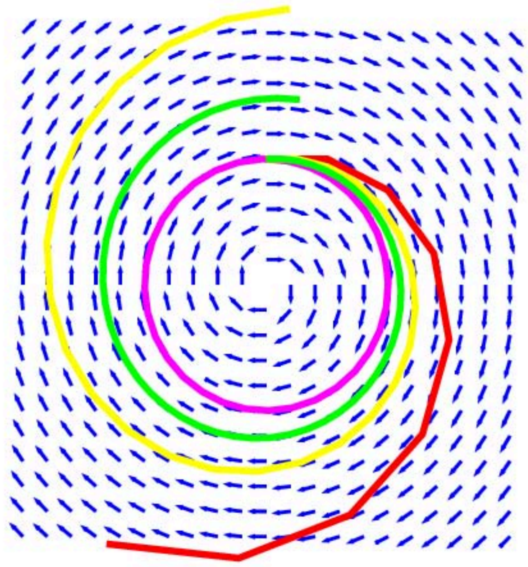
- Euler spirals outward
  no matter how small h is
  – will just diverge more slowly



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More Accurate Alternatives

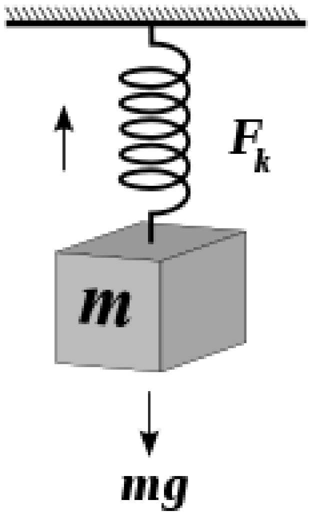
- Midpoint, Trapezoid, Runge-Kutta
  - Also, “implicit methods” (next week)

More on this during next class

- Extremely valuable resource: SIGGRAPH 2001 course notes on physically based modeling

What is a Force?

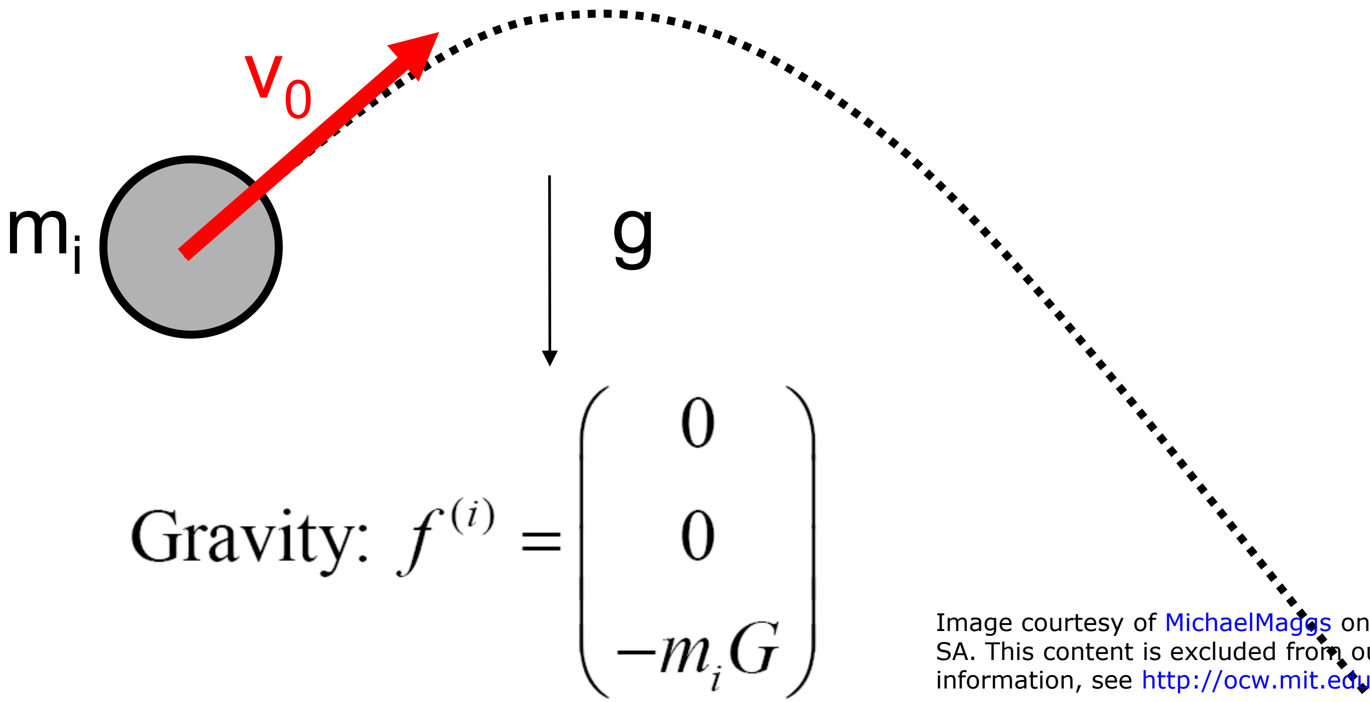
- A force changes the motion of the system
  - Newton says: When there are no forces, motion continues uniformly in a straight line (good enough for us)
- Forces can depend on location, time, velocity
  - Gravity, spring, viscosity, wind, etc.
- For point masses, forces are vectors
  - Ie., to get total force, take vector sum of everything



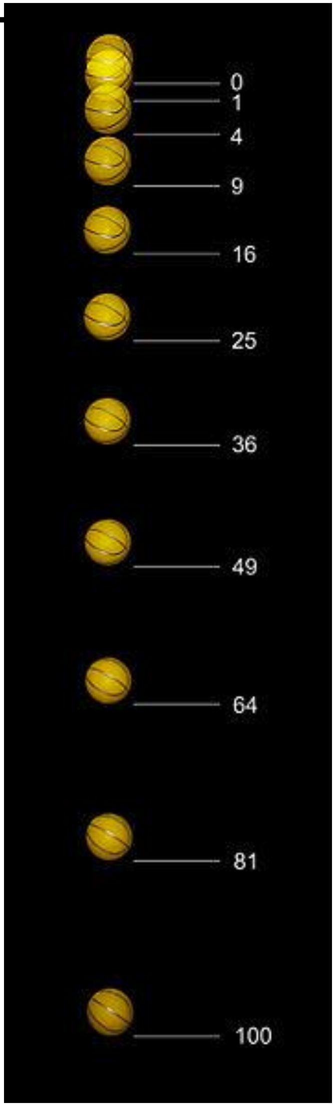
Wikipedia

Forces: Gravity on Earth

- Depends only on particle mass
- $f(\mathbf{X},t)$ = constant
- Hack for smoke, etc: make gravity point up!
  - Well, you can call this buoyancy, too.



$$\text{Gravity: } f^{(i)} = \begin{pmatrix} 0 \\ 0 \\ -m_i G \end{pmatrix}$$

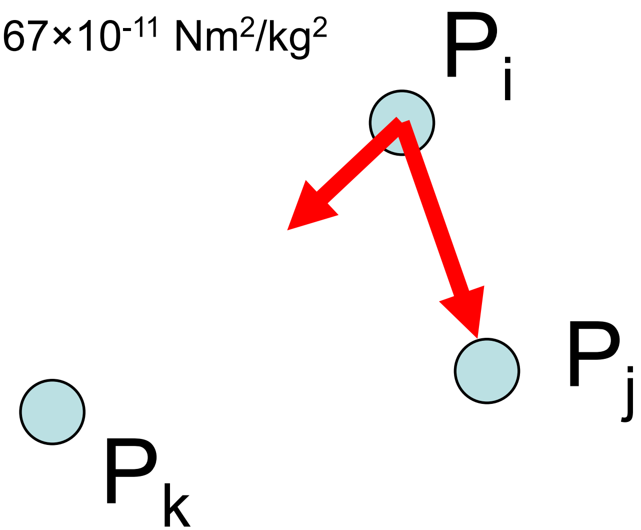


Forces: Gravity (N-body problem)

- Gravity depends on all other particles
- Opposite for pairs of particles
- Force in the direction of p_i-p_j with magnitude inversely proportional to square distance

$$\|\boldsymbol{F}_{ij}\| = \frac{G\, m_i\, m_j}{r^2}$$

where $G=6.67\times10^{-11}\ Nm^2/kg^2$



- Testing all pairs is $O(n^2)$!

Particles are not independent!

Real-Time Gravity Demo

http://www.youtube.com/watch?v=uhTuJZiAG64

NVIDIA

An Aside on Gravity

- That was Brute Force
  - Meaning all O(n^2) pairs of particles were considered when computing forces
  - Yes, computers are fast these days, but this gets prohibitively expensive soon. (The square in n^2 wins.)

- *Hierarchical techniques* approximate forces caused by many distant attractors by one force, yields O(n)!
  - "Fast Multipole Method", Greengard and Rokhlin, J Comput Phys 73, p. 325 (1987)
  - This inspired very cool hierarchical illumination rendering algorithms in graphics (hierarchical radiosity, etc.)

Forces: Viscous Damping

$$f^{(i)} = -dv^{(i)}$$

- Damping force on particle i determined its velocity
  - Opposes motion
  - E.g. wind resistance
- Removes energy, so system can settle
- Small amount of damping can stabilize solver
- Too much damping makes motion like in glue

Forces: Spatial Fields

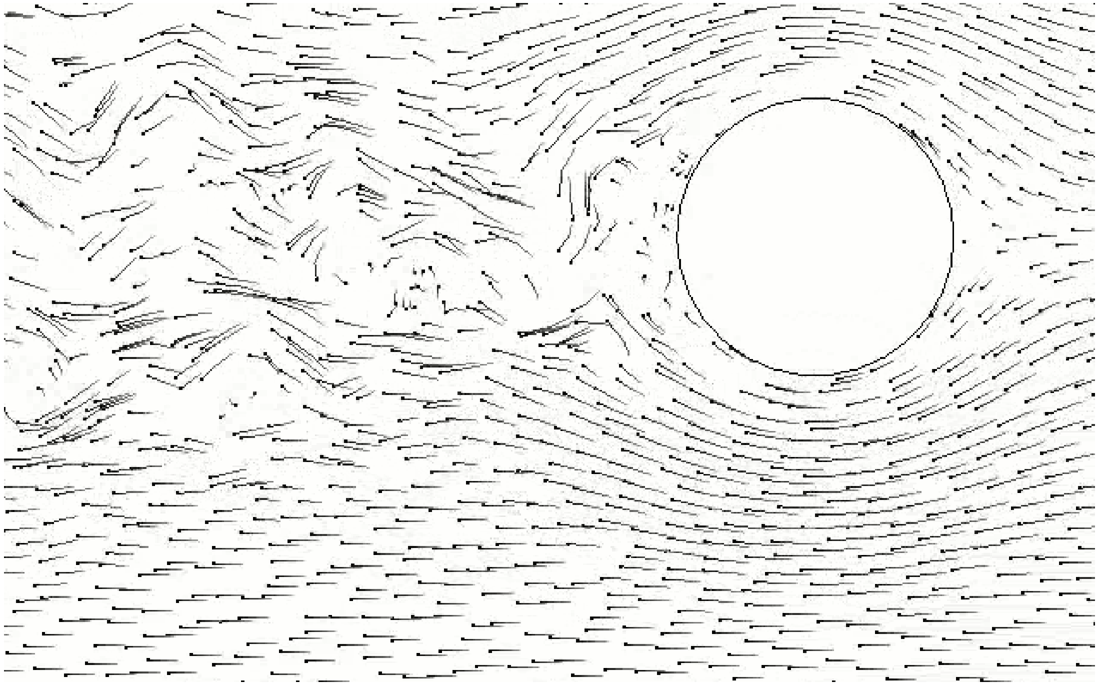
- Externally specified force (or velocity) fields in space
- Force on particle i depends only on its position
- Arbitrary functions
  - wind
  - attractors, repulsors
  - vortices
- Can depend on time
- Note: these add energy, may need damping

Processing demo

- http://processing.org/learning/topics/smokeparticlesystem.html

Example: Procedural Spatial Field

- Curl noise for procedural fluid flow, R. Bridson, J. Hourihan, and M. Nordenstam, Proc. ACM SIGGRAPH 2007.



Plausible, conrollable force fields – just advecting particles along the flow gives cool results!

And it's simple, too!

Curl-Noise for Procedural Fluid Flow

Robert Bridson

Jim Hourihan

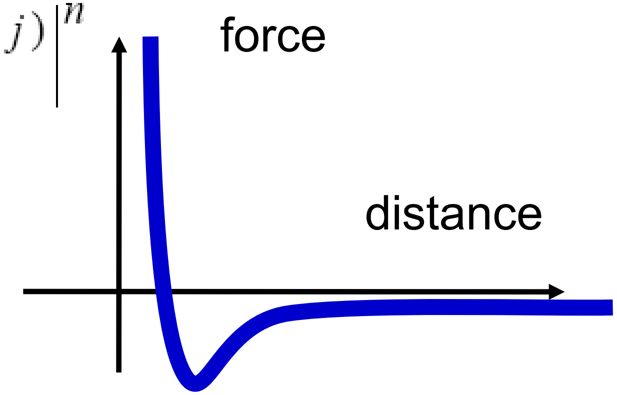
Markus Nordenstam

Forces: Other Spatial Interaction

$$\text{Spatial interaction: } f^{(i)} = \sum_j f(x^{(i)}, x^{(j)})$$

- E.g., approximate fluid using Lennard-Jones force:

$$f(x^{(i)}, x^{(j)}) = \frac{k_1}{\left|x^{(i)} - x^{(j)}\right|^m} - \frac{k_2}{\left|x^{(i)} - x^{(j)}\right|^n}$$



- Repulsive + attractive force
- Again, $O(N^2)$ to test all pairs
  - usually only local
  - Use buckets to optimize. Cf. 6.839

Particles are not independent!

http://www.youtube.com/watch?v=nI7makIgYnI&feature=related

Lennard-Jones forces

http://www.youtube.com/watch?v=XfjYlKxKIWQ&feature=autoplay&list=PL0605C44C6E8D5EDB&lf=autoplay&playnext=2

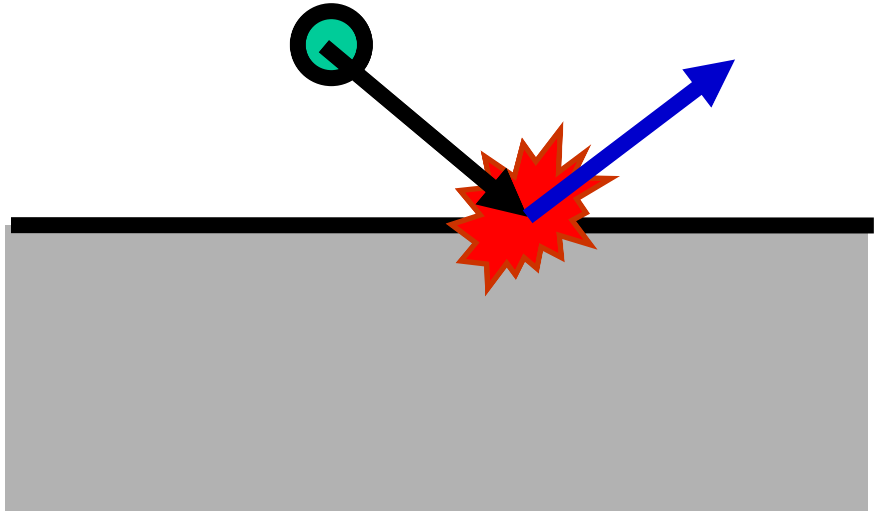
Questions?

http://www.youtube.com/watch?v=dHWCT7RPjPo

Collisions

- Detection
- Response

- Covered later



More Eyecandy from NVIDIA

- Fluid flow solved using a regular grid solver
  - This gives a velocity field
- 0.5M smoke particles advected using the field
  - That means particle velocity is given by field
- Particles are for rendering, motion solved using other methods
- Link to video

NVIDIA

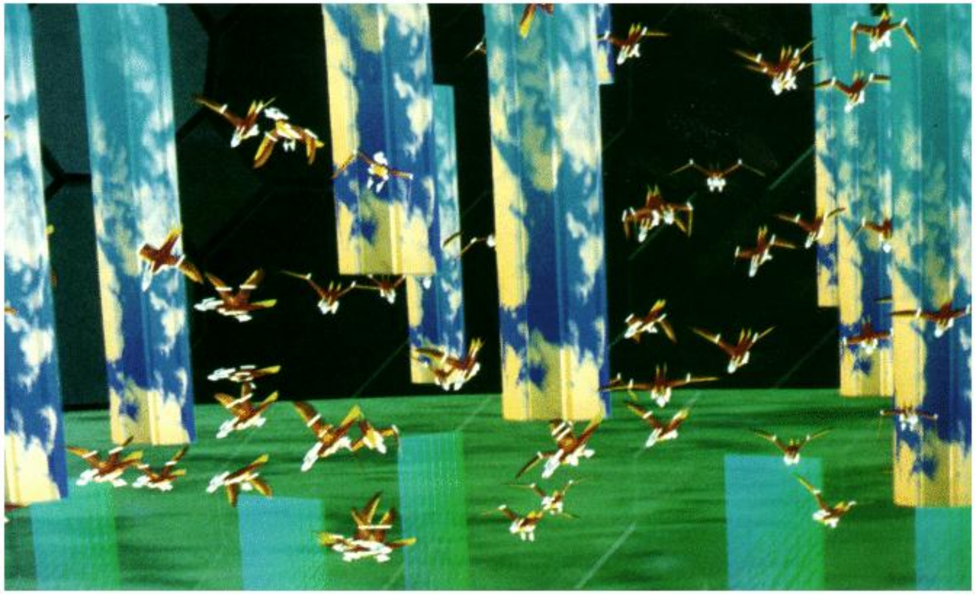
Image removed due to copyright restrictions.

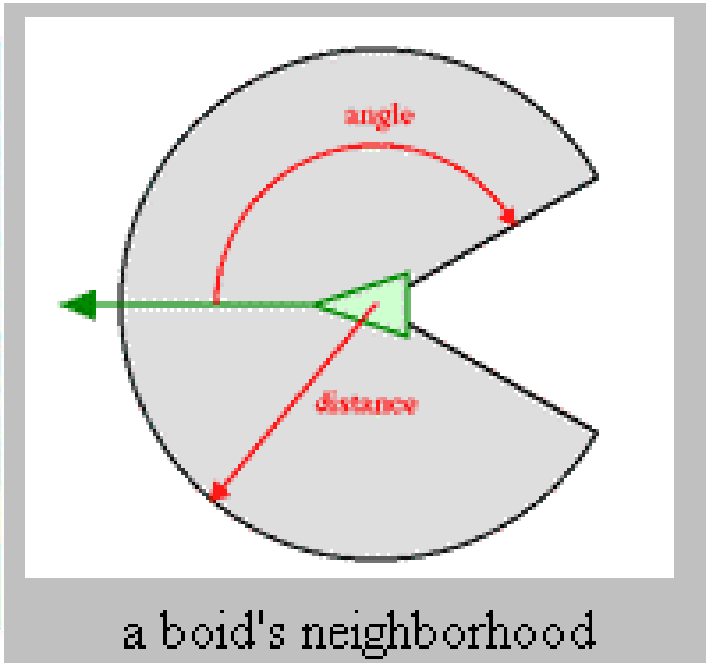
More Advanced "Forces"

- Flocking birds, fish shoals
  - http://www.red3d.com/cwr/boids/
- Crowds (www.massivesoftware.com)

Flocks (“Boids”)

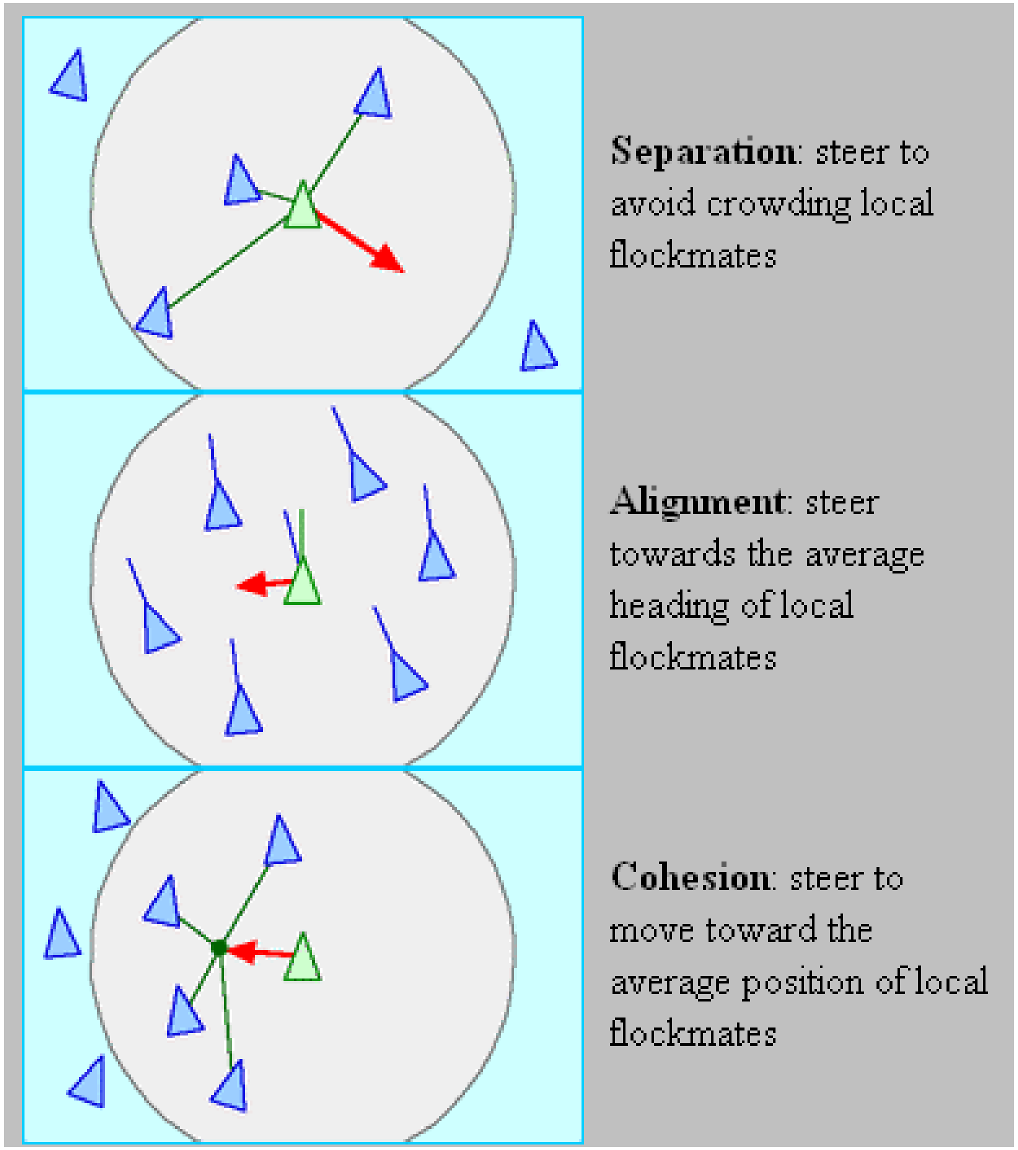
- From Craig Reynolds
- Each bird modeled as a complex particle (“boid”)
- A set of forces control its behavior
- Based on location of other birds and control forces





Flocks ("Boids")

- ("Boid" was an abbreviation of "birdoid". His rules applied equally to simulated flocking birds, and shoaling fish.)



Courtesy of Craig W. Reynolds. Used with permission.

Flocks ("Boids")

Craig Reynolds

Predator-Prey

- http://www.youtube.com/watch?v=rN8DzlgMt3M

Massive software

- http://www.massivesoftware.com/
- Used for battle scenes in the Lord of The Rings

Processing demo

- http://processing.org/learning/topics/flocking.html

Battle of the Helm's deep, LOTR

Image removed due to copyright restrictions.

Questions?

Image removed due to copyright restrictions.

Where do particles come from?

- Often created by generators or **emitters**
  - Can be attached to objects in the model
- Given rate of creation: particles/second
  - record t_{last} of last particle created

$$n = \lfloor (t - t_{last}) rate \rfloor$$

  - create n particles.
    update t_{last} if $n > 0$
- Create with (random) distribution of initial $\boldsymbol{x}$ and $\boldsymbol{v}$
  - if creating $n > 1$ particles at once, spread out on path

Image removed due to copyright restrictions.

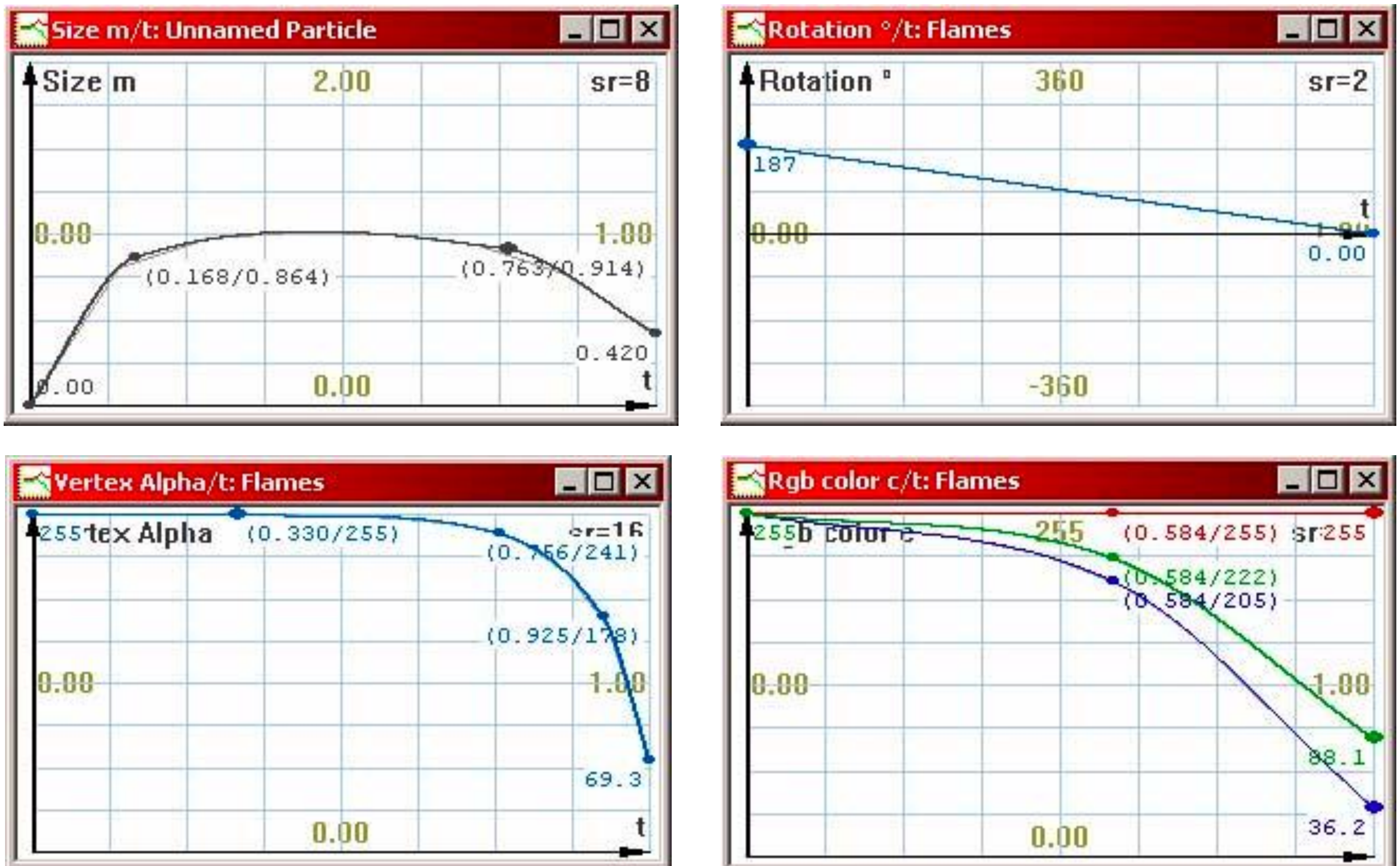
http://www.particlesystems.org/

Particle Controls

- In production tools, all these variables are time-varying and controllable by the user (artist)
  - Emission rate, color, velocity distribution, direction spread, textures, etc. etc.
    - All as a function of time!
  - Example: ParticleFX (Max Payne Particle Editor)
    - Custom editor software
    - You can **download it** (for Windows) and easily create your own particle systems. Comes with examples!
    - This is what we used for all the particles in the game!

Emitter Controls

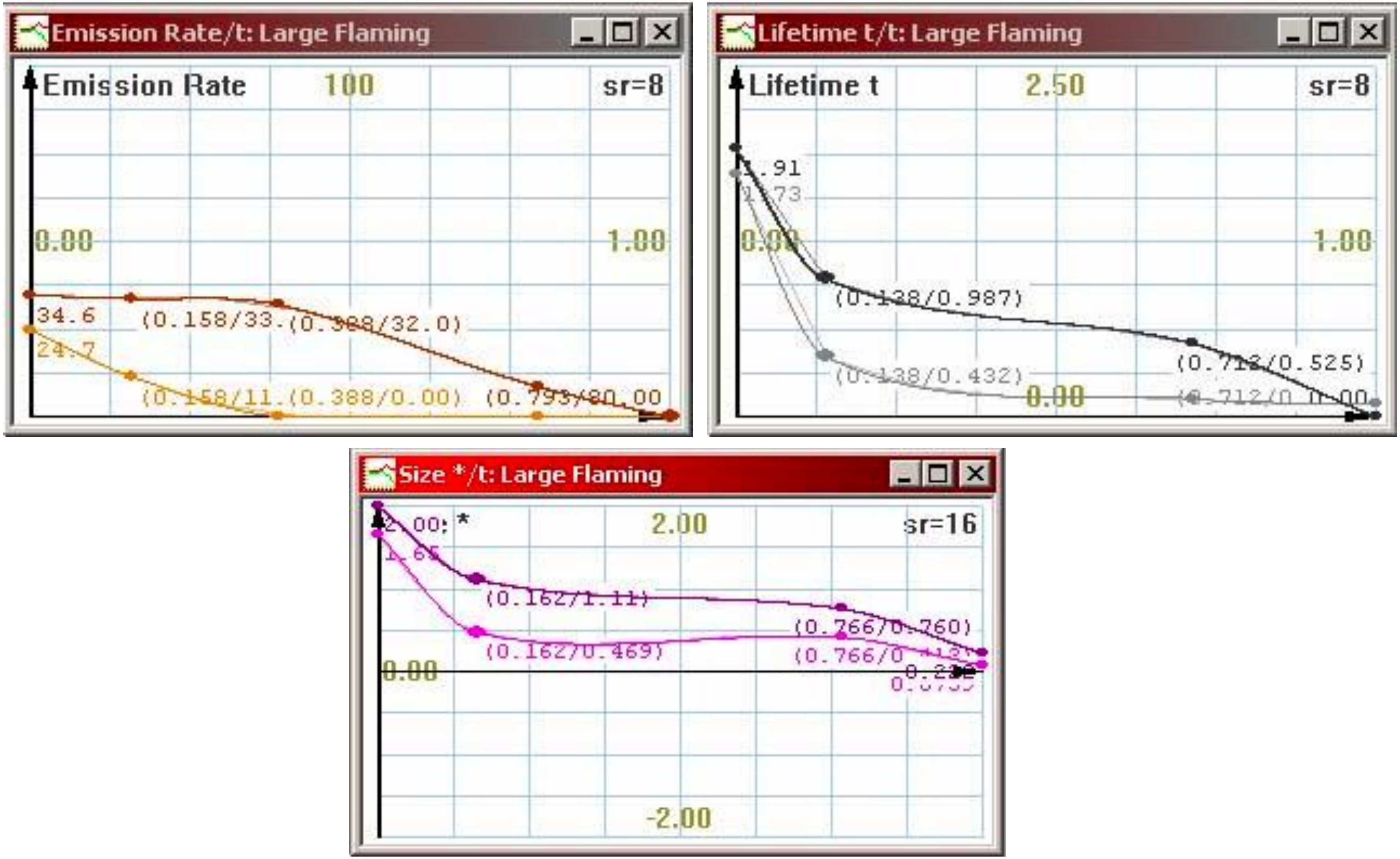
- Again, reuse splines!



Controls from ParticleFX by Remedy Entertainment

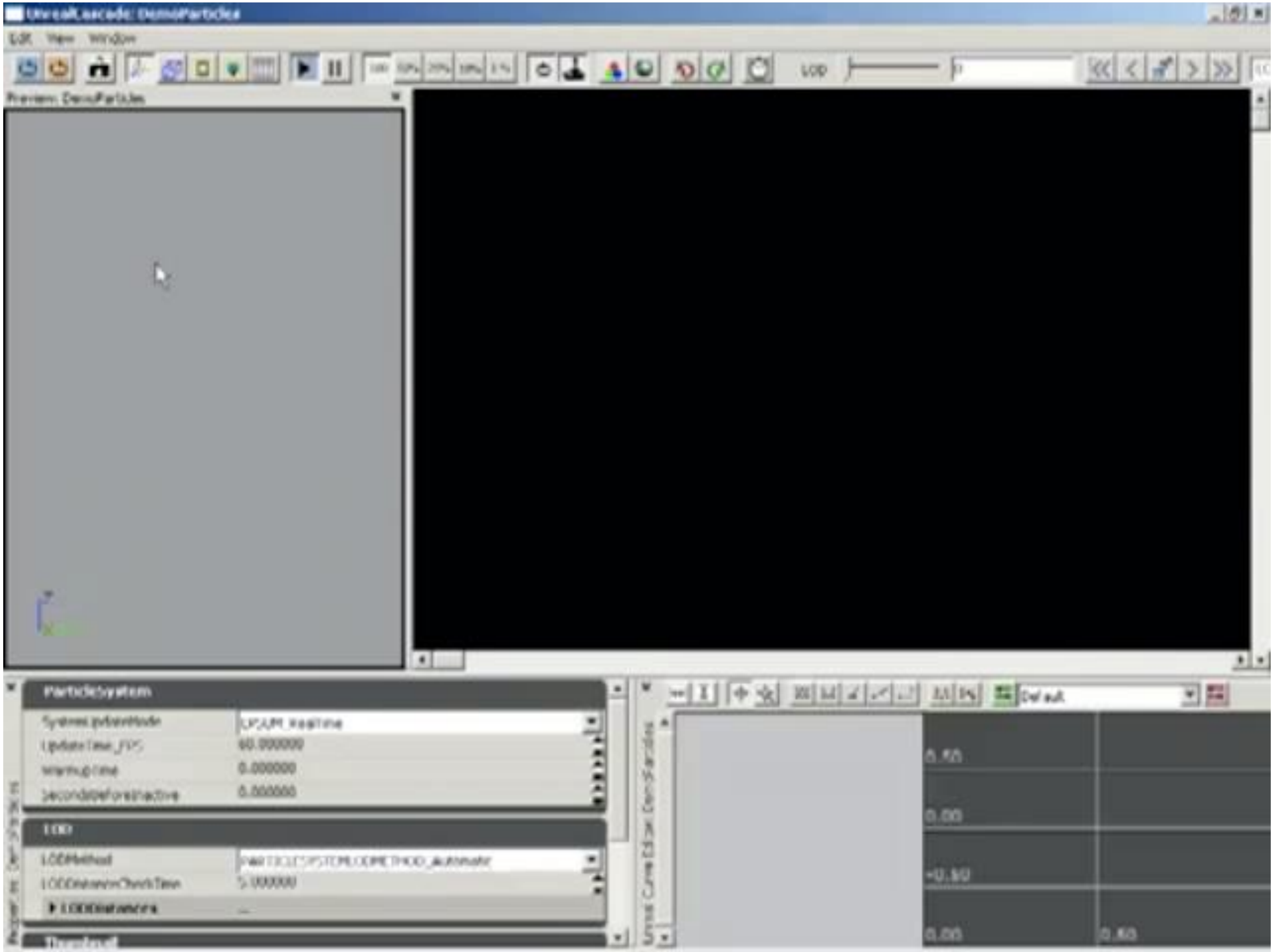
Emitter Controls

- Again, reuse splines!



Controls from ParticleFX by Remedy Entertainment

Unreal Engine



Unreal Engine

Rendering and Motion Blur

- Often not shaded (just emission, think sparks)
  - But realistic non-emissive particles needs shadows, etc.
- Most often, particles don't contribute to the z-buffer, i.e., they do not fully occlude stuff that's behind
  - Rendered with z testing on (particles get occluded by solid stuff)
- Draw a line for motion blur
  - $(\mathbf{x}, \mathbf{x}+\mathbf{v}\, dt)$
  - Or an elongated quad with texture

Metal Gear Solid by Konami

Rendering and Motion Blur

Metal Gear Solid by Konami

Image removed due to copyright restrictions.

- Often use texture maps (fire, clouds, smoke puffs)
  - Called “billboards” or “sprites”
  - Always parallel to image plane

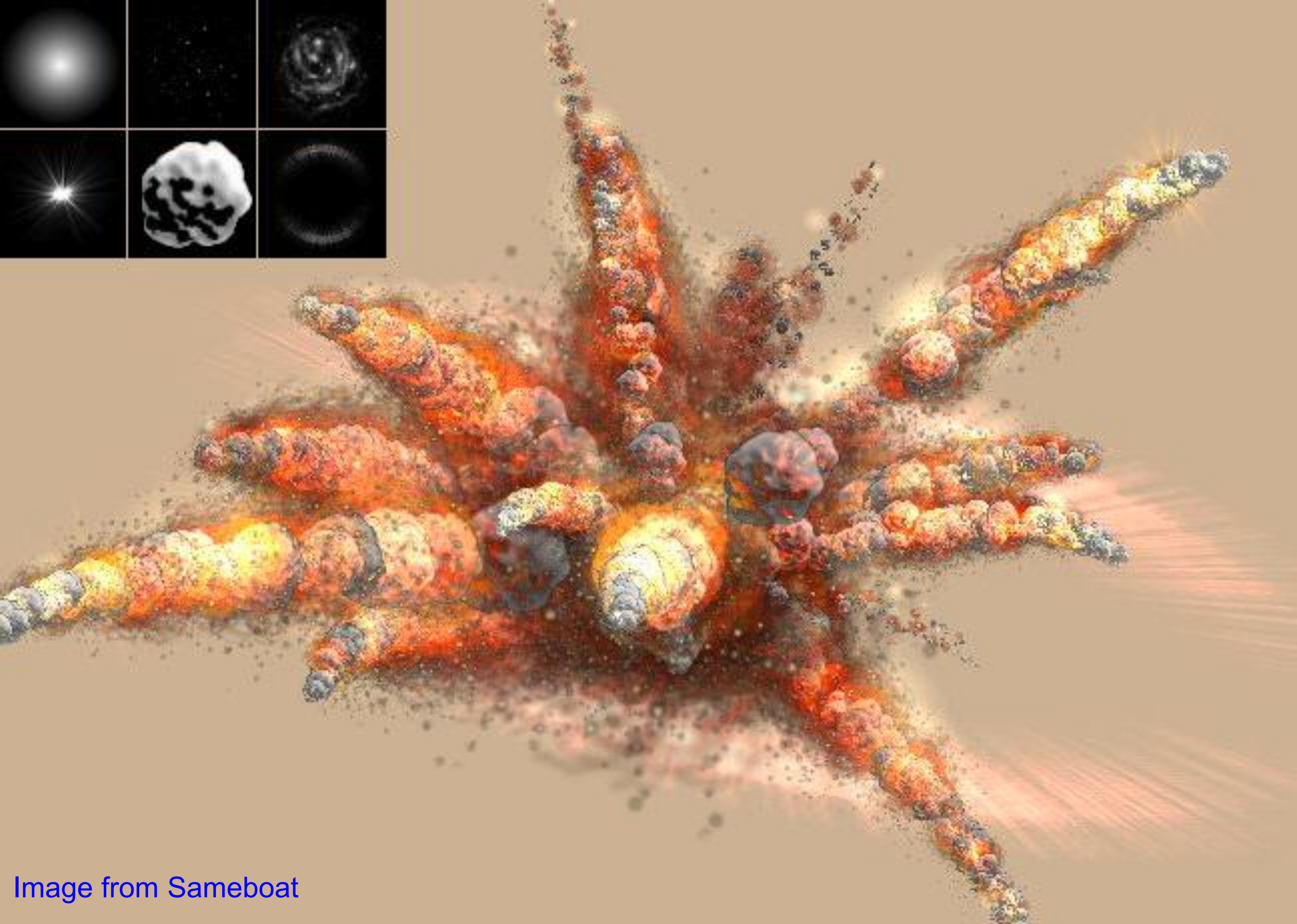


Image from Sameboat

Star Trek 2 – The Wrath of Khan

- One of the earliest particle systems (from 1982)
- Also, fractal landscapes

- Described in [Reeves, 1983]

Paramount Pictures

Particle Modeling [Reeves 1983]

- The grass is made of particles
  - The entire lifetime of the particle is drawn at once.
  - This can be done procedurally on the GPU these days!

William Reeves

Questions?

Courtesy of Karl Sims. Used with permission.

Early particle fun by Karl Sims

That’s All for Today!

- Further reading
  - Witkin, Baraff, Kass: Physically-based Modeling Course Notes, SIGGRAPH 2001
    - **Extremely good, easy-to-read resource. Highly recommended!**
  - William Reeves: Particle systems—a technique for modeling a class of fuzzy objects, Proc. SIGGRAPH 1983
    - The original paper on particle systems
  - particlesystems.org