# I-Structures and Open Lists

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Lecture 11





## Array: An Abstract Datatype

```
module Array (Array, mkArray, (!), bounds)
   where

   infix 9 (!)

   data (Ix a) => Array a t
   mkArray  :: (Ix a) => (a,a) -> (a->t) ->
                                   (Array a t)
   (!)      :: (Ix a) => (Array a t) -> a -> t
    bounds  :: (Ix a) => (Array a t) -> (a,a)
```

Thus,

```
type ArrayI  t = Array Int t
type MatrixI t = Array (Int,Int) t
```

# Index Type Class

pH allows arrays to be indexed by any type that can be regarded as having a contiguous enumerable range

```
class Ix a where
   range    :: (a,a) -> [a]
   index    :: (a,a) -> a -> Int
   inRange  :: (a,a) -> a -> Bool
```

**range:** Returns the list of *index* elements between a lower and an upper bound

**index:** Given a *range* and an *index*, it returns an integer specifying the position of the index in the range based on 0

**inRange** : Tests if an *index* is in the *range*





# Higher Dimensional Arrays

```
x = mkArray ((l1,l2),(u1,u2)) f
```

means `x!(i,j) = f (i,j)`   l1 ≤ i ≤ u1, l2 ≤ j ≤ u2

Type

```
x  :: (Array (Int,Int) t)
```

Assuming

```
f :: (Int,Int) -> t
```

**mkArray** will work for higher dimensional matrices as well.

# The *Wavefront* Example

$$x_{i,j} = x_{i-1,j} + x_{i,j-1}$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 2 | 3 | 4 | 5 | ⊥ | ⊥ | ⊥ |
| 1 | 3 | 6 | 10 | ⊥ | ⊥ | ⊥ | ⊥ |
| 1 | 4 | 10 | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ |
| 1 | 5 | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ |
| 1 | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ |
| 1 | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ |
| 1 | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ |

```
x = mkArray ((1,1),(n,n)) (f x)

f x (i, j) = if  i == 1 then 1
             else if  j == l  then  1
                  else x!(i-1,j) + x!(i,j-1)
```





# Array Comprehension

A special function to turn a list of (index,value) pairs into an array

```
array :: (Ix a) => (a,a) -> [(a,t)] -> (Array a t)
array  ebound
      ([(ie1,e1)  |  gen-pred, ..]
    ++ [(ie2,e2)  |  gen-pred, ..] ++ …)
```

Thus,

```
mkArray (l,u) f =
  array (l,u) [(j,(f j)) | j <- range(l,u)]
```

List comprehensions and function array provide flexibility in constructing arrays, and the compiler can implement them efficiently

duplicates?

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# Array Comprehension: *Wavefront*

x[i,j]= x[i-1,j] + x[i,j-1]

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | | | | | | | |
| 1 | | | | | | | |
| 1 | | | | | | | |
| 1 | | | | | | | |
| 1 | | | | | | | |
| 1 | | | | | | | |
| 1 | | | | | | | |

```
x = array ((1,1),(n,n))
       ([((1,1), 1)]
     ++ [((i,1), 1) |              ]
     ++ [((1,j), 1) |              ]
     ++ [((i,j), x!(i-1,j) + x!(i,j-1))
                      |
                                     ])
```

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# Computed Indices

Inverse permutation
y ! (x ! i) = i

x:

| | | | | | |
|---|---|---|---|---|---|
| 2 | 5 | 6 | 1 | 3 | 4 |

⇓

y:

| | | | | | |
|---|---|---|---|---|---|
| 4 | 1 | 5 | 6 | 2 | 3 |

```
find x i =
     let % find j such that x!j = i
           step j = if x!j== i then j
                  else step j+1
     in
         step 1
y = mkArray (1,n) (find x)
```

*How many comparisons? Can we do better?*

```
y = array (1,n) [(     ,     )| i <- [1..n]]
```

http://www.csg.lcs.mit.edu/6.827

# I-structures

In functional data structures, a *single construct* specifies:

- The *shape* of the data structure
- The value of its components

These two aspects are specified *separately* using I-structures

➔ efficiency
➔ parallelism

I-structures preserve *determinacy* but are *not* functional !





# I-Arrays

- *Allocation* expression

**iArray(1,n) []** ➔

| 1 | 2 | . . . | n |
|---|---|---|---|
| ⊥ | ⊥ | . . . | ⊥ |

- *Assignment*

**iAStore a 2 5**
*or* **a!2 := 5**

| 1 | 2 | . . . | n |
|---|---|---|---|
| ⊥ | 5 | . . . | ⊥ |

provided the previous content was ⊥
*"The single assignment restriction."*

- *Selection* expression

**a!2** ➔ **5**

(⊥ means empty)

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# Computed Indices Using I-structures

Inverse permutation
y ! (x ! i) = i

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 5 | 6 | 1 | 3 | 4 | x |
| 4 | 1 | 5 | 6 | 2 | 3 | y |

```
let
      y = iArray (1,n) []
      _ = for i <- [1..n] do
              _ = iAStore y (x!i) i
          finally ()    % unit data type
in
      y
```

*What if x contains a duplicate ?*

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# Multiple-Store Error

Multiple assignments to an iArray slot cause a multiple store error

A program with exposed store error is suppose to blow up!

Program --> T

The Top represents a contradiction

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# The Unit Type

```
data () = ()
```

means we cannot do much with an object of the unit type. However, it does allow us to drop '_ ='

```
let
   y = iArray (1,n) []
   for i <- [1..n] do
      iAStore y (x!i) i
   finally ()            --  unit data type
in
   y
```

For better syntax replace

`iAStore y (x!i) i` *by* `y!(x!i) := i`





# I-Cell

```
data ICell a = ICell {contents :: . a}
```

I-Structure field

*Constructor*

```
ICell :: a -> ICell a
```

`ICell e` *or* `ICell {contents = e}`

*or create an empty cell and fill it*

```
ic = ICell {}
contents ic := e
```

*Selector*

`contents ic` *or*

```
case ic of
     ICell x -> ... x ...
```

# An Array of ICells

Example: Rearrange an array such that the negative numbers precede the positive numbers

**2 8 -3 14 2 7 -5**

**-3 -5 2 8 14 2 7**

Functional solutions are not efficient

```
let y = array (1,n) [(i,ICell {})| i<-[1..n]]
   (l,r) = (0,n+1)
    final_r = for j <- [1..n] do
                  (l’,r’,k) =



                   contents (y!k) := x!j
                   next l = l’
                   next r = r’
                  finally r
in (y, final_r)
```





# Type Issues

In the previous example

```
x :: Array Int
y :: Array (Icell Int)
```

1. We will introduce an I-Structure array to eliminate an extra level of indirection

2. The type of a functional array (Array) is different from the type of an IArray.

However, an IArray behaves like a functional Array after all its elements have been filled .

We provide a primitive function for this conversion

```
cvt_IArray_to_Array ia -> a
```

# Types Issue *(cont.)*

Hindley-Milner type system has to be extended to deal with I-structures

$\Rightarrow$? *ref type* -- requires new rules
*more on this later...*





*All* functional data structures in pH are implemented as I-structures.

# Array Comprehensions:
*a packaging of I-structures*

```
array dimension
      ([(ie1,e1) | x <- xs, y <- ys]
    ++ [(ie2,e2) | z <- zs] )
```

translated into

```
let   a = iArray dimension []
      for x <- xs do
            for y <- ys do
                  a!ie1 := e1
            finally ()
      finally ()
      for z <- zs do
            a!ie2 := e2
      finally ()
in  cvt_IArray_to_Array a
```





# I-lists

```
data IList t = INil
             | ICons {hd ::t, tl:: .(IList t)}
```

I-Structure field

*Allocation*

```
x = ICons {hd = 5}
```

*Assignment*

```
tl x := e
```

*The single assignment restriction.*
If violated the program will blow up.

*Selection*

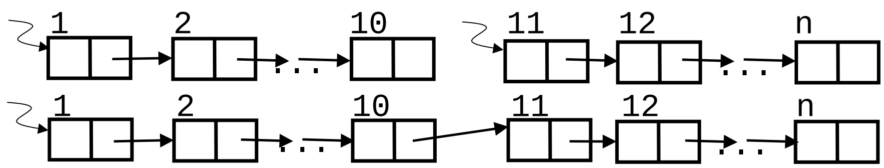
```
case xs of
      INil        -> ...
      ICons h t -> …
```

we can also write `ICons {hd=h, tl=t} -> …`

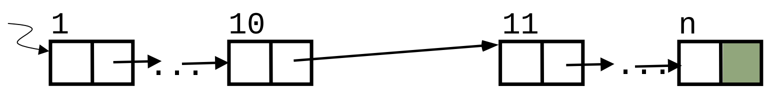
# Open List Operations

A pair of I-list pointers for the *header* and the *trailer* cells.

*joining* two open lists




*closing* an open list








# Open List Operation Definitions

```
type open_list t = ((IList t), (IList t))

nil_ol = (INil, INil)

close (hr,tr) =
       let
            case hr of
               INil -> ()
               ICons _ _ -> {tl tr := INil}
        in  cnv_Ilist_to_list hr

join (hr1,tr1) (hr2,tr2) =
        case hr1 of
            INil ->
            ICons _ _ ->
```

# Map Using Open Lists

```
map f   []   = []
map f (x:xs) = (f x):(map f xs)
```

- *Inefficient* because *it is not tail recursive!*
- A tail recursive version can be written using open lists:
  `map f xs = close (open_map f xs)`
  where

```
open_map f []     = (INil, INil)
open_map f (x:xs) =
      let tr   = ICons {hd=(f x)}
          last = for x' <- xs do



                       finally tr
      in (tr,last)
```





# Implementing List Comprehensions

Functional solution 1

```
[ e | x <- xs, y <- ys] ⇒
      concatMap (\x->
      concatMap (\y-> [e]) ys) xs
```

Functional solution 2

```
[ e | x <- xs, y <- ys] ⇒
      let f []          = []
          f (x:xs')     =
              let g []      = f xs'
                  g (y:ys') = e:(g ys')
              in (g ys)
      in (f xs)
```

# Implementing List Comprehensions Using Open Lists

```
[ e | x <- xs, y <- ys]
```

1. Make n *open lists*, one for each x in xs
2. Join these lists together

```
let
     zs = nil_ol
in
     for x <- xs do
        z' = open_map (\y-> e) ys
        next zs = join zs z'
     finally zs
```





# I-structures are *non functional*

```
f x y = let x!1 := 10
            y!1 := 20
        in ()
```

```
let x = iArray (1,2) []
in  f x x
```

≡

```
f  (iArray (1,2) []) (iArray (1,2) []) ?
```

# The example

```
f x y = let x!1 := 10
            y!1 := 20

        in ()
```

```
let
 x = iArray (1,2) []
in
 f x x
 ⇓
```

```
f   (iArray (1,2) [])
    (iArray (1,2) [])
⇓
```