Implicitly Parallel Programming in pH: Functions and Types

Arvind
Laboratory for Computer Science
M.I.T.

September 9, 2002

http://www.csg.lcs.mit.edu/6.827

Explicitly Parallel Fibonacci

C code

```
int fib (int n)
 {if (n < 2)
    return n;
  else
    return
    fib(n-1)+fib(n-2);
     }
 }
```

Cilk code

```
cilk int fib (int n)
 {if (n < 2)
      return n;
  else
    {int x, y;
      x = spawn fib(n-1);
      y = spawn fib(n-2);
      sync;
      return x + y;
    }
 }
```

C dictates that fib(n-1) be executed before fib(n-2)
$\Rightarrow$ annotations (spawns and sync) for parallelism

Alternative: *declarative languages*

Why Declarative Programming?

- *Implicit Parallelism*
  – language only specifies a partial order on operations
- *Powerful programming idioms and* efficient code *reuse*
  – Clear and relatively small programs
- *Declarative language semantics have good algebraic properties*
  – *Compiler optimizations* go farther than in imperative languages

pH is *Implicitly Parallel* and a *Layered* Language

Non-Deterministic Extensions
- M-structures

Deterministic Extensions
- I-structures

Purely Functional
- higher order
- non strict
- strongly typed + polymorphic

← cleaner semantics

→ more expressive power

Function Execution by Substitution

```
      plus x  y = x + y
1.    plus  2  3  →  2 + 3  →  5
2.    plus  (2*3)  (plus 4 5)
```


Confluence

All Functional pH programs (right or wrong) have *repeatable behavior*

Blocks

```
let
    x =  a * a
    y =  b * b
in
    (x - y)/(x + y)
```

- a variable can have at most one definition in a block
- ordering of bindings does not matter

Layout Convention

This convention allows us to omit many delimiters

```
let
    x =  a * a
    y =  b * b
in
    (x - y)/(x + y)
```

is the same as

```
let
    { x =  a * a ;
      y =  b * b ;}
in
    (x - y)/(x + y)
```

Lexical Scoping

```
let
    y = 2 * 2
    x = 3 + 4
    z = let
          x = 5 * 5
          w = x + y * x
        in
           w
in
    x + y + z
```

Lexically closest definition of a variable prevails.

Renaming Bound Identifiers
(α-renaming)

```
let                               let
  y = 2 * 2                         y = 2 * 2
  x = 3 + 4                         x = 3 + 4
  z = let                           z = let
        x = 5 * 5          ≡              x' = 5 * 5
        w = x + y * x                     w = x' + y * x'
      in                                in
         w                                 w
in                                in
   x + y + z                         x + y + z
```

Lexical Scoping and α-renaming

```
plus  x y = x + y

plus' a b = a + b
```

plus and **plus'** are the same because **plus'** can be obtained by *systematic renaming of bound identifiers* of **plus**

Capture of Free Variables

```
f x = . . .
g x = . . .
foo f x = f (g x)
```

Suppose we rename the bound identifier **f** to **g** in the definition of **foo**

```
foo' g x = g (g x)
```

foo $\equiv$ **foo'** ?

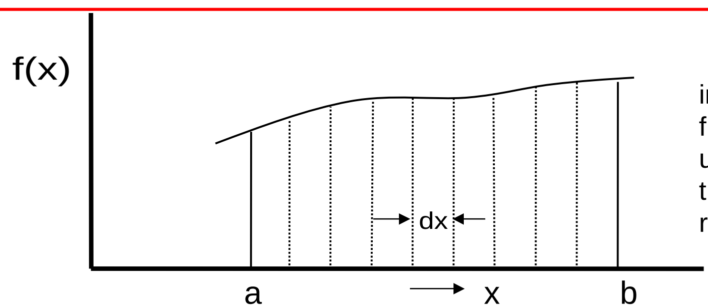
While renaming, entirely new names should be introduced!

Curried functions

```
plus x  y =  x + y

let
    f  = plus 1
in
    f 3
```


Recursion



integrate f(x) from a to b using trapezoidal rule

Integral(a,b) = (f(a + dx/2) + f(a + 3dx/2) + ...) ?dx

```
integrate dx a b f =
  (sum dx b f (a+dx/2) 0) * dx

sum dx b f x tot =
  if x > b then tot
  else sum dx b f (x+dx) (tot+(f x))
```

Local Function Definitions

Improve *modularity* and reduce clutter.

```
integrate dx a b f =
     (sum dx b f (a+dx/2) 0) * dx

sum dx b f x tot =
     if x > b then tot
     else sum dx b f (x+dx) (tot+(f x))
```

```
integrate dx a b f =
     let
        sum x tot =
             if x > b then tot
             else sum (x+dx) (tot+(f x))
     in
        sum (a+dx/2) 0
```

Free variables of `sum` ?

Loops (Tail Recursion)

- Loops or tail recursion is a restricted form of recursion but it is adequate to represent a large class of common programs.
  - Special syntax can make loops easier to read and write
  - Loops can often be implemented with greater efficiency

```
integrate dx a b f =
     let
         x = a + dx/2
         tot = 0
     in
         (while x <= b do
           next    x = x + dx
           next  tot = tot + (f x)
         finally tot) * dx
```

Higher-Order Computation Structures

```
apply_n f n x = if (n == 0) then x
                else apply_n f (n-1) (f x)

succ x =  x + 1

          apply_n succ b a    ?
```

succ can be written as **((+) 1)** also because of the syntactic convention: **x + y** ≡ **(+) x y**

```
apply_n ((+) 1) b a
```

```
mult a b = apply_n                 ?
```


Types

All expressions in pH have a type

23 :: Int

"**23** belongs to the set of integers"
"The type of **23** is **Int**"

```
   true :: Bool
"hello" :: String
```

Type of an expression

```
(sq 529)  :: Int
 sq       :: Int -> Int
```

"**sq** is a function, which when applied to an integer produces an integer."

"**Int -> Int** is the set of functions which when applied to an integer produce an integer."

"The type of **sq** is **Int -> Int**."

Type of a Curried Function

```
        plus x  y =  x + y

 (plus 1) 3     :: Int

 (plus 1)       :: Int -> Int

  plus          ::
```

?

λ- Abstraction

Lambda notation makes it explicit that a value can be a function. Thus,

`(plus 1)` can be written as **`\y -> (1 + y)`**

```
plus x  y =  x + y
```

can be written as

```
    plus = \x -> \y -> (x + y)
```

or as

```
    plus = \x y -> (x + y)
```

(**`\x`** is a syntactic approximation of λ**x** in Haskell)

Parentheses Convention

```
f e1 e2     ≡   ((f e1) e2)
f e1 e2 e3  ≡  (((f e1) e2) e3)
```

application is *left associative*

```
Int -> (Int -> Int) ≡ Int -> Int -> Int
```

type constructor “**`->`**” is *right associative*

Type of a Block

```
(let
   x1 =  e1
   .
   .
   .
   xn  =  en
 in
    e )        ::   t
```

$(\textbf{let}\ x_1 = e_1 \ldots x_n = e_n\ \textbf{in}\ e) :: t$

provided

$e :: t$

Type of a Conditional

$(\textbf{if}\ e\ \textbf{then}\ e_1\ \textbf{else}\ e_2\ ) :: t$

provided

$e :: \text{Bool}$
$e_1 :: t$
$e_2 :: t$

The type of expressions in both branches of conditional must be the same.

Polymorphism

```
twice f x = f (f x)
```

1. `twice (plus 3) 4`

   `twice ::` **?**

2. `twice (appendR "two") "Desmond"`

   `twice ::` **?**

where **`appendR “baz" “foo"`** → **`"foobaz"`**

Deducing Types

`twice f x = f (f x)`
What is the most "general type" for twice?

1. Assign types to every subexpression
   ```
   x :: t0           f :: t1
   f x :: t2     f (f x) :: t3
   ```
   ⇒ `twice :: t1 -> (t0 -> t3)`
2. Set up the constraints
   `t1 = t0 -> t2` because of `(f x)`
   `t1 = t2 -> t3` because of `f (f x)`
3. Resolve the constraints
   `t0 -> t2 = t2 -> t3`
   ⇒ `t0 = t2 and t2 = t3` ⇒ `t0 = t2 = t3`
   ⇒ `twice :: (t0 -> t0) ->(t0 -> t0)`

Another Example: *Compose*

`compose f g x = f (g x)`
What is the type of **`compose`** **?**

1. Assign types to every subexpression

```
    x :: t0     f :: t1       g :: t2
  g x :: t3     f (g x) :: t4
⇒ compose ::
```


Hindley-Milner Type System

pH and most modern functional languages follow the Hindley-Milner type system.

The main source of polymorphism in this system is the *Let block*.

The type of a variable can be instantiated differently within its lexical scope.

much more on this later ...