Noise

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*** With thanks to Steve Senturia, from whose lecture notes some of these materials are adapted.**

Outline

> **Where does noise come from?**
> **Interference and how to deal with it**
> **Noise definitions and characterization**
> **Types of noise**

- **Thermal noise**
- **Shot noise**
- **Flicker noise**

> **Examples**

- **Electronics (diodes, amplifiers)**
- **Resistance thermometer**

> **Modulation**

What limits measurements?

- Fundamental physics of the sensing mechanism, or interactions among parts of your sensor
- Example: using a resistor to sense temperature
  - Apply a voltage across a resistor and measure the current, which tells you resistance
  - Relate change in resistance to change in temperature
  - But measuring resistance dissipates some heat in the resistor, thereby changing the temperature
- Solutions:
  - Make sure that the temperature change that you impose is less than the temperature change that you need to measure
  - Or, use a different sensing technique
- But how low can you lower the voltage and still get enough signal?

An Example: Tunneling Accelerometer

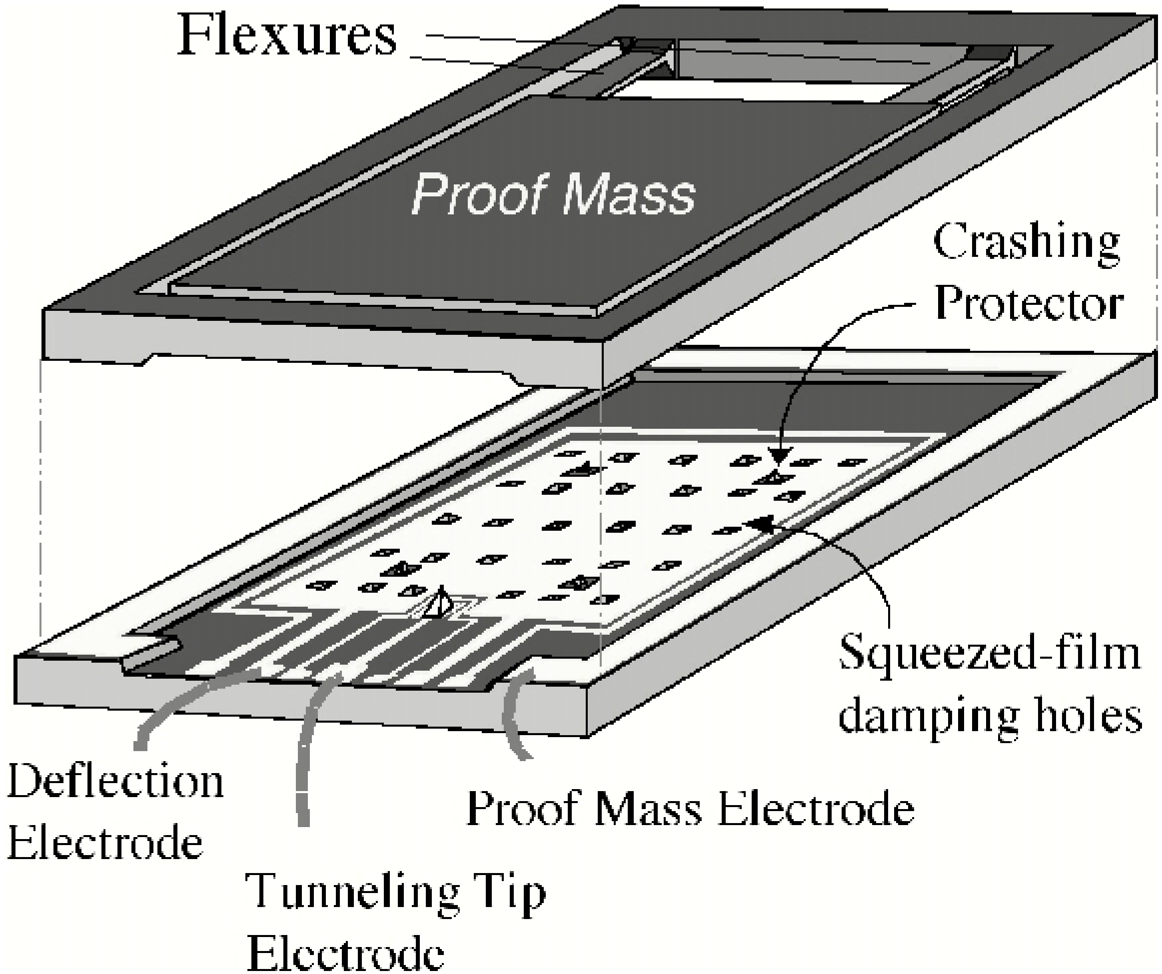


Figure 1 on p. 236 in Liu, C.-H., A. M. Barzilai, J. K. Reynolds, A. Partridge, T. W. Kenny, J. D. Grade, and H. K. Rockstad. "Characterization of a High-sensitivity Micromachined Tunneling Accelerometer with Micro-g Resolution." *Journal of Microelectromechanical Systems* 7, no. 2 (June 1998): 235-244.

Tunneling accelerometer

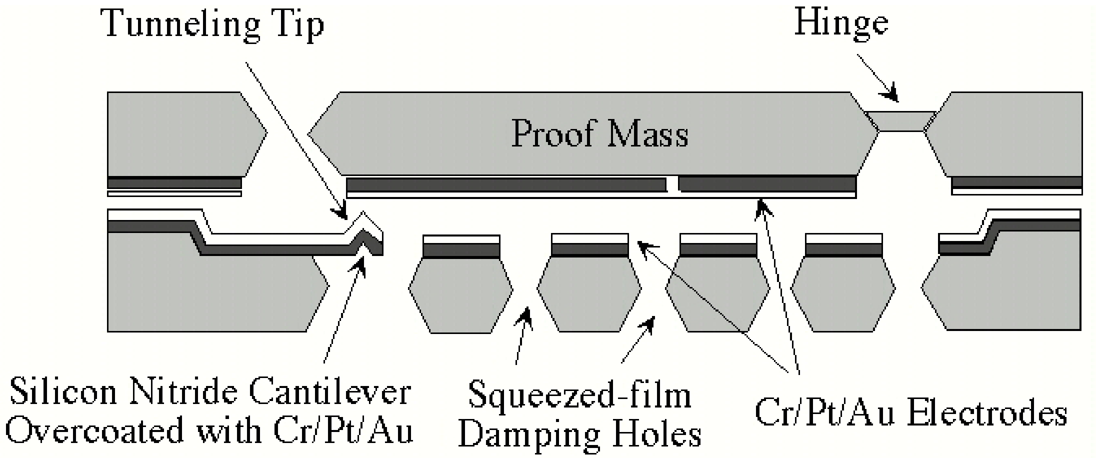


Figure 2 on p. 236 in Liu, C.-H., A. M. Barzilai, J. K. Reynolds, A. Partridge, T. W. Kenny, J. D. Grade, and H. K. Rockstad. "Characterization of a High-sensitivity Micromachined Tunneling Accelerometer with Micro-g Resolution." *Journal of Microelectromechanical Systems* 7, no. 2 (June 1998): 235-244. © 1998 IEEE.

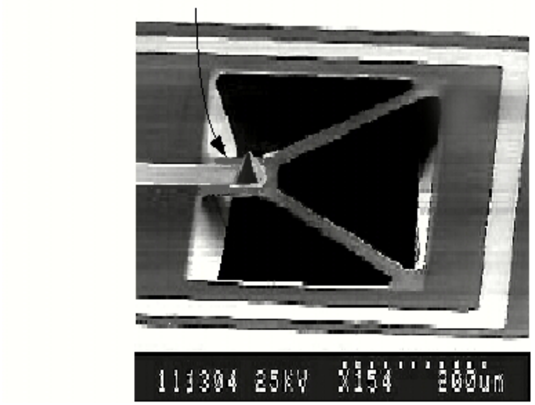


Figure 6 on p. 239 in Liu, C.-H., A. M. Barzilai, J. K. Reynolds, A. Partridge, T. W. Kenny, J. D. Grade, and H. K. Rockstad. "Characterization of a High-sensitivity Micromachined Tunneling Accelerometer with Micro-g Resolution." *Journal of Microelectromechanical Systems* 7, no. 2 (June 1998): 235-244. © 1998 IEEE.

Tunneling current is exponentially sensitive to separation of tip and substrate. WITH FEEDBACK, this offers very sensitive measurements of acceleration.

How sensitive? Must consider sources of NOISE.

Types of Noise Sources

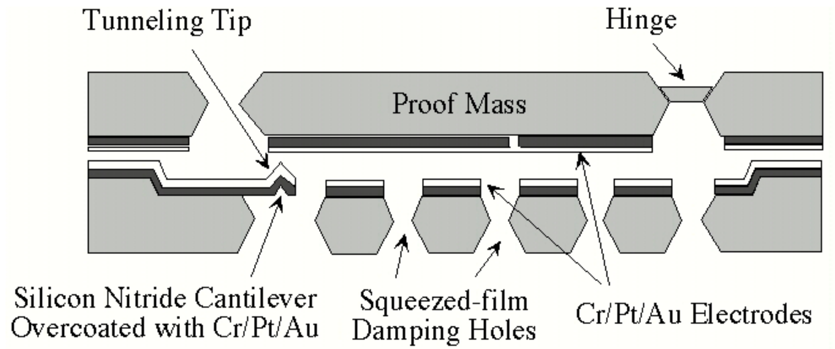
> **Device noise: statistical fluctuations in devices**
- **Johnson noise, shot noise, 1/f or flicker noise**
- **These noise sources seem immensely unlikely in the face of our usual assumptions, but they are real and limit results.**

> **Interference: corruption of the signal of interest by interference from another signal**
- **Power line, radio stations, TV stations, ground loop inductive effects**

> **Drifts: variations in device characteristics**

Sources of noise in the tunneling accelerometer

- Amplifier noise
- Electrons are quantized, so tunneling current is inherently digitized
  - An apparently constant current really isn't… gives noise
- The air molecules in the gap aren't perfectly evenly distributed (Brownian motion)
  - Can build up a force from fluctuations
- Drifts? Interference? Air currents? Package creep?



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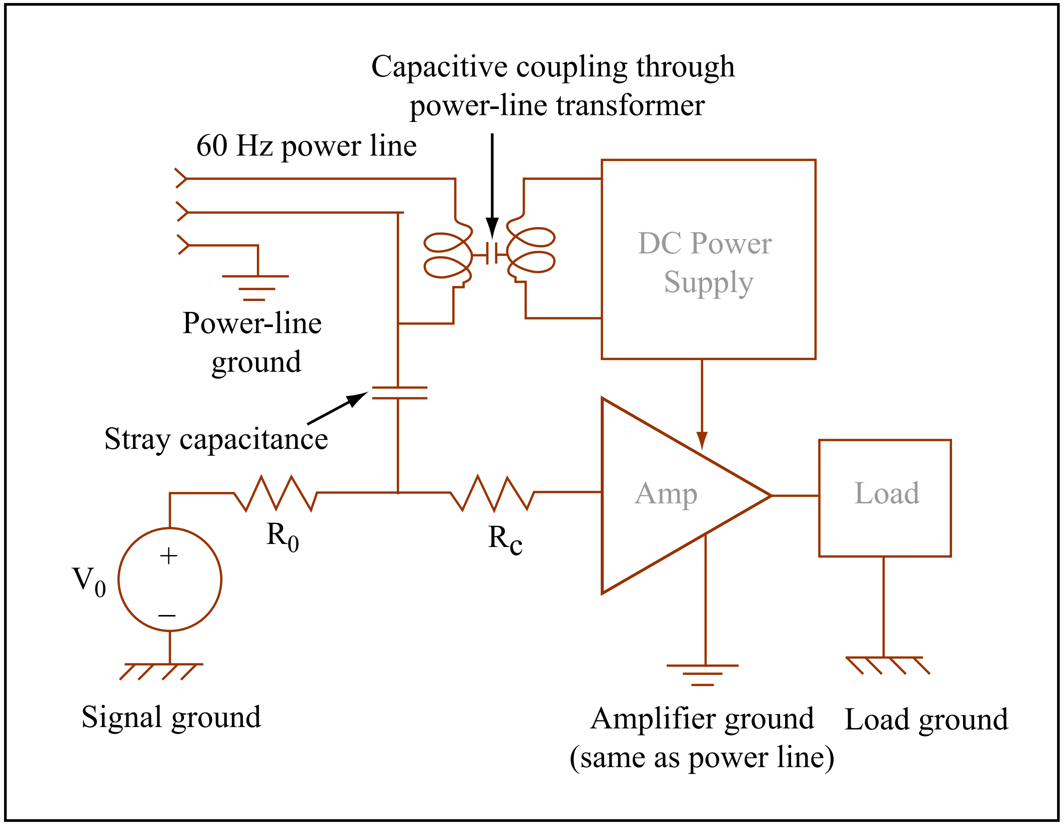
Interference and noise are different

> **Interfering signals:**
  - **When an external signal couples into your circuit in an unintended way**
  - **Capacitive pickup of power line signal, input feeding directly through to output, etc.**
  - **Often reflects a circuit design problem that you can prevent or remedy**
  - **Math looks like superposition and mixing of signals**

> **Random noise:**
  - **Random noise is random!**
  - **Statistical fluctuations in voltages, currents, forces, pressures, etc.**
  - **Math is simpler with random, uncorrelated signals**

The Interference Problem

> **Power-line interference can result from grounding problems or capacitive coupling**



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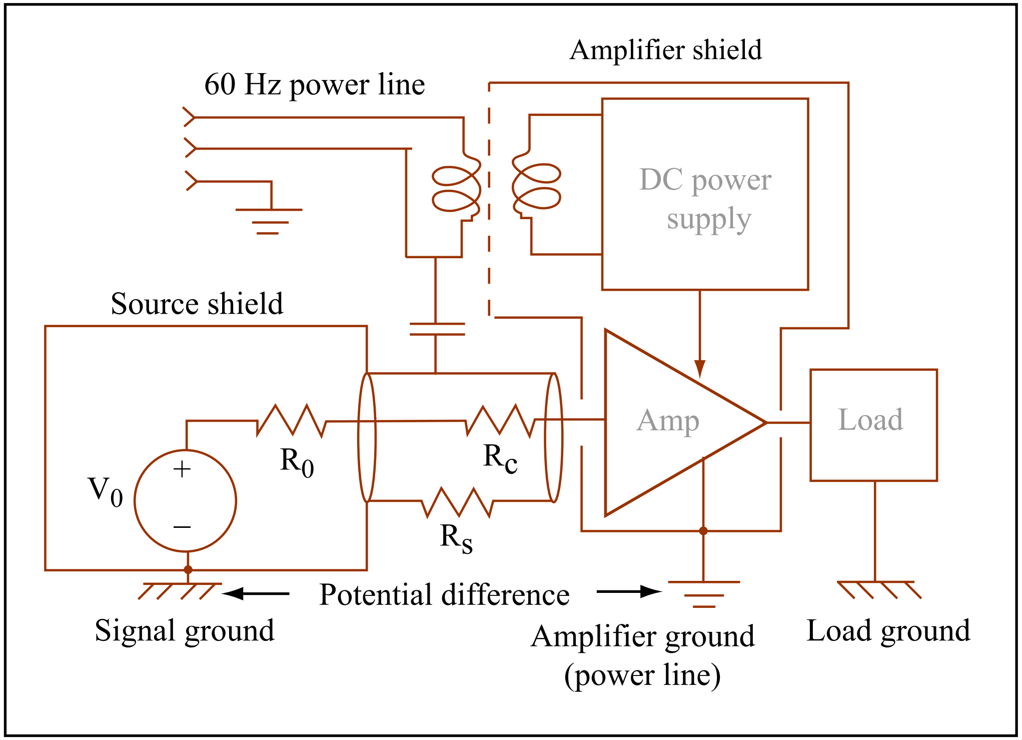
Shields

> A shield is a grounded conductor that surrounds the space being shielded

> But where to connect the ground?



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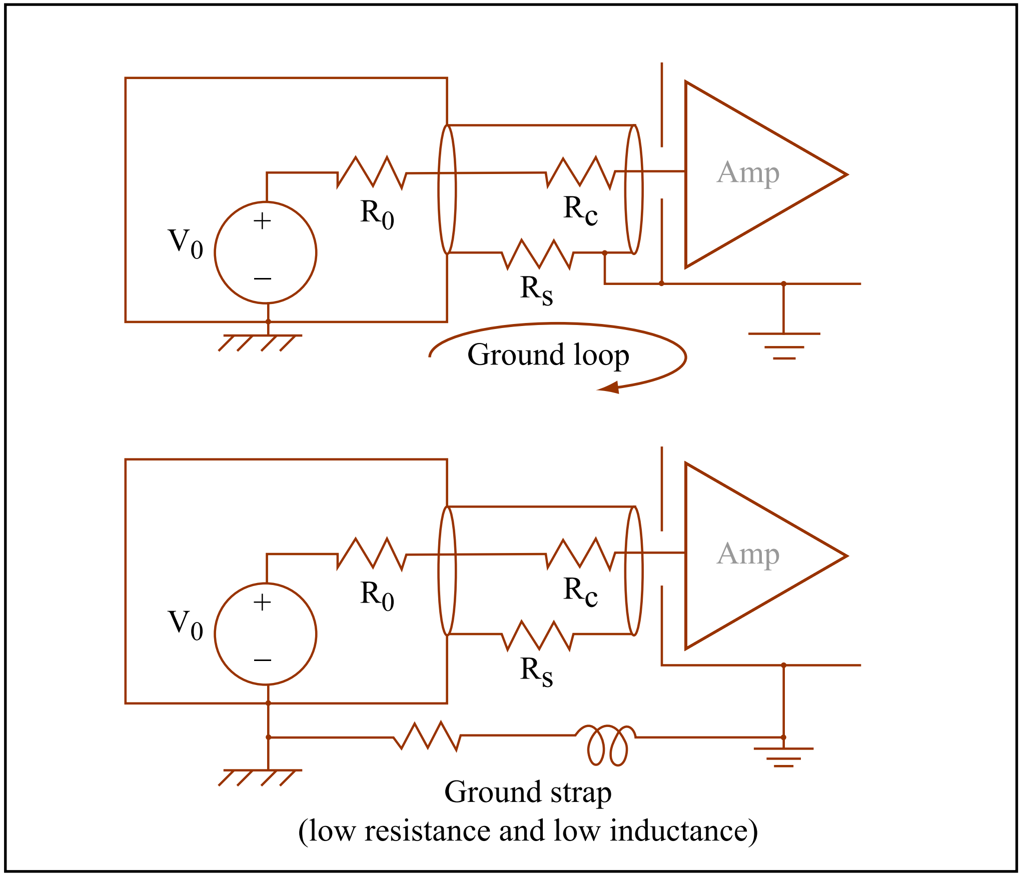
Adapted from Figure 16.2 in Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 427. ISBN: 9780792372462.

Ground Loops

- Multiple grounds create ground loops
- Large inductive EMF’s can be created



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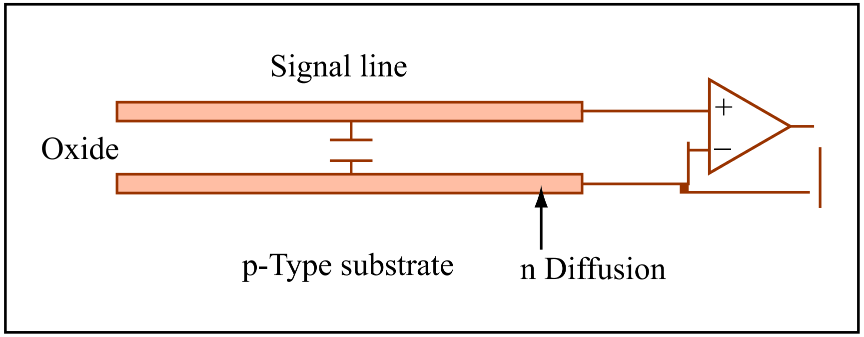
Guards

> **A guard is a shield driven at the common-mode potential of the signal of interest**

> **Guards can be used to intercept surface currents or to nullify the effect of wiring capacitance**



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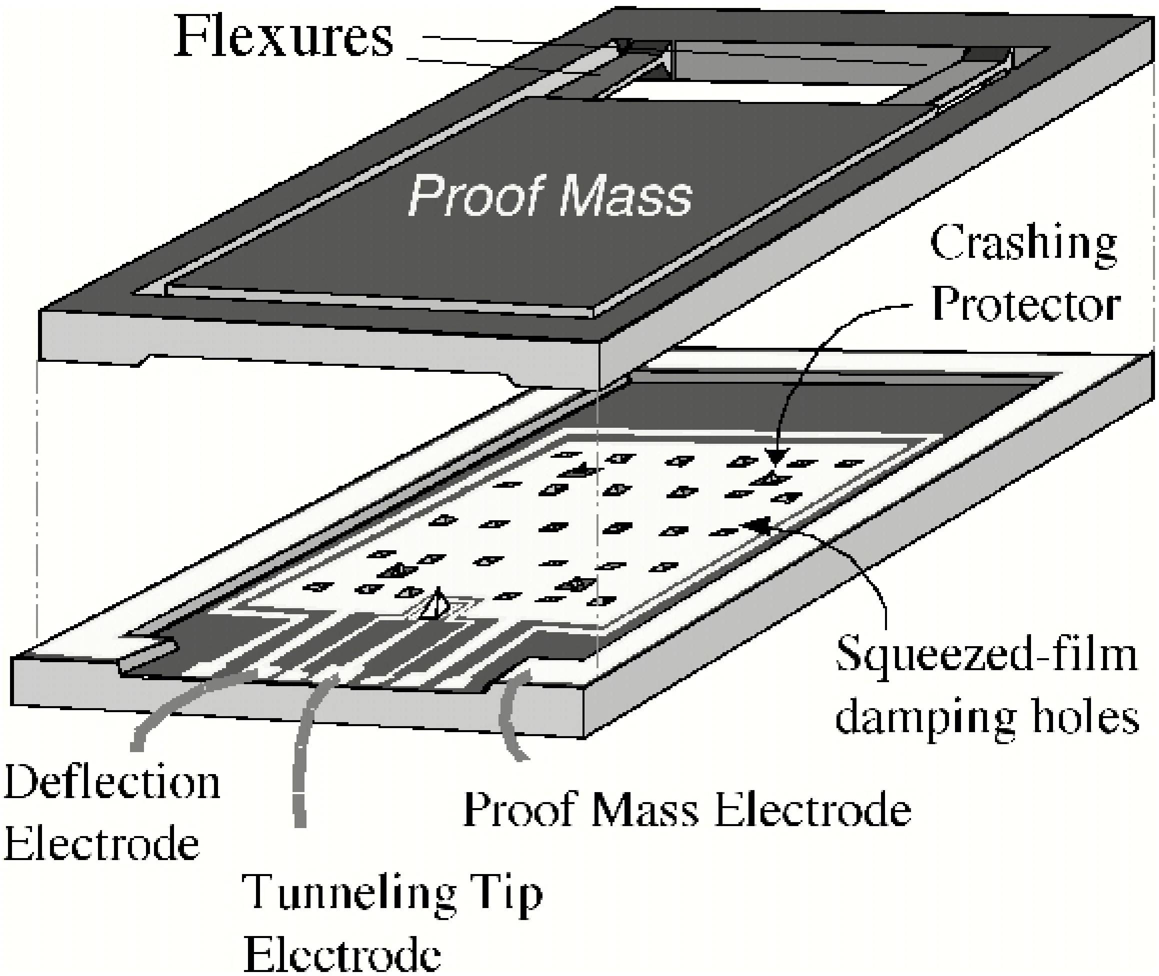
Interference in the tunneling accelerometer?



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Noise characterization

> **Consider a time-varying, random noise voltage** $v_n(t)$

> **Also have a signal** $v_s(t)$

> **Random noise has zero average, but a non-zero mean-square average**

> **The mean square noise from *uncorrelated* noise sources adds**

$$v_s(t) = \text{signal}$$

$$v_n(t) = \text{noise}$$

$$\overline{v}_n = \lim_{\hat{t}\to\infty} \frac{1}{\hat{t}} \int_{-\hat{t}/2}^{\hat{t}/2} v_n(t)\,dt = 0$$

$$\overline{v_n^2} = \lim_{\hat{t}\to\infty} \frac{1}{\hat{t}} \int_{-\hat{t}/2}^{\hat{t}/2} \left[v_n(t)\right]^2 dt \neq 0$$

$$v = v_s + v_n$$

$$\overline{v} = \overline{v}_s$$

$$\overline{v^2} = \overline{v_s^2} + \overline{v_n^2}$$

Signal-to-Noise Ratio

- Signal-to-noise ratio: a power ratio

$$S/N = \frac{\overline{v_s^2}}{\overline{v_n^2}} \text{ (linear scale)}$$

- Normally expressed in logarithmic coordinates
- Decibels is usual unit
- Can use power ratio, or rms amplitude ratio

decibel scale :

$$S/N = 10\log\left(\frac{\overline{v_s^2}}{\overline{v_n^2}}\right)$$

$$S/N = 20\log\left(\frac{v_{s,rms}}{v_{n,rms}}\right)$$

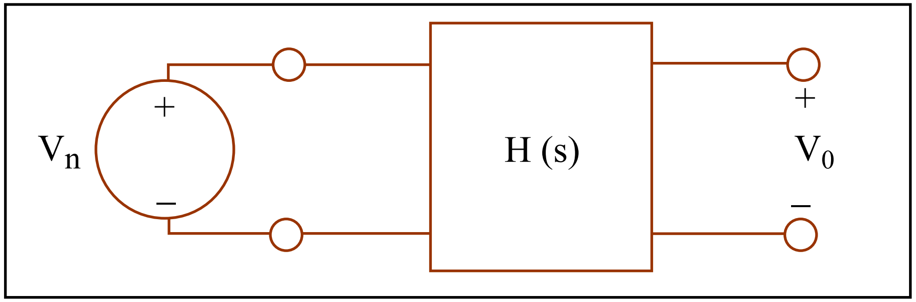
Spectral Density Function

- We imagine an ideal narrow-band filter that allows all frequency content within a small band around a selected frequency to pass
- The mean square output of this filter is assumed to be proportional to the bandwidth, provided the bandwidth is small enough. This is equivalent to saying that the spectral content of the signal of interest has no impulses in it (hence, is not a perfect sine wave)
- The ***spectral density function*** expresses that proportionality

$$\overline{v_o^2(f_o, \delta f)} = S_n(f_o)\delta f \quad \Rightarrow \quad \overline{v_n^2} = \int_0^\infty S_n(f)\, df$$

Noise in Linear Systems

> **The spectral density function at the output is the spectral density at the input times the squared magnitude of the transfer function**



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$$S_o(f) = \left|H(j2\pi f)\right|^2 S_n(f)$$

$$\Downarrow$$

$$\overline{v_0^2} = \int_0^\infty \left|H(j2\pi f)\right|^2 S_n(f)\,df$$

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Thermal Noise

- Imagine a damped harmonic oscillator that is not driven

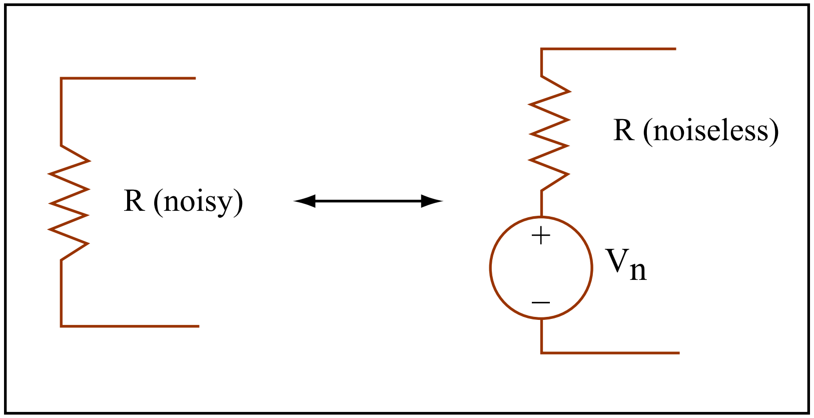
$$m\ddot{x} + b\dot{x} + kx = 0$$

- Any initial motion will be damped out
- BUT this cannot be an accurate description of reality
- Equipartition: in thermal equilibrium, each energy storage mode has an average energy $k_BT/2$
- Energies $mv^2/2$ and $kx^2/2$ cannot be zero!
- Correct equation has a "noise force"

$$m\ddot{x} + b\dot{x} + kx = f_n(b,t)$$

Thermal Noise (Johnson Noise)

- Any dissipative process coupled to a thermal reservoir results in fluctuations, even in equilibrium
- Example: thermomechanical noise force
- Example: electrical resistors have a fluctuating zero-average noise voltage
- The spectrum of Johnson noise is white (at least at all frequencies of interest)



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Adapted from Figure 16.8 in Senturia, Stephen D. *Microsystem Design*.
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$$S_n(f) = 4k_B TR$$

Noise Bandwidth

> **The noise bandwidth, when noise sources are white, measures the magnitude of the mean-square noise.**

$$\overline{v_n^2} = \int_0^\infty S_n(f)\,df$$

$$\text{Consider an ideal filter of bandwidth } \Delta f$$

$$\text{The mean - square noise from a resistor that can pass this filter is}$$

$$\overline{v_n^2} = 4k_B TR\Delta f$$

Thermal Noise: Other Domains

- Our lumped element view of the world makes this very easy to apply to other domains
- Electrical (R = electrical resistance):

$$S_n(f) = 4k_B TR \qquad \overline{v_n^2} = 4k_B TR\Delta f$$

- Mechanical (b = damping):

$$S_n(f) = 4k_B Tb \qquad \overline{f_n^2} = 4k_B Tb\Delta f$$

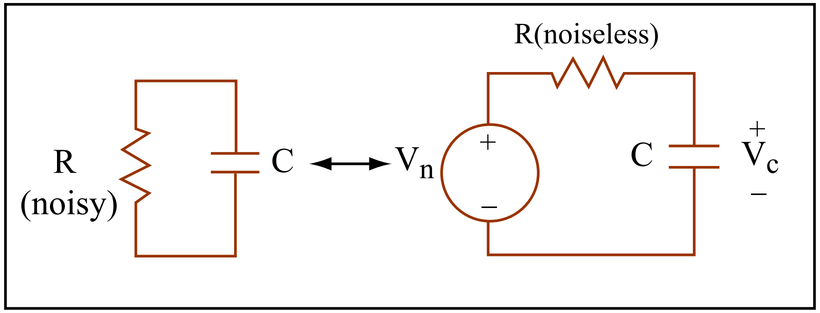
- Fluidic (R = fluidic resistance):

$$S_n(f) = 4k_B TR \qquad \overline{p_n^2} = 4k_B TR\Delta f$$

A good reference: T. Gabrielson, IEEE Trans. Elec. Devices, 40 (1993).

Noise on Capacitors

- When a capacitor is connected to a resistor that is connected to a thermal reservoir, the mean-square noise on the capacitor is $k_B T/C$
- Applies to capacitance equivalents: $k(k_B T)$ noise in a spring



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$$\overline{v_C^2} = \int_0^\infty S_C(f)\, df$$

$$\overline{v_C^2} = \int_0^\infty \left|H(j2\pi f)\right|^2 S_n(f)df$$

$$H(s) = \frac{V_C(s)}{V_n(s)} = \frac{1}{1+sRC}$$

$$\overline{v_C^2} = \int_0^\infty \frac{1}{\left(1+\left(2\pi f RC\right)^2\right)} 4k_B T R df$$

$$\overline{v_C^2} = 4k_B TR\left(\frac{1}{4RC}\right) = \frac{k_B T}{C}$$

Noise bandwidth

- Can define a noise bandwidth for the capacitor noise
- Noise source white
- Transfer function not flat

$$H(s) = \frac{V_C(s)}{V_n(s)} = \frac{1}{1+sRC}$$

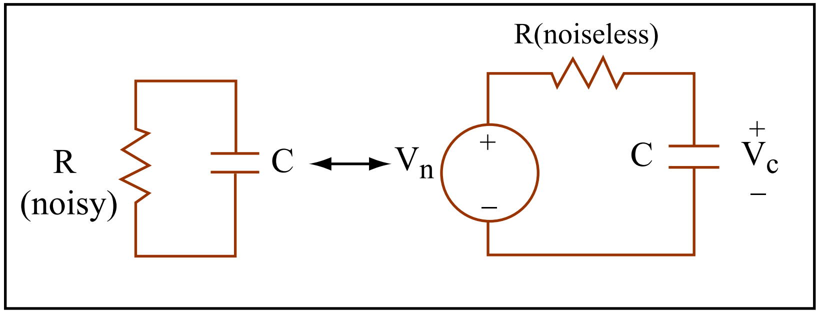
$$\Delta f = \int_0^\infty \frac{1}{1+(2\pi f RC)^2}\, df$$

$$\Downarrow$$

$$\Delta f = \frac{1}{4RC}$$

$$\Downarrow$$

$$\overline{v_C^2} = \frac{k_B T}{C}$$



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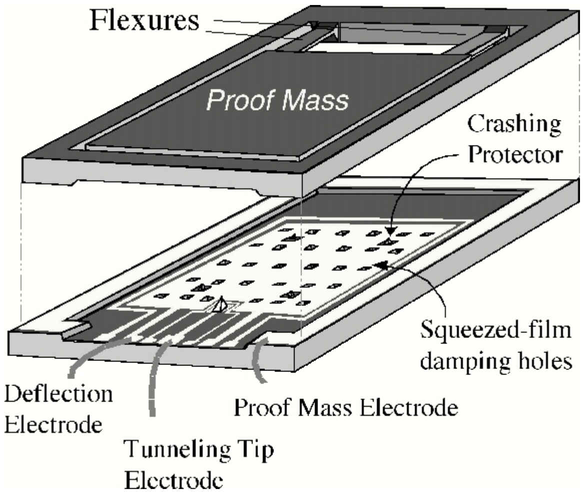
Thermomechanical noise in the tunneling accelerometer

> **Wide plates (7mm x 7mm), narrowly spaced (50 μm): consider squeeze-film damping**

> **Squeeze-film damping holes: estimate about 30 holes, covering between 1% and 10% of the plate area**



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Solid plates, at atmosphere:

$$b_{solid} = \frac{96\eta L W^3}{\pi^4 h_0^3} \approx 0.2 \, {}^{\mathrm{Ns}}\!/_{\mathrm{m}}$$

Disk-shaped plates, one perforated, at 1 atm:

$$b_{perf} = \frac{12\eta A_{plate}^2}{N_{holes}\pi h_0^3} F\left(\frac{A_{holes}}{A_{plate}}\right) \approx 0.01 \, {}^{\mathrm{Ns}}\!/_{\mathrm{m}}$$

Approximate net damping:

$$b_{net} \approx b_{solid} \| b_{perf} = \frac{b_{perf} b_{solid}}{b_{solid} + b_{perf}} \approx 0.01 \, {}^{\mathrm{Ns}}\!/_{\mathrm{m}}$$

Thermomechanical noise in the tunneling accelerometer

> **Estimate noise at room temperature**

$$S_n(f) = 4k_B Tb \approx 1.7\times10^{-22}\ \mathrm{N}^2/\mathrm{Hz}$$

$$\text{So noise is } 1.3\times10^{-11}\ \mathrm{N}/\sqrt{\mathrm{Hz}}$$

> **Given a proof mass of 30 mg, relate noise force to noise acceleration**

$$\text{Thermal noise in acceleration: } 4.4\times10^{-7}\ \mathrm{g's}/\sqrt{\mathrm{Hz}}$$

$$\text{With a 1 kHz bandwidth, this is } 1.4\ \mu\mathrm{g's sensitivity}$$

Thermal noise from the electronics?

> Electronics always contribute noise, and frequently are the dominant noise source

> Amplifiers have some resistive elements in them; hence they have thermal noise

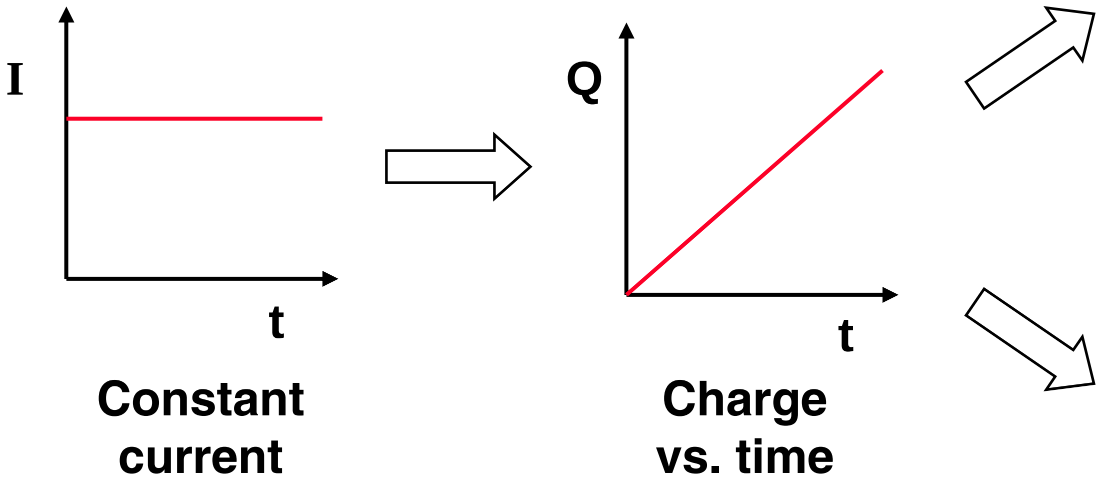
> Amplifiers have other kinds of noise, too, such as 1/f noise: stay tuned until we have the tools

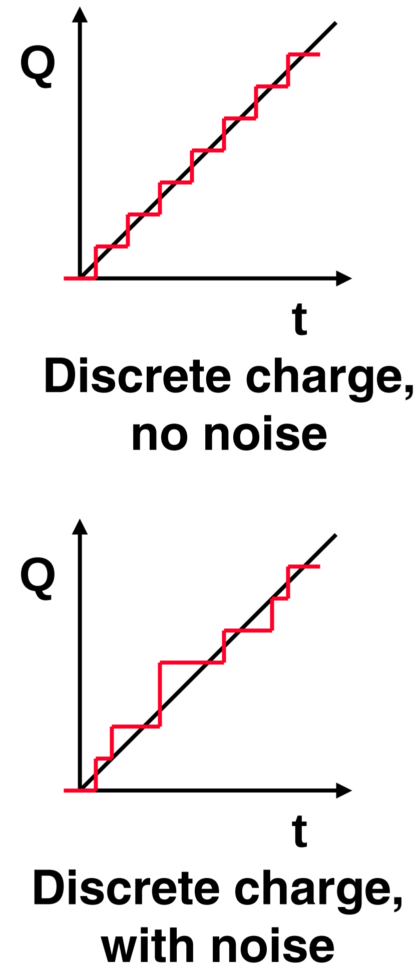
> Electronics noise turns out not to be dominant for the tunneling accelerometer

- Tunneling is exponentially sensitive: high change in current for small change in distance (x3 for 0.1 nm motion near operating point)
- High change in current for small acceleration
- Compare with small change in capacitance for 0.1 nm motion
- Here, electronics introduce noise when signal is already huge

Shot noise

- The granularity of the electronic charge leads to a noise associated with the flow of electric current





Shot noise

> **The spectral density function of shot noise is white (at least at all frequencies of interest)**

$$i(t) = I_{DC} + i_n(t)$$

$$\text{The shot-noise spectral density function is:}$$

$$S_i(f) = 2q_e I_{DC}$$

$$\overline{i_n^2} = \int_0^\infty S_i(f)\,df$$

Shot noise in the tunneling accelerometer

> **Linearize about operating point: output ~ 5 x 10^{-5} A/g**
- **Not grams… acceleration of gravity**
- **Current depends exponentially on tunneling distance**
- **Acceleration changes linearly with position**

> **Tunneling current ~ 1 nA**

$$S_i(f) = 2q_e I_{DC} \approx 3\times10^{-28}\ \mathrm{A}^2\!/\mathrm{Hz}$$

$$\text{Current noise is about } 2\times10^{-14}\ \mathrm{A}\!/\!\sqrt{\mathrm{Hz}}$$

$$\text{Acceleration noise is about } 4\times10^{-10}\ \mathrm{g's}\!/\!\sqrt{\mathrm{Hz}}$$

> **Thermomechanical noise is three orders of magnitude larger than shot noise!**

Flicker Noise (1/f noise)

- Noise with a $1/f^{\alpha}$ spectral density shows up in many systems and is caused by a wide variety of phenomena
  - Diodes, earthquakes, biological systems, material creep/relaxation, sand piles, …
- Although the exponent is not always exactly 1, it is generally called simply "1/f noise" or "flicker noise"
- Important point: 1/f noise shows up a low frequencies
- At higher frequencies, white noise dominates
- The main problem with 1/f noise is that it is often hard to model, except empirically. You may not know that you have a noise problem until you measure it.

1/f noise in the tunneling accelerometer

- The earlier tunneling accelerometer papers classify noise according to type: thermomechanical noise, shot noise, and "excess noise"
- 1/f tunneling noise
  - Migration of atoms on the surface of the tunneling tip
  - Mobility of adsorbed contaminant molecules
- 1/f mechanical noise
  - Package relaxation
  - Thermal creep
  - Thermal bimorph (metal over the silicon hinge of the proof mass)
- How to fix it? Through luck or design, find and control the factors that correlate with the noise.
- In this case, control fluctuations in environmental temperature

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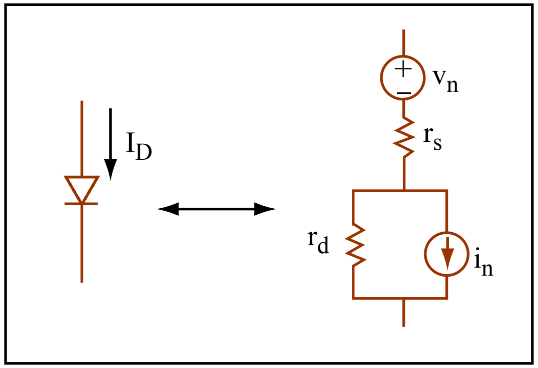
Noise in a Diode

- Charge carriers are trapped in and released from trap states in the semiconductor
- Trap and release times depends on energy of trap state
- A uniform-ish distribution of states leads to a 1/f-ish spectral density function



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r_s is incremental resistance of contacts and diode neutral regions.

r_d is incremental resistance of exponential diode.

$$\frac{1}{r_d} = \frac{dI_D}{dV_D} = \frac{q_e I_D}{k_B T}$$

There is normal Johnson noise associated with r_s

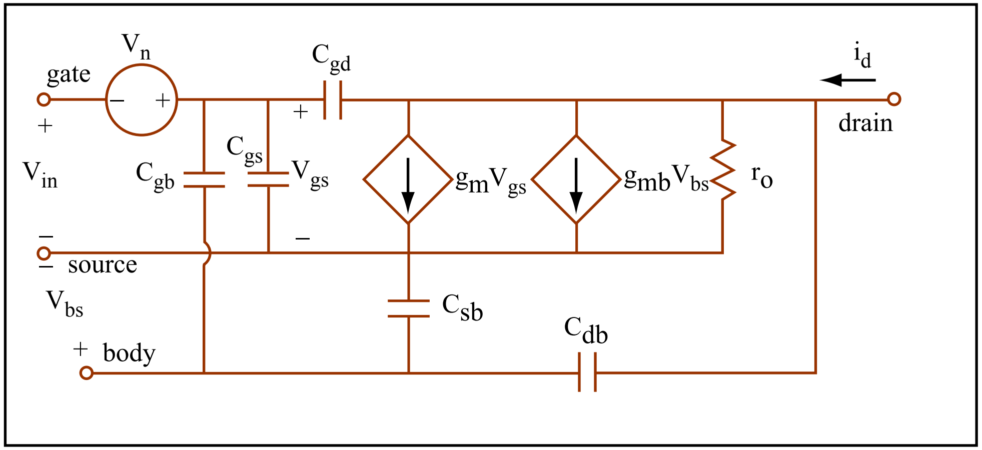
and a combined shot - noise flicker - noise spectrum associated with r_d

$$S_i(f) = 2q_e I_D + K\frac{(I_D)^a}{f}$$

The exponent a is in the range 0.5 - 2

Amplifier Noise

- MOSFETs have relatively high 1/f-noise corner frequencies
- One way to model the noise in a MOSFET is to refer all noise sources back to one equivalent input noise source
- Noise at first stage is most important, because it is amplified in successive stages



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Adapted from Figure 16.11 in Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 441. ISBN: 9780792372462.

$$S_n(f) = 4k_B T\left(\frac{2}{3g_m}\right)(1+F_n) + \frac{K_f}{WL\hat{C}_{ox}f}$$

Example: Resistance thermometer

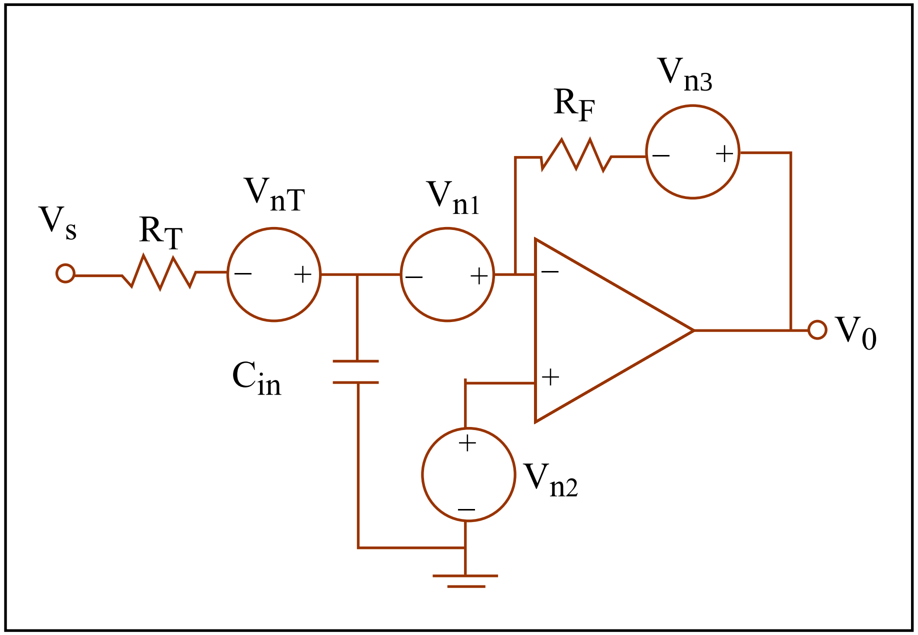
> **"Toy" resistance thermometer system**
- **Use thermometer as one of the resistors for an inverting amplifier**
- **Assume a metal film resistor: $\alpha \sim 3 \times 10^{-3}\ K^{-1}$**

> **We create a noise model for a resistance thermometer by adding**
- **Johnson noise source for the resistor itself**
- **Two input-referred amplifier noise sources for the op-amp**
- **Johnson noise source for the feedback resistor**

> **We calculate the transfer function for each noise source, and using its square, find the contribution to the total mean-square noise**

Transfer function of system

- Contains four terms, one for each noise source.
- We can calculate the minimum detectable temperature change by computing the root-mean-square noise



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Signal portion of output :

$$V_{0,s} = -\frac{R_F}{R_T} V_s$$

Incremental signal in response to a temperature change ΔT

$$v_{o,s} = \frac{R_F V_S \alpha_R}{R_T} \Delta T$$

Need to compare this to mean - square noise and, for that, we need the spectral density function

$$V_0 = -\frac{R_F}{R_T}\left(V_S + v_{nT} + v_{n1}\right) + \left(1 + \frac{R_F}{R_T}\right) v_{n2} + v_{n3}$$

Noise Spectral Density Function

- **We assume uncorrelated noise sources; hence, we can add spectral density functions**

$$S_{n.o}(f)=\left(\frac{R_F}{R_T}\right)^2\left[S_{n,T}(f)+S_{n,1}(f)\right]+\left(1+\frac{R_F}{R_T}\right)^2 S_{n,2}(f)+S_{n,3}(f)$$

$$\text{Assume } R_F = 100R_T \text{ and } R_T = 1\,\text{k}\Omega$$

$$\text{Input transistor noise is as follows:}$$

$$S_{n,1}=S_{n,2}=4k_bT\left(\frac{2}{3g_m}\right)+\frac{K_f}{WL\hat{C}_{ox}f}$$

$$\text{Resistor noise is as follows:}$$

$$S_{n,T}=4k_bTR_T$$

$$S_{n,F}=4k_bTR_F=400k_bTR_T$$

Noise Spectral Density Function

> **Noticing that R_F/R_T >> 1 simplies things**

$$S_{n.o}(f) = \left(\frac{R_F}{R_T}\right)^2 \left[S_{n,T}(f) + S_{n,1}(f)\right] + \left(1 + \frac{R_F}{R_T}\right)^2 S_{n,2}(f) + S_{n,3}(f)$$

For the FET, we assume $L = 2$ microns, $W = 30$ microns, $t_{ox} = 15$ nm

We assume an operating point for the input transistors of 100 microamps,

which sets g_m to 300 microSiemens (about 3 kΩ).

$$S_{n.o}(f) = 10^4 \left[4k_B T\left(R_0 + \frac{4}{3g_m} + \frac{R_0}{100}\right) + \frac{2K_f}{WL\hat{C}_{ox} f}\right]$$

$$S_{n.o}(f) = 8.9 \text{ x } 10^{-13} + \frac{7.2 \text{ x } 10^{-8}}{f}$$

Outline

> **Where does noise come from?**

> **Interference and how to deal with it**

> **Noise definitions and characterization**

> **Types of noise**
- **Thermal noise**
- **Shot noise**
- **Flicker noise**

> **Examples**
- **Electronics (diodes, amplifiers)**
- **Resistance thermometer**

> **Modulation**

Two methods of measuring temperature

> **Use a DC source:**

- **Result is that the minimum detectable temperature change is inversely proportional to the source voltage.**
- **But self-heating increases with source voltage**
- **There clearly will be an optimum value of source voltage to use**

> **Modulation (use an AC source followed by synchronous detection)**

- **Shifts the measurement to the white noise portion of the spectrum**
- **S/N ratio improves by a factor of 100 or so**

DC Source

> **Need to assume some finite bandwidth to filter the signal after amplification. We assume a single-pole filter, with noise bandwidth $1/4\tau_b$ (recall the noise bandwidth for the RC filter.)**

$$\overline{v_{n,o}^2} = \frac{8.9\times10^{-13}}{4\tau_b} + \int_{1/\hat{t}_m}^{\infty} \frac{7.2\times10^{-8}}{f\left[1+\left(2\pi f\tau_b\right)^2\right]} df$$

where the lower limit is set by the measurement time

$$\int_{1/\tau_m}^{\infty} \frac{1}{f\left[1+\left(2\pi f\tau_b\right)^2\right]} df = \frac{1}{2}\ln\left[1+\left(\frac{\hat{t}_m}{2\pi\tau_b}\right)\right] \approx \ln\left(\frac{\hat{t}_m}{2\pi\tau_b}\right)$$

For this example assume $\tau_b = 50$ ms, then $v_{o,rms} = 290\ \mu$V

Assuming $\alpha_R = 3\times10^{-3}$, the final sensitivity for unity S/N is

$$\Delta T_{min} = \frac{9.7\times10^{-4}}{V_S}$$

AC Source

- If we use an AC source, the signal appears within a bandwidth about the source frequency, and limits of integration of the spectral density function to twice the noise bandwidth about that frequency

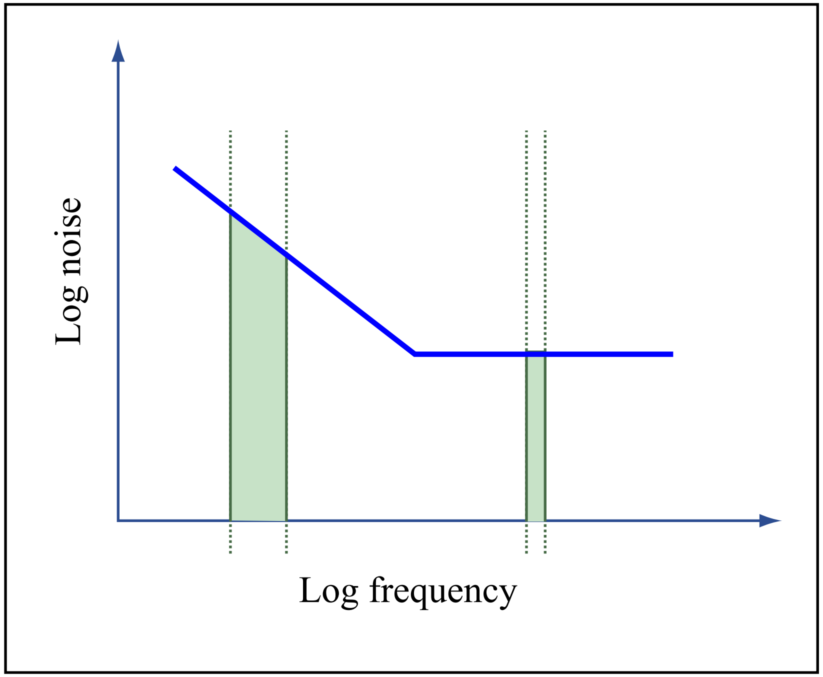

Image by MIT OpenCourseWare.

Same noise bandwidth, but lower spectral density at the higher carrier frequency

$$\overline{v_{o,n}^2} = \frac{S_n(f_o)}{2\tau_b} = 7.2\times 10^{-10}\ \ \mathrm{V}^2 \quad \text{which corresponds to } 27\ \mu\mathrm{V\ rms}$$

Modulation Doesn't Always Work

- **If the resistor itself has a fluctuation in value due, for example, to temperature fluctuations, these fluctuations always show up directly in the same band as the signal**
- **If these fluctuations have 1/f character, then even the modulated and then demodulated signal will show the 1/f spectrum**

$$R_T = R_0\left[1 + \eta(t) + \alpha_R \Delta T\right]$$

$$V_0(t) = -\left(\frac{R_F V_S}{R_0}\right)\frac{1}{\left[1 + \eta(t) + \alpha_R \Delta T\right]}$$

$$\Downarrow$$

$$V_0(t) \cong -\left(\frac{R_F V_S}{R_0}\right)\left[1 - \eta(t) - \alpha_R \Delta T\right]$$

Conclusions

- Many noise calculations are easily integrated into our existing approach to modeling, so you can (and should) consider noise constraints from the earliest stages of design
- The dominant noise sources vary widely from system to system, so don't make any assumptions
- 1/f noise is less easily dealt with in advance, but careful debugging and circuit design can help
- Working at a higher frequency (modulation) can sometimes buy you a lot a device sensitivity