Dissipation and
The Thermal Energy Domain Part I

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***(with thanks to SDS)**

Outline

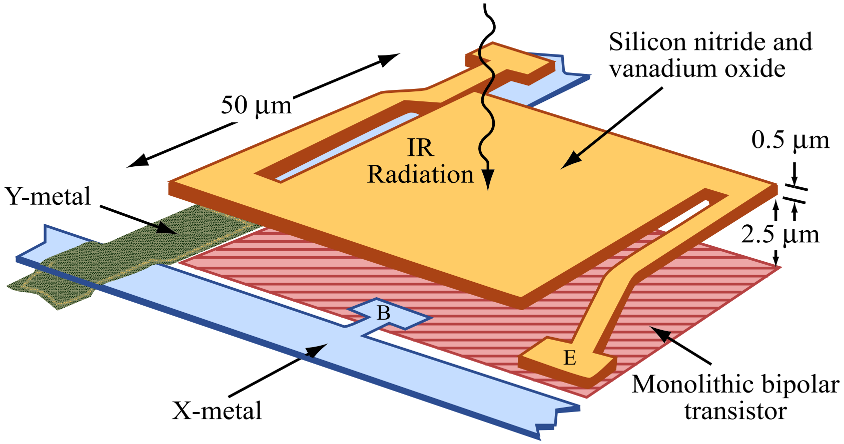
- Thermal energy domain: why we love it and why we hate it
- Dissipative processes
  - Example: Charging a capacitor through a resistor
- The Thermal Energy Domain
  - Governing equations
- Equivalent-circuit elements & the electrothermal transducer
- Modeling the bolometer

Thermal MEMS

- Everything is affected by temperature
- Therefore, anything can be detected or measured or actuated via the thermal domain
- Sometimes this is good…

MEMS Imagers

- A bolometer heats up due to incoming radiation
- This results in a temperature change that changes the resistance across the pixel



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Honeywell

Poor residual stress control

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Figure 1 on p. 55 in: Leonov, V. N., N. A. Perova, P. De Moor, B. Du Bois, C. Goessens, B. Grietens, A. Verbist, C. A. Van Hoof, and J. P. Vermeiren. "Micromachined Poly-SiGe Bolometer Arrays for Infrared Imaging and Spectroscopy." *Proceedings of SPIE Int Soc Opt Eng* 4945 (2003): 54-63.

Better design…

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Figure 2 on p. 56 in: Leonov, V. N., N. A. Perova, P. De Moor, B. Du Bois, C. Goessens, B. Grietens, A. Verbist, C. A. Van Hoof, and J. P. Vermeiren. "Micromachined Poly-SiGe Bolometer Arrays for Infrared Imaging and Spectroscopy." *Proceedings of SPIE Int Soc Opt Eng* 4945 (2003): 54-63.

MEMS Imagers

> **This application illustrates key features of thermal MEMS**

- **Excellent thermal isolation creates excellent sensitivity**
  - » **Response is proportional to thermal resistance**
- **Low thermal mass creates fast response time**
  - » **Response time is proportional to thermal capacitance**
- **Easy integration with sense electronics**

Thermal flow sensing

> A time-of-flight flowrate sensor

> One resistor creates a heat pulse

> A downstream resistor acts as a temperature sensor

> Time for heat pulse to drift downstream is inversely related to flowrate

> This example illustrates the important benefit of MEMS materials

- Large range in thermal conductivities
- From vacuum (~0) to metal (~100's W/m-K)

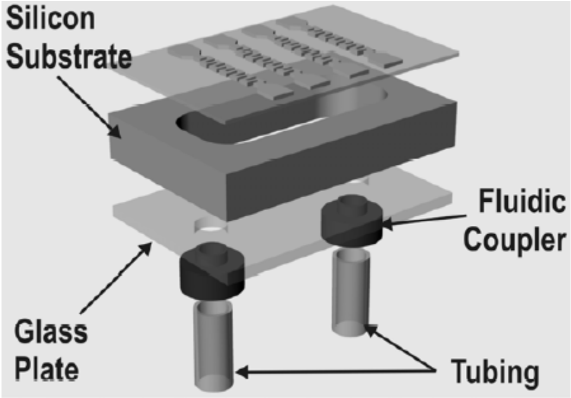


Figure 1 on p. 686 in: Meng, E., and Y.-C. Tai. "A Parylene MEMS Flow Sensing Array." *Technical Digest of Transducers'03: The 12th International Conference on Solid-State Sensors, Actuators, and Microsystems, Boston, June 9-12, 2003.* Vol. 1. Piscataway, NJ: IEEE Electron Devices Society, 2003, pp. 686-689. ISBN: 9780780377318. © 2003 IEEE.

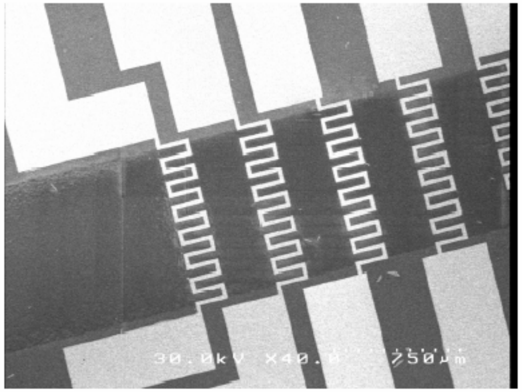
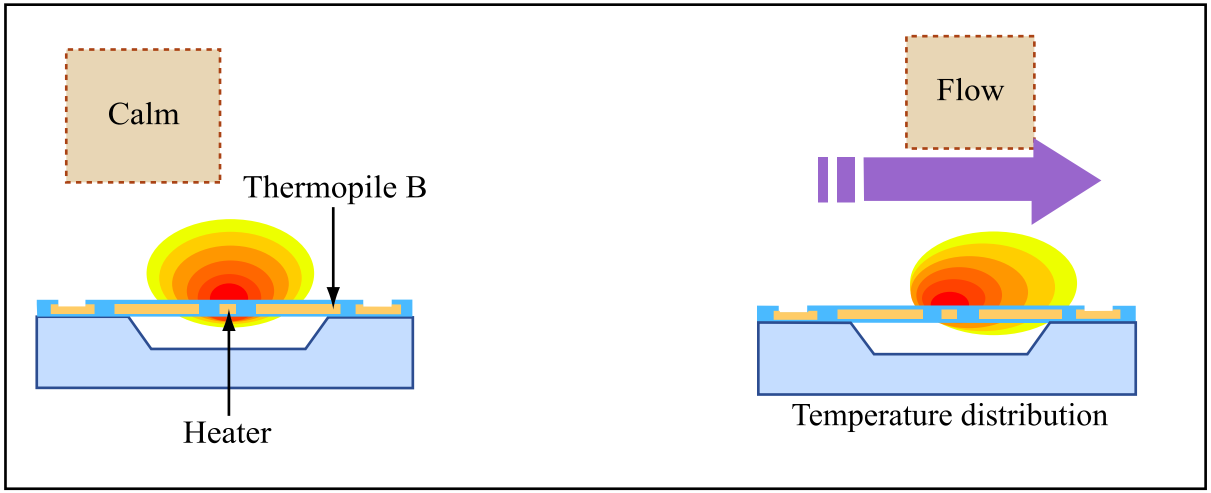


Figure 3 on p. 687 in: Meng, E., and Y.-C. Tai. "A Parylene MEMS Flow Sensing Array." *Technical Digest of Transducers'03: The 12th International Conference on Solid-State Sensors, Actuators, and Microsystems, Boston, June 9-12, 2003.* Vol. 1. Piscataway, NJ: IEEE Electron Devices Society, 2003, pp. 686-689. ISBN: 9780780377318. © 2003 IEEE.

MEMS flow sensing: commercial

- OMRON flow sensor uses measures temperature distribution around a heat source
- Convection alters temperature profile in a predictable fashion

Image removed due to copyright restrictions. OMRON flow sensor.



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The Thermal Domain

- Sometimes this is bad…
- Reason #1
  - Everything is a temperature sensor
  - Evaluation of MEMS devices over temperature is often critical to success
- Reason #2
  - As we will see, we can never transfer energy between domains perfectly
  - The "extra" energy goes into the thermal domain (e.g., heat)
  - We can never totally recover that heat energy

Academic

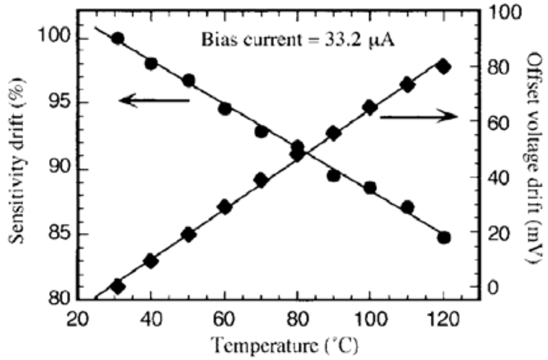
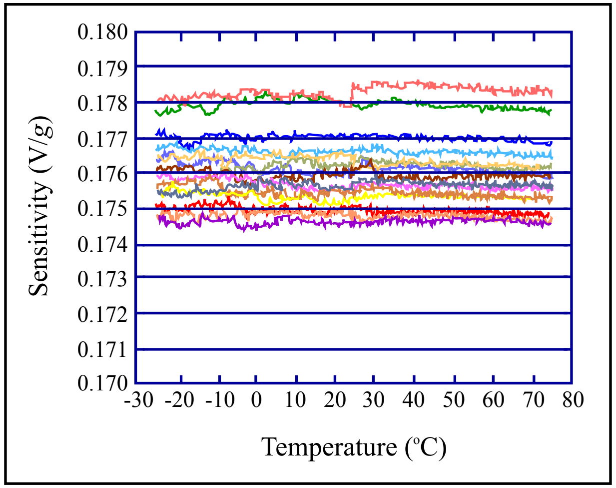


Figure 12 on p. 115 in: Takao, H., Y. Matsumoto, and M. Ishida. "A Monolithically Integrated Three-axis Accelerometer Using CMOS Compatible Stress-sensitive Differential Amplifiers." *IEEE Transactions on Electron Devices* 46, no. 1 (1999): 109-116. © 1999 IEEE.

Commercial



ADI ADXL320

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Charging a capacitor

> We will transfer energy from a power supply to a capacitor

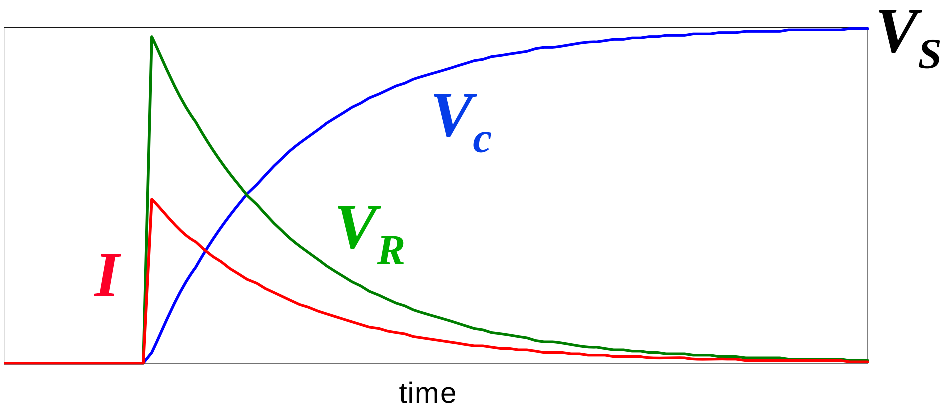
> Ideally, all energy delivered from supply goes to capacitor

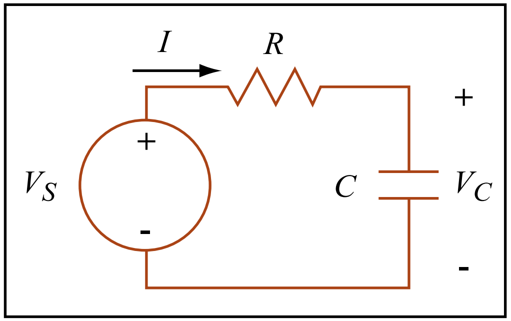
> In actuality, there is ALWAYS dissipation
  - And this is true for ALL domains

> Thus, we will lose some energy

Example: Charging a Capacitor

- Use step input
- Voltage source must supply twice the amount of energy as goes into the capacitor
- One half the energy is dissipated in the resistor, ***independent of the value of R!***





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$$V_C = V_S\left(1 - e^{-t/RC}\right)$$

$$I = C\frac{dV_C}{dt} = \frac{V_S}{R}e^{-t/RC}$$

Energy stored in capacitor: $W_C = \frac{1}{2}CV_S^2$

Energy delivered by power supply:

$$P_S(t) = IV_s = \frac{V_S^2}{R}e^{-t/RC}$$

$$W_s = \int_0^\infty P_s dt = CV_S^2$$

Power Considerations: Joule Heating

- The extra energy is lost to Joule heating in the resistor
- **Globally**, the power entering a resistor is given by the *IV* product.
- **Locally**, there is power dissipation given by the product of the charge flux and the electric field.

$$P_R = IV_R = I^2 R = \frac{V_R^2}{R}$$

For normal "positive" resistors

$$P_R \geq 0$$

This means [ ]/m³

$$\tilde{P}_R = J_e \mathbf{E} = \sigma_e \mathbf{E}^2$$

For normal positive conductivity

$$\tilde{P}_R \geq 0$$

and, one is the integral of the other

$$P_R = \iiint_{Volume} \tilde{P}_R d^3 r$$

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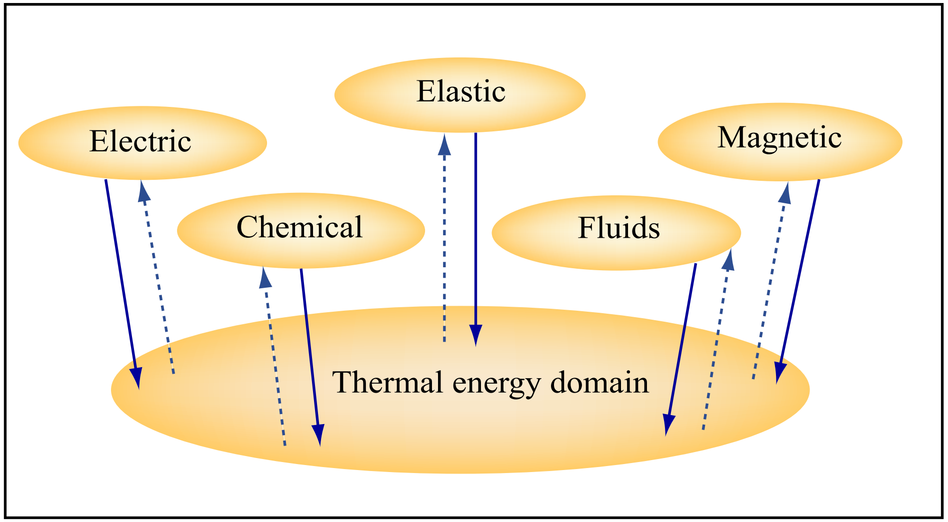
Thermal Energy Domain

- It is easier to put ENERGY INTO than get WORK OUT of thermal domain
- All domains are linked to thermal domain via dissipation
- Thermal domain is linked to all domains because temperature affects constitutive properties



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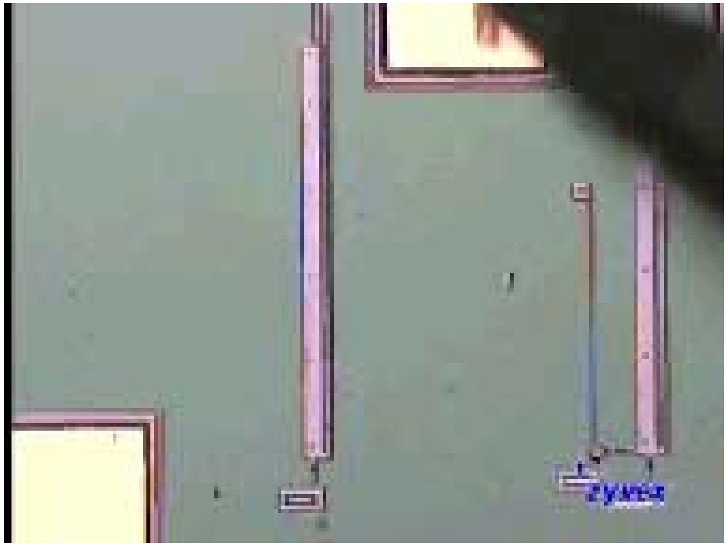
Adapted from Figure 11.2 in Senturia, Stephen D. *Microsystem Design.* Boston, MA: Kluwer Academic Publishers, 2001, p. 271. ISBN: 9780792372462.

Thermal Energy Domain

- Heat engines convert heat into mechanical work, but not perfectly efficiently, just like the charging of a capacitor cannot be done perfectly efficiently
- This is a statement of irreversibility: the 2^{nd} Law of Thermodynamics
- Zyvex heatuator
  - One skinny leg and one fat leg
  - Run a current and skinny leg will heat up
  - Structure will bend in response



Electrical energy

↓

Thermal energy

↓ Lost to thermal reservoir

Strain energy and deflection

Governing Equations

- Some introductory notation
- Be careful with units and normalizations

Q **Thermal energy [J]** $\quad \frac{X}{\mathcal{V}} = \tilde{X}$

$\tilde{Q}$ **Thermal energy/volume [J/m³]**

$\dot{Q} = I_Q$ **Heat flow [W]**

J_Q **Heat flux [W/m²]**

$$C_V = \left.\frac{\partial Q}{\partial T}\right|_{Volume}$$ **Heat capacity at constant volume (J/K)**

$$C_P = \left.\frac{\partial Q}{\partial T}\right|_{Pressure}$$ **Heat capacity at constant pressure (J/K)**

$$C_V = C_P = C$$ **Are the same for incompressible materials**

$$\tilde{C} = C / \mathcal{V}$$ **Heat capacity/unit volume (J/K-m³)**

$$\tilde{C}_m = \tilde{C} / \rho_m$$ **Heat capacity/unit mass (J/K-kg) AKA specific heat**

Governing Equations

- **Like many domains, conservation of energy leads to a continuity equation for thermal energy**

$$\frac{d}{dt}\begin{pmatrix}\text{stuff in}\\\text{volume}\end{pmatrix} = \begin{pmatrix}\text{net stuff}\\\text{entering}\\\text{volume}\end{pmatrix} + \begin{pmatrix}\text{generation}\\\text{of stuff in}\\\text{volume}\end{pmatrix}$$

$$\frac{d}{dt}\int b dV = -\int \mathbf{F}\cdot\mathbf{n} dS + \int g dV$$

Get point relation

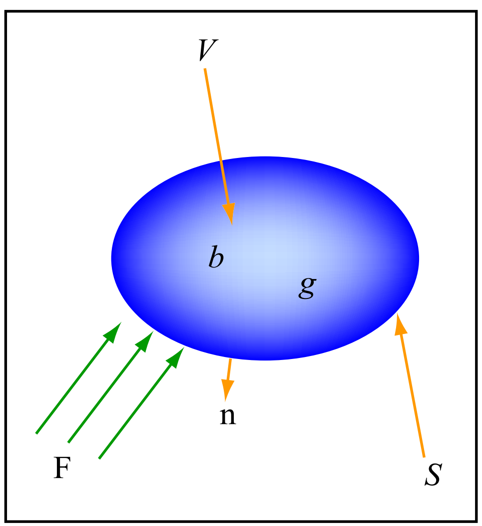
Divergence theorem and bring derivative inside

$$\int \frac{\partial b}{\partial t} dV = -\int \nabla\cdot\mathbf{F} dV + \int g dV$$

$$\frac{\partial b}{\partial t} = -\nabla\cdot\mathbf{F} + g$$

For heat transfer

$$\frac{d\tilde{Q}}{dt} + \nabla\cdot J_Q = \tilde{P}\Big|_{sources}$$



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Types of Heat Flow

> **Flow proportional to a temperature gradient**

- **Heat conduction**

$$J_Q = -\kappa \nabla T$$

> **Convective heat transfer**

- **A subject coupling heat transfer to fluid mechanics**
- **Often not important for MEMS, but sometimes is…**
- **Talk more about this later**

$$J_Q = h_c \left(T_2 - T_1\right)$$

> **Radiative heat transfer**

- **Between two bodies (at T_1 and T_2)**
- **Stefan-Boltzmann Law**
- **Can NEVER turn off**

$$J_Q = \sigma_{SB} F_{12} \left(T_2^4 - T_1^4\right)$$

The Heat-Flow Equation

- If we assume linear heat conduction, we are led to the heat-flow equation

$$J_Q = -\kappa \nabla T$$

$$\Downarrow$$

$$\nabla \cdot J_Q = -\nabla \cdot \left(\kappa \nabla T \right)$$

$$\Downarrow$$

$$\frac{\partial \tilde{Q}}{\partial t} = \nabla \cdot \left(\kappa \nabla T \right) + \tilde{P}_{sources}$$

For homogeneous materials, with $\frac{d\tilde{Q}}{dT} = \tilde{C}$

$$\frac{\partial T}{\partial t} = \frac{\kappa}{\tilde{C}} \nabla^2 T + \frac{1}{\tilde{C}} \tilde{P} \Big|_{sources}$$

Thermodynamic Realities

- **The First law of thermodynamics implies that entropy is a generalized displacement, and temperature is a generalized effort; their product is energy**
- **The Second Law states that entropy production is ≥0 for any process**
  - **In practice it always increases**
- **Entropy is not a conserved quantity….**
- **Thus, it does not make for a good generalized variable**
- **Therefore, we use a new convention, the thermal modeling convention, with temperature as effort and heat flow (power) as the flow. Note that the product of effort and flow is no longer power!!!**
  - **But heat energy (thermal displacement) is conserved**
  - **Just like charge (electrical displacement) is conserved**

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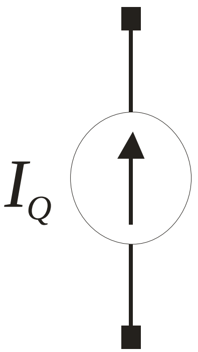
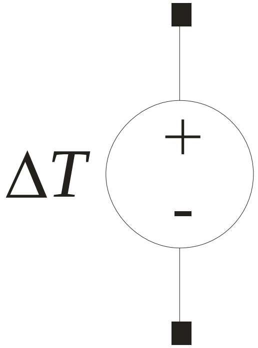
Thermal sources

- Heat current source I_Q is represented with a flow source
- Temperature difference source ΔT is represented with an effort source





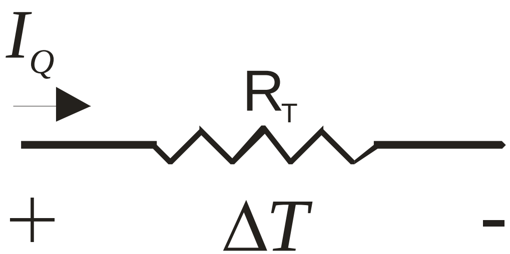
Thermal equivalent-circuit elements

> Use direct analogy again

Thermal		Electrical
Thermal conductivity (κ)		Electrical conductivity (σ_e)
$J_Q = -\kappa \nabla T$	**Relation between effort and flow**	$J_E = \sigma_e E = -\sigma_e \nabla V$
$\dfrac{\partial T}{\partial t} = \dfrac{\kappa}{\widetilde{C}} \nabla^2 T + \widetilde{P}\Big\rvert_{sources}$		
$\nabla^2 T = 0$	**Laplace's Eqn.**	$\nabla^2 V = 0$
$T_2 = T_1$	**Continuity of effort**	$V_2 = V_1$
No heat storage $\left(\mathbf{J}_Q \cdot \mathbf{n}\right)_1 = \left(\mathbf{J}_Q \cdot \mathbf{n}\right)_2$	**Continuity of flow**	$\left(\mathbf{J}_E \cdot \mathbf{n}\right)_1 = \left(\mathbf{J}_E \cdot \mathbf{n}\right)_2$ **No charge storage**

Thermal equivalent-circuit elements

> **Therefore we can derive a thermal resistance**



$$R_T = \frac{1}{\kappa}\frac{L}{A} \quad \text{[K/W]}$$

For bar of uniform cross-section

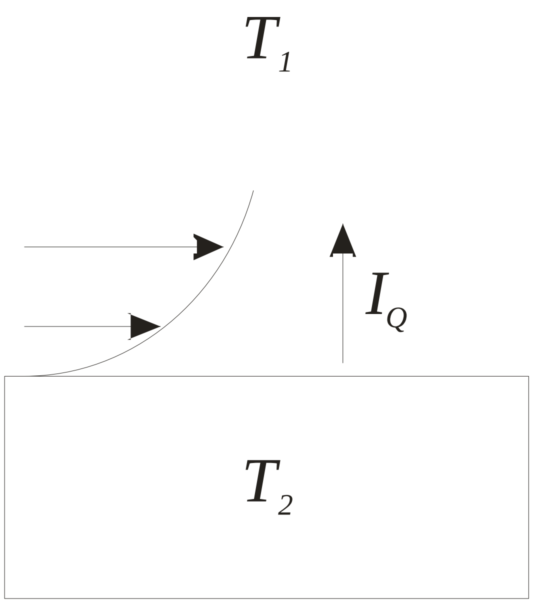
> **Plus, heat conduction and current flow obey the same differential equation**

> **Thus, we can use *exactly* the same solutions for thermal resistors as for electrical resistors**

- **Just change σ to κ**

Thermal equivalent-circuit elements

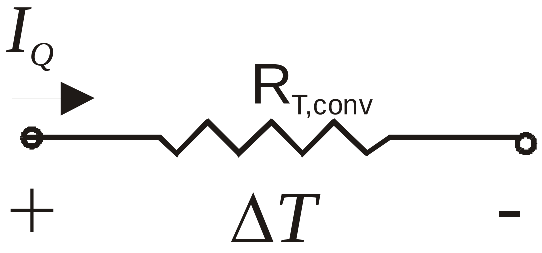
- What about other types of heat flow?
- Convection
- Use linear resistor



$$J_Q = h_c\left(T_2 - T_1\right)$$

$$I_Q = h_c A\left(T_2 - T_1\right) = h_c A \Delta T$$

$$R_{T,conv} = \frac{1}{h_c A}$$

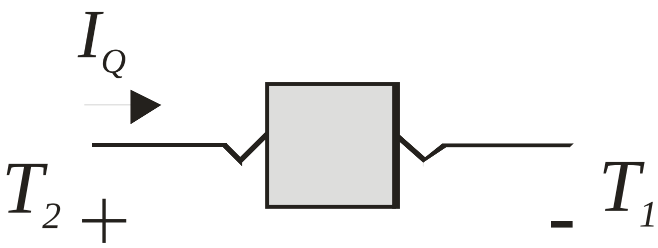


Thermal equivalent-circuit elements

> Radiation

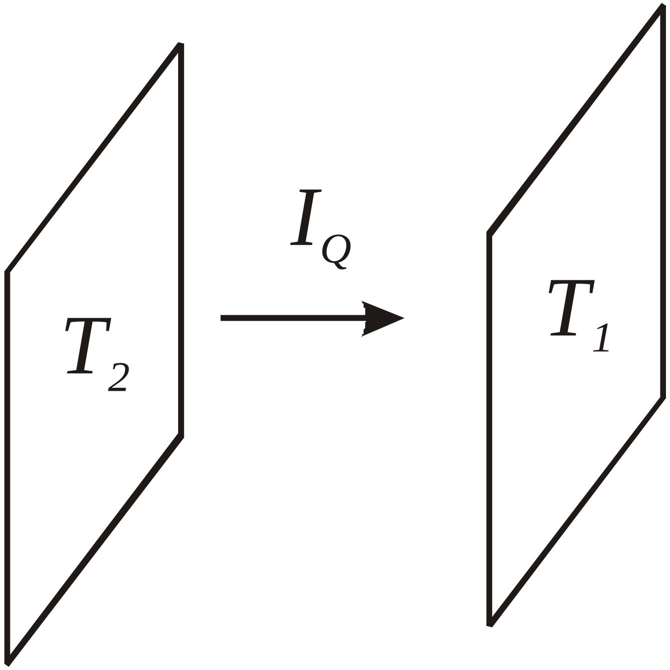
- **Nonlinear with temperature**

Large-signal model



$$J_Q = \sigma_{SB} F_{12} \left(T_2^4 - T_1^4 \right)$$

$$I_Q = \sigma_{SB} F_{12} A \left(T_2^4 - T_1^4 \right)$$



Thermal equivalent-circuit elements

> Radiation

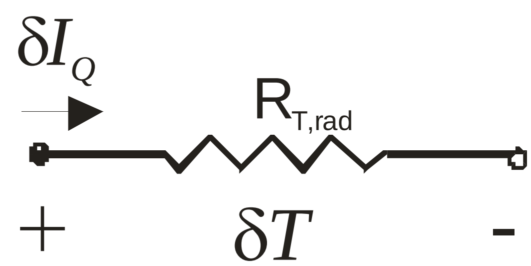
- **Many ways to linearize**
- **We show two approaches**

$$I_Q = \sigma_{SB} F_{12} A\left(T_2^4 - T_1^4\right)$$

$T_2 \gg T_1$

$$I_Q \approx \sigma_{SB} F_{12} A T_2^4$$

$$\delta I_Q = \left(4\sigma_{SB} F_{12} A T_2^3\right)\delta T$$



$T_2 \cong T_1$

$$I_Q \approx \sigma_{SB} F_{12} A\left(\left(T_1 + \delta T\right)^4 - T_1^4\right)$$

$$\delta I_Q = \left(4\sigma_{SB} F_{12} A T_1^3\right)\delta T$$

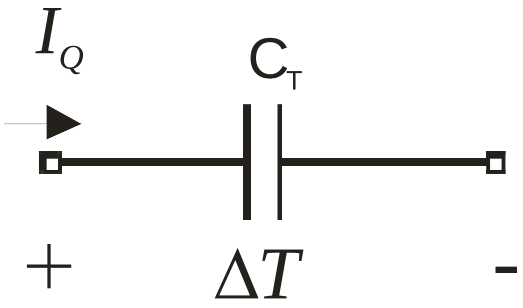
Thermal equivalent-circuit elements

- What about energy storage?
- Just like electrical capacitors store charge (Q),
- We can store thermal energy (Q)
- No thermal inductor ☹

$$C_E = \frac{\partial Q}{\partial V}$$

$$C_T = \frac{\partial Q}{\partial T}$$

$$C_T = \tilde{C}_m \rho_m \mathcal{V}$$



Electro-thermal transducer

- In electromechanical energy transduction, we introduced the two-port capacitor
  - Energy-storing element coupling the two domains
  - Capacitor because it stores potential energy
- What will we use to couple electrical energy into thermal energy?
  - In the electrical domain, this is due to Joule dissipation, a loss mechanism
    - Therefore it looks like an electrical resistor
  - In the thermal domain it looks like a heat *source*
    - Therefore it looks like a thermal current source

Electro-thermal transducer

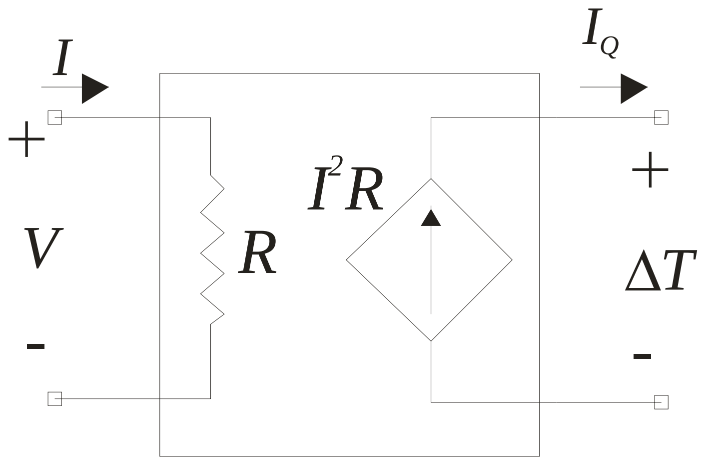
> **Our transducer is a resistor and a *dependent* current source**
- **The thermal current source *depends* on *R* and *I* in the electrical domain**

> **We reverse convention on direction of port variables in thermal domain**
- **OK, because $I_Q \cdot \Delta T$ does not track power**
- **Reflects fact that heat current will always be positive out of transducer**

> **This is not energy-conserving**
- **Dissipation is intrinsic to transducer**

> **This is not reciprocal**
- **Heat current does not cause a voltage**

> **Thermal domain can couple back to the electrical domain**
- **See next time…**



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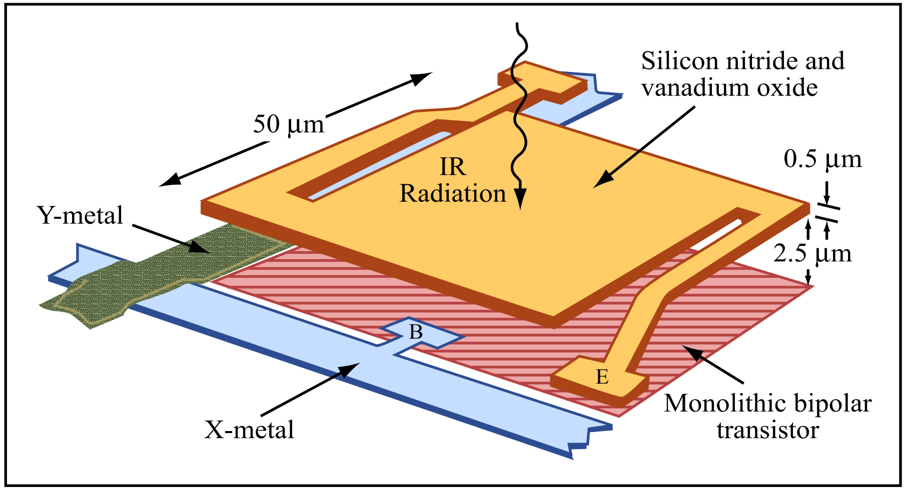
Different heat transfer components

- How does one know which heat transfer processes are important for thermal modeling?



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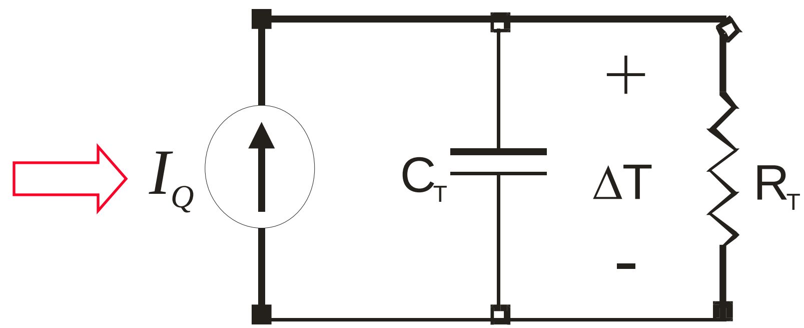


- Heat input is a current
- Heat loss is a resistor
- Heat storage in mass is a capacitor

$$\Delta T = I_Q\left(R_T // \frac{1}{C_T s}\right) = \frac{R_T I_Q}{1 + R_T C_T s}$$

$$\Delta T_{ss} = R_T I_Q$$

$$\tau = R_T C_T$$

Implications

> **Increasing R_T increases response**
  - **This means better thermal isolation**
  - **There is always some limiting value determined by radiation**

> **Given a fixed R_T, decreasing C_T improves response time**
  - **This means reducing the mass or volume of the system**

$$\Delta T_{ss} = R_T I_Q$$

$$\tau = R_T C_T$$

Thermal resistance

> What goes into R_T?

> R_T is the parallel combination of all loss terms

- Conduction through the air and legs
- Convection
- Radiation

> We can determine when different terms dominate

Conduction resistance

> Conduction

- **Si is too thermally conductive**
- **SiO_2 is compressively stressed**
- **Try SiN**

Material	κ (W/m-K)
Silicon	148
Silicon Nitride	20
Thermal Oxide	1.5
Air (1 atm)	0.03
Air (1 mtorr)	10^{-5}

$$R_{T,legs} = \frac{1}{2}\frac{1}{\kappa}\frac{L}{A}$$

$$= \frac{(50\ \mu\text{m})}{2(20\ \text{W/m-K})(0.5\ \mu\text{m})(5\ \mu\text{m})}$$

$$R_{T,legs} = 5 \cdot 10^5\ \text{K/W}$$

$$R_{T,air} = \frac{1}{\kappa}\frac{L}{A}$$

$$= \frac{(2.5\ \mu\text{m})}{(10^{-5}\ \text{W/m-K})(50\ \mu\text{m})(50\ \mu\text{m})}$$

$$R_{T,air} = 10^8\ \text{K/W}$$

Conduction through legs dominates

Other resistances

> **Convection**
- **At low pressure, there will be no air movement**
- **Convection will not exist**

> **Radiation**
- **There is transfer between plate and body**
- **Use case where bodies are close in temp**
- **Radiation negligible**

> **Very fast time constant**
- **~ms is typical for thermal MEMS**

$$R_{T,rad} = \frac{1}{4\sigma_{SB} F_{12} A T_1^3}$$

$$R_{T,rad} = \frac{1}{4\left(5.67\cdot 10^{-8}\ \mathrm{W/m^2K^4}\right)(0.5)(50\ \mu\mathrm{m})(50\ \mu\mathrm{m})(300\ \mathrm{K})^3}$$

$$R_{T,legs} = 1.3\cdot 10^{8}\ \mathrm{K/W}$$

<u>Leg conduction dominates loss</u>

$$\begin{aligned} C_T &= \tilde{C}_m \rho_m \mathcal{V} \\ &= \left(700\ \mathrm{J/kg\text{-}K}\right)\left(3000\ \mathrm{kg/m^3}\right)\left(50\ \mu\mathrm{m}\right)\left(50\ \mu\mathrm{m}\right)\left(0.5\ \mu\mathrm{m}\right) \\ &= 3\cdot 10^{-9}\ \mathrm{J/K} \end{aligned}$$

$$R_T C_T = 1.5\ \mathrm{ms}$$

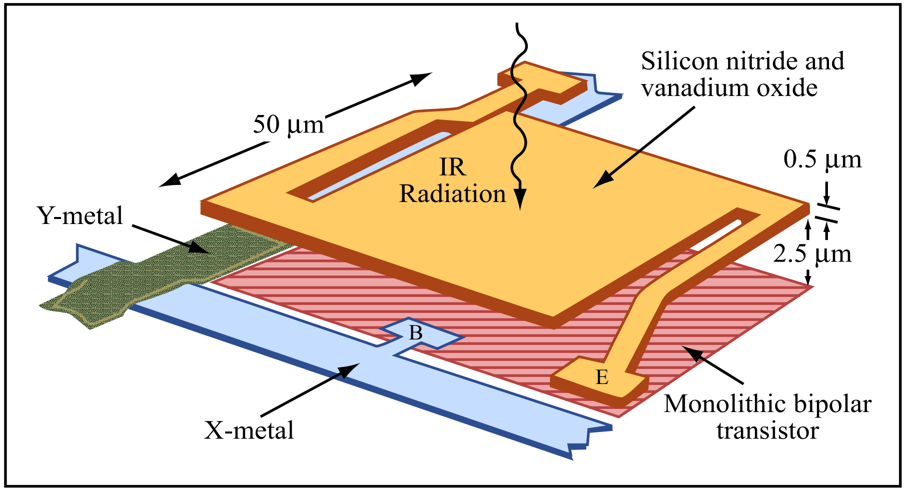
Improving the design

> **How can we make responsivity higher?**

> **Change materials**

> **Decrease thickness or width of legs, or increase length**

- **This reduces mechanical rigidity**



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Does convection ever matter?

- Usually not
- But it can come into play in microfluidics
- The key is whether energy is transported faster by fluid flow or heat conduction
- We'll analyze this a bit better after we do fluids

The Next Step

- When dealing with conservative systems, we found general modeling methods based on energy conservation
- With dissipative systems, we must always be coupled to the thermal energy domain, and must address time-dependence
- This is the topic for the next Lecture