Energy-conserving Transducers

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(*with thanks to SDS)

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Outline

- Last time
- The two-port capacitor as a model for energy-conserving transducers
- The transverse electrostatic actuator
- A look at pull-in
- Formulating state equations

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Last time: equivalent circuits

- Learned how to describe systems as lumped elements and equivalent circuits

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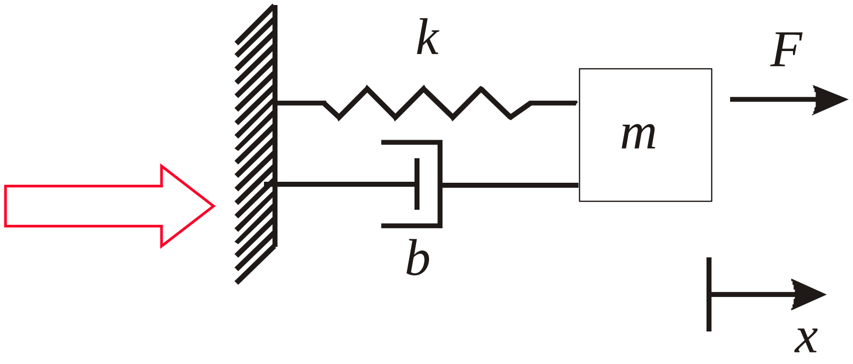

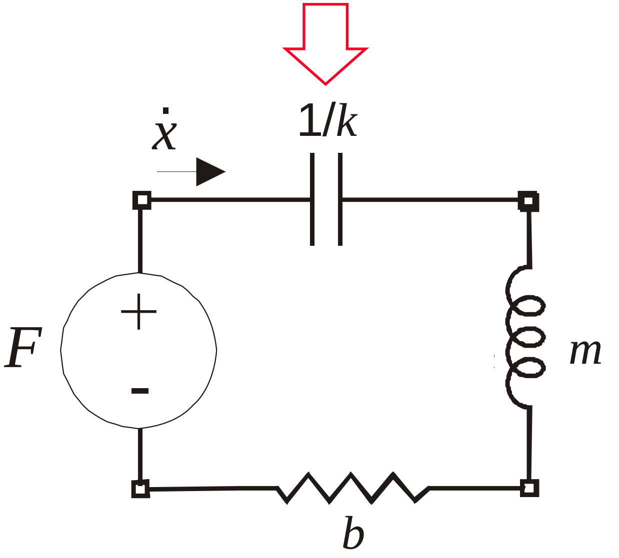

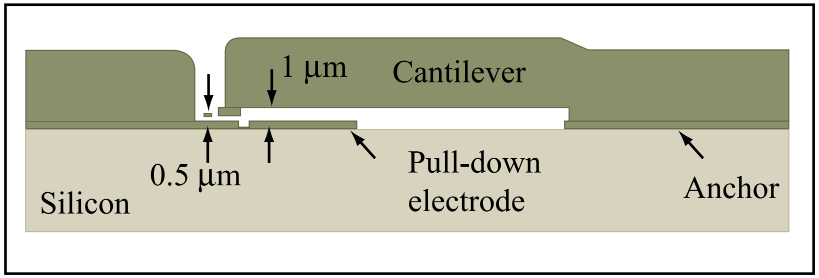

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Last time: equivalent circuits

- Saw that lumped elements in different domains all had equivalent circuits
- Introduced generalized notation to describe many different domains

$$e = \frac{dp}{dt} \qquad f = \frac{dq}{dt}$$

$$p = p_o + \int_0^t e dt \qquad q = q_o + \int_0^t f dt$$

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Equivalent circuit elements

General	Electrical	Mechanical	Fluidic	Thermal
Effort (e)	Voltage, V	Force, F	Pressure, P	Temp. diff., ΔT
Flow (f)	Current, I	Velocity, v	Vol. flow rate, Q	Heat flow, $\dot{Q}$
Displacement (q)	Charge, Q	Displacement, x	Volume, V	Heat, Q
Momentum (p)	-	Momentum, p	Pressure Momentum, Γ	-
Resistance	Resistor, R	Damper, b	Fluidic resistance, R	Thermal resistance, R
Capacitance	Capacitor, C	Spring, k	Fluid capacitance, C	Heat capacity, mcp
Inertance	Inductor, L	Mass, m	Inertance, M	-
Node law	KCL	Continuity of space	Mass conservation	Heat energy conservation
Mesh law	KVL	Newton's 2nd law	Pressure is relative	Temperature is relative

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Today's goal

- How do we model an electrical force applied to the cantilever?
- How can we describe converting energy between domains?
- This leads to energy-conserving transducers

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General Considerations

- In MEMS, we are often interested in sensors and actuators
- We can classify sensors and actuators by the way they handle energy:
  - Energy-conserving transducers
    - Examples: electrostatic, magnetostatic, and piezoelectric actuators
  - Transducers that use a dissipative effect
    - Examples: resistive or piezoresistive sensors
- There are fundamental reasons why these two classes must be treated differently.
  - Energy-conserving transducers depend only on the state variables that control energy storage. Therefore, quasi-static analysis is OK.
  - Dissipative transducers depend, in addition, on state variables that determine the rate of energy dissipation, and are more complex as a result.

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An Energy-Conserving Transducer

- **By definition, it dissipates no energy, hence contains no resistive elements in its representation**
- **Instead, it can store energy from different domains – this creates the transducer action**
- **Because the stored energy is potential energy, we use a capacitor to represent the element, but because there are both mechanical and electrical inputs, this must be a new element: a two-port capacitor**

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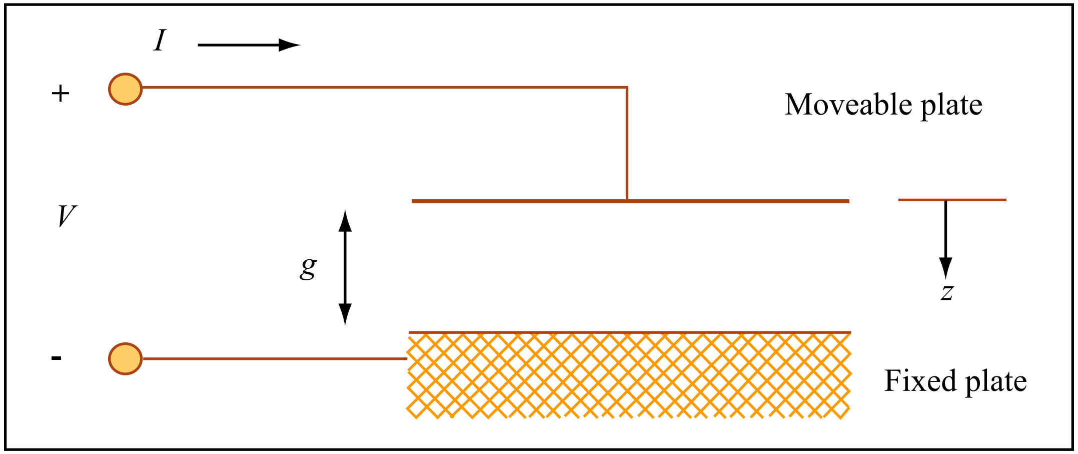
Capacitor with moveable plate

- A charged capacitor has a force of attraction between its two plates
- If one of the plates is moveable, one can make an electrostatic actuator.



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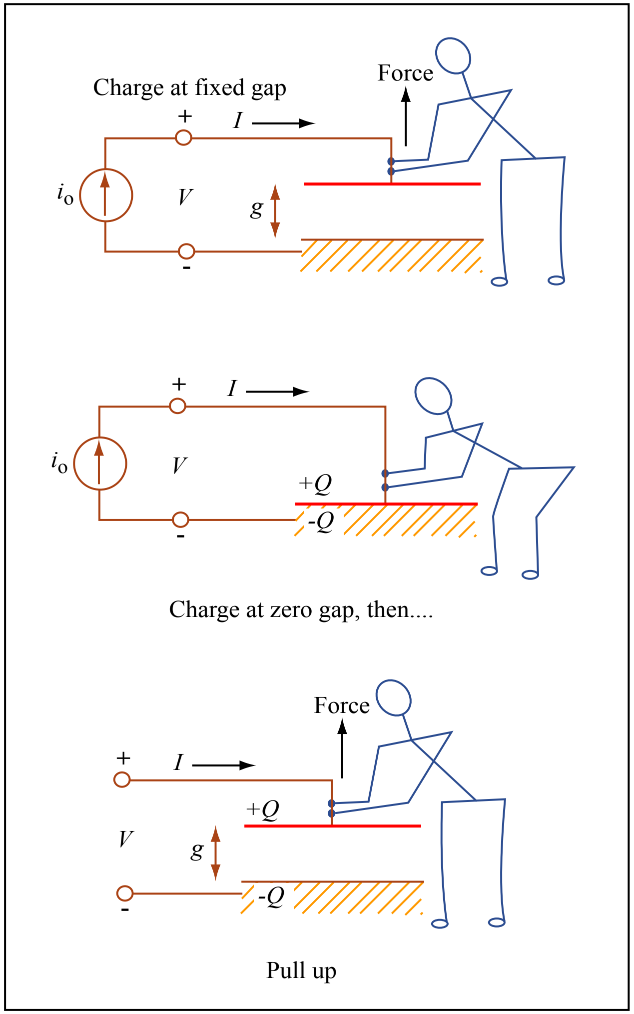
Various ways of charging

- Charging at fixed gap
  - An external force is required to prevent plate motion
  - No movement → No mechanical work
- Charging at zero gap, then lifting
  - No electrical energy at zero gap
  - Must do mechanical work to lift the plate
- Either method results in stored energy



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Charging at Fixed Gap

- The stored energy is obtained directly from the definition for a linear capacitor
- Anticipating that the gap might vary, we now explicitly include the gap as a variable that determines the stored energy

$$\boxed{\begin{matrix} e \rightarrow V \\ q \rightarrow Q \end{matrix}} \qquad \boxed{V = \frac{Q}{C}}$$

$$W = \int_0^q e\,dq = \int_0^Q V\,dQ = \int_0^Q \frac{Q}{C}\,dQ$$

$$W(Q,g) = \frac{Q^2}{2C} = \frac{Q^2 g}{2\varepsilon A}$$

$$\boxed{C = \frac{\varepsilon A}{g}}$$

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Pulling Up at Fixed Charge

- Putting charge at zero gap stores no electrical energy

$$\underset{g\to 0}{C} \to \infty \Rightarrow W = \frac{Q^2}{2C} \to 0$$

- Once charge is applied, determining stored energy is a mechanics problem.

- In determining the force, we must avoid double-counting of charge

E-field of bottom plate

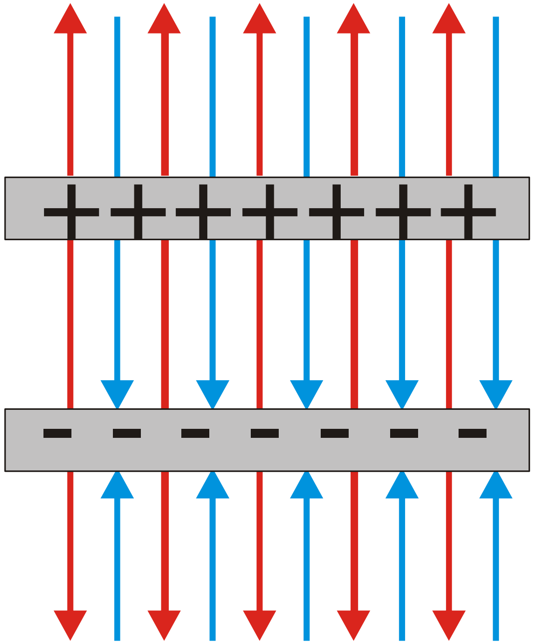
$$E = \frac{Q}{2\varepsilon A}$$

Q on top plate

$$F = QE = \frac{Q^2}{2\varepsilon A}$$

$$\boxed{W(Q,g) = \int_0^g F dg = \frac{Q^2 g}{2\varepsilon A}}$$

The final stored energy is same as before! ONLY depends on Q and g, not the path!



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Lossless transducers

> The energy in the system ONLY depends on the STATE variables (e.g., Q, g) and NOT how we put the energy in

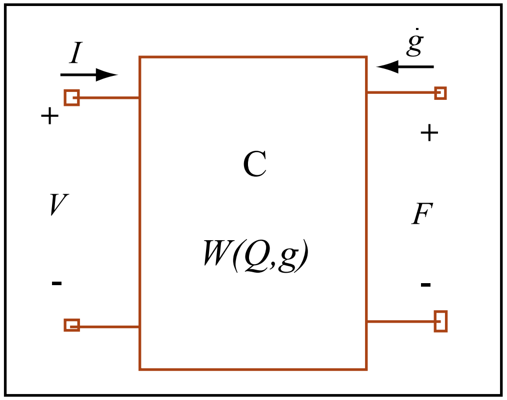
- **The system is lossless/conservative**

$$\frac{dW}{dt} = P_{electrical} + P_{mechanical}$$

$$= VI + F\dot{g}$$

$$\frac{dW}{dt} = V\frac{dQ}{dt} + F\frac{dg}{dt}$$

$$\boxed{dW = VdQ + Fdg}$$



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A Differential Version

- **Since we can modify the stored energy either by changing the charge or moving the plate, we can think of the stored energy as defined differentially**

$$dW = VdQ + Fdg$$

This leads to a pair of differential relations for the force and voltage

$$\boldsymbol{F = \left.\frac{\partial W(Q,g)}{\partial g}\right|_{Q}} \qquad \boldsymbol{V = \left.\frac{\partial W(Q,g)}{\partial Q}\right|_{g}}$$

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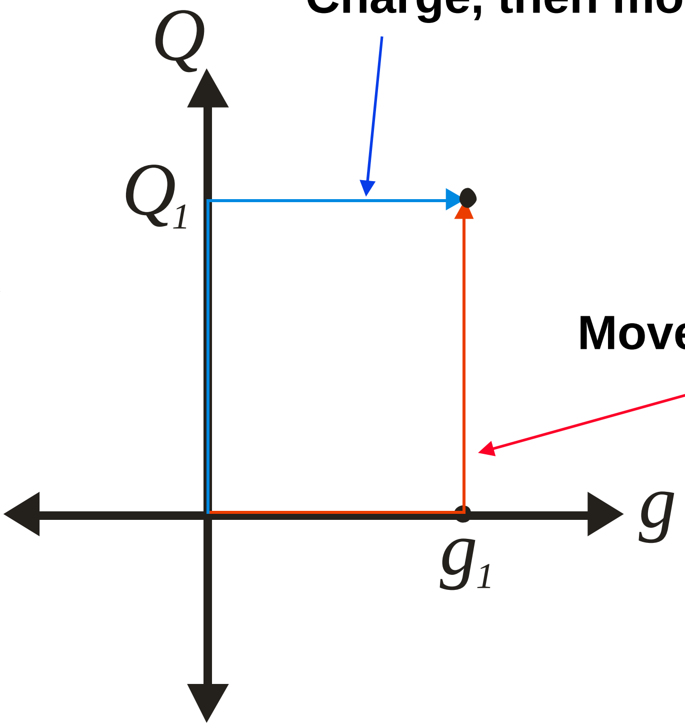
Revisit charging the capacitor

- **The energy only depends on Q, g**
  - **These are thus the STATE variables for this transducer**

$$W(Q,g)=\frac{Q^2 g}{2\varepsilon A}$$



Charge, then move plates

Move plates, then charge

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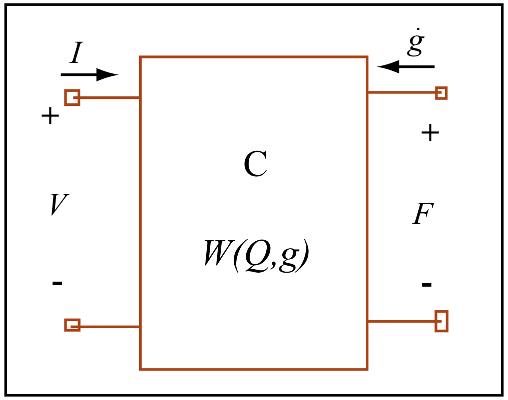
The two-port capacitor

- This transducer is what will couple our electrical domain to our mechanical domain



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$$W(Q,g) = \frac{Q^2 g}{2\varepsilon A}$$

$$V = \left.\frac{\partial W(Q,g)}{\partial Q}\right|_g = \frac{Qg}{\varepsilon A}$$

$$F = \left.\frac{\partial W(Q,g)}{\partial g}\right|_Q = \frac{Q^2}{2\varepsilon A}$$

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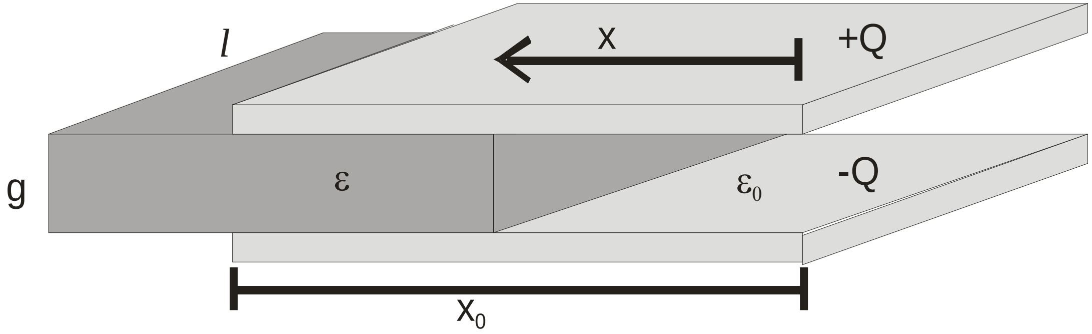
A different example

- What if the material in the gap could move?



$$W(Q,x)=\frac{Q^2}{2C}$$

$$C=\frac{l}{g}\left(\varepsilon_0 x+\varepsilon(x_0-x)\right)$$

$$F=\left.\frac{\partial W(Q,x)}{\partial x}\right|_Q=\frac{Q^2 g}{2l}\frac{\partial}{\partial x}\frac{1}{\left(\varepsilon_0 x+\varepsilon(x_0-x)\right)}$$

$$F=\frac{Q^2 g}{2l}\frac{\varepsilon-\varepsilon_0}{\left(\varepsilon_0 x+\varepsilon(x_0-x)\right)^2}$$

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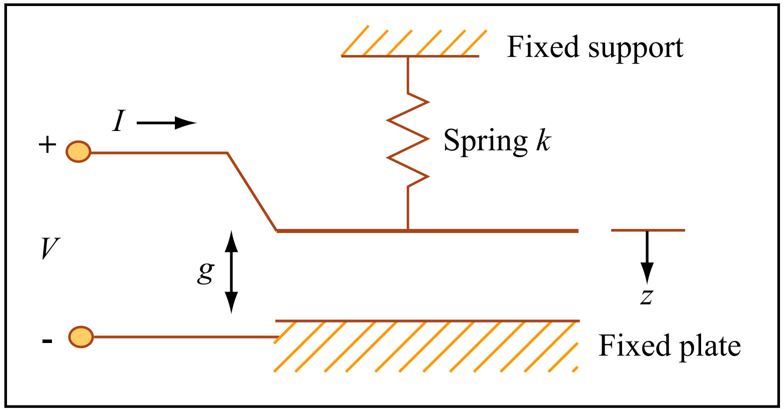
The Electrostatic Actuator

- If we now add a spring to the upper plate to supply the external mechanical force, a practical actuator results

- We are getting closer to our RF switch…



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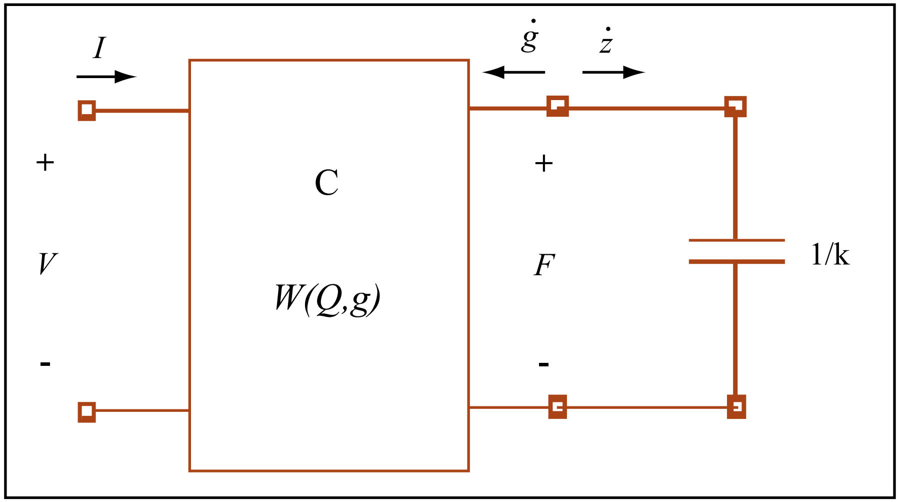

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Two methods of electrical control

> Charge control

- Capacitor is charged from a current source, specifically controlling the charge regardless of the motion of the plate
- This method is analyzed with the stored energy

> Voltage control

- Capacitor is charged from a voltage source, specifically controlling the voltage regardless of the motion of the plate
- This method is analyzed with the stored co-energy

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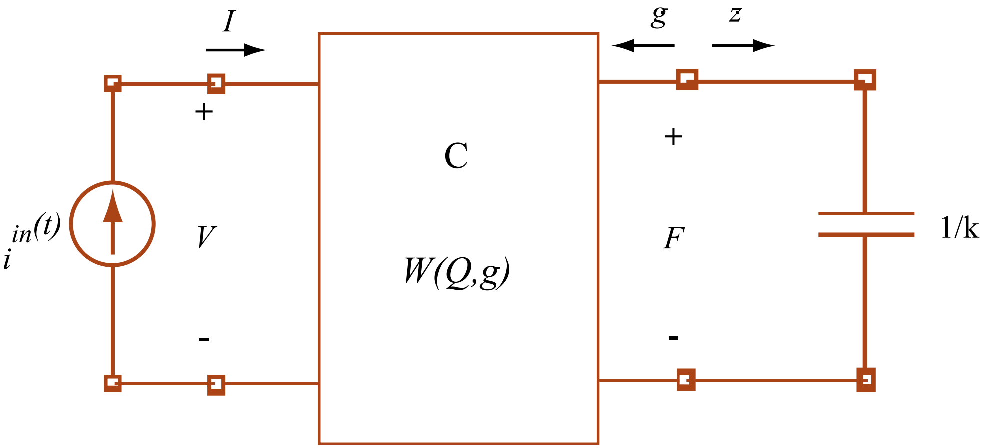
Charge control

- **Following the causal path**
  1. **Current source determines the charge**
  2. **Charge determines the force (at any gap!)**
  3. **Force determines the extension of the spring**
  4. **Extension of the spring determines the gap**
  5. **Charge and gap together determine the voltage**



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1) $$Q = \int_0^t i_{in}(t)dt$$

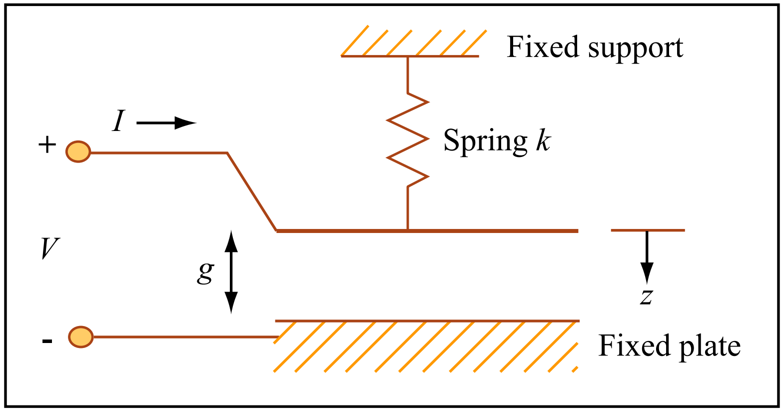
2) $$F = \left.\frac{\partial W}{\partial g}\right|_Q = \frac{Q^2}{2\varepsilon A}$$

3) $$z = \frac{F}{k}$$

initial displacement

4) $$g = g_0 - z$$

$$g = g_0 - \frac{Q^2}{2\varepsilon A k}$$



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Charge control

> **Let's get voltage, normalize and plot**

$$V = \left.\frac{\partial W}{\partial Q}\right|_g = \frac{Qg}{\varepsilon A} = \frac{Q\left(g_0 - \dfrac{Q^2}{2\varepsilon Ak}\right)}{\varepsilon A}$$

> **Normalize variables to make easier to plot**

- **First normalize V *and* Q to some nominal values**
- **Introduce ξ (normalized displacement) that goes from 0 (g=g_0) to 1 (g=0)**

$$v = V/V_0 \qquad q = Q/Q_0 \qquad \xi = z/g_0 = (g_0 - g)/g_0$$

- **Define Q_0 and V_0 using expression above**

$$V_0 = \frac{Q_0 g_0}{\varepsilon A} \qquad Q_0^2 = 2\varepsilon Akg_0$$

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Charge control

- Now, plug in to non-dimensionalize

$$V = \frac{Q\left(g_0 - \frac{Q^2}{2\varepsilon A k}\right)}{\varepsilon A}$$

$$V = \frac{(qQ_0)\left(g_0 - \frac{(qQ_0)^2}{2\varepsilon A k}\right)}{\varepsilon A} = \frac{(qQ_0)(g_0 - q^2 g_0)}{\varepsilon A}$$

$$V = \frac{Q_0 g_0}{\varepsilon A} q(1-q^2) \Rightarrow \boxed{v = q(1-q^2)}$$

$$\xi = 1 - g/g_0 = 1-(1-q^2) \Rightarrow \boxed{\xi = q^2}$$

- Now we get expressions relating voltage and displacement to charge

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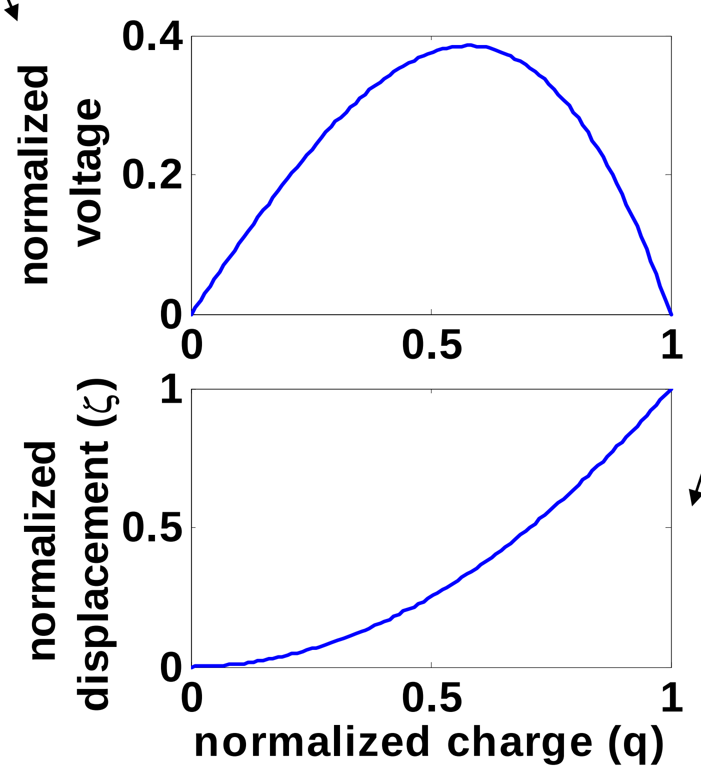
Charge control

- Actuator is stable at all gaps – the voltage goes to zero at zero gap
- The voltage is multivalued→ the charge uniquely determines the state and thus the energy

$$V = \left.\frac{\partial W}{\partial Q}\right|_g = \frac{Qg}{\varepsilon A} = \frac{Q\left(g_0 - \dfrac{Q^2}{2\varepsilon A k}\right)}{\varepsilon A}$$



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Co-Energy

- For voltage control, we cannot use *W(Q,g)* directly, because we cannot maintain constant charge. Instead we use the co-energy
    - So we change variables

Recall: $W^*(e_1) = q_1 e_1 - W(q_1)$

$$W^*(V,g) = QV - W(Q,g)$$

$$dW^*(V,g) = d\left(QV\right) - dW(Q,g)$$

$$dW^*(V,g) = \left[QdV + VdQ\right] - \left[VdQ + Fdg\right]$$

$$dW^*(V,g) = QdV - Fdg$$

$$\boxed{\begin{aligned} &\Rightarrow Q = \left.\frac{\partial W^*(V,g)}{\partial V}\right|_g \\ &\Rightarrow F = -\left.\frac{\partial W^*(V,g)}{\partial g}\right|_V \end{aligned}}$$

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Voltage control

- Following the causal path
  1. Voltage and gap (implicitly) determines the force
  2. Force determines the spring extension
  3. And thus the gap
  4. Voltage and gap together determine the charge

$$W^*(V_{in}, g) = \frac{1}{2} C V_{in}^2 = \frac{\varepsilon A}{2g} V_{in}^2$$

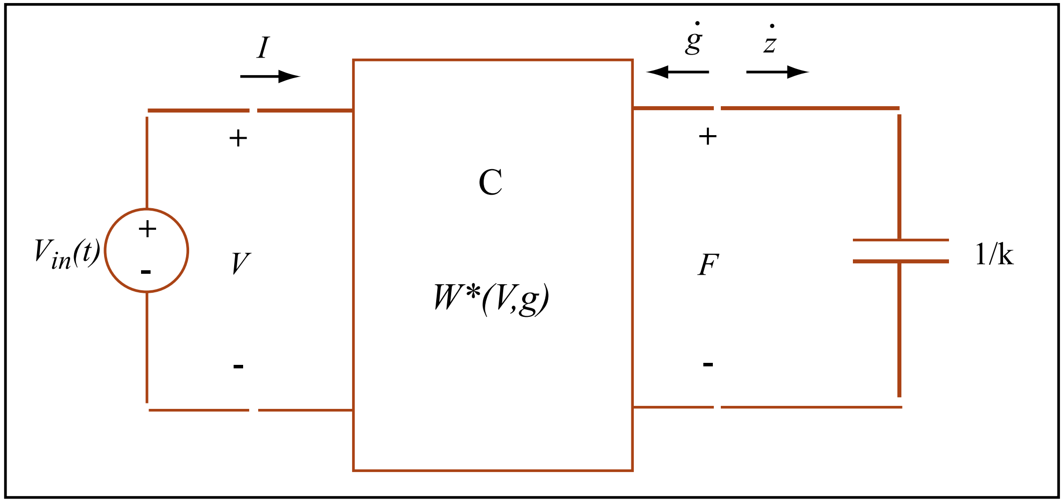
1) $F = -\left.\frac{\partial W^*}{\partial g}\right|_V = \frac{\varepsilon A V_{in}^2}{2g^2}$

2) $g = g_0 - z$

$$z = \frac{F}{k}$$

3) $g = g_0 - \frac{\varepsilon A V_{in}^2}{2kg^2}$

4) $Q = \frac{\varepsilon A}{g} V_{in} = C V_{in}$



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Forces and stability

- Let's examine the net force on the actuator

$$F_{Net} = F_{mech} - F_{elec} = 0$$

$$= k(g_0 - g) - \frac{\varepsilon A V^2}{2g^2} = 0$$

positive force increases gap

- Nondimensionalize again

$$\xi = (g_0 - g)/g_0$$

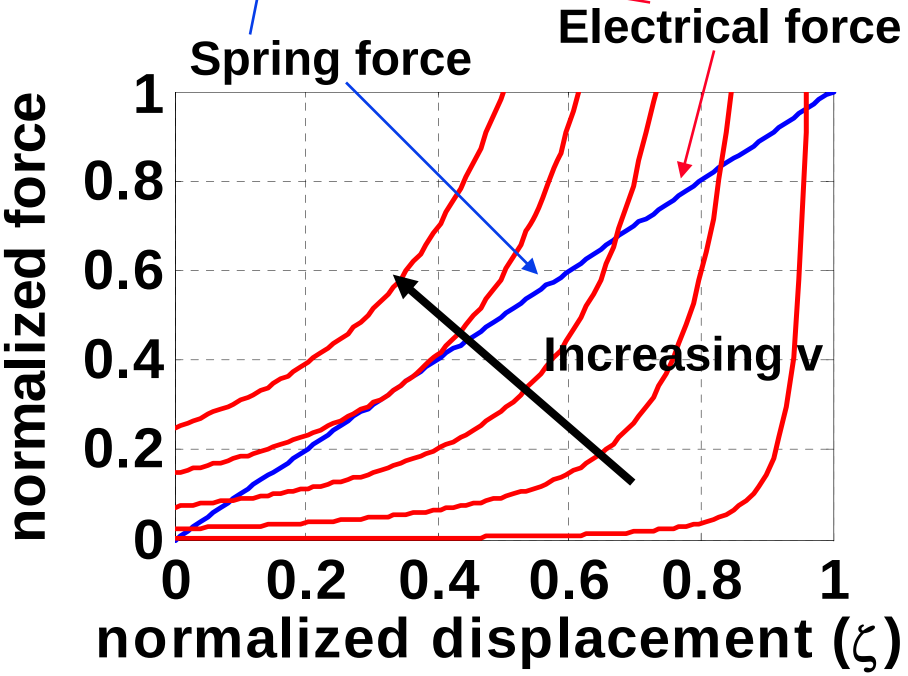
$$v = V / V_{PI}$$

$$V_{PI}^2 = 8kg_0^3 / 27\varepsilon A$$

$$F_{Net} = k\xi g_0 - \frac{\varepsilon A v^2}{2g^2}\frac{8kg_0^3}{27\varepsilon A} = 0$$

$$\xi - \frac{4v^2}{27}\frac{g_0^2}{g^2} = 0$$

$$\xi - \frac{4v^2}{27(1-\xi)^2} = 0$$



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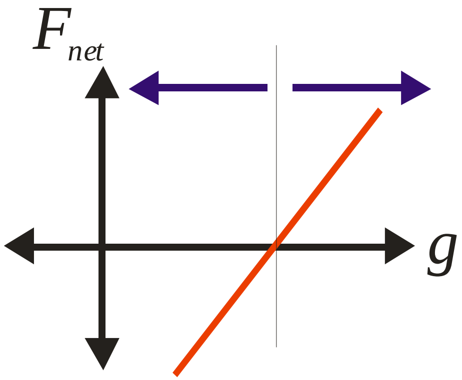
Stability criterion

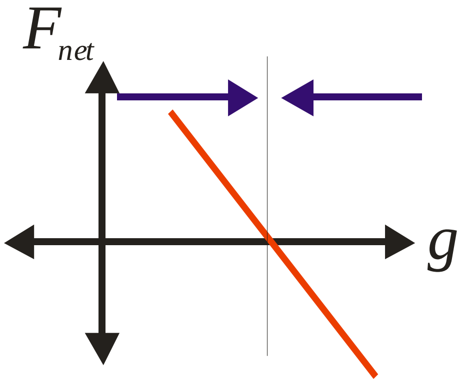
- At low voltage, there are two intersections
  - Which is stable?
- At higher voltages, there are none
  - What is happening?

The position of the actuator is stable only when there is a net restoring force when the system is disturbed from equilibrium

unstable



stable



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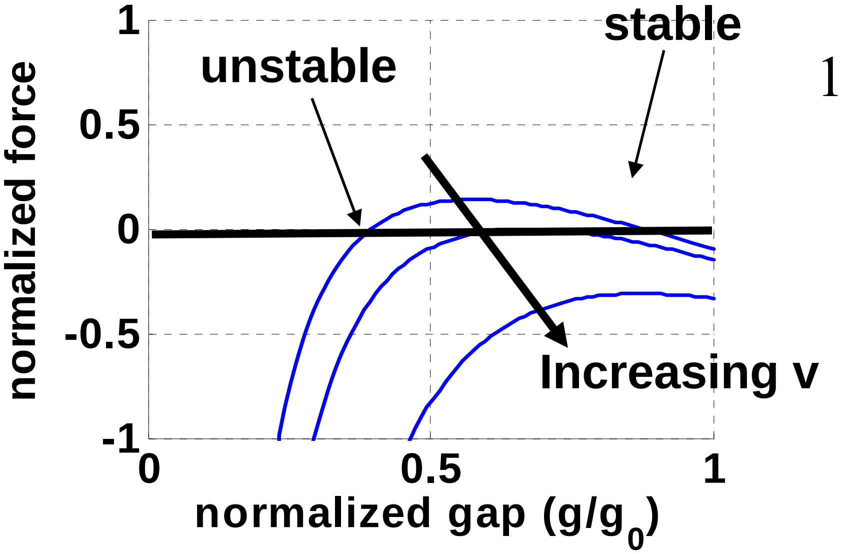
Stability criterion

- We can plot the normalized NET force versus normalized gap and check



$$F_{Net} = F_{mech} - F_{elec}$$

$$= k(g_0 - g) - \frac{\varepsilon A V^2}{2g^2}$$

$$1 - \xi = \frac{g}{g_0}$$

$$= \xi - \frac{4v^2}{27(1-\xi)^2}$$

$$\boxed{f_{net} = \left(\frac{g_0}{g} + 1\right) - \frac{4v^2}{27\left(\frac{g_0}{g}\right)^2}}$$

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Stability criterion

- So what we want is a negative slope
- In this example, this means that the spring constant must exceed a critical value that varies with voltage

$$F_{Net} = k(g_0 - g) - \frac{\varepsilon A V^2}{2g^2}$$

Stability:

$$\boxed{\begin{gathered}\frac{\partial F_{Net}}{\partial g} = \left(-k + \frac{\varepsilon A V^2}{g^3}\right) < 0 \\ \frac{\varepsilon A V^2}{g^3} < k\end{gathered}}$$

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Stability criterion

- If the voltage is too large, the system becomes unstable, and we encounter pull-in
- Right at pull-in, the spring constant is AT the critical value AND static equilibrium is maintained

At pull-in:

$$\boxed{\begin{gathered} k = \frac{\varepsilon A V_{PI}^2}{g_{PI}^3} \\ k(g_0 - g_{PI}) = \frac{\varepsilon A V_{PI}^2}{2 g_{PI}^2} \end{gathered}}$$

$$k(g_0 - g_{PI}) = \frac{k g_{PI}}{2}$$

$$\boxed{\begin{gathered} V_{PI} = \sqrt{\frac{8 k g_0^3}{27 \varepsilon A}} \\ \boxed{g_{PI} = \frac{2}{3} g_0} \end{gathered}}$$

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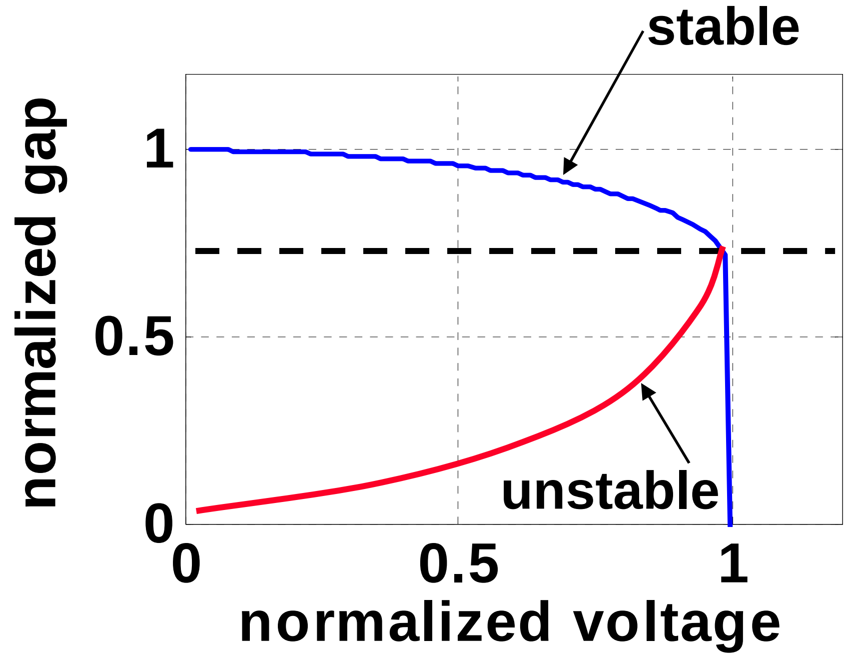
Stability analysis of pull-in

- Plot normalized gap versus normalized voltage

- Solve cubic equation

$$g = g_0 - \frac{\varepsilon A V_{in}^2}{2kg^2}$$



In Matlab: `g = fzero(@(g)(g - g0 + eps*A*V^2/(2*k*g^2)),g0);`

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Release voltage after pull-in

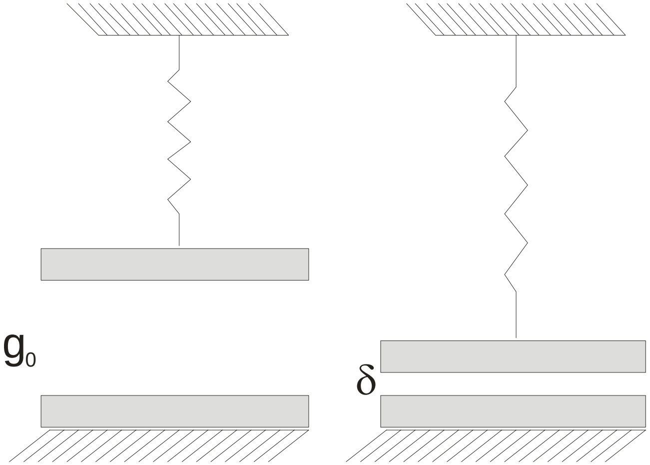
- After pull-in less voltage is needed to keep beam down
- Find force when pulled down
- Equate to mechanical force to get hold-down voltage
- Is usually much less than pull-in voltage

$$F_{elec}\Big|_{g=\delta} = \frac{\varepsilon A V_{in}^2}{2\delta^2}$$

$$F_{mech}\Big|_{g=\delta} = k\left(g_0 - \delta\right) \approx kg_0$$

$$\frac{\varepsilon A V_{HD}^2}{2\delta^2} = kg_0$$

$$V_{HD}^2 = \frac{2\delta^2 kg_0}{\varepsilon A}$$



Normalize to V_{PI}

$$V_{PI}^2 = \frac{8kg_0^3}{27\varepsilon A}$$

$$\boxed{\left(\frac{V_{HD}}{V_{PI}}\right)^2 = \frac{27}{4}\left(\frac{\delta}{g_0}\right)^2 < 1}$$

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Macro pull-in?

- Can we do a macroscopic pull-in demo?
- Use soft spring k = 1 N/m
- Use
  - A = 8.5” x 11” plates
  - g_0 = 1 cm

$$V_{PI} = \sqrt{\frac{8kg_0^3}{27\varepsilon A}}$$

$$= \sqrt{\frac{8(1)(0.01)^3}{27\left(8.85\times10^{-12}\right)\left(8.5\times11\times\left(0.0254\right)^2\right)}}$$

$$\approx 750 \text{ V}$$

- Not easy… this is why pull-in is a MEMS-specific phenomenon

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Outline

- Last time
- The two-port capacitor as a model for energy-conserving transducers
- The transverse electrostatic actuator
- A look at pull-in
- Formulating state equations

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Adding dynamics

- Add components to complete the system:
  - Source resistor for the voltage source
  - Inertial mass, dashpot
- This is now our RF switch!
- System is nonlinear, so we can't use Laplace to get transfer functions
- Instead, model with state equations

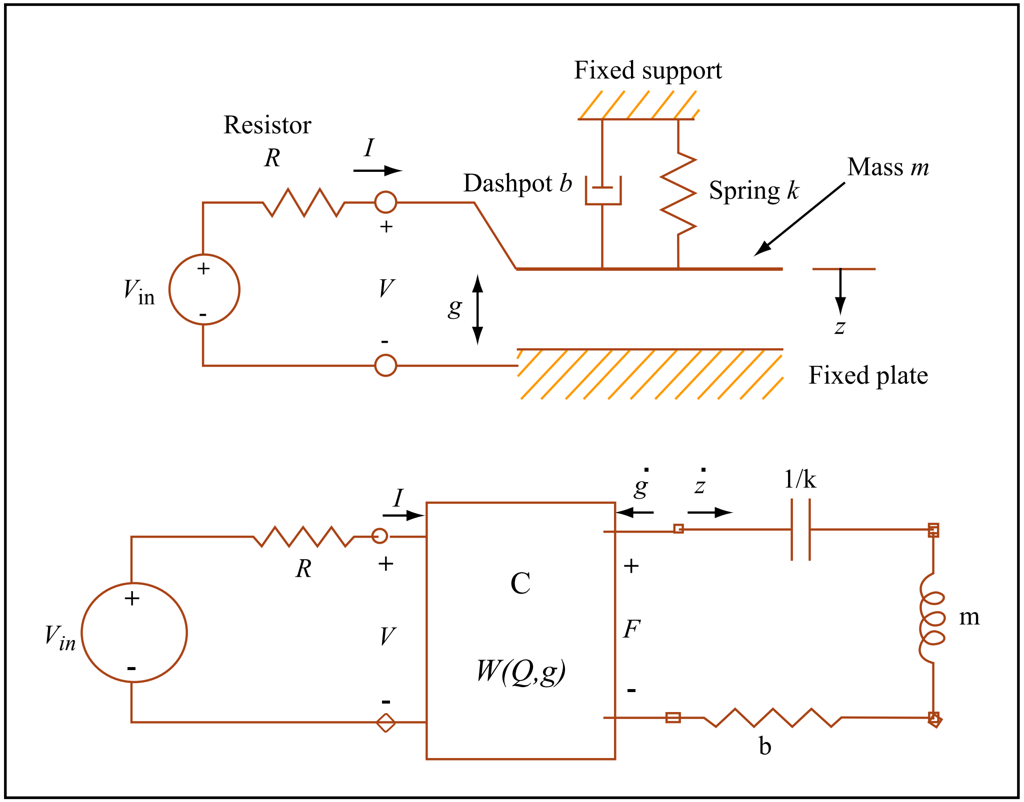

Image by MIT OpenCourseWare.

Adapted from Figure 6.9 in Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 138. ISBN: 9780792372462.

Electrical domain | Mechanical domain

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The System is Now General

- **The addition of the source resistor breaks up the distinction between voltage-controlled and charge-controlled actuation:**
  - **For small R, the system behaves like a voltage-controlled actuator**
  - **For large R, the system behaves like a charge-controlled actuator *at short times* because the "impedance" of the rest of the circuit is negligible ➔ the voltage source delivers a constant current V/R***

***See, for example, Castaner and Senturia, JMEMS, 8, 290 (1999)**

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State Equations

- Dynamic equations for general system (linear or nonlinear) can be formulated by solving equivalent circuit
- In general, there is one state variable for each independent energy-storage element (port)
- Good choices for state variables: the charge on a capacitor (displacement) and the current in an inductor (momentum)
- For electrostatic transducer, need three state variables
  - Two for transducer (Q,g)
  - One for mass (dg/dt)

Goal:

$$\frac{d}{dt}\begin{bmatrix} Q \\ g \\ \dot{g} \end{bmatrix} = \begin{pmatrix} \text{functions of} \\ Q, g, \dot{g} \text{ or constants} \end{pmatrix}$$

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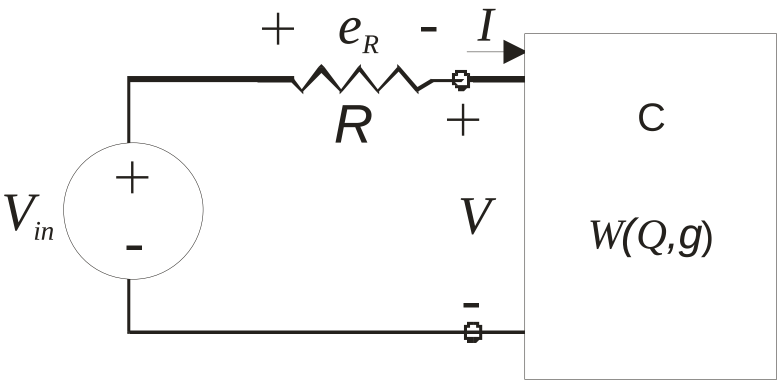
Formulating state equations

- Start with Q
- We know that $dQ/dt=I$
- Find relation between I and state variables and constants



$$\text{KVL}: V_{in} - e_R - V = 0$$

$$e_R = IR$$

$$V_{in} - IR - V = 0$$

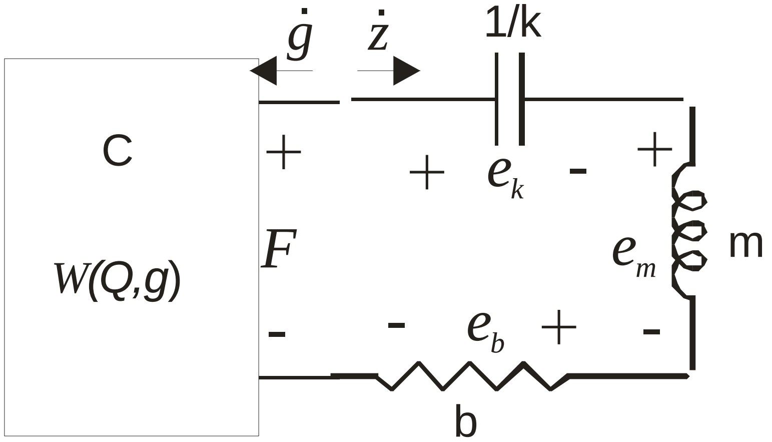
$$\frac{dQ}{dt} = I = \frac{1}{R}\left(V_{in} - V\right)$$

$$V = \frac{Qg}{\varepsilon A}$$

$$\boxed{\frac{dQ}{dt} = \frac{1}{R}\left(V_{in} - \frac{Qg}{\varepsilon A}\right)}$$

Formulating state equations

- Now we'll do $\dot{g}$
- We know that $d\dot{g}/dt = \ddot{g}$



$$\text{KVL}:$$
$$F - e_k - e_m - e_b = 0$$

$$F - kz - m\ddot{z} - b\dot{z} = 0$$

$$e_k = kz$$
$$e_m = m\ddot{z}$$
$$e_b = b\dot{z}$$

$$z = g_0 - g \Rightarrow \dot{z} = -\dot{g}, \ddot{z} = -\ddot{g}$$

$$F - k(g_0 - g) + m\ddot{g} + b\dot{g} = 0$$

$$\ddot{g} = -\frac{1}{m}\left[F - k(g_0 - g) + b\dot{g}\right]$$

$$\boxed{\frac{d\dot{g}}{dt} = -\frac{1}{m}\left[\frac{Q^2}{2\varepsilon A} - k(g_0 - g) + b\dot{g}\right]}$$

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Formulating state equations

- **State equation for g is easy:** $\boxed{\frac{dg}{dt} = \dot{g}}$
- **Collect all three nonlinear state equations**

$$\frac{d}{dt}\begin{bmatrix} Q \\ g \\ \dot{g} \end{bmatrix} = \begin{bmatrix} \frac{1}{R}\left(V_{in} - \frac{Qg}{\varepsilon A}\right) \\ \dot{g} \\ -\frac{1}{m}\left[\frac{Q^2}{2\varepsilon A} - k(g_0 - g) + b\dot{g}\right] \end{bmatrix}$$

- **Now we are ready to simulate dynamics (WED)**

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What have we wrought?

- We have modeled a complex multi-domain 3D structure using
  - Equivalent circuits
  - A two-port nonlinear capacitor

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- What can we now get
  - Actuation voltage: V_{PI}
  - Tip dynamics

Figure 9 on p. 17 in: Nguyen, C. T.-C. "Vibrating RF MEMS Overview: Applications to Wireless Communications." *Proceedings of SPIE Int Soc Opt Eng* 5715 (January 2005): 11-25.

Images removed due to copyright restrictions. Figure 11 on p. 342 in: Zavracky, P. M., N. E. McGruer, R. H. Morrison, and D. Potter. "Microswitches and Microrelays with a View Toward Microwave Applications." *International Journal of RF and Microwave Comput-Aided Engineering* 9, no. 4 (1999): 338-347.

- What have we lost
  - Capacitor plates are not really parallel during actuation
  - Neglected fringing fields
  - Neglected stiction forces when beam is pulled in

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Conclusions

- We can successfully model nonlinear transducers with a new element: the two-port capacitor
- Know when to use energy or co-energy for forces
  - At best a sign error
  - At worst just wrong
- Under charge control, transverse electrostatic actuator is well-behaved
- Under voltage control, exhibits pull-in

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