Computing the time dependent amplitude function

Now that we have $\vec{A}(q)$, how do we find the time dependence of $\vec{\Psi}(x,t)$? Since $\vec{A}(q)$ is the coefficient vector for the eigenfunctions of $\mathcal{H}$, we know the time dependence is accounted for by multiplying each coefficient by $e^{-iEt/\hbar}$. So, we now create a vector **E**, indexed by **q**.

$$\vec{E} = \frac{\hbar^2}{2m} \cdot \overrightarrow{q^2} = \frac{\hbar^2}{2m} \begin{pmatrix} q_1^2 \\ q_2^2 \\ q_3^2 \\ \vdots \\ q_m^2 \end{pmatrix}$$

Now, if we take the exponential of $-it/\hbar$ times each element, where **t** is the time we wish to evaluate $\Psi(x)$ at, we get

$$\vec{\varsigma} = \begin{pmatrix} e^{-iE_1 t/\hbar} \\ e^{-iE_2 t/\hbar} \\ \vdots \\ e^{-iE_m t/\hbar} \end{pmatrix}$$

Now, if we perform element by element multiplication (**MATLAB®** command is ".*") on $\vec{\varsigma}(q)$ and $\vec{A}(q)$ we get:

$$\vec{A}_t(q) = \vec{A}(q) \cdot {*} \vec{\varsigma}(q) = \begin{pmatrix} A(q_1)\varsigma(q_1) \\ A(q_2)\varsigma(q_2) \\ \vdots \\ A(q_m)\varsigma(q_m) \end{pmatrix} = \begin{pmatrix} A(q_1)e^{-iE_1 t/\hbar} \\ A(q_2)e^{-iE_2 t/\hbar} \\ \vdots \\ A(q_m)e^{-iE_m t/\hbar} \end{pmatrix}$$

Now, we have taken account for the time dependence by modifying our amplitude function (note the subscript **t** to denote that this is $A(q)$ at a particular time **t**). The last chore is to now compute the wave function in x-space from the modified amplitude function.